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NON-PERTURBATIVE APPROACH TO SCALAR-FERMION THEORIES

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The σ-model with Wilson lattice fermions and an explicitly chiral symmetric extension of it is considered. A lattice regularized $SU(2)_L \otimes U(1)_Y$ symmetric electro-weak model is briefly described. The rôle of the infra-red fixed point structure of the Callan-Symanzik renormalization group equations in the continuum limit is discussed.

1. INTRODUCTION

In the Standard Model the masses of elementary particles arise due to spontaneous symmetry breaking via the Higgs mechanism. The main motivation for the non-perturbative study of the Higgs-sector of the Standard Model is a better understanding of the spontaneous symmetry breaking. A few of the interesting questions are:

- the continuum limit (or, more generally, the large cut-off behaviour) of non-asymptotically free couplings, like the scalar quartic self-coupling;
- the bounds on the mass of the Higgs boson;
- the upper bound on fermion masses;
- the possibility of a strongly interacting Higgs-sector (strong quartic and/or Yukawa-couplings) etc.

The non-perturbative features of the scalar sector (without fermions) were intensively studied in recent years (for a summary see the contribution of R. Shrock to this conference and [1]). A prototype model of the scalar sector describes an SU(2) gauge field interacting with a scalar doublet matter field ("standard Higgs model"). The general properties of this and some other similar models are by now reasonably well understood (but the work is still going on).

The inclusion of fermions (quarks and leptons) in the non-perturbative investigations, however, has some conceptual as well as practical difficulties. The conceptual difficulty is related to chirality and to chiral anomalies, whereas the practical difficulty is due to the fact that the numerical simulation of fermionic systems is very slow. A prototype model for the Higgs sector with fermions is the Gell-Mann-Lévy σ-model [2] of a scalar doublet and a fermion doublet. This model has a chiral $SU(2)_L \otimes SU(2)_R \equiv O(4)$ symmetry which is simply related to the $SU(2)_L \otimes U(1)_Y$ symmetry of the standard electro-weak model.

2. THE σ-MODEL WITH WILSON FERMIONS

Keeping the field normalizations arbitrary and using the Wilson [3] fermion field discretization, the lattice action of the σ-model is:

$$S = \sum_x \Big\{ \mu\phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx})^2 - \kappa\sum_\mu \phi_{Sx+\hat\mu}\phi_{Sx} + M(\bar\psi_x\psi_x) - K\sum_\mu(\bar\psi_{x+\hat\mu}[r+\gamma_\mu]\psi_x) + G\phi_{Sx}(\bar\psi_x\Gamma_S\psi_x)\Big\} \tag{1}$$

The scalar field is here: $\varphi_x \equiv \phi_{0x} + i\tau_s\phi_{sx} \equiv \sigma_x + i\tau_s\pi_{sx}$ with ($S = 0,1,2,3$; $s = 1,2,3$). The 8⊗8 matrices Γ_S ($S = 0,1,2,3$) are defined as: $\Gamma_S \equiv (1, -i\gamma_5\tau_s)$. The Wilson fermion parameter is $0 < r \le 1$. The normalization of the fields can be chosen by convenience: in perturbation theory the simplest choice is $\kappa = K = \frac{1}{2}$, whereas in numerical studies one uses $\mu = 1 - 2\lambda$ and $M = 1$. Taking into account the freedom of field normalizations, the number of independent relevant bare parameters in the above action is 4.

A first important task is to find the *critical set* of points in the bare parameter space where the masses vanish (the correlation length in lattice units is infinite). These are the points where a (non-perturbative) continuum limit can, in principle, be defined. A qualitative description of the critical set can be obtained at small couplings λ, G in *lattice perturbation theory*, and for $G \to \infty$ by the *hopping parameter expansion* in powers of K and κ. The result is [4] that at small bare Yukawa-coupling G the points with $m_\pi = 0$ and $m_\sigma = 0$ do not coincide (except for $K = 0$, where the fermions decouple). The same happens also at $G = \infty$ where, in addition, the set of points with vanishing π-mass and the set of points with vanishing fermion mass $m_F = 0$ are disjoint. At $G = \lambda = \infty$ the hopping parameter expansion implies that besides the original fermion there is a dynamically produced opposite parity fermion. In the points where the fermion mass is zero the opposite parity fermion mass is also zero.

The 1-loop perturbation theory and the hopping parameter expansion imply that there are some difficulties with the above Wilson fermion formulation of the σ-model:

- The contribution of the fermion tadpole diagrams to the vacuum expectation value of σ_x is non-zero. Therefore, one has to add to the action a counterterm linear in σ_x. The coefficient of this term has to be appropriately tuned, in order to find the required spontaneous symmetry breaking pattern.

- In the whole bare parameter space there are no points with $m_\pi = m_\sigma = m_F = 0$, except for the point with $\lambda = G = 0$ where perturbation theory is defined.

- The chiral $SU(2)_L \otimes SU(2)_R$ symmetry cannot be gauged because of the explicit chiral symmetry breaking of the Wilson-term in the fermion action.

- A dynamical parity doubling of the fermion occurs.

On the basis of these difficulties the above action cannot be considered as a satisfactory non-perturbative formulation of the σ-model.

3. CHIRAL FERMION MODELS

All the above difficulties are connected to the lack of chiral symmetry in the Wilson fermion formulation. In order to have an explicit chiral symmetry let us introduce [5] an additional "mirror fermion" field $\chi_x, \bar{\chi}_x$ with exchanged left-right transformation properties. The lattice action with chiral $SU(2)_L \otimes SU(2)_R$ symmetry is:

$$\begin{aligned} S = \sum_x \Big\{ & \mu \phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx})^2 - \kappa \sum_\mu \phi_{Sx+\hat\mu}\phi_{Sx} \\ & + \mu_{\psi\chi}\left[(\bar\chi_x\psi_x) + (\bar\psi_x\chi_x)\right] + \mu_\psi(\bar\psi_x\psi_x) + \mu_\chi(\bar\chi_x\chi_x) \\ & - \sum_\mu \left[K_\psi(\bar\psi_{x+\hat\mu}\gamma_\mu\psi_x) + K_\chi(\bar\chi_{x+\hat\mu}\gamma_\mu\chi_x)\right] \\ & + r\sum_\mu \left[(\bar\chi_x\psi_x) - (\bar\chi_{x+\hat\mu}\psi_x) + (\bar\psi_x\chi_x) - (\bar\psi_{x+\hat\mu}\chi_x)\right] \\ & + G_\psi\phi_{Sx}(\bar\psi_x\Gamma_S\psi_x) + G_\chi\phi_{Sx}(\bar\chi_x\Gamma_S^+\chi_x)\Big\} \end{aligned} \qquad (2)$$

In the chiral limit μ_ψ and μ_χ vanish, but the ψ-χ-mixing mass $\mu_{\psi\chi}$ can be non-zero.

The opposite contribution from the mirror fermion cancels all the chiral anomalies of the original fermion [6]. The global SU(2) anomalies found by Witten [7] are also absent. The physical spectrum of the fermions consists of two fermion states which are, in general, mixtures of the ψ- and χ-components. The additional lattice fermion states at the non-zero corners of the Brillouin-zone have masses proportional to the cut-off, similarly to the situation for Wilson fermions. This can be seen from the inverse fermion propagator in momentum space:

$$\tilde{G}_k^{-1} = \begin{pmatrix} \mu_\psi + i\gamma\cdot\bar{k} & \mu_{\psi\chi} + r\hat{k}^2 \\ \mu_{\psi\chi} + r\hat{k}^2 & \mu_\chi + i\gamma\cdot\bar{k} \end{pmatrix} \qquad (3)$$

Here the notations $\bar{k}_\mu = \sin k_\mu$ and $\hat{k}_\mu = 2\sin(k_\mu/2)$ were used.

In case of spontaneous symmetry breaking the ψ- and χ-masses are proportional to the vacuum expectation value v of the scalar field: $\mu_\psi = G_\psi v$; $\mu_\chi = G_\chi v$

(the chiral invariant ψ-χ-mixing mass $\mu_{\psi\chi}$ can be arbitrary). The question naturally arises whether the mirror-fermion can be removed from the spectrum by taking the limit $G_\chi \to \infty$? The answer is no, because the above tree-level formulae for the masses are valid only for sufficiently small Yukawa-couplings. There is an upper bound for the physical fermion masses, in the same way as in the limit $\lambda \to \infty$ for the Higgs-boson mass. (For a discussion of the question of anomalies in case of very heavy fermions see also [8].) With a more complicated Higgs-sector (e. g. two Higgs doublets) it is possible to perform a continuum limit where $\mu_\psi \to 0$; $\mu_\chi \neq 0$, but in this case after gauging the $SU(2)_L$ symmetry the mass of the W-boson stays also finite in lattice units, hence the W-boson will also be removed from the spectrum. The reason of the impossibility to remove by spontaneous symmetry breaking the mirror fermion alone is, that in the symmetric phase there is a degenerate fermion parity doublet representing the unbroken chiral symmetry. Near the critical point it is possible to describe both the unbroken and broken phases by an expansion around the critical point, and in the critical theory there are both the ψ- and χ-fermions present.

The gauging of the $SU(2)_L \otimes SU(2)_R$ symmetry is straightforward [5]. In order to define a chiral $SU(2)_L \otimes U(1)_Y$-symmetric model, the L- and R-handed gauge fields $U_{L,R}(x,\mu)$ have to be replaced by

$$
\begin{aligned}
U_L(x,\mu) &\Longrightarrow U_L(x,\mu)U_Y(x,\mu) \\
U_R(x,\mu) &\Longrightarrow U_Y(x,\mu) \qquad (4)
\end{aligned}
$$

The hypercharge quantum number Y is given by

$$Y = 2T_{R3} + B - L \qquad (5)$$

where T_{R3} is the third component of $SU(2)_R$ isospin. The vector-like $(B-L)$ quntum number is -1 for leptons and $\frac{1}{3}$ for quarks. The Yukawa-couplings can now break the global $SU(2)_R$ symmetry, for instance as a real diagonal matrix G_ψ in

$$(\tilde{\psi}_{Rx} G_\psi \varphi_x^+ \psi_{Lx}) + (\tilde{\psi}_{Lx} \varphi_x G_\psi \psi_{Rx}) \qquad (6)$$

In the case of three standard fermion generations the mass matrix entries μ_ψ, μ_χ, $\mu_{\psi\chi}$ (for a given quark- or lepton-type) are $3\otimes 3$ matrices. The physical fermion states are obtained by diagonalizing the $6\otimes 6$ mass matrix. In the simple case, when the chiral invariant mass mixing parameter is negligible: $\mu_{\psi\chi} \ll \mu_\psi, \mu_\chi$, the ψ-components can be separately diagonalized according to

$$\mu_\psi^D = F_\psi^{(L)-1} \mu_\psi F_\psi^{(R)} \qquad (7)$$

and the Kobayashi-Maskawa matrix between u- and d-quarks is given by

$$KM \equiv F_{\psi d}^{(L)-1} F_{\psi u}^{(L)} \qquad (8)$$

The fermion part of the action in Eq. (2) has (for $\mu_\psi = \mu_\chi = 0$) a global $U(2)_L \otimes U(2)_R$ symmetry. As a consequence, in the $SU(2)_L \otimes U(1)_Y$ model there is lepton- and quark-number conservation. In addition, in the fermion sector there is also an exact $U(1)_{Peccei-Quinn}$ global chiral symmetry [9], which is explicitly broken in the Higgs-Yukawa-sector. In a model with two Higgs-doublets this breaking can, however, be very small.

At low energies the $SU(2)_L \otimes U(1)_Y$ model with mirror fermions can be very similar to the standard electroweak model: by chosing the Yukawa-couplings like G_ψ and the ψ-χ mixing masses $\mu_{\psi\chi}$ small one can make the mirror-fermion components of the light fermions small (the mirror-fermion components have $V+A$ couplings to the W-boson). At higher energies there is, however, a marked difference due to the occurence of the physical mirror fermion states. The masses of the mirror fermions have to be roughly below ~500 GeV, corresponding to the unitarity limit for Yukawa-couplings [10]. The only possible manifestation of the mirror fermions at low energies is a small $V+A$ admixture to the dominant $V-A$ weak couplings. The present experimental limits on the $V+A$ admixtures are, however, not very strong [11]: in the best cases roughly of the order of 10^{-2}.

4. CONTINUUM LIMIT AND IRFP STRUCTURE

A non-perturbative continuum limit can be defined by tuning the mass parameters (like κ and $\mu_{\psi\chi}$) for fixed values of the couplings (λ, G_ψ and G_χ) to a critical point where all the physical masses tend to zero. The behaviour of the renormalized couplings as a function of the scale variable $\tau \equiv \log(am)^{-1}$ (where am is some mass in lattice units) is governed for $\tau \to \infty$ by the infrared fixed point (IRFP) structure of the Callan-Symanzik β-functions. In the case of the lattice action in Eq. (2) the 1-loop β-functions are:

$$\beta_{G\psi} = \frac{1}{16\pi^2} \cdot 4G_{\psi r}(G^2_{\psi r} + G^2_{\chi r})$$

$$\beta_{G\chi} = \frac{1}{16\pi^2} \cdot 4G_{\chi r}(G^2_{\psi r} + G^2_{\chi r})$$

$$\beta_\lambda = \frac{1}{16\pi^2} \cdot$$

$$\cdot \left(96\lambda_r^2 + 16G^2_{\psi r}\lambda_r + 16G^2_{\chi r}\lambda_r - 4G^4_{\psi r} - 4G^4_{\chi r}\right) \quad (9)$$

These have an IRFP at $\lambda_r = G_{\psi r} = G_{\chi r} = 0$, therefore the continuum limit of the model is trivial, unless there is some other non-perturbative non-trivial IRFP.

It can be easily seen that in the limit $\tau \to \infty$, among the ratios

$$x \equiv \frac{G^2_{\psi r}}{G^2_{\chi r}}; \qquad y_\psi \equiv \frac{\lambda_r}{G^2_{\psi r}}; \qquad y_\chi \equiv \frac{\lambda_r}{G^2_{\chi r}} \quad (10)$$

x is an arbitrary constant, and

$$y_\chi \to \frac{1}{6}; \qquad y_\psi \to \frac{1}{6x} \quad (11)$$

The arbitrariness of the ratio x is important, because in the continuum limit in the spontaneously broken phase it allows to fix the mass ratio μ_χ/μ_ψ.

After including the gauge couplings, in particular also the colour SU(3) coupling, in the renormalization group equations the IRFP structure becomes non-trivial. (See [1] and references therein.) In order to see the qualitative behaviour let us consider a simple model with a heavy (colour triplet) quark doublet and a scalar doublet. (The SU(2) gauge coupling will be neglected here, and no mirror fermions are considered.) The 1-loop renormalization group (RG) equations for the renormalized SU(3) gauge coupling g_r and the renormalized Yukawa- and quartic couplings (G_r and λ_r) are:

$$16\pi^2\frac{dg_r^2}{d\tau} = \frac{58}{3}g_r^4$$

$$16\pi^2\frac{dG_r}{d\tau} = -12G_r^3 + 8g_r^2G_r$$

$$16\pi^2\frac{d\lambda_r}{d\tau} = -96\lambda_r^2 - 48\lambda_rG_r^2 + 12G_r^4 \quad (12)$$

The solution of the first equation with an initial value g^2_{r0} at $\tau = 0$ is:

$$g_r^2(\tau) = \left(g_{r0}^{-2} - \tau\frac{29}{24\pi^2}\right)^{-1} \quad (13)$$

Substituting this to the second equation one obtains:

$$G_r^2(\tau) = g_r^2(\tau)\left(1 - \tau g^2_{r0}\frac{29}{24\pi^2}\right)^{\frac{5}{29}} \cdot$$

$$\cdot \left[\frac{g^2_{r0}}{G^2_{r0}} + \frac{36}{5} - \frac{36}{5}\left(1 - \tau g^2_{r0}\frac{29}{24\pi^2}\right)^{\frac{5}{29}}\right]^{-1} \quad (14)$$

The third equation cannot, in general, be solved explicitly. However, in the limit $g_r/G_r \to \infty$ the asymptotic behaviour of the solution is:

$$\lambda_r(\tau) \to \frac{18G_r^4(\tau)}{19g_r^2(\tau)} \quad (15)$$

The above RG-equations describe the behaviour of the renormalized couplings in the case of tuning the hopping parameters, for fixed bare couplings (g, G, λ), to the critical hypersurface. Since the above β-functions are taken from perturbation theory, one has to assume, however, that the renormalized couplings are small enough. In order to have a complete description, the dependence of the $\tau = 0$ initial data on the fixed bare couplings has to be given:

$$g_{r0} = g_{r0}(g, G, \lambda); \qquad G_{r0} = G_{r0}(g, G, \lambda)$$

$$\lambda_{r0} = \lambda_{r0}(g, G, \lambda) \quad (16)$$

For a qualitative description one can take $g_{r0} \simeq g$; $G_{r0} \simeq G$; $\lambda_{r0} \simeq \lambda$, at least if the bare couplings themselves are not very large.

According to Eqs. (13-15), for fixed (g, G, λ), the renormalized couplings grow for increasing τ. Even if the initial couplings are small, at some point the perturbative β-functions are not applicable any more. This is quite different from the behaviour in pure ϕ^4 or in the standard SU(2) Higgs model. Nevertheless, similarly to the case of the standard SU(2) Higgs model, one can consider the *"curves of partially constant physics"* (CPCP's [1,12]) with fixed renormalized gauge coupling (g_r) and fixed bare Yukawa- and quartic couplings (G, λ). Along these curves the bare gauge coupling goes to zero ($\beta \equiv 6g^{-2} \to \infty$) and for sufficiently small g_r^2 the above solutions imply:

$$G_r^2(\tau) = g_r^2 \left(1 + \tau g_r^2 \frac{29}{24\pi^2}\right)^{-\frac{5}{29}} \cdot$$

$$\cdot \left[G^{-2} \left(g_r^{-2} + \tau \frac{29}{24\pi^2} \right)^{-1} + \frac{36}{5} - \right.$$

$$\left. - \frac{36}{5} \left(1 + \tau g_r^2 \frac{29}{24\pi^2}\right)^{-\frac{5}{29}} \right]^{-1} \tag{17}$$

Therefore, in the limit of zero lattice spacing $\tau \to \infty$, the renormalized Yukawa-coupling tends to zero (and due to Eq. (15) the same is true also for the renormalized quartic coupling λ_r). In other words, the continuum limit is a "trivial" QCD-like gauge theory. The non-trivial IRFP structure of the above RG equations does not imply a non-trivial continuum limit (different from QCD) for the whole theory with Yukawa and quartic couplings. Still, the qualitative behaviour is quite different from the case of the standard SU(2) Higgs model without fermions (see [1] and references therein). The vanishing of the Yukawa-coupling along the CPCP's is also very-very slow: it goes to zero as a small inverse power (actually $-\frac{5}{58}$) of the logarithm of the cut-off! Therefore one can expect, that in the case of strong enough initial Yukawa couplings the strong interaction can persist also for rather high cut-off's. According to the above discussion this must be the case in the vicinity of the critical surface for non-zero bare gauge couplings, where strong Yukawa- and quartic couplings are supported by the strong renormalized gauge coupling. Since the perturbative RG structure is so much different from the pure ϕ^4 theory or from the standard Higgs model without fermions, it cannot be excluded, that for stronger gauge couplings the perturbative treatment breaks down completely and there exists a non-trivial continuum limit also in the mathematical sense.

The most interesting question is, of course, what is the situation in nature: is the colour interaction at the W-boson mass scale strong enough to support a strongly interacting Higgs-Yukawa sector for very high cut-off's? In any case, a necessary condition for this is the existence of heavy fermions. In the case of the above chiral $SU(2)_L \otimes U(1)_Y$ model the rôle of the heavy fermions with strong Yukawa-couplings can be taken over by the mirror fermions.

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