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by

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I discuss the idea that quarks and leptons may be fermionic partners of Goldstone bosons, arising from the spontaneous breakdown of some global symmetry in a supersymmetric theory. The special role that the complex extension of the symmetry group has for these considerations is emphasized. Some semirealistic examples, involving both ordinary preon models as well as exceptional chains, are given.

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I discuss the idea that quarks and leptons may be fermionic partners of Goldstone bosons, arising from the spontaneous breakdown of some global symmetry in a supersymmetric theory. The special role that the complex extension of the symmetry group has for these considerations is emphasized. Some semirealistic examples, involving both ordinary preon models as well as exceptional chains, are given.

1. MOTIVATION AND INTRODUCTORY REMARKS

Although the standard SU(3) x SU(2) x U(1) model for the strong and electroweak interactions is eminently successful phenomenologically, there remain a variety of deep questions to be answered. Perhaps among the most puzzling of these is why the pattern of fermions we observe is chiral and why the fermions appear in generational repetitions. In this talk, I would like to discuss some recent ideas which may have a bearing on this question.

The quantum numbers of quarks and leptons are quite varied and asymmetric under SU(3) x SU(2) x U(1). For instance, under the standard model group, u_L transforms as (3,2,1/6) and e_L transforms as (1,2,-1/2) while u_L^c transforms as $(\bar{3},1,-2/3)$ and e_L^c as (1,1,1)*. Because ψ_L^c does not transform as $(\psi_L)^*$, it is clear that the fermions are in chiral representations.

The variety of SU(3) x SU(2) x U(1) fermion representations can be reduced if one classifies the fermions according to some grand unified (GUT) groups /1/. Under the rank 4 SU(5) group the fermions of one generation transform as $\bar{5}$ plus 10. In the rank 5 SO(10) group the

* I detail only the transformation laws of ψ_L since those of ψ_R follows from those of ψ_L^c.

fermions, including now a right handed neutrino, transform as a 16. This sequence can be extended to a rank 6 group, E_6, if one admits even more additional states for each family. Quarks and leptons then transform according to the 27 of E_6, but one needs 11 extra states. In terms of an SO(10) decomposition, one has

$$27 = 16 + 10 + 1 \qquad (1.1)$$

It is reasonable to ask if there is any dynamical significance to the appearance of these particular representations in nature. From the point of view of grand unified theories, the answer is obviously no. Although GUTS unify the dynamics of gauge fields, they do not fix how fermions must transform under the GUT group. That is, to be a grand unified theory SU(5) must contain SU(3) x SU(2) x U(1). However, the fermions of the theory can, in principle, sit in any SU(5) representation. It is only phenomenology which tells one that quarks and leptons transform as $\bar{5} + 10$ in SU(5).

If one wants to fix the representation content of fermions dynamically, it is probably necessary to relate fermions to bosons. Obviously, the most direct way to do this is by invoking the existence of supersymmetry at some level. Even then, one remains with two possible options. Either the quarks and leptons are related to gauge fields, (1/2,1) multiplet, or these fermions are related to scalar excitations, (1/2,0) multiplet. At first sight, both these options look pretty hopeless. Supersymmetry implies that fermions and bosons are in the same G representation. Gauge fields transform, by definition, as the adjoint of G. However, quarks and leptons do not sit in the adjoint of SU(5) or SO(10) or of E_6. Similarly, although one can imagine scalar fields transforming as $\bar{5} + 10$ of SU(5), it is not obvious what dynamics would associate these fields to the quarks and leptons.

Deeper considerations can, however, relate fermions, via supersymmetry, to the dynamics of either gauge fields or of scalar excitations. An example of the first kind is provided by the suddenly very popular superstring theories /2/. These theories, in the zero slope limit, reduce to a supergravity theory in 10 dimensions, interacting with a supersymmetric Yang Mills theory with a group G = SO(32) or $E_8 \times E_8$. The connection with four dimensional physics comes about because it is thought that, near the Planck scale, the 10 dimensional manifold compactifies down to $M_4 \times K$, leaving in the process an N = 1 supersymmetry unbroken /3/ (For this to occur, K has to be a Calabi-Yau manifold /4/ with SU(3) holonomy). In this compactification process, chiral fermions can survive as long as they transform non trivially under the holonomy group /3/. From these considerations, it is apparent that the $E_8 \times E_8$ superstrings are particularly interesting. E_8 contains E_6 x SU(3) as a subgroup and, with proper identification, this SU(3) can be taken as the holonomy group. The E_8 gauginos transform according to the adjoint, which under the E_6 x SU(3) subgroup decomposes as

$$248 = (78,1) + (1,8) + (27,3) + (\overline{27},\bar{3}) \qquad (1.2)$$

Clearly, therefore these superstring theories allow the appearance of chiral fermions in the 27 of E_6 and provide a raison d'être for the existence of fermions transforming according to this representation and no other.

Scalars can also have a dynamical role, if they are the by products of the spontaneous breakdown of a symmetry. In these circumstances their fermionic partners, induced by supersymmetry, are again present in the spectrum for purely dynamical reasons. The elaboration of this line of thinking will occupy the rest of my considerations.

2. PREON DYNAMICS AND QUASI GOLDSTONE FERMIONS

If G is a global symmetry of a theory but not of the vacuum state, the symmetry suffers a spontaneous breakdown. If H is the subgroup of G which respects the vacuum, then $G \to H$ and in the spectrum of the theory there appear dimG - dimH massless J = 0 excitations (Nambu-Goldstone bosons) /5/. In a supersymmetric theory, where there is a natural pairing of bosons with fermions, the spontaneous breakdown of a global symmetry G causes also the appearance of massless spin 1/2 excitations. These are nothing else but the fermionic partners of the dynamically required Goldstone bosons, which in Ref. 6 we dubbed Quasi Goldstone Fermions (QGF).

Quasi Goldstone fermions may be good candidates for quarks and leptons in models in which the states are composite (preon models). Since we see, at present, no departure from elementarity, the scale of compositeness must be very high, certainly very much greater than the actual masses of leptons and quarks:

$$\Lambda_c \gg m_{q,\ell} \qquad (2.1)$$

Thus preon models must have a dynamics which, to a good approximation, generates massless fermion bound states, irrespective of the scale Λ_c of the binding. Clearly if quarks and leptons were nearly QGF, their masslessness with respect to Λ_c would be dynamically understood. Furthermore, by choosing G and H appropriately one may indeed insure that the light bound states of the theory have the quantum numbers of quarks and leptons.

This last point is nicely illustrated by the original example used by Buchmüller, Yanagida and me /7/ to motivate the QGF idea phenomenologically. Consider a supersymmetric confining theory with a global symmetry G = SU(5), broken to H = $SU(3)_c \times U(1)_{em}$. The number of Goldstone bosons in the breakdown is clearly 24-9 = 15. By assigning charge and color appropriately in the SU(5), it is easy to check that the 15 Goldstone bosons have precisely the same quantum numbers under $SU(3)_c \times U(1)_{em}$ that the quarks and leptons do. That is, under $SU(3)_c \times U(1)_{em}$, one finds /7/:

$$GB \sim (3,2/3)+(\bar{3},-2/3)+(3,-1/3)+(\bar{3},1/3)+(1,-1)+(1,1)+(1,0) \qquad (2.2)$$

Since by supersymmetry these Goldstone bosons must have fermionic partners, it is natural to assume that the QGF in the model transform also according to (2.2). Hence the model provides dynamically massless fermionic bound states with the quantum numbers of quarks and leptons (of one generation).

These nice features of the model rely on the assumption that the QGF transform in the same way as the GB. In fact, if this is so, it is necessary that in the theory there be also other massless bosonic excitations, quasi Goldstone bosons (QGB). Supersymmetry requires that there be the same number of bosonic and fermionic degrees of freedom. Since fermions have two degrees of freedom, it is thus necessary that in the model there should appear also a set (2.2) of QGB. It is important to emphasize, however, that supersymmetry only requires that the equation

$$2n_{QGF} = n_{GB} + n_{QGB} \tag{2.3}$$

be satisfied. The assumption $n_{QGF} = n_{GB}$ (and hence also $n_{QGF} = n_{QGB}$) needs dynamical justification. In contrast to Goldstone bosons, whose number and transformation properties are fixed by the G → H breakdown, the pattern of emerging QGF is a dynamical issue.

There is a further important point that can be gleaned from this SU(5) model, or any other model in which $n_{QGF} = n_{GB} = n_{QGB}$ (Total doubling). Goldstone bosons sit in the G/H piece of the adjoint representation of G, and thus they transform as a real (r) or vector-like (i + ī) representation of H. Thus in the case of total doubling, the massless fermions are never in a chiral representation. There is no way, therefore, to introduce in these models the weak interactions at the fundamental level!

Even in these circumstances the idea of quarks and leptons as QGF is not totally without physical motivation. Recall that for Goldstone bosons the dynamics of their interactions is specified entirely by an effective non linear Lagrangian, whose structure is essentially determined by the G → H breakdown /8/. Specifically, the non linear Lagrangian for a set of Goldstone bosons π_i arising from the breakdown of G → H is given by /9/,

$$\mathcal{L}^{NL}_{GB} = -\partial^\mu \pi_i \, g^{ij}(\pi/f_\pi)\, \partial_\mu \pi_j \tag{2.4}$$

where g^{ij} is the metric of the G/H coset space. (The only undetermined parameter in (2.4) is the scale f_π*, which is fixed by the underlying dynamics.) In a supersymmetric theory the QGF dynamics is also (partially) determined. The supersymmetric generalization of (2.4) was first constructed by Zumino /10/. The piece of this Lagrangian which contains only the QGF fields has the form of a 4-Fermi interaction

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* If there are various representations of GB, there are further scales f_π.

$$\mathcal{L}^{NL}_{QGF} = \frac{c_{ijkl}}{f_\pi^2} (\bar\psi_i \gamma^\mu \psi_j)(\bar\psi_k \gamma_\mu \psi_l) \tag{2.5}$$

The coefficients C_{ijkl}, in contrast to what happens in (2.4), are fixed both by the coset space G/H and by the underlying dynamics.

Because (2.5) has the form of an effective current-current interaction one can speculate that, in models where the QGF are in non chiral representations, the weak interactions occur as residual interactions among the QGF. Of course, this requires both that the coefficients C_{ijkl} be such that (2.5) reproduce the standard model at $q^2 = 0$, and that dynamically one is able to generate low mass composite W-bosons. This is very unlikely to be the case for the SU(5) model /11/, but could well occur in some more realistic model.

Putting aside these speculations, it is clear that if one wants to retain the notion that the weak interactions are given by a chiral SU(2) x U(1) gauge theory acting at the preon level, it is necessary to have quarks and leptons as QGF which are not totally doubled (i.e. $n_{QGF} < n_{GB}$). I discuss below the circumstances for which this may occur.

3. THE ROLE OF THE COMPLEX EXTENSION

Although for any breakdown G → H the number of Goldstone bosons is fixed, the pattern of QGF in a supersymmetric theory is only subject to the constraint of Eq. (2.3). It is the underlying dynamics which fixes the number of QGB and of QGF. However, for a given G → H breakdown it is possible to discuss the pattern of QGF which can arise. Which of these patterns eventually obtains in a given model is then a dynamical question.

The pattern of QGF emerging from a given breakdown was studied by Lee and Sharatchandra /12/ and by Lerche /13/. Subsequently Kugo, Ojima and Yanagida /14/ clarified the role that the complex extension of G, Ĝ, plays in these considerations. The pattern of QGF which can arise from a breakdown G → H can be most simply* characterized by considering the breakdown in a sequential fashion: $G \to G_1 \to G_2 \to \ldots G_n \to H$. At each stage in this chain the GB are either in a real (r) or vector-like (i + ī) representation. If the GB are in a real representation, GB ~ r, then necessarily also the QGF ~ r and there is a further set of QGB ~ r. If the GB are in a vector-like representation, GB ~ i + ī, then again one can have total doubling, QGB ~ QGF ~ i + ī. However, if G_i/G_{i+1} is a Kähler manifold, then one can have QGF ~ i and no QGB are needed /12/. This last case is obviously the most interesting.

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* In general there can be many different sequential paths. All paths must be considered.

One can understand rather simply why chiral fermions can arise if the manifold is Kählerian. The supersymmetric generalization of Eq. (2.4), first detailed by Zumino /10/, involves the D-term of an arbitrary function of the chiral and antichiral Goldstone superfields ϕ and $\bar{\Phi}$

$$\mathcal{L}^{NL}_{susy} = K(\bar{\Phi},\phi)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{3.1}$$

From (3.1) it is simple to find what the effective Lagrangian for the scalar sector looks like:

$$\mathcal{L}^{NL}_{scalar} = -\partial^{\mu}\varphi^{*i}\left[\frac{\partial^2 K(\varphi^*,\varphi)}{\partial\varphi^{*i}\partial\varphi^{j}}\right]\partial_{\mu}\varphi^{j} \tag{3.2}$$

One sees that the relevant metric $g_{ij}(\varphi^*,\varphi)$ from Eq. (3.2) obeys the Kähler condition, characteristic of certain complex manifolds:

$$\frac{\partial g_{ij}}{\partial\varphi^{*k}} = \frac{\partial g_{kj}}{\partial\varphi^{*i}} \quad ; \quad \frac{\partial g_{ij}}{\partial\varphi^{k}} = \frac{\partial g_{ik}}{\partial\varphi^{j}} \tag{3.3}$$

Thus supersymmetry requires that the scalar excitations be coordinates of a Kähler manifold. Hence if G_i/G_{i+1} is a Kähler manifold, one needs no other scalar excitations but GB ($n_{QGB} = 0$ $n_{QGF} = 1/2\ n_{GB}$). On the other hand, if G_i/G_{i+1} is not Kählerian, then one needs to add other scalar excitations to the GB to be able to have a set of complex coordinates of a Kähler manifold (which is obviously not G_i/G_{i+1}). This requires $n_{QGB} = n_{GB}$ and thus there is total doubling. Of course, this option is always open, even if G_i/G_{i+1} is Kählerian.

The above remarks explain the origin of the possible QGF patterns. Of course, for any given model, what particular pattern of QGF emerges is a purely dynamical question. For a Lagrangian field theory, this issue can be analyzed by studying the properties of the superpotential W/13//14/. Consider a general supersymmetric field theory of chiral superfields ϕ_i, invariant under a global symmetry G. Then the relevant Lagrangian density is just

$$\mathcal{L}_{susy} = \bar{\Phi}_i\phi_i\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + W(\phi_i)\Big|_{\theta\theta} + W(\bar{\Phi}_i)\Big|_{\bar{\theta}\bar{\theta}} \tag{3.4}$$

where W is the superpotential. Note that W is a function only of ϕ_i and not of both ϕ_i and $\bar{\Phi}_i$. For $\mathcal{L}_{SUSY}$ to be G invariant it is necessary that both the kinetic energy and W be G invariant. For W, invariance under G implies:

$$\delta_a W(\phi_i) = \frac{\partial W}{\partial\phi_i}\,\delta_a\phi_i = 0 \tag{3.5}$$

Note, however, that since W depends only on ϕ_i, if (3.5) holds then $W(\phi_i)$ is actually also invariant under the complex extension of G, which I shall denote by $\bar{G}$

*If one writes $\delta_a\phi_i = i\,T_a\phi_i$, then for transformations of $\bar{G}$, $T_a \neq T_a^{\dagger}$

Suppose that a symmetry breakdown of $G \to H$ occurs, because various of the superfields have non zero vacuum expectation values, $\langle\phi_i\rangle$. Then it is easy to show that the pattern of QGF is detemined by /13/

$$\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\Bigg|_{\phi_i=\langle\phi_i\rangle}\delta_a\langle\phi_j\rangle = \left(M_{QGF}\right)_{ij}\delta_a\langle\phi_j\rangle = 0 \tag{3.6}$$

Since this equation involves only the superpotential, the transformations in (3.6) can be extended to those of $\bar{G}$. Hence one sees that one gets zero mass fermions for transformations $T \in \bar{G}$ for which $T_a\langle\phi_i\rangle \neq 0$ /13/ /14/. Because T_a is non hermitian, one can have chiral patterns of QGF.

A nice example /15/ is provided by the breakdown $U_2 \to U_1$ due to doublet breaking: $\langle\phi\rangle = \binom{1}{0}$. The generators of U(2) and of its complex extension GL(2, $\mathbb{C}$) are, respectively,

$$T = \begin{pmatrix} a & b \\ b^* & d \end{pmatrix} \qquad a,d \in \mathbb{R}\ ;\ b \in \mathbb{C} \tag{3.7a}$$

$$\tilde{T} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a,b,c,d \in \mathbb{C} \tag{3.7b}$$

Clearly the conserved symmetries in the breakdown are larger for $\tilde{T}$ than T, since $T\langle\phi\rangle = 0$ implies

$$T = \begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix} \tag{3.8a}$$

while $\tilde{T}\langle\phi\rangle = 0$ implies

$$\tilde{T} = \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} \tag{3.8b}$$

Hence, in this* breakdown, there are 3 GB ~ (0,+,-) but only 2 QGF ~ (0,+). Note, however, that this result depends on the dynamics. In a model where $U_2 \to U_1$ is accomplished by two doublet vacuum expectation values: $\langle\phi_1\rangle = (1,0)$ and $\langle\phi_2\rangle = \binom{1}{0}$, it is easy to check that one needs 3 QGF. That is, in this case there is total doubling.

Independent of the detailed dynamics, it is possible to establish the following general results on QGF, if one starts from a Lagrangian field theory:

* Supersymmetry (Eq. 2.3) implies also a QGB ~ (0)

1. If G/H is a symmetric space (e.g. SU(m+n)/SU(m)xSU(n)xU(1)) then there is always full doubling: $n_{GB} = n_{QGB} = n_{QGF}$ /13/

2. For any $G \to H$ breakdown there will be at least one QGB /13/ /16/.

The second result is certainly somewhat puzzling. One would think, naively, that if G/H is a Kähler manifold (e.g. $E_6/SO(10) \times U(1)$) then the dynamics should allow $n_{QGB} = 0$, $n_{QGF} = 1/2\, n_{GB}$. This in fact is not allowed in (linear) Lagrangian models because the superpotential is invariant under both G and $\bar{G}$. Of course, Result 2, does not imply that one cannot construct non linear supersymmetric Lagrangians, based on a G/H which is Kählerian, which have QGF in chiral representations. These Lagrangians arise from putting symmetry breaking constraints in by hand which are invariant under G but not $\bar{G}$, so the complex extension plays no role at all.

I will sketch the proof /15/, /16/ of the second statement above; the first is proved analogously. It is convenient for these purposes to represent the Lie algebra of $\bar{G}$ in a step generator basis:

$$[E_\alpha, E_{-\alpha}] = \rho_i(\alpha) H_i \qquad i = 1, \ldots, \text{rank } G \qquad \alpha = 1, \ldots, \tfrac{1}{2}(\dim - \text{rank})G \tag{3.9}$$

Here H_i are diagonal generators and the $\rho_i(\alpha)$ are the root vectors. Obviously one has a chiral QGF if for some α

$$E_\alpha \langle\phi\rangle \neq 0 \quad \text{but} \quad E_{-\alpha}\langle\phi\rangle = 0 \tag{3.10}$$

so that one generator is broken and the other is not. Applying the commutator in (3.9) on $\langle\phi\rangle$ and using (3.10) it follows that there must be some H_i such that $H_i \langle\phi\rangle \neq 0$. But the breaking of H_i - since it is real - leads always to doubling. Hence $n_{QGB} \geq 1$.

Besides describing the pattern of QGF for a given model, it is of course very interesting to study the non linear interactions among the QGF. This was done, in an approximate manner, for the SU(5) model and other simple models, in Ref. 11. More general methods were developed by a number of authors /12/ /14/, but the most comprehensive treatment was given by Bando, Muramoto, Maskawa and Uehara /17/. This method has been applied to several interesting examples in Ref. 18. Here I would like to summarize in a qualitative way the results found and indicate clearly why these Lagrangians are not totally specified by the geometry of G/H, in contrast to what happens in the non supersymmetric case.

In a breakdown $G \to H$ one expects that the QGF contain, in general, both doubled and non doubled pieces:

$$\text{QGF} \sim i + 1 + (i + \bar{i} + r) \tag{3.11}$$

The non doubled pieces involve chiral representations, i, for which no QGB are needed. The doubled pieces contain both singlets, 1, as well as vector-like representations, $i + \bar{i} + r$, and have $n_{QGF} = n_{QGB} = n_{GB}$. Letting Π stand generically for a Goldstone superfield, one sees that the scalar content of Π is different depending on whether its QGF is doubled or not doubled.
One has:

$$\Pi^{\text{scalar}}_{\text{doubled}} = (g,p) \quad ; \Pi^{\text{scalar}}_{\text{non doubled}} = (g,g) \tag{3.12}$$

That is the scalar fields in the doubled case contain both pure Goldstone bosons (g) and quasi Goldstone bosons (p). In the non doubled case the scalar components of Π are all Goldstone bosons. In terms of symmetries of the superpotential W after the breakdown $G \to H$, if there is total doubling then the invariance group is just the complex extension of H, $\bar{H}$. If, however, there are some non doubled representations then the invariance group H is bigger, $\tilde{H} \supset \bar{H}$.

The full set of scalar fields in Π parametrize the coset space $\tilde{M} = \bar{G}/\tilde{H}$. If one restricts the scalar fields to be just the Goldstone excitations, g, then these fields parametrize the compact manifold M = G/H. The set p is associated with broken non unitary symmetries and thus serves to parametrize the non compact directions in $\tilde{M}$. The non linear Lagrangian describing the interaction of Goldstone superfields is given as in (3.1), in terms of a functional of the Goldstone superfields, the Kähler potential, $K(\bar{\pi}, \pi)$. The metric g_{ij} derived from K /10/

$$g_{ij} = \frac{\partial^2 K(\pi^*, \pi)}{\partial \pi^*_i \, \partial \pi_j} \tag{3.13}$$

is, however, the metric of a manifold M^*, which is not identical to $\tilde{M}$. M^* is also parametrized by the scalar fields in Π, but its isometries are those of G, not those of $\bar{G}$ which was the case for $\tilde{M}$. Physics is, after all, G not $\bar{G}$ invariant! The manifold M^* is a topological deformation of $\tilde{M}$. However, the shape of M^* along the non compact directions (those related to the QGB p) is not fixed by symmetry /15/. It is this feature that makes the Kähler potential depend on the dynamnics and not only on the geometry of G/H. The more QGB p there are, the less fixed K will be. These considerations are nicely illustrated in the example given below.

4. A SEMI REALISTIC EXAMPLE -THE NOVINO MODEL

The novino model, developed in collaboration with Buchmüller and Yanagida /19/, is based on an underlying supersymmetric SU(2) gauge theory. The preon supermultiplet consists of 6 preons ϕ^r_α which are doublets under SU(2). The global symmetry of the model is $G = SU(6) \times U_X(1)$, where X is a combination of preon number and R symmetry which has no SU(2) anomalies. The formation of the SU(2) singlet condensate

*One can give dynamical arguments supporting the formation of this condensate /19/

$$V = \langle \epsilon^{\alpha\beta} \phi^1_\alpha \phi^2_\beta \rangle \qquad (4.1)$$

breaks $G \to H = SU(4) \times SU(2) \times U(1)_{X'}$. The 17 GB that ensue are easily seen to transform as:

$$GB \sim (4,2) + (\bar{4},2) + (1,1) \qquad (4.2)$$

If the dynamics of the model were such that the QGF transformed in a not totally doubled way:

$$QGF \sim (4.2) + (1,1) \qquad (4.3)$$

then one would have a semirealistic model for one generation of left-handed fermions. Assigning color to ϕ^p_α (p = 3,4,5) and charge as Q = (-1/2,1/2,1/6,1/6,1/6,-1/2), it is easy to see that the (4,2) multiplet above indeed has the correct quantum number of quarks and leptons. The (1,1) state, the novino, is an extra excitation of the model. In fact, one can check in two ways that the pattern of QGF of Eq. (4.3) arises from the dynamics /19/. The set (4.3), but not the totally doubled set, matches the chiral anomalies at the preon level, satisfying 't Hooft's consistency condition /20/. Furthermore, examining the model in the Higgs phase and applying complementarity /21/ also yields (4.3).

Knowing how the QGF transform then allows one to compute the effective Lagrangian describing their interactions. Because of the presence of the doubled novino state, this Lagrangian will have some arbitrariness. I detail below only the piece which involves the quarks and leptons, since it has an interesting structure

$$\mathcal{L}^{eff}_{NL} = \frac{1}{v_1^2}\{(\bar{\psi}_L \gamma^\mu \vec{\tau} \psi_L)\cdot(\bar{\psi}_L \gamma_\mu \vec{\tau} \psi_L)\} + \frac{v_1^2 - v_2^2}{v_1^4}\{(\bar{\psi}_L \gamma^\mu \psi_L)\cdot(\bar{\psi}_L \gamma_\mu \psi_L)\} \qquad (4.4)$$

Here ψ_L is the (4,2) field and v_1, v_2 are scales associated with the (4,2) and (1,1) multiplets, respectively, whose values are fixed by the underlying dynamics.

If the coset space G/H had been Kählerian (e.g. SU(6)/SU(4) x SU(2) x U(1)) with no novino, then $v_2 = 0$ and $\mathcal{L}^{eff}_{NL}$ would have been totally fixed, apart from an overall scale. The presence of novino - which must be there because of the general theorem of Sec. 3 - affects the residual interaction (4.4). Only a knowledge of the underlying dynamics fixes the ratio of v_2/v_1. In Ref. 19, we

argued that most probably $v_2 \simeq v_1$, so that (4.4), after γ - Z^0 mixing, can in fact reproduce the form of the $q^2 = 0$ weak interaction. However, it is not necessary here to imagine that the weak interactions are residual interactions. The quarks and leptons are chiral and one can gauge SU(2) x U(1) at the preon level.

The novino model can be extended rather simply /22/, so that also right-handed quarks and leptons emerge as QGF. One can also incorporate a family structure in the model, but this is done rather unnaturally. Basically, one just changes the number of preons from 6 to $4n_f + 2$, where n_f is the number of families. The relevant breakdown, due to the condensate (4.1), now produces QGF transforming as $(4n_f,2)$ under H, which can be taken as n_f repetitions of (4,2).

Greenberg, Mohapatra and Yasuè /23/, in a model quite similar to the novino model, have a somewhat better way to incorporate families. Basically they introduce an SU(6) gauge theory and then, in addition to the (4,2) QGF arising from the $G \to H$ breakdown, they need two more families of massless fermions to match the 't Hooft conditions.

Both of the above examples of generating families are quite artificial* . It would be nice if families of QGF came out directly out of the group theory. This can occur, rather naturally, if the global symmetry is based on exceptional groups, /24/ /25/ /26/ /27/ /28/ as I discuss in the next section.

5. FAMILIES AND EXCEPTIONAL CHAINS

I already noted earlier how groups in the exceptional chain (E_4 = SU(5), E_5 = SO(10), E_6) were useful as classification groups for the quarks and leptons. Some of the coset spaces involving exceptional groups are also equally well suited. For instance E_6/SO(10) x U(1) has GB ~ 16 + $\overline{16}$ of SO(10); E_7/E_6 x U(1) has GB ~ 27 + $\overline{27}$ of E_6, etc. Clearly, non doubled QGF arising from these coset spaces will have precisely the wanted quantum numbers of quarks and leptons (of one generation, for the above examples).

In the literature /24/ - /28/ there exist various multifamily models based on exceptional coset spaces. This discussion has been systematized recently by Buchmüller and Napoly /29/ and by Itoh, Kunimoto and Kugo /18/ (see also Ref. 30) and I shall describe briefly their findings. These authors catalogue all coset spaces involving exceptional groups which have the following properties:

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* Perhaps less so for the suggestion of Ref. 23. However, here the dynamics is on a more shaky ground.

1. G/H is a Kähler manifold
2. $n_f \geq 3$
3. $H = H' \times K$, with H' = SU(5), SO(10), E_6
4. The resulting $\mathcal{L}_{eff}$ is well defined

Let me make a few remarks on points 1 and 4, since these conditions are perhaps questionable or not obvious. One could object to restricting oneself only to Kähler manifolds, since one knows that for a (linear) Lagrangian theory $n_{QGB} \geq 1$. However, these coset spaces could well arise from a deeper theory - not necessarily one based on a linear Lagrangian - and it could well be that $n_{QGB} = 0$. Furthermore, Kähler manifolds G/H always have $H = H' \times U(1)$. So one expects that, up to a novino, their QGF patterns are similar to that of the non Kähler manifolds G/H'.

Point 4 is also related to having purely chiral fermions in the manifold. In these circumstances, one knows that at times it is not possible to write a consistent non linear Lagrangian for the chiral superfields, because the fermion determinant is ill defined /31/. Because of this reason it may not be sensible to consider these coset spaces. This constraint eliminates possible candidate coset spaces like $E_7/SU(5) \times SU(3) \times U(1)$ /25/. However, as Yanagida /30/ points out, one may always add matter fields to the original model which serve to cancel these anomalies. Hence, from this point of view, constraint 4 may be too restrictive.

Using the constraints 1-4 yields only 5 viable coset spaces /29/ /18/. They all have $G = E_8$ and $H' = E_6$ or SO(10). However, two of these coset spaces, although they give rise to three repetitions of E_6 representations, contain two families plus an antifamily. Hence they are phenomenologically useless. The interesting coset spaces that remain are

i) $E_8/SO(10) \times SU(3) \times U(1)$
ii) $E_8/SO(10) \times SU(2) \times U(1)^2$
iii) $E_8/SO(10) \times U(1)^3$

The coset space i) was discussed already by Ong /24/ and by Iriê and Yasui /28/, and it contains 3 families plus an antifamily:

$$\mathrm{QGF(i)} : (16,3)_1 + (\overline{16},1)_3 + (10,\bar{3})_2 + (1,3)_4 \qquad (5.1)$$

The coset space ii) has two different QGF patterns - differing by how the U(1) quantum numbers are assigned - which also lead to 3 16's of SO(10) plus a $\overline{16}$. Finally the last coset space has many QGF patterns, but none of them have 4 16's. Here again one has always at least one antifamily.

There are both technical and physical remarks that I wish to make on these results. On the technical side, the way in which the Kähler

manifolds were obtained, and their complex structure analyzed, is very nice. Basically there is a simple way to find out which coset space is Kählerian, which is due to Bordermann, Forger and Römer /32/. The manifold $G/H' \times U(1)^p$ is a Kähler manifold if the H' group has a Dynkin diagram which can be obtained from the Dynkin diagram of G by crossing out p dots. The complex structures in $G/H' \times U(1)^p$ are then gotten by selecting all roots which are positive with respect to any chosen U(1) generator in $U(1)^p$ /18/ /29/. For $p=1$, obviously, there is a unique set of QGF. For $p > 1$, on the other hand there can be distinct patterns of QGF even for a given Kähler manifold. This is not surprising, since in these cases there are distinct ways in which to pair up the Goldstone boson fields to fermions.

Physically, the most interesting result of this analysis is that there appear always at least one antifamily in these exceptional coset spaces. This suggests, if these ideas are correct, that there should be a fourth generation with V+A interactions. Although predicting the existence of an antifamily is nice, it is difficult for me to understand what will happen dynamically when mass is generated for the QGF (by breaking G and gauging H). One would imagine that one 16 and the $\overline{16}$ would combine together to form very massive states, leaving then just two relatively light chiral families. If this is so, then this approach is in trouble phenomenologically.

I should remark that by relaxing the restriction 4 one can obtain models with 3 families and no antifamilies. For instance this happens in the $E_7/SU(5) \times SU(3) \times U(1)$ model, studied by Kugo and Yanagida /25/. Hence, serious thought should be given whether the requirement of no σ-model anomalies cannot be circumvented. Another possibility to avoid antifamilies may be not to have purely Kähler manifolds. By removing some of the U(1) factors in the coset spaces in i) - iii) might it not be possible to reverse a 16 into a $\overline{16}$? Naively one may think this might be possible, since after all the U(1) factors were the ones used to select the QGF structures. Buchmüller /33/, however, thinks that this will not help, and that no $\overline{16}$ can be turned into a 16 this way.

6. CONCLUDING REMARKS

I hope to have convinced you that the idea of trying to associate quarks and leptons with coordinates of a coset space is an interesting speculation. Of course, one is still very far from having arrived at a convincing realistic model. In fact, also the original idea that QGF were bound states of preons has somewhat changed. If the global symmetry is that of an exceptional group, it is clear that the underlying theory cannot be just an ordinary non Abelian gauge theory. In effect, these exceptional coset spaces are much closer in spirit to superstring ideas than preon models.

Even if one were to become convinced that a given G/H coset space correctly is to be associated with the quarks and leptons we know, the most difficult part of the program still remains ahead. One has a dynamics (supersymmetry plus spontaneous breakdown of a global symmetry) which gives massless fermions. How does one get the quarks and leptons to have small masses, and how does one remove the unwanted scalar states? Clearly one must break the supersymmetry and break the global symmetry, but what triggers these breakings and how do the quark and lepton mass hierarchies come about? There is plenty of room left for good ideas!

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REFERENCES

/1/ P. Langacker, Phys. Repts. 72C (1981) 185
/2/ M.B. Green and J.H. Schwarz, Phys. Lett. 149B (1984) 117; Nucl. Phys. B243 (1984) 285
/3/ P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46; see also Proceedings of the Symposium on Anomalies, Geometry and Topology, Ed. by W.A. Bardeen and A.R. White, (World Scientific, Singapore, 1985)
/4/ E. Calabi in Algebraic Geoemtry and Topology: A symposium in Honor of S. Lefschetz (Princeton University Press, Princeton 1957); S.-T. Yau, Proc. Natl. Acad. Sci. 74 (1977) 1798
/5/ Y. Nambu, Phys. Rev. Lett. 4 (1960) 380; J. Goldstone, Nuovo Cimento 19 (1961) 154
/6/ W. Buchmüller, S.T. Love, R.D. Peccei and T. Yanagida, Phys. Lett. 115B (1982) 233
/7/ W. Buchmüller, R.D. Peccei and T. Yanagida, Phys. Lett. 124B (1983) 67
/8/ S. Weinberg, Phys. Rev. 166 (1968) 1568
/9/ S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; G.C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247
/10/ B. Zumino, Phys. Lett. 87B (1979) 203
/11/ W. Buchmüller, R.D. Peccei, and T. Yanagida, Nucl. Phys. B227 (1983) 503
/12/ C.K. Lee and H.S. Sharatchandra, MPI-PAE/PTh 54/83
/13/ W. Lerche, Nucl. Phys. B238 (1984) 582
/14/ T. Kugo, I. Ojima and T. Yanagida, Phys. Lett. 135B (1984) 402
/15/ W. Lerche, Thesis University of Munich 1985
/16/ G. Shore, Nucl. Phys. B244 (1984) 157
/17/ M. Bando, T. Kuramoto, T. Maskawa and S. Uehara, Phys. Lett. 138B (1984) 94; Prog. Theor. Phys. 72 (1984) 313
/18/ K. Itoh, T. Kugo and H. Kunimoto, Univ. of Kyoto preprint, KUNS-792 and 793 (1985)
/19/ W. Buchmüller, R.D. Peccei and T. Yanagida, Nucl. Phys. B231 (1984) 53
/20/ G. 't Hooft in Recent Developments in Gauge Theories, Cargese Summer School 1979, ed. by G. 't Hooft et al. (Plenum Press, New York 1980)

/21/ S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B173 (1980) 208
/22/ W. Buchmüller, R.D. Peccei and T. Yanagida, Nucl. Phys. B244 (1984) 186
/23/ O.W. Greenberg, R.N. Mohapatra and M. Yasue, Phys. Rev. Lett. 51 (1983) 1737
/24/ C.L. Ong, Phys. Rev. D27 (1983) 911, 3044; D31 (1985) 3271
/25/ T. Kugo and T. Yanagida, Phys. Lett. 134B (1984) 313
/26/ L. Ibañez, Phys. Lett. 150B (1985) 127
/27/ Y. Achiman, S. Aoyama and J.W. Van Holten, Phys. Lett. 141 (1984) 54
/28/ S. Iriê and Y. Yasui, Tu/85/278, Zeit. für Physik to be published
/29/ W. Buchmüller and O. Napoly, CERN preprint CERN TH 4197 (1985)
/30/ T. Yanagida in Proceedings of the 1985 INS Symposium on Composite Models of Quarks and Leptons, Tokyo, Japan
/31/ G. Moore and P. Nelson, Phys. Rev. Lett. 53 (1984) 1519; E. Cohen and C. Gomez, Nucl. Phys. B254 (1985) 235
/32/ M. Bordemann, M. Forger and H. Römer, CERN preprint, CERN TH 4111 (1985)
/33/ W. Buchmüller, private communication