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Quantum Physics and Gravitation

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A problem which has been around for a good fifty years and has now reached a virulent stage is the synthesis between General Relativity and Quantum Physics. I cannot offer the solution here but shall limit myself to a few remarks pinpointing the problem and suggesting a direction of approach. These remarks are mathematically low-brow and have no strings attached.

The relativity theories developed by an increasing insistence on the principle of locality which has originated in the Faryday-Maxwell approach the Electrodynamics replacing the notion of an action at distance between material bodies by the idea that all physical effects propagate from point to neighboring point in space and time. In general relativity this is applied in its most stringent form: all laws of nature pertain only to infinitesimal neighborhoods of points in the 4-dimensional space-time continuum. In special relativity we still have one remnant of global laws namely the rigid, a priori given metric structure. Closely related to the strict insistence on locality is the principle of general covariance, saying that there can be no preferred (finite parametric) class of coordinate systems, and the laws must be formulated in an intrinsic, coordinate-independent way. In the classical domain these requirements are met by stipulating that the laws are differential equations for field quantities and that among these fields there is the metric field g which governs the metric (and causal) structure in the neighborhood of each point.

In Quantum Theory on the formal level the fields become noncommuting objects and we need commutation relations in addition to field equations to define the theory. Since commutativity of two observables is tied to their causal independence the usual formulation of quantum field theory needs global knowledge of the causal structure of space-time. This is available in special relativity and also if the metric field is treated as a classical background field ("quantum field theory in curved space-time") but not if the metric field itself undergoes quantum fluctuations. Therefore my first remark will be concerned with the question as to whether and how a strictly local formulation of the quantum laws is possible.

Mathematical terminology

Before proceeding I should make clear the few mathematical terms I shall be using.

a) An "algebra", denoted by $\mathfrak{A}$, shall mean an abstract, non commutative *-algebra over the complex field, i.e. we have within $\mathfrak{A}$ the operations

$$A + B,\ A \cdot B,\ c\,a \text{ where } A, B \in \mathfrak{A};\ c \in \mathbb{C};$$

and the "involution" $A \to A^*$. $\mathfrak{A}$ is equipped with a topology. It is immaterial for the moment whether this is defined by a single norm or a system of seminorms. $\mathfrak{A}$ is called simple if it does not possess any closed, two-sided *-ideal.

b) Positive elements of $\mathfrak{A}$ are those of the form $A^* \cdot A$. A state ω over $\mathfrak{A}$ is a normalized, positive, linear form, i.e. $\omega(A)$ is a linear function from $\mathfrak{A}$ to the complex numbers with $\omega(A^* A) \geq 0$ and $\|\omega\| = 1$. [1)]

c) Given a state ω we have a canonical construction of a representation π_ω of $\mathfrak{A}$ by operators acting on a Hilbert space $\mathcal{H}_\omega$. The operator representing $A \in \mathfrak{A}$ is denoted by $\pi_\omega(A)$. The state can then be described by a vector Ω in $\mathcal{H}_\omega$ so that

$$\omega(A) = (\Omega, \pi_\omega(A)\Omega).$$

1) If $\mathfrak{A}$ has a unit element 1 then $\|\omega\|$ equals $\omega(1)$. The definition of the norm $\|\omega\|$ under more general situations need not concern us here.

This is the Gelfand-Naimark-Segal construction. We can then pass over from $\pi_\omega(\mathfrak{A})$ to the afiliated von Neumann algebra R (a weakly closed algebra of bounded operators). R' denotes its commutant. The state is called primary and ω factorial if the center of R is trivial, i.e. $R \cap R' = \{\lambda 1\}$.

The special relativistic setting

The conceptually most transparent way to describe the synthesis of special relativity and quantum physics is to say that a specific theory defines a correspondence between open space-time regions $\mathcal{O}$ and algebras $\mathfrak{A}(\mathcal{O})$

$$\mathcal{O} \to \mathfrak{A}(\mathcal{O}). \tag{1}$$

One may interpret the set of selfadjoint elements of $\mathfrak{A}(\mathcal{O})$ as the "observables" which can be measured in $\mathcal{O}$ or, perhaps better, the projectors in $\mathfrak{A}(\mathcal{O})$ as the possible events which can occur in $\mathcal{O}$. It appears that for bounded regions $\mathcal{O}$ it is sufficiently general to take $\mathfrak{A}(\mathcal{O})$ to be a W^*-algebra, i.e. isimorphic to a von Neumann algebra $R(\mathcal{O})$ acting on a Hilbert space.

In the setting of special relativistic theory the "net of algebras" $R(\mathcal{O})$ should satisfy a few general requirements:

(i) Additivity

$$R(\cup O_i) = \bigvee_i R(O_i)$$

where the right hand side means the W^*-algebra generated by the $R(O_i)$.

(ii) Causality

a) $R(O_1) \subset R'(O_2)$ if O_1 space-like to O_2,

b) $R(O_1) \subset R(O_2)$ if O_1 is within the causal dependence region of O_2.

(iii) Poincarè invariance

The Poincarè group acts on the net by automorphisms and

$$\alpha_g R(O) = R(gO)$$

where g denotes an element of the Poincarè group, α_g the corresponding action on the algebraic elements and gO is the transformed region.

(iv) Stability ("positive of the energy")

This is usually formulated by demanding that there should exist a "vacuum state" ω_o which is Poincarè invariant and primary for every R(O). In the GNS-representation of the net, constructed from this state we have an implementation of the Poincarè group by unitary operators. The generators of the translation subgroup are called the energy-momentum operators P_μ. The requirement that the vacuum state is the state of lowest energy is then expressed by the "spectral condition": the simultaneous spectrum of the operators P_μ must be confined to the forward cone.

(v) Nuclearity

This property is crucial for the existence of states corresponding to stable particles and the formulation of a collision theory for particles within this setting. It demands roughly that the set of states which have bounded energy and which deviate strongly from the vacuum only in a finitely extended volume at a given time is essentially finite dimensional. A precise formulation is given in /1/, a weaker form is the "compactness criterion" of /2/.

The main message: It can be shown that a local net of algebras, i.e. a corresponcence (1) satisfying the general requirements (i) - (v) defines a complete

theory including physical interpretation. In other words, once such a correspondence is given one can work out what types of particles, what charge quantum numbers occur, what the collision cross sections are etc. /3/, /4/, /5/.

General covariance and strict locality.

If we want to incorporate ideas from general relativity with minimal change we can continue to base the theory locally on a 4-dimensional manifold and retain the formulation of the quantum theory in terms of a local net (1). But all the properties have to be modified since the manifold and in particular its metric structure cannot be given a priori. Rather, in analogy to manifold theory, the description has to be given in terms of a system of charts which are combined to an atlas with the help of transition precriptions. Here a single chart consists of an open region $\hat{O} \subset R^4$, a net of algebras $\mathfrak{A}(O)$ for $O \subset \hat{O}$ and a representation π of these algebras by operators on a Hilbert space $\mathcal{H}$. The net should separate the points in $\hat{O}$, i.e. for sufficiently small neighborhoods of two distinct points the algebras should be disjoint (apart from multiples of the identity if the algebras have a unit element). Transition between "adjoining" charts $\Gamma_1 = (\hat{\mathcal{O}}_1, \mathfrak{A}_1, \pi_1)$ and $\Gamma_2 = (\hat{\mathcal{O}}_2, \mathfrak{A}_2, \pi_2)$ is described by a transition operator U_{21}, a unitary operator mapping the representation space $\mathcal{H}_1$ of π_1 on $\mathcal{H}_2$ such that it provides an identification of a subnet of Γ_1 in a region $\mathcal{O}_1^{(2)} \subset \hat{\mathcal{O}}_1$ with a subnet of Γ_2 in $\mathcal{O}_2^{(1)} \subset \hat{\mathcal{O}}_2$ by

$$\mathcal{U}_{21}\, \pi_1(\mathfrak{A}_1(\mathcal{O}))\, \mathcal{U}_{21}^{-1} = \pi_2(\mathfrak{A}_2(\varphi_{21}\mathcal{O})) \quad \text{for } \mathcal{O} \subset \mathcal{O}_1^{(2)} \subset \hat{\mathcal{O}}_1 \qquad (2)$$

where φ_{21} is an ordinary transition function i.e. a diffeomorphism from $\mathcal{O}_1^{(2)}$ to $\mathcal{O}_2^{(1)}$. The transition operator determines the transition function due to the separation property of the nets. Thus an atlas of such charts with transition operators satisfying the apropriate compatibility conditions will define a manifold together with a net of operator algebras acting on a single Hilbert space. It will define a "theory" but such a definition cannot, in general, be regarded as a local formulation because the transition operators between two local charts are not, in general, fixed by the identification of the common parts of Γ_1, Γ_2 but may include global information. This is related to the problem mentioned in the introduction that commutativity relates the causal independence.

Suppose we have two charts and an identification of the subnets in $\mathcal{O}_1^{(2)}$ and $\mathcal{O}_2^{(1)}$. This is essentially procided by giving the ordinary transition function φ_{21} because the remaining ambiguity is only an "inner symmetry" of the subnet in $\mathcal{O}_1^{(2)}$ i.e. an automorphism of $\mathfrak{A}_1(\mathcal{O}_1^{(2)})$ transforming each subalgebra $\mathfrak{A}_1(\mathcal{O})$ into itself for $\mathcal{O}\subset\mathcal{O}_1^{(2)}$. If there are no inner symmetries then φ_{21} fixes the identification completely otherwise we have to add more information. But anyway this will be strictly local information. Now let U_{21} be a transition operator which gives this prescribed identification between $\pi_1(\mathfrak{A}_1(\mathcal{O}_1^{(2)}))$ and $\pi_2(\mathfrak{A}_2(\mathcal{O}_2^{(1)}))$ by (2). Then if V is any unitary operator in the commutant of $\pi_1(\mathfrak{A}_1(\mathcal{O}_1^{(2)}))$ the transition operator

$$U'_{21} = U_{21}V$$

gives the same idenification. The choice of V will determine, however, the causal relations between observables in $\hat{0}_1$, $\hat{0}_2$ outside the identified parts and, if we have a chain of charts, this ambiguity will affect the causal structure at large.

In /6/ K. Fredenhagen and I have suggested one way to construct the theory from local information. It turns out that if instead of $\Gamma = (\hat{\mathcal{O}}, \mathfrak{A}, \pi)$ one prescribes $\Lambda = (\hat{\mathcal{O}}, \mathfrak{A}, \omega)$ where ω is a state on $\mathfrak{A}(\hat{0})$ with certain correlation properties then the identification of the substructres of Λ_1, Λ_2 in the overlap regions given by a transition function φ_{21} suffices to determine the transition operator U_{21} uniquely (the representation spaces are given by the GNS-construction).

It is, however, also possible to work with the charts Γ. But then the topology with which the algebras $\mathfrak{A}$ are equipped will play an important rôle. While it is still reasonable to assume that $\pi(\mathfrak{A})$ contains bounded operators (since we would like to have projectors) it may be unreasonable to assume that the von Neumann algebra resulting from the weak closure of $\pi(\mathfrak{A})$ will contain the same information. In fact, if in the full theory there exists no strict causal disjointness between different regions (a situation which should be expected if the classical gravitation theory is "quantized" so that the metric field has quantum fluctuations) then it may happen that the commutant of $\pi_1(\mathfrak{A}_1(\mathcal{O}_1^{(2)}))$ is trivial and hence the transition operator U_{21} is uniquely determined by the identification in the overlap region. In this case the weak closure of $\pi(\mathfrak{A}(\mathcal{O}))$ will be the algebra of all bounded operators, independent of 0 and will carry no information.

The purpose of the previous discussion was to show that it is possible to generate a quantum theory based on a 4-dimensional manifold from striclty local information i.e. to incorporate the principles of strict locality and general covariance. Nothing has been said so far about the other structural principles: what guarantees that causality and metric structure at large builds up, what are the stability and nuclearity requirements? I cannot say much on these questions but shall mention one attempt we made in /6/ to get some feeling for the nature of $\pi(\mathfrak{A}(\mathcal{O}))$ in the small.

Starting with the tensor algebra over the scalar C^{∞}-functions on $\hat{0}$ on which the action of local diffeomorphisms is naturally defined (and which corresponds kinematically to a theory with a scalar quantum field) we assumed that the allowed physical states have scaling limits under semigroups of diffeomorphisms $\varphi_{\bar{x}}(\lambda)$ contracting $\hat{0}$ to a point $\bar{x}$ as $\lambda \to 0$ and such that $d\varphi|_{\lambda=1}$ = id. Starting from a state ω the scaling limit at a point $\bar{x}$ gives a net of operator algebras labeled by regions in the tangent space of $\bar{x}$ and it gives a distinguished state ω_o on this net. This tangent space theory has some remarkable properties. First, it is independent of the choice of the contracting semigroup of diffeomorphisms. Secondly, it (including ω_o) is independent of the choice of the starting state ω as long as ω varies within one primary folium [1]. Third, if the dependence of the scaling limit on the contraction point is smooth then ω_o is invariant under translations in tangent space. Thus one can define energy-momentum operators in the tangent space theory. ω_o has to be invariant under some subgroup of the linear transformations in tangent space, otherwise the starting state (or rather folium) defines a flat connection in $\hat{O}$. One interesting possibility is that this stability subgroup is isomorphic to the Lorentz group. In this case the folium defines a (pseudo)-Riemannian structure with a Levi-Civita connection on $\hat{O}$ and the tangent space theories are Poincaré-invariant, massless theories (with respect to the metric obtained by the scaling limit at the respective point). The properties (i) - (v) can then be formulated and imposed on the tangent space theories. This structure is relevant and also useful for the description of a quantum field theory on a Riemann space (gravitational background field). It leads for instance in a simple way to the Hawking temperature of a black hole. Whether the scaling assumption remains reasonable if quantum fluctuations of the gravitational field are considered is not clear. If so then the tangent space states ω_o must have a higher symmetry, perhaps under all SLR(4) and the breaking of this symmetry to the Lorentz group must be obtained as a cooperative effect analogous to the breaking of rotational symmetry by spontaneous magnetization.

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1) ω and ω' are in the same folium if ω' can be described by a density matrix in the GNS-representation of the original tensor algebra obtained from ω.

Many obvious questions I must leave unanswered. On the positive side my remarks do show that a synthesis between quantum physics and general relativity within a rather conservative conceptual frame does not meet with unsurmountable obstacles. The challenge is to construct a tractable model incorporating the features mentioned.

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