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by '

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# Heavy Flavour Production at HERA

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## Abstract

We present explicit formulae for the total differential cross sections of heavy flavour production at HERA including  $\gamma$ -Z<sup>0</sup>-interference and W-exchange. We evaluate total top production rates and discuss theoretical uncertainties. Assuming an integrated luminosity of 100 pb<sup>-1</sup> we estimate that a top quark mass up to 100 GeV can be probed.

### **1** Introduction

One of the main topics of forthcoming electron proton colliders like HERA [1] is heavy flavour production. Production and decay of heavy fermions are ideal reactions to test the standard model [2]. In the electroweak sector they allow for a determination of, for example, the electroweak coupling constants, the Cabibbo-Kobayashi-Maskawa matrix elements and the quark masses. Furthermore, reactions involving heavy fermions are of importance in order to get information about possible extensions of the standard model. These reactions are in competition with reactions of models with supersymmetric particles or colour excited leptons and quarks [3].

In electron-proton colliders like HERA a heavy quark pair is created via the boson-gluon fusion mechanism [4] where the boson is either a W-boson, a photon, or a  $Z^0$ -boson. Total production rates will not only tell us which heavy quark mass is accessible at a given center of mass energy but also will give us information about the gluon structure function  $G(x, Q^2)$ for small values of x. Furthermore, heavy flavour production will influence the value of  $R_L = \sigma_L/\sigma_2$  as predicted by massless QCD. As a last point, the thresholds for heavy flavour productions lead to a power-like scaling violation [6]. This scaling violation is stronger than the logarithmic scaling violation [5] of massless QCD. Thus the latter can only be extracted after correcting for the scaling violation due to heavy flavours [6,11]

There has been a great effort in the study of boson-gluon fusion <sup>1</sup> into heavy flavours. Several authors have calculated the process in the Weiszäcker-Williams approximation [7,9], for which Monte Carlo event generators even exist [12,19]. Numerical studies for charm production at fixed target experiments have been done as well [4]. Explicit formulae for the deep-inelastic structure functions  $F_i$  for heavy flavour production have existed for a long time [10]. They have been recently updated by [11,12]. Also some work has been done on the transverse momentum distribution of massive quarks. [13].

However, in order to study the topology of heavy flavour production at HERA, the fully (five-fold) differential cross section is needed. Incorporating the fully differential cross section formula in a Monte Carlo event generator allows not only for an inclusion of the experimental setup but also for a study of the various dependences of the cross section and how far the theoretical input may be reconstructed. As a first step in this direction we present a calculation of heavy flavour production in electron-proton scattering, leading to an explicit formula for the total differential cross section. We inculde both W-exchange and  $\gamma$ -Z<sup>0</sup> interference. Furthermore we allow for electron or positron scattering and include longitudinal polarization of the incoming lepton.

The organization of the paper is as follows: After defining the kinematical variables in section 2 and the electroweak contents in section 3, we give an explicit representation of the particle momenta and a discussion of the physical region boundaries in section 4. In section 5 we present the final formula for the total differential cross section. There we also perform the two integrations which can be done analytically and which yield the usual deep-inelastic structure functions  $F_1, F_2$  and  $F_3$ . Section 6 contains a short description of the Weiszäcker-

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<sup>&</sup>lt;sup>1</sup>The authors of [7,8] also investigated higher order contributions. The corrections to  $t\bar{t}$  production were found to be negligible. However, contributions coming from W production with subsequent decay into heavy flavours seem to produce rather large 20% (25%) corrections to the  $\bar{t}b$  rate for  $m_t = 50(70)$ GeV [11]. These calculations [8] were done in the Weiszäcker-Williams approximation and should be confirmed. We will neglect these contributions to the  $\bar{t}b$  production in the following.

Williams approximation [WWA], whose applicability we investigate in the last section. There we present results for total rates of top production and discuss possible uncertainties of the results. Since the massive  $O(\alpha_s)$  calculation contains additional kinematic and electroweak dependences compared to the massless case, we desribe in the appendix the calculation of the matrix element in some length.

We find a top production cross section of about  $0.03 \text{ pb}^{-1}$  for  $m_t = 100 \text{ GeV}$ . Assuming an integrated luminosity of  $100 \text{ pb}^{-1}$  and requiring at least 3 events for detection we estimate that a top mass up to 100 GeV should be observable at HERA.

### 2 Notation

We consider positron (electron) proton scattering into quarks of heavy flavours f and f':

$$e^{\pm}(l_e) + p(P) \rightarrow l(l) + q_f(p_f) + \overline{q}_{f'}(p_{f'}) + X \tag{1}$$

allowing for longitudinal polarized positrons (electrons). Here l is either a positron (electron) or an (anti-) neutrino. We neglect the lepton masses whenever it is safe (see below), thus:

$$l_e^2 = l^2 = 0 (2)$$

whereas

$$P^2 = m_p^2, \quad p_f^2 = m_f^2, \quad p_{f'}^2 = m_{f'}^2.$$
 (3)

Experimentally, no heavy quarks are seen within the proton-sea. The lowest order QCD realization of process (1) is assumed to happen via gluon emission out of the proton which in turn goes into a heavy quark-antiquark pair. We thus consider the subprocess:

$$e^{\pm}(l_e) + \text{gluon}(p) \rightarrow l(l) + q_f(p_f) + \overline{q}_{f'}(p_{f'}) \tag{4}$$

where we take the gluon to be on-shell  $(p^2 = 0)$ .

To lowest order in the electroweak coupling  $\alpha$  the process (4) happens via one-boson exchange, the boson being either the photon, the  $Z^0$  or the W. This subprocess

$$boson(q) + gluon(p) \rightarrow q_f(p_f) + \overline{q}_{f'}(p_{f'})$$
(5)

is the so called boson-gluon fusion [4]. The momentum q of the boson is given by

$$q := l_e - l \tag{6}$$

satisfying  $p + q = p_f + p_{f'}$ .

Let us define the following invariants (here  $\hat{}$  denotes variables of the subsystem (5), e.g.  $\hat{s}$  is the cms energy of (5) ):

$$Q^{2} := -q^{2}$$

$$\hat{s} := s_{12} \equiv (p_{f} + p_{f'})^{2}$$

$$\hat{t} := (p - p_{f})^{2}$$

$$\hat{u} := (p - p_{f'})^{2}$$
(7)

 $\mathbf{2}$ 

yielding the relations

$$2p \cdot q = \hat{s} + Q^{2}$$

$$\hat{u} = m_{f}^{2} + m_{f'}^{2} - Q^{2} - \hat{s} - \hat{t}$$
(8)

As usually, we denote by s the (total) squared cms-energy:

$$s := (P + l_e)^2 \approx 2P \cdot l_e \tag{9}$$

and by W the final hadronic mass:

$$W^2 := (P+q)^2 \tag{10}$$

In the following we assume the gluon to carry a fraction  $\eta$  of the proton momentum:

$$p = \eta P$$
, i.e.  $p \cdot q = \eta P \cdot q$ , ... (11)

Then the squared cms-energy  $s_G$  of the electron-gluon sub-system (4) equals

$$\beta_G := (p + l_e)^2 = 2p \cdot l_e = \eta s$$
 (12)

It is convenient to define the following dimensionless variables:

$$x := \frac{Q^2}{2P \cdot q}$$

$$x_G := \frac{Q^2}{2p \cdot q}$$

$$y := \frac{p \cdot q}{p \cdot l_e}$$

$$z := \frac{p \cdot p_f}{p \cdot q}$$
(13)

We find the following relations

$$x = \eta x_{G} = \eta \frac{Q^{2}}{\hat{s} + Q^{2}}$$

$$y = \frac{\hat{s} + Q^{2}}{\eta s} = \frac{Q^{2}}{xs}$$

$$Q^{2} = x_{G}ys_{G} = xys$$

$$W^{2} = \frac{1 - x}{x}Q^{2}$$

$$\hat{s} = (1 - x_{G})ys_{G} = ys(\eta - x) = Q^{2}\frac{\eta - x}{x} = Q^{2}\frac{1 - x_{G}}{x_{G}}$$

$$\hat{t} = m_{f}^{2} - zys_{G}$$
(14)

## 3 Cross section formula

We define electromagnetic and weak couplings by the following interaction Lagrangean densities:

$$\mathcal{L}_{e.m.} = -\epsilon \left[ Q_{\epsilon} \bar{\Psi}_{\epsilon} \gamma^{\mu} \Psi_{\epsilon} + Q_{f} \bar{\Psi}_{f} \gamma^{\mu} \Psi_{f} \right] A_{\mu}$$

$$\mathcal{L}_{n.c.} = -\frac{\epsilon}{2 \sin 2\theta_{W}} \left[ \bar{\Psi}_{\epsilon} \gamma^{\mu} (v - a\gamma_{5}) \Psi_{\epsilon} + \bar{\Psi}_{f} \gamma^{\mu} (v_{f} - a_{f} \gamma_{5}) \Psi_{f} \right] Z_{\mu}$$

$$\mathcal{L}_{c.c.} = -\frac{\epsilon}{2\sqrt{2} \sin \theta_{W}} \left[ \bar{\Psi}_{\epsilon} \gamma^{\mu} (1 - \gamma_{5}) \Psi_{\nu} + \bar{\Psi}_{f} V_{ff'} \gamma^{\mu} (1 - \gamma_{5}) \Psi_{f'} \right] W_{\mu}$$

$$(15)$$

where the Dirac fields  $\Psi_e, \Psi_\nu, \Psi_f(\Psi_{f'})$  and the vector fields  $A_\mu, Z_\mu$   $(W_\mu)$  describe, respectively, the electron  $e^-$ , the electron-neutrino, the quark with flavour f(f'), the photon  $\gamma$  and the neutral intermediate boson  $Z^0(W)$  of mass  $m_Z(m_W)$ ;  $Q_e$  and  $Q_f$  give the electric charge of  $e^-$  and quark of flavour f in units of  $e = \sqrt{4\pi\alpha}$ , where  $\alpha$  is the fine structure constant.  $V_{ff'}$  is the Kobayashi-Maskawa quark mixing matrix and  $\theta_W$  is the Weinberg angle. The parameters  $v(v_f)$  and  $a(a_f)$  determine the strength of the vector and axial vector couplings of the  $Z^0$  at the electron (quark) vertex. One has for:

(i) charged leptons

$$Q_e = -1$$

- (ii) neutral leptons
  - $Q_e = 0 \qquad v = +1 \qquad a = +1 \tag{16}$

a = -1

 $v=-1+4\sin^2 heta_W$ 

(iii) u,c,t, ... quarks

$$Q_f = 2/3$$
  $v_f = +1 - 8/3 \sin^2 heta_W$   $a_f = +1$ 

(iv) d,s,b, ... quarks

$$Q_f = -1/3$$
  $v_f = -1 + 4/3 \sin^2 heta_W$   $a_f = -1$ 

The masses of  $Z^0$  and W are given by:

$$m_W^2 = \frac{\pi \alpha}{G_F \sin^2 \theta_W (1 - \Delta r) \sqrt{2}}$$

$$m_Z^2 = \frac{m_W^2}{\cos^2 \theta_W}$$
(17)

where  $G_F$  is the Fermi weak coupling constant and  $\Delta r$  an electroweak radiative correction.

The cross section for producing a quark-antiquark pair of flavour f (ff' in the charged sector) is given by the formula<sup>2</sup>:

$$d\sigma_{f}^{\pm} = \frac{1}{4p \cdot l} \frac{1}{2} \frac{1}{2} \theta_{l} G(\eta, \hat{s}) d\eta dP S^{(3)} \sum_{\text{spins}} |\mathcal{M}_{f}^{\pm}|^{2}$$
(18)

Here  $G(\eta, \hat{s})$  is the gluon distribution (normalized to 1/2), and gluon spin- and colour- average factors (1/2 and 1/2) have been included.  $\theta_l$  equals 1/2 (1) for an incoming electron (neutrino). The momentum scale of the gluon density has been chosen to correspond to the total invariant energy  $\hat{s}$  of the produced heavy quark system. This scale  $\hat{s}$  has also been used as argument of the running QCD coupling constant  $g_s$ :

$$\alpha_{s}(\hat{s}) = \frac{12\pi}{(33 - 2N_{f})\ln\frac{\hat{s}}{\Lambda^{2}}}$$
(19)

In section 7 we will further comment on this choice as well as on the choice of  $\Lambda$  and the number of flavours  $N_f$ .

<sup>&</sup>lt;sup>2</sup>"+" and "-" refer to positron and electon scattering, respectively.

#### G.A. Schuler: Heavy flavours

In the appendix we will show that the most general cross section can be written as:

$$d\sigma_{f}^{\pm} = \frac{(2\pi)^{3} \alpha_{s}(\hat{s}) \alpha^{2} 2\theta_{l}}{yQ^{4}} G(\eta, \hat{s}) d\eta dP S^{(3)} \left\{ g_{11}^{f;\pm} A_{11} + g_{12}^{f;\pm} A_{12} + g_{44}^{f;\pm} A_{44} \right\}$$
(20)

The coefficients  $g_{ij}^{ff';\pm}(Q^2)$  in (20) depend solely on  $Q^2$  and on the electroweak coupling constants. In the neutral sector they contain all the information on  $\gamma$ - $Z^0$  interference and the  $Z^0$ -boson itself. Explicitly we have:

$$g_{11}^{f;\pm} = Q_f^2 + 2Q_f v_f \Re(\chi_Z)(-v \mp \rho a) + (v_f^2 + a_f^2) |\chi_Z|^2 (v^2 + a^2 \pm 2va\rho)$$
(21)  

$$g_{12}^{f;\pm} = Q_f^2 + 2Q_f v_f \Re(\chi_Z)(-v \mp \rho a) + (v_f^2 - a_f^2) |\chi_Z|^2 (v^2 + a^2 \pm 2va\rho)$$
(21)  

$$g_{44}^{f;\pm} = 2Q_f a_f \Re(\chi_Z)(\pm a + v\rho) + 2v_f a_f |\chi_Z|^2 (\mp 2va - \rho(v^2 + a^2))$$

Here  $\chi_Z(Q^2)$  is the ratio of the Z<sup>0</sup>-propagator to the photon propagator times a coupling strength factor:

$$\chi_Z(Q^2) = \frac{1}{(2\sin 2\theta_W)^2} \frac{Q^2}{Q^2 + m_Z^2 - im_Z\Gamma_Z}$$
(22)

and  $\rho$  is the degree of longitudinal polarization of the electron (positron) with  $\rho \in [-1, 1]$ .

In the charged current case the electroweak couplings are:

$$g_{11}^{ff';\pm} = 4 |V_{ff'}|^2 |\chi_W|^2 (1 \pm \rho), \qquad (23)$$

$$g_{12}^{ff';\pm} = 0$$

$$g_{44}^{ff';\pm} = 4 |V_{ff'}|^2 |\chi_W|^2 (\mp 1 - \rho)$$

Here  $\chi_W(Q^2)$  is defined by:

$$\chi_W(Q^2) = \frac{1}{(2\sqrt{2}\sin\theta_W)^2} \frac{Q^2}{Q^2 + m_W^2 - im_W\Gamma_W}$$
(24)

The quantities  $A_{11}, A_{12}, A_{44}$  in (20), finally, are given by (i=1,2; see appendix):

$$A_{1i} \equiv \frac{y}{2s_G} L_1^{\mu\nu} H_{\mu\nu}^i = x_G y^2 G_1^{(i)}(x_G, z, y, s_G) + (1-y) G_2^{(i)}(x_G, z, y, s_G) + (2-y) \sqrt{1-y} \cos \Phi G_A^{(i)}(x_G, z, y, s_G) + 2x_G (1-y) \cos 2\Phi G_B^{(i)}(x_G, z, y, s_G) A_{44} \equiv \frac{y}{2s_G} L_4^{\mu\nu} H_{\mu\nu}^4 = -x_G y (2-y) G_3(x_G, z, y, s_G) - 2y \sqrt{1-y} \cos \Phi G_C(x_G, z, y, s_G) (25)$$

The (partonic) structure functions  $G_l^{(i)} \equiv G_l^{(i)}(x_g, z; y, s_G)$  are given explicitly in the appendix. Here we want to emphasize that the  $G_l^{(i)}$  depend explicitly on the variable y. This differs from what is found in the massless quark case [14,15] where  $G_l^{(i)}$  depend on z and  $x_G$  solely. Thus the decomposition of the total cross section in different y-dependent terms a la (25) does not exhibit the full y-structure in the massive case.

### 4 Phase space

In this section we present an explicit representation of the particle momenta and investigate the physical region boundaries. We find it convenient to work in the final-state-quark (or boson-gluon) cms defined by

$$\vec{p_f} + \vec{p_{f'}} = 0 = \vec{p} + \vec{q}$$
(26)

where the gluon momentum  $\vec{p}$  and thereby the proton momentum  $\vec{P}$  points into the positive z-direction. We define  $\Phi$  to be the angle between the lepton plane  $(\vec{p} - \vec{l_e})$  and the hadron plane  $(\vec{p} - \vec{p_f})^3$ :

$$\cos \Phi := \frac{(\vec{p} \times \vec{l_e}) \cdot (\vec{p} \times \vec{p_f})}{|\vec{p} \times \vec{l_e}| |\vec{p} \times \vec{p_f}|}$$
(27)

An explicit representation of the particle momenta is provided by:

$$p = (E_p, 0, 0, E_p)$$

$$q = (Q_0, 0, 0, -E_p)$$

$$l_e = E_l(1, \sin\beta, 0, \cos\beta)$$

$$p_f = (E_1, q_1 \sin\theta \cos\Phi, q_1 \sin\theta \sin\Phi, q_1 \cos\theta)$$

$$p_{f'} = (E_2, -q_1 \sin\theta \cos\Phi, -q_1 \sin\theta \sin\Phi, -q_1 \cos\theta)$$
(28)

where

$$E_{p} = E(\hat{s}, 0, -Q^{2})$$

$$Q_{0} = E(\hat{s}, -Q^{2}, 0)$$

$$E_{l} = E(\hat{s}, 0, t_{3})$$

$$E_{1} = E(\hat{s}, m_{f}^{2}, m_{f'}^{2})$$

$$E_{2} = E(\hat{s}, m_{f'}^{2}, m_{f}^{2})$$

$$q_{1} = P(\hat{s}, m_{f}^{2}, m_{f'}^{2})$$

$$\cos \beta = \frac{2E_{p}E_{l} - s_{G}}{2E_{p}E_{l}}$$

$$\cos \theta = \frac{2E_{p}E_{1} - m_{f}^{2} + \hat{t}}{2E_{p}q_{1}}$$
(29)

 $\mathbf{and}$ 

$$t_{3} = (p - l_{e})^{2} = -s_{G} + \hat{s} + Q^{2}$$

$$E(x, y, z) = \frac{x + y - z}{2\sqrt{x}}$$

$$P(x, y, z) = \sqrt{\frac{x^{2} + y^{2} + z^{2} - 2xy - 2xz - 2yz}{4x}}$$
(30)

The 3-particle phase space is then given by

$$128\pi^3 dPS^{(3)} = \frac{d\hat{s}dQ^2 d\hat{t}}{s_G(\hat{s}+Q^2)} \frac{d\Phi}{2\pi} = s_G y dy dx_G dz \frac{d\Phi}{2\pi}$$
(31)

Note that

$$dz \frac{d\Phi}{2\pi} = 8\pi \, dPS^{(2)} \tag{32}$$

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<sup>&</sup>lt;sup>3</sup>The authors of [14] consider the hadronic cms  $\vec{P} + \vec{q} = 0$ , where the proton momentum points in the negative z-direction. To coincide with their notation simply replace  $\cos \Phi$  by  $(-\cos \Phi)$  in our final results and leave terms proportional to 1 and  $\cos 2\Phi$  unchanged.

where  $dPS^{(2)}$  is the two particle phase space for the process (5)  $p + q \rightarrow p_f + p_{f'}$ .

In total there are 5 independent integration variables. Looking at the restraints provided by individual angles we can find the physical region boundaries. The integration region for the  $\Phi$ -integration is simply  $[0, 2\pi]$ . The condition  $-1 \leq \cos \theta(p, p_f) \leq +1$  yields the  $\hat{t}$ -integration bounds:

$$\hat{t}_{min} := m_f^2 - \frac{\hat{s} + Q^2}{2\hat{s}} (\gamma^2 + \sqrt{\lambda}) 
\hat{t}_{max} := m_f^2 - \frac{\hat{s} + Q^2}{2\hat{s}} (\gamma^2 - \sqrt{\lambda})$$
(33)

or equivalently:

$$z_{max} := \frac{\gamma^2 + \sqrt{\lambda}}{2\hat{s}}$$
$$z_{min} := \frac{\gamma^2 - \sqrt{\lambda}}{2\hat{s}}$$
(34)

Here we have defined:

$$\begin{aligned} \gamma^{2} &:= \hat{s} + m_{f}^{2} - m_{f'}^{2} = s_{G}y(1 - x_{G}) + m_{f}^{2} - m_{f'}^{2} \\ \lambda &:= (\hat{s} - m_{f}^{2} - m_{f'}^{2})^{2} - 4m_{f}^{2}m_{f'}^{2} \\ &= \left(s_{G}y(1 - x_{G}) - m_{f}^{2} - m_{f'}^{2}\right)^{2} - 4m_{f}^{2}m_{f'}^{2} \end{aligned}$$
(35)

Considering  $(p_f + p_{f'})^2$  gives us

$$\hat{s} \ge \hat{s}_0 := (m_f + m_{f'})^2$$
 (36)

whereas baryon number conservation imposes the cut

$$W^{2} := (P+q)^{2} = m_{p}^{2} + \frac{\hat{s} + Q^{2}}{\eta} - Q^{2}$$
  

$$\geq W_{0}^{2} := (m_{f} + m_{f'} + m_{p})^{2}$$
(37)

Examination of the lower limit of  $Q^2$  gives [16]:

$$Q^2 \ge m_e^2 rac{y^2}{1-y} \equiv Q_{min}^2$$
 (38)

However, in order to investigate the dependence of the cross section on the lower limit  $Q_{min}^2$  of  $Q^2$  we will present numerical values for the total cross section for various values of  $Q_{min}^2$ . The last constraint arising from

$$t_3 = (p-l)^2 \le t_{3_{max}} = 0 \tag{39}$$

can be written as

$$\eta s - \hat{s} - Q^2 \ge 0$$
 (40)

Collecting these constraints we obtain for the physical region boundaries:

$$128\pi^{3} \int d\eta \, dPS^{(3)} = \Theta \left( s + 2m_{e}^{2} - 2\sqrt{m_{e}^{2}(s + m_{e}^{2})} - W_{1}^{2} \right)$$

$$\int_{y_{min}}^{y_{max}} dy \int_{Q_{min}^{2}(y)}^{Q_{max}^{2}(y)} dQ^{2} \int_{\eta_{min}(y,Q^{2})}^{1} d\eta \int_{z_{min}(y,Q^{2},\eta)}^{z_{max}(y,Q^{2},\eta)} dz \int_{0}^{2\pi} \frac{d\Phi}{2\pi}$$
(41)

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where

$$\eta_{min}(y,Q^2) := \frac{\hat{s}_0 + Q^2}{ys}$$

$$Q^2_{max}(y) := ys - W_1^2$$

$$y_{max,min} := \frac{s + W_1^2 \pm \sqrt{(s - W_1^2)^2 - 4m_e^2 W_1^2}}{2(s + m_e^2)}$$

$$W_1^2 := W_0^2 - m_p^2$$
(42)

Using x instead of  $Q^2 = xys$  as second integration variable we obtain the following limits:

$$x_{min} = \frac{m_e^2}{s} \frac{y}{1-y}$$

$$x_{max} = 1 - \frac{W_1^2}{ys}$$
(43)

We finally mention that the transverse momentum of the quark (antiquark) with respect to the proton direction is defined by

$$p_T = \pm |\vec{p}_f^{(12)}| \sin \theta^{(12)} \tag{44}$$

where (12) denotes the cms  $\vec{p_{f}} + \vec{p_{f'}} = 0$  with  $\vec{p}$  pointing into the positive z direction. We find

$$p_T^2 = \hat{s}z(1-z) + z(m_f^2 - m_{f'}^2) - m_f^2$$
(45)

# 5 Total cross section

The total, five-fold differential cross section is now given by:

$$d\sigma_{f}^{\pm} = dy \, dQ^{2} \, d\eta \, dz \, \frac{d\Phi}{2\pi} \, \frac{\alpha_{s}(\hat{s})\alpha^{2}2\theta_{l}}{16yQ^{4}} G(\eta, \hat{s})$$

$$\left[g_{11}^{f;\pm} \left\{x_{G}y^{2}G_{1}^{(1)}(x_{G}, z; y, s_{G}) + (1-y)G_{2}^{(1)}(x_{G}, z; y; s_{G}) + (2-y)\sqrt{1-y}\cos\Phi G_{A}^{(1)}(x_{G}, z; y, s_{G}) + 2x_{G}(1-y)\cos2\Phi G_{B}^{(1)}(x_{G}, z; y, s_{G})\right\}$$

$$+g_{12}^{f;\pm} \left\{x_{G}y^{2}G_{1}^{(2)}(x_{G}, z; y, s_{G}) + (1-y)G_{2}^{(2)}(x_{G}, z; y, s_{G})\right\}$$

$$-g_{44}^{f;\pm} \left\{x_{G}y(2-y)G_{3}(x_{G}, z; y, s_{G}) + 2y\sqrt{1-y}\cos\Phi G_{C}(x_{G}, z; y, s_{G})\right\}$$

$$\left. \left. -g_{44}^{f;\pm} \left\{x_{G}y(2-y)G_{3}(x_{G}, z; y, s_{G}) + 2y\sqrt{1-y}\cos\Phi G_{C}(x_{G}, z; y, s_{G})\right\}\right]$$

$$\left. \left. \left. -g_{44}^{f;\pm} \left\{x_{G}y(2-y)G_{3}(x_{G}, z; y, s_{G}) + 2y\sqrt{1-y}\cos\Phi G_{C}(x_{G}, z; y, s_{G})\right\}\right\} \right]$$

The integration regions are given in (41). This is the most general form of the cross section  $d\sigma^{\pm}$  for positron (electron) scattering on protons into heavy flavours. If the couplings  $g_{ij}^{f_{i\pm}}$  are given by (21) and we take  $m_{f'} = m_f$  in the partonic structure functions  $G_l^{(k)}$ , then (46) gives the cross section for producing a heavy quark-antiquark pair of flavour f in the neutral sector including the  $\gamma$ - $Z^0$  interference. In the charged current sector we have to be aware of the assignment of the quark masses: We arrive at the cross section for producing a heavy up-like quark of flavour f and a heavy down-like antiquark of flavour f' in  $e^+p$ -scattering by using  $g_{ij}^{ff'_{i+}}$  as defined in (23) and taking  $m_f(m_{f'})$  as the mass of the up- (down-) like quark and a

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heavy up-like antiquark in  $e^-p$ -scattering we have to use  $g_{ij}^{ff';-}$  and the opposite assignment of the masses. For example, in  $e^+p \to t\bar{b}$  we need  $m_f = m_t, m_{f'} = m_b$ , whereas in  $e^-p \to \bar{t}b$ we have to take  $m_f = m_b, m_{f'} = m_t$ . The parameter  $\rho$  in (21) and (23) accounts for a possible longitudinal polarization of the incoming lepton.

From (46) many interesting differential distributions can be calculated, such as the  $p_T$  distribution of the heavy quarks or the asymmetry in the angle  $\Phi$ . Here, however, we restrict ourselves to the total integrated cross section. We find that two of the five integrations can be done analytically. These integrations will just eliminate the inner structure of the final state (see (32)) and yield the contributions of the heavy flavours to the deep-inelastic (DIS) structure functions.

The  $\Phi$ -integration is trivial, it just eliminates the  $\Phi$ - dependent terms in (46). Besides the  $\Phi$ -integration the z-integration can be done analytically. Let us define:

$$\int_{z_{min}}^{z_{max}} dz \int_{0}^{2\pi} \frac{d\Phi}{2\pi} G_{j}^{(i)} =: R_{j}^{(i)}(x_{G}; y, s_{G})$$
(47)

Then we obtain:

$$R_{1}^{(1)} = -\frac{16\sqrt{\lambda}}{ys_{G}(1-x_{G})}(4x_{G}^{2}-4x_{G}+1)$$

$$+8\Psi(2x_{G}^{2}-2x_{G}+1+2m_{-}^{4}-4x_{G}m_{-}^{2}+2m_{-}^{2})$$

$$+8\tilde{\Psi}(2x_{G}^{2}-2x_{G}+1+2m_{-}^{4}+4x_{G}m_{-}^{2}-2m_{-}^{2})$$

$$(48)$$

$$egin{array}{rcl} R_1^{(2)}&=&-rac{64m_0\sqrt{\lambda}}{ys_G}+32m_0(1-x_G-m_+^2)(\Psi+ ilde{\Psi})\ R_2^{(1)}&=&2x_G\,R_1^{(1)}+rac{64\sqrt{\lambda}}{ys_G}(2x_G^2-x_Gm_+^2+m_-^4)\ &+16\Psi(-2x_Gm_+^4+2m_+^2m_-^4+m_+^2(-6x_G^2+2x_G+1)\ &+2m_-^4(3x_G-1)+4m_-^2x_G(1-2x_G)\ &+16 ilde{\Psi}(m_A\,\leftrightarrow\,m_A) \end{array}$$

$$R_2^{(2)} ~=~ -32m_0(\Psi+ ilde{\Psi})$$

$$egin{array}{rcl} R_3 &=& \displaystylerac{32m_-^2\sqrt{\lambda}}{ys_G} \ &&+8\Psi(-2x_G^2+2x_G-1+4x_Gm_-^2+2m_-^2m_+^2-2m_-^2) \ &&-8 ilde{\Psi}(-2x_G^2+2x_G-1-4x_Gm_-^2-2m_-^2m_+^2+2m_-^2) \end{array}$$

Here we have defined:

$$egin{array}{rcl} \Psi &=& \ln rac{\gamma^2 + \sqrt{\lambda}}{\gamma^2 - \sqrt{\lambda}} \ ilde{\Psi} &=& \Psi(m_f \leftrightarrow m_{f'}) \end{array}$$

 $\gamma$  and  $\lambda$  have already been defined in (35), and  $m_{\pm}^2, m_0$  are defined in (75).

(49)

Note that  $R_1^{(i)}$  and  $R_2^{(i)}$  (i = 1, 2) are symmetric under the exchange of  $m_f$  and  $m_{f'}$ , whereas  $R_3$  is antisymmetric under this exchange. Thus  $R_3$  is zero in the neutral case.

The  $\Phi$ - and z-integrated cross section can now be written as:

$$\sigma_{f}^{\pm} = \int_{y_{min}}^{y_{max}} dy \int_{Q_{min}^{2}}^{Q_{max}^{2}} dQ^{2} \int_{\eta_{min}}^{1} d\eta \frac{\alpha_{s}(\hat{s})\alpha^{2}2\theta_{l}}{16yQ^{4}} G(\eta, \hat{s})$$

$$\left[g_{11}^{f;\pm} \left\{x_{G}y^{2}R_{1}^{(1)}(x_{G}; y, s_{G}) + (1-y)R_{2}^{(1)}(x_{G}; y, s_{G})\right\} + g_{12}^{f;\pm} \left\{x_{G}y^{2}R_{1}^{(2)}(x_{G}; y, s_{G}) + (1-y)R_{2}^{(2)}(x_{G}; y, s_{G})\right\} - g_{44}^{f;\pm}x_{G}y(2-y)R_{3}(x_{G}; y, s_{G})\right] \Theta\left(s + 2m_{e}^{2} - 2\sqrt{m_{e}^{2}(s + m_{e}^{2})} - W_{1}^{2}\right)$$

$$(50)$$

We finally want to express the usual DIS structure functions  $F_i$  (i = 1, 2, 3) by the partonic structure functions  $R_i^{(j)}$ . We start from the usual definitions for  $F_i$ :

$$d\sigma_{f}^{\pm} = dxdy \frac{4\pi\alpha^{2}}{Q^{2}xy} \Theta(y - y_{min})\Theta(y_{max} - y)\Theta(x - x_{min}(y))\Theta(x_{max}(y) - x)$$
(51)  
$$\Theta\left(s + 2m_{e}^{2} - 2\sqrt{m_{e}^{2}(s + m_{e}^{2})} - W_{1}^{2}\right) \\\left[g_{11}^{f;\pm}\left\{xy^{2}F_{1}(x,Q^{2}) + (1 - y)F_{2}(x,Q^{2})\right\} - g_{44}^{f;\pm}xy(2 - y)F_{3}(x,Q^{2})\right]$$

For the charged current sector it is sometimes more convenient to write (51) in the following way:

$$d\sigma_{ff'}^{\pm} = dxdy \frac{G_F^2 s}{2\pi} \frac{|V_{ff'}|^2 (1 - \Delta r)^2}{(1 + Q^2/m_W^2)^2} \Theta(y - y_{min})\Theta(y_{max} - y)\Theta(x - x_{min}(y))$$
(52)  
$$\Theta(x_{max}(y) - x)\Theta\left(s + 2m_e^2 - 2\sqrt{m_e^2(s + m_e^2)} - W_1^2\right) \\\left[(1 \pm \rho)\left\{xy^2F_1(x, Q^2) + (1 - y)F_2(x, Q^2)\right\} + (\pm 1 + \rho)xy(2 - y)F_3(x, Q^2)\right]$$

We then find the following relations  $(a = x + \hat{s}_0/(ys))$ :

$$F_{1}(x,Q^{2}) = \frac{\alpha_{s}(\hat{s})}{2\pi} \frac{2\theta_{l}}{32} \int_{a}^{1} \frac{d\eta}{\eta} G(\eta,\hat{s}) \left[ R_{1}^{(1)}(x_{G},y,s_{G}) + \frac{g_{12}^{f;\pm}}{g_{11}^{f;\pm}} R_{1}^{(2)}(x_{G},y,s_{G}) \right]$$
(53)  

$$F_{2}(x,Q^{2}) = \frac{\alpha_{s}(\hat{s})}{2\pi} \frac{2\theta_{l}}{32} \int_{a}^{1} d\eta G(\eta,\hat{s}) \left[ R_{2}^{(1)}(x_{G},y,s_{G}) + \frac{g_{12}^{f;\pm}}{g_{11}^{f;\pm}} R_{2}^{(2)}(x_{G},y,s_{G}) \right]$$
(53)  

$$F_{3}(x,Q^{2}) = \frac{\alpha_{s}(\hat{s})}{2\pi} \frac{2\theta_{l}}{32} \int_{a}^{1} \frac{d\eta}{\eta} G(\eta,\hat{s}) R_{3}(x_{G},y,s_{G})$$

However, several comments should be added. Firstly, in the NC sector the cross section develops a new dependence on the standard electroweak couplings via the  $g_{12}^{f;\pm}/g_{11}^{f;\pm}$  terms in (53) ( $g_{12}^{CC} = 0$ ). Secondly, we emphasize that the hadronic structure functions  $F_i$  are explicitly y-dependent for massive quarks:  $F_i = F_i(y)$ . This differs from the  $O(\alpha_s)$  result of massless QCD where the partonic structure functions  $R_i^{(i)}$  depend solely on  $x_G$  [14]. There the cross section becomes  $Q^2$ -dependent only via the use of a  $Q^2$ -dependent gluon density (and/or a  $Q^2$ -dependent  $\alpha_s$ ). The third comment concerns the use of (51) as a starting function to generate the variables (x, y) via a Monte Carlo event generator program. Contrary to the massless case one can not avoid to perform an integration, see (53). Finally we mention that the  $e^-p$ - and  $e^+p$ -cross sections are the same as long as the leptons are unpolarized ( $\rho = 0$ ). The " $\pm$ " sign in (52) in front of  $F_3$  cancels against the antisymmetry of  $F_3$  in  $m_f \leftrightarrow m_{f'}$ .

# 6 Weizsäcker-Williams approximation

The Weizsäcker-Williams approximation (WWA) provides a convenient framework to derive simple approximate forms of the previous formulae. These may be useful for several reasons. On the one hand the above expressions are unnecessarily complicated for many purposes. On the other hand, approximate formulae have existed for quite a long time [9]. These were used as guess for the total rates expected for heavy quarks at HERA. By comparison with the exact calculation of the previous sections we shall be able to specify the accuracy limits of these approximations.

For an inelastic process like (1) the double pole at  $Q^2 = 0$  in the rate arising from the photon propagator is reduced to a simple pole by a corresponding zero in the numerator. The remaining singularity leads to a logarithmic term in the cross section. When the logarithmic term from photon exchange is large enough, the cross section is well approximated by only taking this term correctly into account. This is precisely what is done in the WWA where the cross section (1) is described by a convolution of the probability for emitting a photon from the lepton vertex with the corresponding real photon cross section at the hadron vertex.

For process (1) the WWA gives:

$$\sigma_f^{\pm}(s) = \int_{y_{min}}^1 dy \ P_{\gamma}(y) \int_{\eta_{min}}^1 d\eta \ G(\eta, M^2) \hat{\sigma}_f(\hat{s})$$
(54)

where  $P_{\gamma}$  is related to the probability of finding a photon in the positron (electron):

$$P_{\gamma} = \frac{\alpha}{2\pi} \frac{1 + (1 - y)^2}{y} \ln \frac{Q_2^2}{Q_1^2}$$
(55)

 $\hat{\sigma}_f(\hat{s})$  is the cross section for production of the heavy quark-antiquark pair by a real photon of energy  $\frac{1}{2}\sqrt{\hat{s}} = \frac{1}{2}\sqrt{y\eta s}$  (see (14) with  $Q^2 = 0 = x = x_G$ ).

Note that  $\sigma_f^{\pm}$  depends on  $Q_1^2$  which appears in (55) for  $P_{\gamma}$ . Also note that, as it is the case for all leading approximations, there is some ambiguity on the choice of  $Q_2^2$ , which also appears in (55) for  $P_{\gamma}$ . In the next section we shall discuss the dependences of the cross section on  $Q_1^2$  and  $Q_2^2$ . As in the full  $Q^2$ -formula there still is another ambiguity. Whereas it might be reasonable to take Q as the mass scale M for the gluon density  $G(\eta, M^2)$  and for the strong coupling constant  $\alpha_s(M^2)$  in the full  $Q^2$  formula (at least for high  $Q^2$ ), this choice clearly can not be maintained for low  $Q^2$ . Thus we will take  $M^2 = \hat{s}$  as in the full  $Q^2$  case.

We now shortly describe the calculation of  $\hat{\sigma}_f$ . Two possibilities are open. On the one hand we can start from the real process  $\gamma + \text{gluon} \rightarrow q + \bar{q}$ . Or we can take the  $Q^2 \rightarrow 0$  limit in the previous exact formulae. As a consistency check we will pursue both methods. Let us start on the first way.

The real photon process  $\gamma + \text{gluon} \rightarrow q\bar{q}$  is given by:

$$\frac{d\hat{\sigma}_f}{d\hat{t}} = \frac{\pi Q_f^2 \alpha \alpha_s}{\hat{s}^2} \left( \frac{v_1}{v_2} + \frac{v_2}{v_1} + \frac{4m_f^2 \hat{s}}{v_1 v_2} - \frac{4m_f^4 \hat{s}^2}{v_1^2 v_2^2} \right)$$
(56)

where  $v_1 = m_f^2 - \hat{t}$ ,  $v_2 = m_f^2 - \hat{u}$  and  $m_f$  is the heavy quark mass. Integration over  $\hat{t}$  from  $\hat{t}_{min}$  to  $\hat{t}_{max}$  (see (33)) yields:

$$\hat{\sigma}_f = \frac{\pi Q_f^2 \alpha \alpha_s}{y \eta s} \left\{ \Psi(2 + 2w - w^2) - 2\chi(1 + w) \right\}$$
(57)

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Here  $w = 4m_f^2/\hat{s} = 4m_f^2/(y\eta s)$ ,  $\chi = \sqrt{1-w}$  and  $\Psi = \ln \frac{1+\chi}{1-\chi} = 2\ln(1+\chi)/\sqrt{w}$ . Inserting (57) in (54) gives the *ep* cross section in the WWA.

The same result is arrived at by considering the  $Q^2 \rightarrow 0$  limit (or equivalently  $x_G \rightarrow 0$  limit) in the formulae of the last section. First note that in the pure electromagnetic case:

$$g_{11}^{f;\pm} = g_{12}^{f;\pm} = Q_f^2 \quad ; \quad g_{44}^{f;\pm} = 0 \tag{58}$$

Thus (i = 1, 2):

$$L_{i} \equiv g_{11}^{f;\pm} R_{i}^{(1)} + g_{12}^{f;\pm} R_{i}^{(2)} = Q_{f}^{2} (R_{i}^{(1)} + R_{i}^{(2)})$$

$$L_{3} \equiv g_{44}^{f;\pm} R_{3} = 0$$
(59)

Taking now the  $Q^2 = 0$  limit we find:

$$L_1 = L_2 = 8Q_f^2 \left\{ \Psi(2 + 2w - w^2) - 2\chi(1 + w) \right\}$$
(60)

Finally we obtain:

$$\frac{\alpha_{s}(\hat{s})\alpha^{2}2\theta_{l}}{16yQ^{4}} \left[ g_{11}^{f;\pm} \left\{ x_{G}y^{2}R_{1}^{(1)}(x_{G};y,s_{G}) + (1-y)R_{2}^{(1)}(x_{G};y,s_{G}) \right\} + g_{12}^{f;\pm} \left\{ x_{G}y^{2}R_{1}^{(2)}(x_{G};y,s_{G}) + (1-y)R_{2}^{(2)}(x_{G};y,s_{G}) \right\} - g_{44}^{f;\pm}x_{G}y(2-y)R_{3}(x_{G};y,s_{G}) \right] \\ = \frac{1}{Q^{2}} \frac{\alpha}{2\pi} \frac{y^{2} + 2(1-y)}{y} \hat{\sigma}$$
(61)

which shows that both methods coincide.

### 7 Numerical results

As an application of the derived formulae for heavy flavour production we present in this section results for the total rates of top production at HERA and discuss their uncertainties. We restrict ourselves to unpolarized electron scattering and take  $\sin^2 \Theta_W = 0.226$ ,  $\alpha = 1/137$ ,  $\Delta r = 0.0696$ ,  $\sqrt{s} = 314 \text{ GeV}$ ,  $V_{tb} = 1$ ,  $V_{td} = V_{ts} = 0$  and  $m_b = 5 \text{ GeV}$ .

The uncertainty of the integrated cross section is now determined by essentially two <sup>4</sup> facts: On the one hand we are somewhat free in the choice of the mass scale for the gluon density and the mass scale for the strong coupling constant  $\alpha_s$ . On the other hand we can use various parametrizations of the gluon density. However, in order to be consistent with the low energy data from which the parametrizations of the gluon density were extracted we choose  $\Lambda$  and the number of flavours  $N_f$  in the formula (19) for  $\alpha_s$  according to the values used in the respective parametrization. Furthermore, we find it plausible to use the same mass scale M for the gluon density and for  $\alpha_s$ .

Let us first present the total cross section for top production at HERA using a definite gluon parametrization [6] and a fixed choice of the above mentioned mass scale:  $M^2 = \hat{s}$ . In Table 1 we give the number of top events for various top quark masses assuming an integrated luminosity of 100 pb<sup>-1</sup>. The first two columns, denoted by "CC" and "NC", give the total number of  $\bar{t}b$  and  $t\bar{t}$  events, respectively. In the next three columns of Table 1 we show

<sup>&</sup>lt;sup>4</sup>Using a running coupling constant  $\alpha(Q^2)$  instead of  $\alpha(0)$  will change the result by at most 12%.

the individual contributions to the  $t\bar{t}$ -rates coming from  $\gamma$ -exchange, Z-exchange and  $\gamma$ -Z interference. The last column of Table 1 denoted by "total rate top" gives the total number of expected top events (i.e. t plus  $\bar{t}$  events), again for  $\int Ldt = 100 \text{ pb}^{-1}$ .

A graphical representation of Table 1 can be found in Figure 1. Here the upper full line represents the total number of top events, whereas the lower lying full line, the dotted line, the dashed line and the dashed-dotted line denote the contributions coming from  $\gamma$ -exchange, Z-exchange,  $\gamma$ -Z-interference and W-exchange, respectively.

We find that for a top mass of 50 GeV (i.e. just beyond the reach of SLC and LEP I) we expect  $\approx 116$  top events ( $\int Ldt = 100 \,\mathrm{pb^{-1}}$ ) and even for a top mass of 100 GeV still  $\approx 3$  top events. Thus the top quark can be searched for up to masses of  $\approx 100 \,\mathrm{GeV}$ . It is also remarkable [11] that for a top mass above  $\approx 50 \,\mathrm{GeV}$  the cross section is dominated by *W*-exchange. We can also see that both the *Z*-exchange and the pure  $\gamma$ -*Z* interference contributions are negligible for the total rate of top production. For comparison we give the number of *b* events ( $m_b = 5 \,\mathrm{GeV}$ ) to be expected at HERA in Table 2. Here we assume a top mass of 50 GeV. All other parameters have the same values as for Table 1. We find that the total number of *b* events is clearly dominated by  $\gamma$ -exchange and is fairly large.

Next we investigate the influence of the choice of the mass scale M on the total rates. We have checked that the changes in the cross sections are rather small as long as one chooses a scale like  $\hat{s}$ ,  $\hat{t}$  or  $\hat{u}$ . To give a feeling of the uncertainty due to different values for M we plot the total number of top events as a function of the top mass for two different choices of the mass scale M in Figure 2. (Again  $\int Ldt = 100 \text{ pb}^{-1}$  and the gluon density parametrization is from [6]). The full line in Figure 2 results from a calculation with  $M^2 = \hat{s}$  whereas the dashed line is obtained using  $M^2 = (m_Q + m_{\bar{Q}})^2$ . Figure 2 demonstrates that the uncertainty in the total rate is less than 10% over the whole mass range.

We now discuss the second uncertainty in the cross section which arises from the use of various parametrizations of the gluon density. We already mentioned that we always choose  $\Lambda$  and the number of flavours  $N_f$  according to the values used in the respective parametrization of the gluon density. In Figure 3 we plot the total rate of top production versus the top mass using five different parametrizations of the gluon density. Here we use  $M^2 = \hat{s}$  and assume again  $\int L dt = 100 \text{ pb}^{-1}$ . The parametrizations set 1 and 2 of [18] are represented by the lower lying full and the lower lying dotted line, the parametrization of [6] is represented by the upper lying full line, respectively. In Figure 3 we also give the result using the scale independent gluon density  $xG(x) = 3(1-x)^5$  with  $N_f = 5$  and  $\Lambda = 0.2 \text{ GeV}$ . It corresponds to the upper lying dotted curve. We find that the uncertainty due to different parametrizations of the gluon density is  $\leq 25\%$  over most of the mass range and tends to increase for increasing top mass. However, the scale independent gluon density overestimates the rate of top production by a factor of 2-5.

For experimental purposes it might be necessary to impose a lower cutoff in  $Q^2$ . It is therefore interesting to check how much one will lose in the total rate of top production. Since the photon exchange cross section is dominated by the low  $Q^2$  region we expect the corresponding rate to depend strongly on the lower limit of  $Q^2$ . This should also be true for the total rate of  $t\bar{t}$  production since the total NC rate is dominated by photon exchange. The dependence of the top production rates on a lower cutoff  $Q_l^2$  in  $Q^2$ , i.e.

$$\sigma = \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 rac{d\sigma}{dQ^2} \Theta(Q^2-Q_l^2) \quad {
m vs.} \quad Q_l^2$$

can be seen in Figure 4 for a top quark of 40 GeV. Here we give the number of top events versus  $Q_l^2$  where we take the gluon density of [6],  $M^2 = \hat{s}$  and  $\int Ldt = 100 \text{ pb}^{-1}$ . The contributions from  $\gamma$ -exchange, Z-exchange,  $\gamma$ -Z interference and W-exchange are represented by the full, dotted, dashed and dashed-dotted line, respectively.

We recognize the described  $Q_l^2$ -behaviour of the photon exchange contribution. In fact, increasing the lower limit  $Q_l^2$  from the kinematical bound  $\approx m_e^2$  up to  $2 \,\text{GeV}^2$  decreases the photonic rate by a factor of three. We also see that the contributions from Z-exchange and  $\gamma$ -Z interference are more or less unchanged but negligible. We thus conclude that for a top quark mass up to 50 GeV the total top production rate will be diminished by a factor of three when imposing a  $Q^2$ -cut of  $2 \,\text{GeV}^2$ .

The situation, however, gets reversed for a top quark of mass greater than  $\approx 50 \text{ GeV}$  because then the total rate of top production is dominated by *W*-exchange. From Figure 4 we note that the rate of the *W*-exchange contribution is stable against a  $Q^2$ -cut. We thus conclude that the total rate of top production will be almost unchanged by imposing a lower cut in  $Q^2$  for  $m_t \geq 50 \text{ GeV}$ . In particular, we still expect  $\approx 3$  top events for a 100 GeV top quark (if, again,  $\int Ldt = 100 \text{ pb}^{-1}$ ).

Let us now investigate the range of validity of the WWA. Clearly, since W-exchange becomes more and more important as the heavy quark mass increases we expect the WWA to be valid only for quarks with relatively low masses. However, in regions where the cross section is dominated by photon exchange ( $m_Q \leq 30 \text{ GeV}$ ) the WWA is certainly applicable.

We already mentioned that the WWA does not only depend on the lower cutoff  $Q_1^2$  of  $Q^2$  but also on the upper limit  $Q_2^2$  via the  $\ln(Q_2^2/Q_1^2)$  term in (55). We note that we would overestimate the cross section if in (55) we choosed a scale for  $Q_2^2$  of the order of the total cms energy squared s (this is the prefered choice in  $e^+e^-$  collisions). This overestimation still remains if the cuts  $Q_1^2$  and  $Q_2^2$  are given by the values of the exact  $Q^2$ -formula, i.e. if  $Q_1^2 = Q_{min}^2$  and  $Q_2^2 = Q_{max}^2$  where  $Q_{max,min}^2$  are given in (38,42). The reason is that the logarithmic increase of the WWA is valid only for moderate values of  $Q_2^2$ . The deviation of the WWA from the exact formula for high  $Q^2$  is demonstrated in Figure 5 where we plot the number of  $t\bar{t}$  events ( $m_t = 40 \text{ GeV}$ ) coming from pure  $\gamma$ -exchange against an upper limit  $Q_u^2$  of  $Q^2$ :

$$\sigma = \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 rac{d\sigma}{dQ^2} \Theta(Q^2_u - Q^2) \quad {
m vs.} \quad Q^2_u$$

The full (dashed) line results from the full  $Q^2$  (WWA) formula. Here we have chosen the above mentioned cuts  $Q_1^2 = Q_{min}^2$  and  $Q_2^2 = Q_{max}^2$  for the WWA and used the gluon density parametrization of [6]. The decline of the WWA curve at the upper edge of  $Q_u^2$  stems from the fact that the effective upper limit  $Q_{2,eff}^2$  in (55) for the WWA formula is given by  $Q_{2,eff}^2 = \min(Q_2^2, Q_u^2)$ . In total we observe that the deviation between the exact formula and the WWA is less than 7%.

Finally we want to determine the accuracy limits of the WWA over a wide range of the top mass. Note, however, that this investigation only makes sense if we restrict ourselves to  $t\bar{t}$  production since for  $m_t \geq 50 \text{ GeV}$  the  $\bar{t}b$  production will dominate. To this end we plot in Figure 6 the number of  $t\bar{t}$  events resulting from pure  $\gamma$ -exchange versus the heavy

quark mass. The full line results from the exact formula whereas the dotted line, the dashed line and the dashed dotted line stem from the WWA using as lower and upper bound on  $Q^2$ :  $(Q_1^2, Q_2^2) = (m_e^2, m_\pi^2), (m_e^2, \hat{s}), (m_e^2, s - 4m_t^2)$ , respectively. The gluon density parametrization is from [6] and  $\int Ldt = 100 \text{ pb}^{-1}$ . We find that the WWA curves are in good shape with the full  $Q^2$ -formula. The discrepancy is less than 20% except for the case where  $(Q_1^2, Q_2^2) = (m_e^2, m_\pi^2)$  which slightly underestimates the  $t\bar{t}$ -rate.

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### 8 Appendix

In this section we exhibit the angular correlations between the initial and final state and the hadronic QCD effects. We recast the final expression for the total differential cross section in a form similar to the quark-parton model (QPM) formula. However, since the massive case possesses a richer electroweak and kinematic structure than the massless case, our discussions will be quite lengthy.

It is convenient to expand the squared matrix element for process (4) in the following way:

$$\sum_{\text{spins}} |\mathcal{M}_{ff'}^{\pm}|^2 = g_s^2 \frac{e^4}{Q^4} \sum_{i,j=1}^4 g_{ij}^{ff';\pm}(Q^2) L_i^{\mu\nu} H_{\mu\nu}^j$$
(62)

The expansion is a linear superposition of the various parity conserving (p.c.) and parity violating (p.v.) terms in the hadron tensors  $H_{\mu\mu}^{JJ'}$  and lepton tensors  $L_{\mu\mu}^{JJ'}$ : We define four hermitian hadron tensors  $H_{\mu\nu}^{r}$  (r = 1, ...4) by:

$$\begin{aligned}
H^{1}_{\mu\nu} &= \frac{1}{2} (L^{VV}_{\mu\nu} + L^{AA}_{\mu\nu}) &= H^{RR}_{\mu\nu} + H^{LL}_{\mu\nu} \\
H^{2}_{\mu\nu} &= \frac{1}{2} (H^{VV}_{\mu\nu} - H^{AA}_{\mu\nu}) &= H^{LR}_{\mu\nu} + H^{RL}_{\mu\nu} \\
H^{3}_{\mu\nu} &= \frac{1}{2} (H^{VA}_{\mu\nu} - H^{AV}_{\mu\nu}) &= i (H^{LR}_{\mu\nu} - H^{RL}_{\mu\nu}) \\
H^{4}_{\mu\nu} &= \frac{1}{2} (H^{VA}_{\mu\nu} + H^{AV}_{\mu\nu}) &= H^{RR}_{\mu\nu} - H^{LL}_{\mu\nu}
\end{aligned}$$
(63)

and similarly four lepton tensors  $L^j_{\mu\nu}$ . The superscripts V and A denote the contributions of the vector  $(V_{\mu} = \langle f | J^V_{\mu}(0) | \rangle)$  and axial-vector  $(A_{\mu} = \langle f | J^A_{\mu}(0) | \rangle)$  currents according to  $H^{JJ'}_{\mu\nu} = \langle J_{\mu}J'_{\nu} \rangle$ , (J, J' = V, A), where  $\langle \ldots \rangle$  means sum over the (final state) polarizations. The superscript L and R, on the other hand, refer to the contributions of the left-handed  $(2L_{\mu} = V_{\mu} - A_{\mu})$  and right-handed  $(2R_{\mu} = V_{\mu} + A_{\mu})$  currents used especially in neutrino reactions. Similar definitions apply to the lepton tensors  $L^j_{\mu\nu}$ . The (finally surviving) electroweak coupling coefficients  $g^{ff';\pm}_{ij}(Q^2)$  appearing in (62) are given in the main text.

Since we neglect the lepton masses the lepton tensors are simply given by

$$L^{1}_{\mu\nu} = 4 \left( 2l_{e\mu}l_{e\nu} - q_{\mu}l_{e\nu} - l_{e\mu}q_{\nu} + g_{\mu\nu}l_{e} \cdot q \right)$$

$$L^{2}_{\mu\nu} = 0$$
(64)

(70)

$$egin{array}{rcl} L^3_{\mu
u}&=&0\ L^4_{\mu
u}&=&4i\epsilon_{\mu
u
ho\sigma}l^
ho_eq^\sigma \end{array}$$

The hadron tensors are given by (X, Y = V, A):

$$H_{\mu\nu}^{XY} := -Tr\left\{ (\not p_f + m_f) (T_{\mu}^{X(1)} + T_{\mu}^{X(2)}) (\not p_{f'} - m_{f'}) (\overline{T_{\nu}^{Y(1)}} + \overline{T_{\nu}^{Y(2)}}) \right\}$$
(65)

which the amplitudes  $(X_{\mu} = \gamma_{\mu}, \gamma_{\mu}\gamma_{5} \text{ for } X = V, A)$ :

$$T_{\mu}^{X(1)} = \overline{u(p_f)} \left\{ X_{\mu} \frac{\not p - \not p_{f'} + m_{f'}}{(p - p_{f'})^2 - m_{f'}^2} \gamma_{\alpha} \right\} v(p_{f'}) \epsilon_{\alpha}(p)$$

$$T_{\mu}^{X(2)} = \overline{u(p_f)} \left\{ \gamma_{\alpha} \frac{\not p_{f} - \not p + m_{f}}{(p_{f} - p)^2 - m_{f}^2} X_{\mu} \right\} v(p_{f'}) \epsilon_{\alpha}(p)$$
(66)

We use

$$\sum_{spins} u(p_f) \overline{u(p_f)} = p_f + m_f \tag{67}$$

which is consistent with our phase space measure

$$d\bar{p_f} \equiv \frac{d^3 p_f}{2E_f (2\pi)^3} \tag{68}$$

The most general expansion of the (parity even) hadron tensors  $H^1_{\mu\nu}$  and  $H^2_{\mu\nu}$  is given by (i = 1, 2):

$$H^{i}_{\mu\nu} = \sum_{k=1}^{10} H^{(i)}_{k} B^{(k)}_{\mu\nu}$$
(69)

where

16

The analogous expansion of the two further (parity odd) hadron tensors  $H^3_{\mu\nu}$  and  $H^4_{\mu\nu}$  is (i = 3, 4):

$$H^{i}_{\mu\nu} = \sum_{k=1}^{6} H^{(i)}_{k} A^{(k)}_{\mu\nu}$$
(71)

where

$$A_{\mu\nu}^{(1)} := \frac{i}{ys_G} \epsilon_{\mu\nu\rho\sigma} p^{\rho} q^{\sigma}$$

$$A_{\mu\nu}^{(2)} := \frac{i}{ys_G} \epsilon_{\mu\nu\rho\sigma} p^{\rho} p_f^{\sigma}$$

$$A_{\mu\nu}^{(3)} := \frac{i}{ys_G} \epsilon_{\mu\nu\rho\sigma} q^{\rho} p_f^{\sigma}$$

$$A_{\mu\nu}^{(4)} := \frac{i}{ys_G} (p_{f\mu} F_{\nu} + p_{f\nu} F_{\mu})$$

$$A_{\mu\nu}^{(5)} := \frac{i}{ys_G} (p_{\mu} F_{\nu} + p_{\nu} F_{\mu})$$

$$A_{\mu\nu}^{(6)} := \frac{i}{ys_G} (q_{\mu} F_{\nu} + q_{\nu} F_{\mu})$$
(72)

and  $F_{\mu} = \epsilon_{\mu\nu\rho\sigma} p^{\nu} q^{\rho} p_{f}^{\sigma}$ .

However, we are only interested in the  $O(\alpha_s)$  cross section. Then not all of the terms in (69) and (71) contribute to the cross section. First note that the (tree) hadron tensors are real. Thus there is no contribution to  $B_{\mu\nu}^{(i)}$  for (i = 8, 9, 10) and to  $A_{\mu\nu}^{(i)}$  for (i = 4, 5, 6). Further, since we neglect the lepton masses, all terms proportional to  $q_{\mu}$  and  $q_{\nu}$  in (69) can be dropped when contracting with the lepton tensor using

$$q^{\mu}L_{\mu\nu} = 0 = q^{\nu}L_{\mu\nu} \tag{73}$$

Also we find that  $H^3_{\mu\nu}$  is identically zero. For the remaining contributing terms one obtains  $(H^{(j)}_i \equiv H^{(j)}_i(x_G, z; y, s_G))^{-5}$ :

$$H_{1}^{(1)} = \frac{4}{z(1-z)} \{ 2z(1-z) + 2x_{G}(1-x_{G}) - 1 \} + \frac{4}{z^{2}(1-z)^{2}}$$

$$\left\{ -m_{+}^{4} + m_{+}^{2}m_{-}^{2}(2z-1) + m_{+}^{2}(2z(z-1)(x_{G}-1) - x_{G}) + m_{-}^{2}x_{G}(2z-1) \right\}$$

$$H_{2}^{(1)} = 16 \left\{ \frac{x_{G}}{z(1-z)} + \frac{m_{+}^{2} + (1-z)m_{-}^{2}}{z^{2}(1-z)} \right\}$$

$$H_{4}^{(1)} = \frac{32x_{G}}{z(1-z)} + \frac{16}{z^{2}(1-z)^{2}} \left\{ m_{+}^{2} + (1-2z)m_{-}^{2} \right\}$$

$$H_{6}^{(1)} = -\frac{32x_{G}}{z(1-z)} - \frac{32}{z^{2}(1-z)} \left\{ m_{+}^{2} + (1-z)m_{-}^{2} \right\}$$

$$H_1^{(2)} = -\frac{16m_0(1-x_G)}{z(1-z)} + \frac{8m_0}{z^2(1-z)^2} \left\{ m_+^2 + (1-2z)m_-^2 \right\}$$
$$H_2^{(2)} = -\frac{32m_0}{z(1-z)}$$

<sup>5</sup>Note that the two p.c. hadron tensors are not gauge invariant, only  $q^{\mu}H^4_{\mu\nu} = 0$ .

(75)

$$\begin{aligned} H_4^{(2)} &= 0\\ H_6^{(2)} &= 0\\ H_1^{(4)} &= -\frac{8}{z^2(1-z)} \left\{ -z^2 + zx_G + m_+^2 + (1-z)m_-^2 \right\}\\ H_2^{(4)} &= 0\\ H_3^{(4)} &= 8\frac{1-2x_G}{z(1-z)} - 8\frac{m_+^2 + (1-2z)m_-^2}{z^2(1-z)^2} \end{aligned}$$

Here we have defined:

$$m_{+}^{2} = \frac{m_{f}^{2} + m_{f'}^{2}}{ys_{G}}$$
$$m_{-}^{2} = \frac{m_{f}^{2} - m_{f'}^{2}}{ys_{G}}$$
$$m_{0} = \frac{m_{f}m_{f'}}{ys_{G}}$$

Let us now perform the contractions of the lepton tensors with the hadron tensors  $L_i^{\mu\nu} H_{\mu\nu}^j$  (i = 1, 2; j = 1, 2, 4). Since to  $O(\alpha^2 \alpha_s) L_1^{\mu\nu}, H_{\mu\nu}^1$  and  $H_{\mu\nu}^2$  are symmetric under the exchange  $\mu \leftrightarrow \nu$  whereas  $L_4^{\mu\nu}$  and  $H_{\mu\nu}^4$  are antisymmetric (under this exchange), only the following contractions contribute (i=1,2):

$$\frac{y}{2s_G} L_1^{\mu\nu} H_{\mu\nu}^i = x_G y^2 G_1^{(i)}(y; x_G, z) + (1-y) G_2^{(i)}(y; x_G, z) + (2-y) \sqrt{1-y} \cos \Phi G_A^{(i)}(y; x_G, z) + 2x_G (1-y) \cos 2\Phi G_B^{(i)}(y; x_G, z)$$
(76)

 $\mathbf{and}$ 

$$\frac{y}{2s_G}L_4^{\mu\nu}H_{\mu\nu}^4 = -x_G y(2-y)G_3(y;x_G,z) - 2y\sqrt{1-y}\cos\Phi G_C(y;x_G,z)$$
(77)

The (partonic) structure functions  $G_l^{(i)} \equiv G_l^{(i)}(x_g, z; y, s_G)$  are given by:

$$\begin{aligned}
G_{1}^{(1)} &= -2H_{1}^{(1)} + f(x_{G}, z; y, s_{G})H_{4}^{(1)} \\
G_{2}^{(1)} &= 2x_{G}f(x_{G}, z; y, s_{G})H_{4}^{(1)} + H_{2}^{(1)} + g(x_{G}, z; y, s_{G})\left(H_{6}^{(1)} + g(x_{G}, z; y, s_{G})H_{4}^{(1)}\right) \\
G_{A}^{(1)} &= \sqrt{x_{G}f(x_{G}, z; y, s_{G})}\left(H_{6}^{(1)} + 2g(x_{G}, z; y, s_{G})H_{4}^{(1)}\right) \\
G_{B}^{(1)} &= f(x_{G}, z; y, s_{G})H_{4}^{(1)} \\
G_{B}^{(2)} &= -2H_{1}^{(2)} \\
G_{2}^{(2)} &= H_{2}^{(2)} \\
G_{A}^{(2)} &= 0 \\
G_{B}^{(2)} &= 0 \\
G_{B}^{(2)} &= 0 \\
G_{3} &= H_{1}^{(4)} + zH_{2}^{(4)} - g(x_{G}, z; y, s_{G})H_{3}^{(4)} \\
G_{C} &= \sqrt{x_{G}f(x_{G}, z; y, s_{G})}\left(H_{2}^{(4)} - 2x_{G}H_{3}^{(4)}\right)
\end{aligned}$$
(78)

#### G.A. Schuler: Heavy flavours

Here we have defined

$$f(x_G, z; y, s_G) := (1 - x_G)z(1 - z) + \frac{z(m_f^2 - m_{f'}^2) - m_f^2}{ys_G}$$

$$g(x_G, z; y, s_G) := (1 - x_G)(1 - z) + x_G z + m_-^2$$
(79)

The  $\Phi$ -independent structure functions are given explicitly by:

$$G_{1}^{(1)} = \frac{8}{z(1-z)} \left\{ \left( x_{G}^{2} + (1-x_{G})^{2} \right) \left( z^{2} + (1-z)^{2} \right) + 2 \left( (1-2z)(1-2x_{G})m_{-}^{2} - x_{G}m_{+}^{2} \right) \right\} \\ + \frac{8}{z^{2}(1-z)^{2}} \left\{ (x_{G} - m_{+}^{2})(1-2z)m_{-}^{2} + (4z-4z^{2}-1)m_{-}^{4} + x_{G}m_{+}^{2} \right\}$$
(80)  
$$G_{2}^{(1)} = 2x_{G}[G_{1}^{(1)} + 64x_{G}(1-x_{G})] - \frac{16}{z^{2}(1-z)^{2}} \left\{ (1-2z)m_{-}^{6} + m_{-}^{4}m_{+}^{2} + 3x_{G}(2z-1)m_{-}^{2}m_{+}^{2} \\ - x_{G}m_{+}^{4} - 4m_{-}^{2}(2z-1)z(z-1)x_{G}(2x_{G}-1) - 2m_{-}^{4}(z(1-z)(1-5x_{G}) + x_{G}) \right\}$$

$$\begin{aligned} &+\frac{10m_{+}^{2}}{z(1-z)}(6x_{G}^{2}-2x_{G}-1) \\ G_{3} &= \frac{8}{z(1-z)}(2z-1)(2x_{G}^{2}-2x_{G}+1) + \\ &= \frac{8m_{-}^{4}(1-2z)+8m_{-}^{2}m_{+}^{2}+8m_{-}^{2}(-6z^{2}x_{G}+2z^{2}+6zx_{G}-2z-x_{G})+8m_{+}^{2}x_{G}(2z-1)}{z^{2}(1-z)^{2}} \end{aligned}$$

Note that the partonic structure functions  $G_i^{(l)}$  are still y-dependent as long as the quark masses are nonzero. The representations (76,77) of the y-dependence are thus not unique in the massive case. These decompositions were chosen to allow for a simple comparison with the usual (massless) results [14,15].

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Table 1: Top production rates at HERA										
${ m for} \ {{\int Ldt}} = 100 { m pb}^{-1} \ { m and} \ m_b = 5  { m GeV}$										
	$\overline{t}b$ -rate		$t\bar{t}$	total rate						
$m_t$	CC	NC	$\gamma$	Z	$\gamma$ -Z	$\operatorname{top}$				
30	82	974	973	1.2	0.36	2030				
40	53	188	187	0.42	0.09	428				
50	34	41	41	0.14	0.02	116				
60	22	9.2	9.2	0.04	0.006	40				
70	14	2.0	2.0	0.01	0.001	• 18				
80	8.3	0.44	0.44	0.003	3E-4	9.2				
90	5.0	0.09	0.09	5E-4	6E-5	5.2				
100	3.0	0.01	0.01	1E-4	1E-5	3				

						•			
Table 2: Bottom production rates at HERA									
for $\int Ldt = 100 \text{pb}^{-1}$ and $m_t = 50 \text{ GeV}$									
	$ar{t}b ext{-rate}$		total rate						
$m_b$	CC	NC	$\gamma$	Z	$\gamma$ -Z	bottom			
5	34	3.3 E5	3.3 E5	23	25	$6.6\mathrm{E}5$			

## **Figure Captions**

#### Figure 1: Total top production rate.

Number of top events versus top quark mass m for an integrated luminosity of  $\int Ldt = 100 \text{ pb}^{-1}$ . The upper full line represents the total number of top events, whereas the lower lying full line, the dotted line, the dashed line and the dashed-dotted line denote the contributions coming from  $\gamma$ -exchange, Z-exchange,  $\gamma$ -Z-interference and W exchange, respectively. The gluon density G parametrization is from [6] and the mass scale M for G and  $\alpha_s$  is  $M^2 = \hat{s}$ . All other parameters are given in the main text.

#### Figure 2: Mass scale dependence of the cross section.

Number of top events versus top quark mass m for an integrated luminosity of  $\int Ldt = 100 \text{ pb}^{-1}$ . The two lines result from different mass scales M used for  $\alpha_s$  and for the gluon density of [6]  $(M^2 = \hat{s}:$  full line;  $M^2 = (m_f + m_{f'})^2$ : dashed line). All other parameters are given in the main text.

#### Figure 3: Dependence on the parametrizations of the gluon density.

Number of top events versus top quark mass m for an integrated luminosity of  $\int Ldt = 100 \text{ pb}^{-1}$ . The six lines result from different parametrizations of the gluon density (set 1 of [18]: lower lying full line; set 2 of [18]: lower lying dotted line; set 1 of [17]: dashed line; set 2 of [17]: dashed-dotted line; [6]: upper lying full line). The upper lying dotted curve has been calculated using a scale independent gluon density  $xG(x) = 3(1-x)^5$  and with  $N_f = 5$ ,  $\Lambda = 0.2$ GeV. The mass scale M is  $M^2 = \hat{s}$ . All other parameters are given in the main text.

#### Figure 4: Influence of a lower cutoff in $Q^2$ .

Individual contributions (number of events for  $\int Ldt = 100 \text{ pb}^{-1}$ ) to the top production cross section as a function of the lower cutoff  $Q_l^2$  in  $Q^2$ :

$$\sigma = \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 rac{d\sigma}{dQ^2} \Theta(Q^2-Q_l^2) \quad {
m vs.} \quad Q_l^2$$

The four lines represent the contributions from  $\gamma$ -exchange: full line; Z-exchange: dotted line;  $\gamma$ -Z interference: dashed line; W-exchange: dashed-dotted line. The gluon density parametrization is from [6],  $M^2 = \hat{s}$  and the top quark mass was taken to be m = 40GeV. All other parameters are given in the main text.

Figure 5:  $Q_{cut}^2$  dependence of the WWA.

Number of  $t\bar{t}$  events for pure  $\gamma$ -exchange  $(\int Ldt = 100 \text{ pb}^{-1})$  as a function of an upper cutoff  $Q_u^2$  in  $Q^2$ :

$$\sigma = \int_{Q^2_{max}}^{Q^2_{max}} dQ^2 rac{d\sigma}{dQ^2} \Theta(Q^2_u - Q^2) \quad {
m vs.} \quad Q^2_u$$

The full (dashed) line results from the full  $Q^2$  (WWA) formula where we used the upper bound and lower bound of  $Q^2$  in (55) for the WWA as given by the exact values in (38) and (42). The gluon density parametrization is from [6] and the top quark mass was taken to be m = 40GeV. All other parameters are given in the main text.

### Figure 6: Accuracy limits of the WWA.

Number of  $t\bar{t}$  events for pure  $\gamma$ -exchange  $(\int Ldt = 100 \,\mathrm{pb^{-1}})$  versus top quark mass m. The full line results from the full  $Q^2$  formula. The dotted line, the dashed line and the dashed-dotted line result from the WWA using as lower and upper bound on  $Q^2$ :  $(m_e^2, m_\pi^2), (m_e^2, \hat{s}), (m_e^2, s - 4m^2)$ , respectively. The gluon density parametrization is from [6]. All other parameters are given in the main text.









# Figure 2

#events



Figure 3

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Figure 4



Figure 5

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Figure 6