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A Scan for Models with Realistic Fermion Mass Patterns

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Abstract

We consider models which have no small Yukawa couplings unrelated to symmetry. This situation is generic in higher dimensional unification where Yukawa couplings are predicted to have strength similar to the gauge couplings. Generations have then to be differentiated by symmetry properties and the structure of fermion mass matrices is given in terms of quantum numbers alone. We scan possible symmetries leading to realistic mass matrices.

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Most of the free parameters of the standard SU(3) x SU(2) x U(1) model are Yukawa couplings between quarks, leptons and the Higgs scalar. Ideas of further unification of all forces aim for an explanation of those Yukawa couplings and thereby a resolution of the old puzzle about the origin of the difference between muon and electron. In particular, unification in more than four dimensions relates the number of generations to topological properties of internal space [1]. As a consequence, the differentiation between generations should also be explained by symmetries and topology of internal space (including ground state configurations of other bosonic fields). [2)-4)]

In this letter we describe a computerized search for realistic fermion mass matrices whose structure is entirely explained by quantum numbers of quarks and leptons. Although motivated by higher dimensional theories, the framework of our discussion is in four dimensions. Our central assumption is that the theory has no small Yukawa couplings unless they are protected by some symmetry. In the limit of unbroken symmetry all dimensionless cubic couplings are assumed to be of the same order as the gauge coupling g or to vanish for reasons of symmetry or topology. (This is the generic situation resulting from higher dimensional unification). If generations are not distinguished by the order of magnitude of their Yukawa couplings, they must be differentiated by some symmetry G larger than SU(3) x SU(2) x U(1). Such a symmetry G may consist of local or global continuous symmetries or be discrete.

In the limit of unbroken G the top quark and the electron must couple to scalar doublets d_i in different representations of G - otherwise their mass would be of the same order of magnitude. The Higgs doublet responsible for weak symmetry breaking will be some linear combination of d_i, $\phi = \sum_i \gamma_i^* d_i$. The vacuum expectation values (vev) are

$$\langle d_i \rangle = \gamma_i \langle \phi \rangle \qquad (1)$$

Masses of quarks and leptons are given by the product of Yukawa couplings (of order g) and the vev $\langle d_i \rangle$ which couples to them. The apparent small Yukawa coupling for the electron originates in a small mixing coefficient γ for the corresponding doublet! In the limit of unbroken G the various doublets cannot mix and all γ_i vanish except one ($\gamma_0 = 1$). (This "leading" doublet should only couple to the top quark in the three generation case.) Nonvanishing mixings γ_i are induced by symmetry breaking of G. Let us denote by M the typical scale of mass terms for the doublets d_i in the limit of unbroken G and by M_G the scale of G symmetry breaking. The dimensionless γ_i are then of order

$$\gamma_i \approx \left(\frac{M_G}{M} \right)^{P_i} \qquad (2)$$

with P_i some integer calculable from symmetry considerations [4]. A small ratio M_G/M should be responsible for all small quantities in the fermion mass matrices.

For the purpose of this letter the choice of M is arbitrary. It could be a very high unification scale - the compactification scale in higher

dimensional theories, the string tension for superstrings or the GUT scale for some extended version of grand unification/family unification. In this case the structure of fermion mass matrices is related to a fine structure of scales around the unification scale [3]. Only one Higgs doublet ϕ survives at low energies. The other extreme case is a "low energy" ($\approx$ TeV) scale M only somewhat above the weak scale $M_W \approx M_G$. This scenario requires several doublets in the range below a few TeV. It may be realized in supersymmetric theories with M the gravitino mass. Between these extreme scenarios one can of course consider possible combinations or a scale M in some intermediate range.

There is a first necessary criterion for these ideas to work: The symmetry G must differentiate enough between various quarks and leptons to allow for realistic structures of the mass matrices. There should be at least one choice of $\langle d_i \rangle$ (the calculation of these vev's is not attempted at this stage) which produces correctly all orders of magnitude for the various entries to the quark and lepton mass matrices M_U, M_D and M_L. This requirement leads to many restrictions on possible quantum numbers with respect to G. Consider a three generation example: The doublet responsible for the top quark mass should be forbidden by G symmetry to couple to down quarks or leptons. Otherwise the largest eigenvalue in M_D or M_L would generically be of the same order as m_t. We may illustrate this for G a continuous abelian symmetry $U(1)^n$ with charges Q_k ($k = 1,\dots,n$). Denoting by Q_k (q_i) the charges for the i-th quark doublet and similarly by Q_k (u_i^c), Q_k (d_i^c) the charges for u^c and d^c, the charges for the doublet responsible for the top quark mass,

ϕ_{tt^c}, are

$$Q_k(\phi_{tt^c}) = - Q_k(t) - Q_k(t^c) \tag{3}$$

If for some pair (i,j) one finds for all k

$$Q_k(\phi_{tt^c}) = Q_k(d_i^c) + Q_k(q_j) \tag{4}$$

this doublet can also couple to $d_i^c q_j$ and the model should be discarded. (Note the difference in sign between (2) and (3) which reflects the fact that doublets coupling to $d_i^c q_j$ or $e_i^c L_j$ have the opposite hypercharge from doublets coupling to $u_i^c q_j$. If d_i couples to $u_i^c q_j$, only d_i^* can couple to $d_i^c q_j$ or $e_i^c L_j$.) For a given symmetry G and given representations for quarks and leptons (given $Q_k(q_i)$ etc.) a computerized scan for acceptable mass matrices becomes possible. For this we require that all small quantities below an order of magnitude in the fermion mass matrices should be explained by a symmetry G and different scales $\langle d_i \rangle$. This concerns small ratios of mass eigenvalues as well as the small mixing angles. We allow for the possibility that factors ≈ 5 are attributed to group theoretical Clebsch Gordan coefficients or other details of dynamics. There is of course some arbitrariness in the choice of a boundary for small quantities not explained by G. Given the fact that quantities as small as 10^{-5} (m_e/m_t) appear, this does not change the overall scheme.

Our scanning program is based on the observation that for the three generation case the structure of fermion mass matrices is well described

by four (or five) scales. These scales themselves are separated by about an order of magnitude. The highest scale is the top quark mass, which we assume to be several ten GeV. The second scale is a few GeV at which level we have the bottom quark, the charm quark and the tau lepton.*) The strange quark and the muon constitute the third level of a few hundred MeV. The fourth level consists of the up quark, the down quark and the electron. Since the electron mass and the down quark mass differ by about **) an order of magnitude it can be argued that we need a fifth scale here. The fourth scale is below a few ten MeV. The only other information about the mass matrices comes from the measured mixing angles. We have (a fairly large) Cabibbo angle and a few percent of mixing between the 2nd and 3rd generation. The limit on mixing between the first and the third generation is somewhat less than one percent. No information about the lepton mass matrix besides its eigenvalues is available. We will denote these scales by n_s, every scale being a few 10^{n_s} MeV. So the n_s are

---

*) We use a normalization for the quark and lepton mass matrices at the weak scale M_W. If we assume that these entries are generated at a very high scale (M_{GUT} or M_P) we multiply the lepton mass matrix by a factor of 2.5 - 3 in order to account for different renormalization of quarks and leptons.

**) This ratio is reduced to a factor of five for the case where masses get generated at a very high scale.

$$
\begin{array}{ll}
m_t & : 4 \\
m_b, m_c, m_\tau & : 3 \\
m_\mu, m_s & : 2 \\
m_u, m_d, m_e & : 1\ (0)
\end{array}
\qquad (5)
$$

Upper bounds on the sizes of entries in the mass matrices are given by

$$M_U \leqslant \begin{pmatrix} 4 & 3 & 2 \\ 4 & 3 & 2 \\ 4 & 3 & 1 \end{pmatrix} \qquad (6a)$$

$$M_D \leqslant \begin{pmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{pmatrix} \qquad (6b)$$

$$M_L \leqslant \begin{pmatrix} 3 & 2 & 2 \\ 3 & 2 & 2 \\ 3 & 2 & 1(0) \end{pmatrix} \text{ or } \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 2 & 2 & 1(0) \end{pmatrix} \qquad (6c)$$

The mass matrices (5a,b,c) have been ordered here in the standard way. The bounds on the M_{11} elements come from the maximal size of mass eigenvalues. This entry is always required to generate the largest mass. The bounds on M_{12} and M_{13} come from the observed small values of the mixing with the third generation. This does not provide a bound on these elements in M_L though. The bounds on M_{21} and M_{31} only come from the

size of the largest mass since these elements can always be removed by a left multiplication of a unitary matrix which is unobservable. The bound on M_{22} and M_{32} is from the 2nd eigenvalue and the one on M_{23} comes from the smallness of the Cabibbo angle. *) In the lepton matrix limits only come from the eigenvalues. Hence, whenever M_L is acceptable, M_L^T (transposed) is too. Finally the bounds on M_{33} reflect the smallness of first generation masses. These masses could also be generated by paired off diagonal elements [5] and we have to impose additional "quadratic" constraints. In terms of n_s they read

$$(M_U)_{13} + (M_U)_{31} \leqslant 5 \qquad (7a)$$

$$(M_U)_{23} + (M_U)_{32} \leqslant 4 \qquad (7b)$$

$$(M_D)_{23} + (M_D)_{32} \leqslant 3 \qquad (7c)$$

$$(M_L)_{13} + (M_L)_{31} \leqslant 4 \quad (3) \qquad (7d)$$

$$(M_L)_{23} + (M_L)_{32} \leqslant 3 \quad (2) \qquad (7e)$$

The sizes in brackets are those where we implement a fifth scale for the electron mass. **)

*) This is however a limiting case. If we would allow the Cabibbo angle be of order 1 these entries could be an order of magnitude higher.

**) As a limiting case we would allow $(M_L)_{11} \approx (M_L)_{12} \approx (M_L)_{21} = 3$ in the lepton mass matrix. This would require 3 factors of O(1) to conspire to produce a difference in order of magnitude. A similar argument would increase all the bounds in eq. (7) by 1. In this case no new information (assuming the Cabibbo angle of $O(10^{-1})$) is contained in the quadratic constraints. This however requires a set of O(1) factors to all go in the right direction and we will discard this possibility in this letter.

We now can describe our scanning procedure: The fermion masses are generated level by level. At each step we try all possible assignments of the required scales n_s to suitable entries in the corresponding mass matrix. If the same doublet d_i is allowed to couple to more than one entry in M_U, M_D or M_L we assign the same scale n_s. Consistency is then checked by comparing the scale pattern with the bounds (6a)-(6c). A model is rejected if at some level no consistent assignment is found. We will use the rather mild consistency vetos discussed above where we only require that scale ratios smaller as around an order of magnitude are generated by different scales of entries and therefore by doublets with different G quantum number. Stronger vetos are easily implemented. We also note that we arbitrarily can permute all rows in fermion mass matrices as well as columns in M_L in order to bring them to the standard form (6a) to (6c). For the quark mass matrices, permutations of columns have to be done simultaneously in M_U and M_D in order to keep track of mixing angles.

At the first level we look for an entry only appearing in one column of M_U and not in M_D or M_L. This defines the top mass with $n_s = 4$. At level two we first assign a candidate for m_b with $n_s = 3$. We veto if the label $n_s = 3$ appears in more than one column in M_D or M_L or if it appears in more than one column outside the top column in M_U. If m_τ is not generated by the m_b entry, we try additional $n_s = 3$ entries in M_L. The same procedure then applies to m_c which can be generated either by diagonal or paired off diagonal $n_s = 3$ entries. The combined set of all $n_s = 3$ entries is subject to the consistency veto described for m_b. At the third level we first generate m_μ by an $n_s = 2$ entry. We again

allow for diagonal or paired off diagonal entries. We veto if this entry appears in the last column of M_D or in $(M_U)_{33}$ or $(M_L)_{33}$ or if one of the quadratic bounds (7a) - (7c) is violated. If m_s is not yet generated, we assign additional $n_s = 2$ entries in M_D with the same veto. At the end of level three all generation numbers t,c,u etc. are assigned to the various rows and columns. It is now easy to check by inspection of the various labels 4, 3 and 2 in the mass matrices if sufficient mixings θ_{23} and θ_{12} are already generated. If not, we have to assign for θ_{23} an appropriate $n_s = 2$ or 3 entry in M_U or an $n_s = 1$ or 2 entry in M_D. The same holds for θ_{12} with $n_s = 2$ or 1 for M_U and M_D, respectively. Of course, possible $n_s = 3$ or 2 entries are subject to the appropriate consistency vetos of level two or three, respectively. Finally, we check if all first generation masses can be generated by $n_s = 1$ entries. This will always be the case unless "topological reasons" enforce the absence of certain doublets coupling to the first generation bilinears. We account for such topological restrictions by setting appropriate entries to zero ($n_s = -10$).

As our first example we take G = U(1) x U(1) with two anomaly free abelian symmetries in four dimensions. The first U(1) is a linear combination of the three generation hypercharges and the second is a linear combination of the generation B-L symmetry. A generation hypercharge has zero charges for all generations except one for which we have the standard assignment of Y. For three generations one has two independent generation hypercharges Y_1, Y_2 in addition to the standard weak hypercharge $Y = Y_1+Y_2+Y_2$. The same definition applies to generation B-L symmetry. These symmetries have no mixed anomalies with SU(3) x SU(2) x U(1) or mixed gravitational anomalies and are by themselves anomaly free. The fermion quantum numbers under G are

$$q_{Li}(\tfrac{1}{3}y_i,\ \tfrac{1}{3}b_i);\ u^c_{Li}(-\tfrac{4}{3}y_i,\ -\tfrac{1}{3}b_i);\ d^c_{Li}(\tfrac{2}{3}y_i,\ -\tfrac{1}{3}b_i);$$

$$L_{Li}(-\ y_i,\ -\ b_i);\ e^c_{Li}(2y_i,\ b_i). \tag{8}$$

In this example we assume that every fermion bilinear can couple to a scalar doublet. Many four dimensional models are more restricted because there will not be a scalar doublet with the right G quantum numbers to couple to every possible fermion bilinear, but even at this level the reduction in possible solutions is significant. To see this, let us estimate the total number of possibilities for assigning scales n_s to entries in mass matrices for the case of a symmetry G large enough to discriminate between all fermion bilinears. We first have all possible reorderings of rows and columns. The columns of M_U and M_D are linked via the observed mixing angles so this is a factor of $(3!)^5$. For every assignment of vev's in M_L the transposed assignment is also valid (a factor of 2). The 2nd generation masses can be fed down from the 3rd generation via off diagonal entries or can be directly generated (a factor of 2^3) and similarly the first generation masses can be generated directly or fed down from the 2nd or 3rd generation (3^3). Some uncertainty comes from the way how mixing between generations is generated and assigning values or not to irrelevant entries (those that do not

generate masses or mixings but violate none of the vetos). So the total number of possibilities is

$$n_{sol} = 0(6^5\ 2\ 2^3\ 3^3) = 0(3\ 10^6) \tag{9}$$

We will not distinguish between the different ways of generating the first generation masses so that the maximum number of different assignments is

$$n_{sol} = 0(6^5\ 2^4) = 0(10^5). \tag{10}$$

It is this latter number that our program calculates and the actual number found for maximally differentiated mass matrices is 124416.

We now can compare the number of possible scale assignments for our U(1) x U(1) example. Results for several values of (y_i, b_i) are given in table 1. Even some fairly simple restrictions on the mass matrices can reduce the number of solutions significantly. The maximal differentiation from a U(1) x U(1) symmetry of this type is that all off diagonal elements have different quantum numbers whereas the diagonals are the same in the three mass matrices. This general form allows for 18768 solutions (the exact number is somewhat lower since our program has some double counting from always treating M_L and M_L^T as different solutions.) But for some simple assignments the reduction is much more dramatic as can be seen in table 1. For certain U(1) x U(1) groups no solutions are found at all and such models are inconsistent with our assumptions.

As our second example we discuss a simple higher dimensional model, namely monopole solutions of the six dimensional SO(12) theory [2]. (This can be considered as a subgroup analysis for the $E_8 \times E_8$ superstring for appropriate deformation classes of the ground state [4].) Monopole solutions with SU(3) x SU(2) x U(1) symmetry are characterized by three integers n, m and p with n+p even. We list [2] the numbers of chiral fermions with given charge $q = \pm 1/2$ (corresponding to the abelian subgroup of SO(12) commuting with SO(10)):

$$
\begin{aligned}
q_L &: \left[\tfrac{1}{2}(n+p)\right]_{1/2} + \left[\tfrac{1}{2}(n-p)\right]_{-1/2} \\
u_L^c &: \left[\tfrac{1}{2}(n-p+2m)\right]_{1/2} + \left[\tfrac{1}{2}(n+p-2m)\right]_{-1/2} \\
d_L^c &: \left[\tfrac{1}{2}(n-p-2m)\right]_{1/2} + \left[\tfrac{1}{2}(n+p+2m)\right]_{-1/2} \\
L_L &: \left[\tfrac{1}{2}(n-3p)\right]_{1/2} + \left[\tfrac{1}{2}(n+3p)\right]_{-1/2} \\
e_L^c &: \left[\tfrac{1}{2}(n+3p-2m)\right]_{1/2} + \left[\tfrac{1}{2}(n-3p+2m)\right]_{-1/2}
\end{aligned}
\tag{11}
$$

Negative integers in the brackets correspond to the corresponding number of mirror particles $q_L, u_L^c \ldots$ with charge q opposite to the indicated index. Possible mirrors q_L have therefore the same $U(1)_q$ charge as q_L and mass terms between standard fermions and mirrors require breaking of $U(1)_q$.

Besides $U(1)_q$ and $U(1)_{B-L}$ within SO(12) there is another possible abelian group $U(1)_G$ commuting with $SU(3)_C \times SU(2)_L \times U(1)_Y$. This comes from an isometry of rotations on two dimensional internal space. The charge I with respect to $U(1)_G$ for q_L is given by ($n+p > o$, $n-p > o$)

$$
I = \begin{cases} \frac{1}{4}(n+p) - \frac{1}{2},\ \frac{1}{4}(n+p) - \frac{3}{2},\ \ldots,\ -\frac{1}{4}(n+p) + \frac{1}{2} & (q = 1/2) \\[2ex] \frac{1}{4}(n-p) - \frac{1}{2},\ \frac{1}{4}(n-p) - \frac{3}{2},\ \ldots,\ -\frac{1}{4}(n-p) + \frac{1}{2} & (q = -1/2) \end{cases}
\tag{12}
$$

and correspondingly for mirrors and the other fermions. (I is the third component of $SU(2)_G$ spin for $SU(2)_G$ representations with dimension given in the brackets in the list (11)). We want to know if the abelian charges q and I can differentiate sufficiently between various quarks and leptons to allow for realistic mass matrices. (This is the abelian part of a more complete nonabelian analysis as sketched in ref. 4.) We restrict ourselves to the three generation case n = 3.

In the first column of table 2 we give the number of acceptable solutions for various values of m and p for n = 3. We observe that no assignments of scales produces acceptable mass matrices for low values of m and p where we have no mirror particles. For high m and p the number of solutions increases rapidly. This is due to a large number of mirror particles. In fact, for this first investigation, we have treated the mechanism giving mass to mirrors as independent from the mechanism mixing the various doublets coupling to the "surviving" chiral fermions. This means, for n_q quarks and n_q-3 mirror quarks we have independently counted all solutions picking arbitrarily three "surviving" quarks out of n_q quarks and assuming that the remaining n_q-3 quarks form heavy masses with the n_q-3 mirror quarks.

In a more detailed analysis the SU(3) x SU(2) x U(1) singlet operators responsible for the heavy masses of mirror quarks also lead to the mixing between different doublets by G symmetry breaking. (In our case $G \equiv U(1)_q \times U(1)_I$.) As a first step towards a more detailed scanning we have implemented an additional constraint: The leading singlet operator responsible for the mixing between the doublets coupling to tt^c and bb^c should not induce a mass term between mirrors and "surviving" fermions. Results of this additional veto are shown in the second column of table 2. We note that for m = 3 p = 1 all solutions are eliminated. It becomes obvious that it is easy to impose additional requirements in the scanning process and thus to select models with particular properties. As an example we give in the third column of table 2 the number of solutions where the bottom quark and the tau lepton obtain their mass from the same doublet - motivated by the approximate equality of m_b and m_c at the unification scale. As an example, we have found the following solution for m = 1, p = 3:

$$M_U: \quad \begin{pmatrix} (1,-1);\ (1,1);\ (1,0) \\ (0,-1/2);\ (0,3/2);\ (0,1/2) \\ (0,-3/2);\ (0,1/2);\ (0,-1/2) \end{pmatrix};\quad \begin{pmatrix} 4 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad (12a)$$

$$M_D: \quad \begin{pmatrix} (0,-3/2);\ (0,1/2);\ (0,-1/2) \\ (0,1/2);\ (0,5/2);\ (0,3/2) \\ (0,-1/2);\ (0,3/2);\ (0,1/2) \end{pmatrix};\quad \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad (12b)$$

$$M_L: \quad \begin{pmatrix} (0,-3/2);\ (0,1/2);\ (0,-1/2) \\ (0,1/2);\ (0,5/2);\ (0,3/2) \\ (0,-1/2);\ (0,3/2);\ (0,1/2) \end{pmatrix};\quad \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad (12c)$$

Here the first matrix exhibits the quantum numbers (q,I) for the various bilinears in the mass matrices whereas the second matrix gives the choosen assignment of n_s.

We conclude that a computerized scan for mass matrices whose structure is only determined by G quantum numbers is possible - at least for abelian G. This analysis should certainly be extended to a more complete connection between the mirror masses and the mass matrices M_U, M_D and M_L as well as to an inclusion of the calculable powers P_i in eq. (2) [4]. Nevertheless, we find that already at this stage of the scanning many models are excluded since the symmetry G does not differentiate enough the generations to account for realistic mass matrices.

Table 1

Number of solutions for $G^2 = U(1)^2$ with $Q_1 = \sum_{i=1}^{3} y_i Y_i$ and $Q_2 = \sum_{i=1}^{3} b_i (B-L)_i$

y_1	y_2	y_3	b_1	b_2	b_3	n_{sol}
1	0	0	0	0	1	2256
1	-1	0	0	0	1	18768
1	-1	0	0	0	0	464
0	0	0	1	-1	0	0
0	1	-2	0	0	0	822
1	3	0	0	0	0	2256
1	6	0	0	0	0	13118
1	1	0	1	1	0	0
-1	3	0	0	0	0	3012

Table 2

Number of solutions for various compactifications of a six dimensional model.

n_{sol}: number of solutions

n'_{sol}: " " " with the partial veto on mixing with mirrors (see text)

n''_{sol}: " " " with m_τ and m_b generated by the same scalar doublet

n'''_{sol}: " " " with both requirements

m	p	n_{sol}	n'_{sol}	n''_{sol}	n'''_{sol}
-3	1	4750	1456	1386	68
-2	1	0	0	0	0
-1	1	0	0	0	0
0	1	0	0	0	0
1	1	0	0	0	0
2	1	0	0	0	0
3	1	72	0	72	0
-3	3	$\geqslant$ 100000	$\geqslant$ 100000	0	0
-2	3	$\geqslant$ 100000	$\geqslant$ 100000	58724	58724
-1	3	26856	884	0	0
0	3	0	0	0	0
1	3	3132	24	1796	24
2	3	22636	17280	13324	9272
3	3	12840	9250	8780	5580

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