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Moment Matching for Bayesian Inference in the Baseline New–Keynesian Model

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Abstract

Contrary to claims in studies on financial economics, a sparse database often obscures the identification of parameters in macroeconomic models. These identification problems originate from the poorly defined mapping between a structural model and reduced-form parameters. Hence, researchers rely on prominent estimation methods, such as Bayesian approaches, which require sound knowledge of prior distributions on parameters. These approaches, however, are characterized by a flat likelihood and/or a posterior distribution driven mainly by prior information. To alleviate identification issues, we apply approximate Bayesian computation combined with the choice of specific moment conditions. This estimation approach not only allows for circumventing high dimensional likelihood functions but also avoids parameter identification problems given the use of a bootstrap method. Our estimation method is successfully applied to a hybrid version of the New Keynesian model.

Keywords: Approximate Bayesian Computation; Identification; Moment Conditions; New–Keynesian model.

JEL classification: C11, C14, E12

1 Introduction

The identification and estimation of structural models are challenging tasks in empirical research. Improvements in data availability and mathematical methods have provided researchers with reliable models of economic systems, but the estimation of associated structural parameters remains confronted with many issues. Examples of well-known studies date back to those on simultaneity problems found in simple demand-and-supply analysis (Hood & Koopmans, 1953; Manski, 1995), wherein possible combinations of structural parameters in a model's reduced-form representation are associated with the observational equivalence linked to a likelihood function (Hsiao, 1983; Koopman, 1949). Other examples are structural models wherein the complexity of a non-convex and high-dimensional parameter space makes finding a global optimum difficult. These problems appear to have been alleviated by the development of optimization techniques, but identifying a global optimum for objective functions would entail considerable computational costs (Gilli & Winker, 2003; Goffe et al., 1994). Even when such attempts succeed, researchers are faced with a daunting process.

It is not uncommon to find that advanced techniques for estimating structural parameters in macroeconomics models have been proposed (Andreasen, 2010; Fernández-Villaverde & Rubio-Ramırez, 2007). However, their exact identification in dynamic stochastic general equilibrium (DSGE) models is challenging, with difficulties arising even in small-scale DSGE models. These difficulties have been extensively documented, starting from the works of Canova and Sala (2009) and Iskrev (2010). In a generic sense, identification problems originate from the poorly defined mapping between structural and reduced-form parameters, and even a flat likelihood occurs when working with a sparse dataset. The consequent lack of identification prevents the efficient use of optimization-based estimators in macroeconomic models, thus giving rise to the popularity

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of estimation techniques, such as Bayesian approaches, in practice (Fernández-Villaverde, 2010). Indeed, the numerical integration involved in Bayesian analysis has a certain advantage over the extremum estimator applied in a maximum likelihood procedure, but the use of a sparse dataset can trigger criticism given that a posterior distribution is driven mainly by prior information (Chan et al., 2019; Lombardi & Nicoletti, 2012). Hence, results from applying standard Bayesian estimation to the evaluation of the typically high-dimensional integral of a marginal likelihood can be sensitive to the selection of prior information.

Recognizing the identification problems encountered in DSGE models, we take a humble step in finding a way to alleviate this issue, seeking to navigate the aforementioned challenges by implementing approximate Bayesian computation (ABC). ABC reflects the use of Bayesian inference, wherein a high-dimensional likelihood – which is often analytically intractable – is approximated on the basis of moment conditions and, as demonstrated in this paper, is very generally applicable. The ABC method also has the potential to mitigate computational burdens compared with other standard estimation approaches. Nevertheless, although ABC is a popular technique for estimating agent-based models in biology (Csilléry et al., 2010), chemistry (Burr & Skurikhin, 2013), medicine (Minter & Retkute, 2019), and finance (Lux, 2023), to the best of our knowledge, it has been minimally investigated with respect to macroeconomics. A rare exception is the study of Lux (2024), who applied the Hamiltonian Markov chain Monte Carlo algorithm to a behavioral macroeonomic model and found that certain parameters are difficult to identify. In the present research, therefore, we demonstrate the ABC method's potential for estimating the baseline New-Keynesian model (NKM) in its log-linearized representation. As it turns out, ABC enables the efficient use of second moments, and it can be employed under small sample sizes and orders far beyond the reach of numerical likelihood methods.

The merit of our contribution lies in its facilitation of a hybrid strategy that combines the extremum estimator approach and Bayesian analysis, thereby circumventing high-dimensional likelihood functions while also avoiding the parameter identification problems plaguing sparse datasets. We discuss how the ABC approach ties into the established estimation procedure of DSGE models via conventional Bayesian techniques, that is, based on the evaluation of a likelihood function with an updating scheme for prior beliefs. Instead of converting a DSGE model into a likelihood-grounded representation, however, ABC employs a likelihood-free rejection sampler to derive inferences on the distribution of parameter estimates. Therefore, the approach is related to the discussion of sequential Monte Carlo samplers—algorithms used for Bayesian estimation—bringing about the need to modify likelihood iteratively to build a particle approximation of a posterior distribution (see Creal, 2007; Del Moral et al., 2006; Herbst & Schorfheide, 2014).

Our study also focuses on the reliability of parameter estimates of the trade-off between prior and posterior distributions because data tend to be insufficiently informative to enable a distinction. In particular, we compare the estimation results derived via extremum estimator and Bayesian approaches before contrasting these with the findings obtained via the hybrid version of the baseline NKM under rational expectations. We claim, however, that applying the procedure is not limited to this specific linear macroeconomic model but that it finds wide applicability to nonlinear DSGE models as well.

One drawback of using ABC to estimate DSGE models, nonetheless, is that although deep parameters stemming from microfoundations shape macroeconomic dynamics, underlying economic processes have to be inferred using small sample sizes. This problem increasingly stands out in Bayesian approaches because ABC requires resampling from prior distributions. That is, a relatively large number of parameters imposes a burden on the identification of DSGE models involving possibly many combinations of sampling from prior densities. To circumvent these difficulties, we adopt an experimental approach based on block bootstrapping to exogenously set the selection criterion in the ABC sampling process. This alternative approximation of the data generation process suggests an efficient application of ABC to macroeconomic models. Interestingly, a stringent criterion enables the improved approximation of data generation. Correspondingly, we compare the results obtained through this criterion with those derived via standard Bayesian estimation on the grounds of historical US macroeconomic data spanning the period 1954 to 2021.

The rest of the paper is structured as follows. We highlight the three-equation representation of the hybrid NKM for estimation in Section 2. The ABC estimation approach, the selection of moment conditions, and the bootstrap method are discussed in Section 3. The results of an empirical application of standard Bayesian versus various specifications of the ABC approach are presented in Section 4. Section 5 concludes. Technical details and additional findings derived from a Monte Carlo experiment can be found in the Appendix.

2 The Hybrid New-Keynesian Model

We consider the three-equation representation of the NKM in its log-linearized form for a closed economy, following De Grauwe and Ji (2020). This discrete-time framework belongs to the class of DSGE models and is presented in quarterly terms. As standard in the corresponding literature, focus is directed toward a hybrid version of the model, leading to the incorporation of lag terms that indicate past realizations in the output gap and the inflation rate into equations describing the development of both variables. This incorporation rules out monotonic dynamics, consistent with the occurrence of humped-shaped patterns in adjustments made over time in empirical data.

$$y_t \;=\; a_1\,\mathbb{E}_t\{y_{t+1}\} \;+\; (1-a_1)\,y_{t-1} \;-\; a_2\,(r_t - \mathbb{E}_t\{\pi_{t+1}\}) \;+\; v_t. \tag{1}$$

Eq. (1) is the dynamic IS equation, which is the outcome of a representative household's intertemporal optimization approach to consumption and saving. In particular, consumption smoothing is conducted with consideration for the real interest rate denoted by $r_t - E_t\pi_{t+1}$. The degree of intrinsic persistence is measured using $0 \le a_1 \le 1$ and accounts for habit formation in consumption. The parameter $a_2 \ge 0$ denotes the inverse intertemporal elasticity of consumption behavior. In a general equilibrium context, changes in consumption coincide with movements in the output gap (y_t), that is, the difference between actual and potential output levels, in all periods.

$$\pi_t \;=\; b_1\,\mathbb{E}_t\{\pi_{t+1}\} \;+\; (1-b_1)\,\pi_{t-1} \;+\; b_2\,y_t \;+\; \eta_t. \tag{2}$$

Eq. (2) represents the New-Keynesian Phillips curve (NKPC). The degree of intrinsic persistence is measured using $0 \le b_1 \le 1$ and accounts for price indexation on the supply side. The output gap y_t is the driving force of inflation π_t under the assumption of monopolistic competition combined with the Calvo price-setting scheme. Hence, the slope is given by the parameter $b_2 \ge 0$, which measures the degree of price stickiness (Calvo, 1983).

$$r_t \;=\; (1-c_3)\,[\,c_1\,(\pi_t - \pi^*) + c_2\,y_t\,] \;+\; c_3\,r_{t-1} \;+\; u_t. \tag{3}$$

Finally, the Taylor rule is given by Eq. (3), which reflects a central bank's effort to minimize the inflation rate and output gap fluctuations by adjusting the nominal interest rate (r_t). Therefore, the rule responds directly to contemporaneous movements in the deviation of the inflation rate from its target value (where we assume that $\pi^* = 0$) and the output gap. The corresponding values of the policy coefficients are derived from $c_1 \ge 0$ and $c_2 \ge 0$. The central bank smoothens the interest rate to avoid rapid changes in the monetary policy instrument, which align with empirical observations otherwise. The corresponding parameter is denoted by $0 \le c_3 \le 1$.

We assume that the exogenous driving forces of the model variables follow specific shocks v_t, η_t, and u_t to demand, supply, and the monetary policy instrument, respectively. These shocks are independent and identically distributed around mean zero and variance σ_s^2 with the indices $s = \{y, \pi, r\}$. The NKM includes forward-looking terms that reflect the expected single period ahead realizations of the variables in Eqs. (1) and (2). $\mathbb{E}_t$ represents the expectation operator conditional on the information at time t.

3 Bayesian Inference and Moment Conditions

In this section, we discuss the ABC estimation method for Bayesian inference in the NKM with consideration for specific moment conditions. Estimating DSGE (or other) models via traditional maximum likelihood estimation techniques requires searching for the global optimum of a likelihood function over a high-dimensional parameter space. By contrast, a Bayesian approach uses prior information about parameter distribution for the integration of a likelihood function while updating the distribution of parameters with new data observations. In practice, then, the estimation and identification of structural models can be circumvented by specifying the prior distributions of parameters.

The problem is that this method likely gives rise to the possibility of dealing with a relatively sparse macroeconomic dataset, which renders robust inferencing on parameters difficult. We augment the Bayesian technique with a set of moment conditions when estimating a DSGE model and subsequently mitigate the identification problems caused by a small sample size. To these ends, we implement ABC, in which a rejection sampler linked to the moment conditions is used to find parameter values without directly evaluating a likelihood function. The block bootstrap method is applied to tackle the problem under a small sample size as the model is applied to the data. We comprehensively address the issues of interest separately throughout the remainder of this section.

3.1 Approximate Bayesian Computation

At its core, the ABC approach makes use of algorithmic frameworks consisting of two building blocks that reduce the computational costs incurred during the numerical integration of likelihood functions (Lux, 2023). The first building block draws a set of initial parameters from prior distributions, which satisfy tolerance levels to match empirical moments. The second building block consists of the selection process in which Monte Carlo methods are used to update the sequence of draws. The updating mechanism approximates the posterior distribution of parameters from initial prior distributions. The weighting scheme is updated from the parameters drawn from the preceding pool in a transition kernel, thereby enabling us to avoid the tedious process of selecting a random draw of parameters and their evaluation, as is the case with the traditional Bayesian approach.

ABC builds on the principle of Bayes' rule, in which both sample outcomes and parameters are treated as random variables. This rule is designed to improve efficiency in the computation of likelihood functions. We apply the rule to show how a likelihood function related to the NKM is approximated. Given a parameter set θ, the posterior distribution $\varphi(\theta|x)$ is computed contingent on the prior distribution $\varphi(\theta)$:

$$\varphi(\theta|x) = \varphi(\theta)\frac{f(x|\theta)}{\int f(x|\theta)\varphi(\theta)d\theta} \tag{4}$$

where the evaluation of the likelihood of sample x is based on the integration of sampling probability into observations with certain parameter values denoted by $f(x|\theta)$. In what follows, we demonstrate how ABC is used to avoid the integration of high–dimensional likelihood functions on the grounds of structural parameters.

Applying ABC allows for the selection of parameter values from parameters' prior distributions, which satisfy a specific criterion for the likelihood of data:

$$\varphi_{\epsilon}(\theta, z|x) = \varphi(\theta) \frac{f(z|\theta)\, I_{obj.\leq\epsilon}(z)}{\int f(z|\theta)\, I_{obj.\leq\epsilon}\, \varphi(\theta)\, d\theta} \tag{5}$$

where I is an indicator function based on an objective function; some of the simulated samples z are screened when they do not satisfy a certain criterion ϵ. Therefore, the posterior function becomes the function of the selection criterion, which is grounded in a function of distance between simulated and observed data (i.e., moments).[1] If this type of matching is sufficient to reach a minimum of the distance function, then the corresponding rejection sampler accepts a better candidate among simulated draws for the parameters from the prior distribution.

Selecting appropriate moments to match is therefore crucial for the identification of structural parameters – an issue extensively discussed in the literature on econometrics. Here, we regard auto- and cross–covariances as moments to identify whether the NKM effectively explains the business cycles of the economy. Accordingly, we explore 78 moment conditions (Franke et al., 2015; Jang and Sacht, 2016). In each iteration of the ABC, the objective function measures the weighted distance between empirical moments and model-generated moments:

$$\text{obj.} \equiv \arg\min_{\theta}\, (m(\theta) - m_T^{emp})'\, \widehat{W}\, (m(\theta) - m_T^{emp}) \tag{6}$$

where m is a vector of the moment conditions, and $\widehat{W}$ denotes the estimated weighting matrix. Hence, the estimation method is based on the minimization of the objective function: The smaller the objective function value according to expression (6), the better the estimated model's approximation of the data generation process. This study employs the Newey–West estimator to construct $\widehat{W}$, wherein we ignore the off-diagonal terms of the latter.[2] This helps us avoid sampling errors coming from a fairly large number of moment conditions being considered.

In many cases, the distance function is chosen as the summary statistics of the data generation process. This procedure is called a likelihood-free method because it avoids the direct evaluation of a likelihood function. Thus, summary statistics should be carefully chosen to approximate the likelihood of observations given true parameter values. This implies that the efficient use of ABC is anchored in the appropriate selection of prior distributions and summary statistics.

3.2 Bootstrap Method

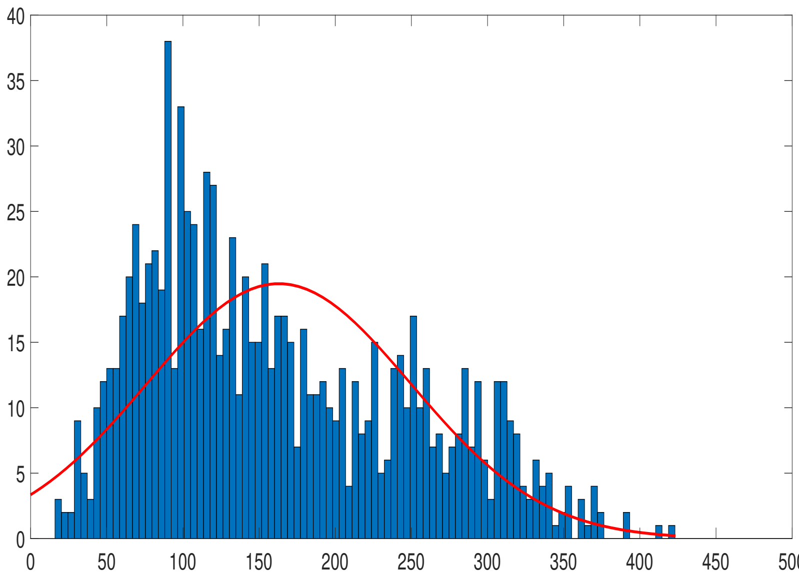
Structural models often exhibit fairly numerous deep parameters, but in reality, the observed sample size presented in quarterly terms is limited, which hinders robust inferencing on parameter values. Observational equivalence is often caused by the flatness of the likelihood arising from different parameter combinations.

Alternatively, we apply a block bootstrap method to examine the identification problems caused by a small sample size. This method is based on a non-parametric approximation of the data generation process. In our case, the data of interest consist of three variables: the nominal interest rate, output gap, and inflation rate. As we consider an appropriate block window size of, say, 5

1. See also Creal (2007) for the application of the important sampling algorithm to the NKM, wherein particle approximation is performed to calculate the posterior likelihood. The sequential Bayesian method has an advantage over DSGE models with many structural parameters.

2. If many moment conditions are used, however, estimates may be inconsistent (or, in other words, biased); hence, several weighting schemes are proposed to remedy the sampling error in the finite sample in GMM (Cheung et al., 2023). For this purpose, we conduct Monte Carlo experiments using the ABC approach based on the underlying model. In Section 3.3, we show that the exercise helps recover true parameter values. This verifies the absence of possible bias stemming from a large number of moment conditions under consideration.

Figure 1. Distribution of block bootstrapped samples for output, inflation, and interest rate



years (i.e., 20 observations in quarterly terms), the data are reshuffled 1,000 times according to these blocks. The moment conditions are then be computed using the reshuffled dataset, that is,

$$\hat{m}_i^s := m(x^{BB}) \tag{7}$$

where $i = 1, \cdots, I$; I indicates the simulation size for a random reshuffling from the bootstrapped samples x^{BB}. This clears the way for a potential distribution of the data generation process and serves as a criterion for measuring the uncertainty of summary statistics. Then, the model is aimed at matching the dimensions of the bootstrapped distribution, which can engender a better perspective of the model validation.

As an example, Figure 1 shows the distribution of block bootstrapped samples for all the three NKM variables studied here. The 90% and 95% quantiles are 293.23 and 317.28, respectively, suggesting that the parameter estimates with the objective function values that are greater than these criteria are rejected as 'true' parameters drawn from the underlying data generation process. The bootstrapped samples follow a χ^2 distribution, with the degrees of freedom determined from the number of moment conditions examined (Winker et al., 2007). However, these criteria are very generous for a small sample size, which means that the power of statistical inferencing on the selection of a true model is non-exhaustive. As an alternative, we can adopt a more stringent experimental criterion for selecting redrawn samples – a value from the 10% quantile, which is 65.06 in this case.

3.3 Monte Carlo Experiments

We numerically test the validity of the ABC estimator with regard to its finite sample properties. We conduct extensive Monte Carlo experiments to determine ABC's effectiveness in consistently recovering pseudo-true parameters in a controlled simulation environment. We distinguish between the conventional ABC method from the ABC incorporated with Bootstrap I and II (BQ I and BQ II, respectively). The difference in the latter two stems from the application of the selection criterion that entails plugging the exogenous criterion for selection from the quantiles under the bootstrapped samples.

We choose the 95% and 10% quantiles as selection criteria for BQ I and BQ II, respectively. These are experimental approaches involving the comparison of the MC results obtained from the stringent criterion imposed in the ABC method and the determination of the degree of quality information needed to infer the data generation process from finite samples. The selection criterion from the 10% quantile is very strict in a statistical sense, but we later illustrate that avoiding substantial variations from relatively many structural parameters in the NKM is advisable.

The results are presented in Tables A1 to A3 in the Appendix.[3] The ABC+BQ II method exhibits satisfactory estimation accuracy and asymptotic tendencies under large sample sizes compared with the other two approaches. Table A3 shows that at a $T = 1,000$ (i.e., with 1,000 artificial data points under consideration), nearly all 'true' parameter values can be recovered with high precision.

For the ABC+BQ II case, low values for the criterion (10% quantile from bootstrapped samples) and $\widehat{\text{obj}}$. indicate a good fit with the data. Even at a small sample size of T = 100, all 'true' parameter values can be recovered with moderate precision. This observation holds, except for parameters a_1 and a_2, which account for habit formation in consumption and the inverse intertemporal elasticity in consumption behavior, respectively (Eq. (1)). This is also true for both the traditional ABC and ABC+BQ I, wherein both parameters are poorly identified even with a large dataset at hand. In general, a_1 (a_2) tends to be downward (upward) biased, except in the BQ II case – at least under $T = 1,000$.

The results derived using traditional ABC confirm our conjecture that parameter re-estimation exercises are not entirely consistent and perhaps driven primarily by prior distributions, especially under small sample sizes. The poor performance in the MC experiments with small sample sizes indicates that empirical results conditional on ABC estimates should be carefully interpreted. These problems can be overcome as researchers collect more data with a stronger fit with the data generation process. This suggests that our experimental approach to establishing the criterion from the 10% quantile points to the need for a large sample size, albeit this does not eliminate criticism of the arbitrary choice of moments in approximating likelihood functions.

4 Empirical Application

4.1 Data

Data for the US are given in quarterly values from the period 1954 Q3 to 2021 Q3. The data come from the Federal Reserve Bank of St. Louis.[4] Throughout transformation, all observations are expressed in logs to match the log-linearized equations in the NKM considered in this work. In this manner, data on the output gap are de-trended, compelling us to apply the Kalman implementation of the one-sided HP filter (with the smoothing parameter set to its default value of 1600), according to Stock and Watson (1999). Since the time series for inflation and the nominal interest rate are stationary, these have not been de-trended. After transformation, all observations are scaled by a factor of 100.

The time series for the real gross domestic product (GDPC1, in billions) of chained (2012) dollars is given as a seasonally adjusted annual rate. To measure inflation, we use the consumer price index for all urban consumers with all items (CPIAUCSL). Because the seasonally adjusted time series is given monthly, we first compute the geometric mean of three months to obtain its counterpart in

3. We refrain from testing the Bayesian estimation approach's validity, as this has already been discussed in the literature for several decades; see, e.g., Vovk & V'Yugin (1993) and Petrova (2024). The former discuss the degree of enumerable compression for the parameter-typical data linked to the algorithmic randomness of a parameter. The latter shows that estimation results obtained from a DSGE-based inference is robust if the structural shocks are assumed to be Gaussian distributed even when their true counterparts are not.

4. See: `https://fred.stlouisfed.org`.

quarterly terms. We then obtain the gross inflation rate in each quarter as we calculate the ratio of the current price index value to that for period $t-1$. As the log-linearized NKPC (Eq. (2)) displays the dynamics of the net inflation rate, our observation for the latter stems from taking the difference of the expression in each point in time from its long-run mean value (all given in logs). The observations for the nominal interest rate are computed on the basis of the seasonally unadjusted time series for the effective federal funds rate (FEDFUNDS). Again, all data are given monthly, which means the computation of the geometric mean of three months to obtain its counterpart in quarterly terms. Subsequently, we transform the net annualized gross rates into quarterly values by approximating the correct geometric mean value. Finally, we de-mean all observations in logs on the basis of the long-run mean value, which yields the net quarterly nominal interest rate that corresponds to the Taylor instrument rule (Eq. (3)).

4.2 Bayesian Estimation of the Model: Numerical Setup

Estimations are conducted using Matlab (version 9.9.0.1467703; R2020b) and its add-on Dynare (version 5.3). We use Chris Sims's "csminwel" optimizer in implementing mode computation ($modecompute = 4$). The number of parallel chains and replications for the Metropolis–Hastings algorithm is set to 3 ($mhnblocks = 3$) and 2,000,000 ($mhreplic = 2,000,000$), respectively. The fraction of initially generated parameter vectors to be removed as burn-ins before carrying out posterior simulations is set to 0.5 ($mhdrop = 0.5$). The corresponding acceptance ratio for the draw candidates (or proposals) within the Metropolis–Hastings algorithm is obtained using the covariance matrix of the proposal density.[5] This matrix must be scaled in such a way that the appropriate acceptance ratio of proposals goes from 25% to 40%. As a reliable compromise, an acceptance ratio of approximately 30% ($mhjscale = 0.66$) is adopted in this work. We consider the 95% highest posterior density interval used for the computation of the parameter distributions ($conf_{sig} = .95$).

With regard to prior information, the prior mean values follow the parametrization in De Grauwe and Ji (2020, Table 1 in these authors' paper). These values, together with prior standard deviations (except for uniformly distributed parameters), are the norm in the literature. The specifications of prior distributions are taken from Herbst und Schorfheide (2016), except for b_2, which is obtained following Franke et al. (2015) since the slope of the NKPC has no upper bound.

In the estimations, we are confident that the convergence checks applied by Brooks and Gelman (1998), which are summarized in univariate and multivariate diagnostics (data not shown here), prevent deviations in the sampling procedure. Diagrams that contrast the posterior distribution of the single parameters with their prior counterparts are shown in Figure A1 in the Appendix. These diagrams illustrate that for all periods, (1) the two distributions fairly differ, (2) the posterior mean values vary (significantly) from the prior ones, and (3) the posterior distribution tends to be Gaussian in nature. These observations indicate that the data are informative about the values of the parameters, thus safely ruling out the possibility of poorly identified parameters.

4.3 Empirical Results

We examine the empirical performance of the ABC approach and compare the results obtained from applying the standard Bayesian estimation technique. We make use of the acronyms SBC, ABC+BQ I, and ABC+BQ II for the Bayesian approach and the ABC methods with Bootstrap I and II, respectively. The estimations of the parameters and shocks found in the hybrid NKM are displayed in Table 1.

5. The Metropolis–Hastings approach belongs to the class of Markov chain Monte Carlo algorithms. It is, by default, incorporated in Dynare and therefore applied in a vast majority of studies on estimating macroeconomic DSGE models.

Table 1. Empirical results

	Prior Information	Posterior Values			
		SBC	ABC	ABC+BQ I	ABC + BQ II
$\hat{a}_1$	Uni ~ (0, 1)	0.510 (0.484–0.537)	0.480 (0.041–0.919)	0.641 (0.178–1.105)	0.434 (0.145–0.723)
$\hat{a}_2$	Gamma ~ (0.2, 0.1)	0.020 (0.007–0.032)	0.523 (0.000–1.210)	0.469 (0.000–1.089)	0.424 (0.000–1.199)
$\hat{b}_1$	Uni ~ (0, 1)	0.550 (0.523–0.577)	0.172 (0.000–0.402)	0.215 (0.000–0.482)	0.164 (0.000–0.350)
$\hat{b}_2$	Gamma ~ (0.05, 0.025)	0.003 (0.001–0.005)	0.051 (0.001–0.102)	0.078 (0.000–0.168)	0.035 (0.006–0.064)
$\hat{c}_1$	Normal ~ (1.3, 0.2)	1.369 (1.142–1.592)	1.588 (1.000–2.193)	1.665 (1.000–2.340)	1.348 (1.110–1.586)
$\hat{c}_2$	Gamma ~ (0.5, 0.25)	0.584 (0.381–0.780)	0.604 (0.000–1.351)	1.023 (0.000–2.271)	0.293 (0.069–0.518)
$\hat{c}_3$	Uni ~ (0, 1)	0.907 (0.883–0.930)	0.409 (0.000–0.910)	0.410 (0.000–0.816)	0.459 (0.000–0.925)
$\hat{\sigma^2}_v$	InvGamma ~ (0.5, 1)	0.379 (0.349–0.409)	0.791 (0.067–1.515)	0.660 (0.000–1.507)	0.656 (0.370–0.942)
$\hat{\sigma^2}_\eta$	InvGamma ~ (0.5, 1)	0.276 (0.252–0.296)	0.175 (0.058–0.292)	0.209 (0.026–0.392)	0.130 (0.069–0.191)
$\hat{\sigma^2}_u$	InvGamma ~ (0.5, 1)	0.187 (0.173–0.201)	0.533 (0.000–1.118)	0.643 (0.000–1.423)	0.455 (0.076–0.833)
Selection criterion			162.8	317.3	65.1
Obj.			129.6	221.6	59.1

Note: SBC stands for standard Bayesian computation. The selection criterion of ABC is chosen from the top 2% quintile of 1,000 sampled parameters from prior distribution. The criteria of ABC methods with Bootstrap I (ABC+BQ I) and II (ABC+BQ II) are selected from the 95% and 10% quartiles in the 1,000 block bootstrapped dataset, respectively.

When interpreting the empirical results for the parameter estimates in economic terms, we identify some noteworthy differences regarding the posterior values obtained from the application of SBC and ABC+BQ II (in the following, denoted simply by the subscript ABC for convenience) on the basis of the performance of the latter in our Monte Carlo experiments. While the degree of backward-looking behavior is moderate when it comes to the dynamic IS equation under an estimated mean value of $\hat{\alpha}_{1,SBC} = 0.510$ versus $\hat{\alpha}_{1,ABC} = 0.434$, in the ABC case, the NKPC leans heavily on the past realization of the inflation rate under $\hat{b}_{1,ABC} = 0.164$ (versus a rather moderate estimate of $\hat{b}_{1,SBC} = 0.550$). However, the lower bound of the corresponding confidence interval for $\hat{b}_{1,ABC}$ indicates that this estimate is nonsignificant. Estimates of the intertemporal elasticity in consumption a_2 point to an almost non-existent influence of monetary policy interventions in response to output gap development given a nonsignificant estimate of $\hat{a}_{2,SBC} = 0.020$. In this case, the transmission channel affecting the household's consumption smoothing procedure is limited. The response to changes in the real interest rate, $r_t - E_t\pi_{t+1}$, tends to be moderate under ABC with $\hat{a}_{2,ABC} = 0.424$, but this response is also nonsignificant. With respect to the parameter

b_2, the pass-through of output gap fluctuations in the inflation rate is virtually nonsignificant for both methods ($\hat{b}_{2,SBC} = 0.003$ versus $\hat{b}_{2,ABC} = 0.035$). This observation reflects findings in the literature stating that given quarterly data, the NKPC is flat given the estimation of its slope close to zero in numerous studies.

The central bank's reaction to the deviation of the inflation rate from its target value does not differ across both cases. The Taylor coefficient, c_1, is estimated above unity, implying a counter-reactionary movement against inflation rate volatility by the monetary authority ($\hat{c}_{1,SBC} = 1.369$ versus $\hat{c}_{1,ABC} = 1.348$). The SBC application hints that changes in the output gap are twice as strongly tackled by the central bank compared with the implementation of ABC ($\hat{c}_{2,SBC} = 0.584$ versus $\hat{c}_{2,ABC} = 0.293$). Interestingly, these actions against inflation rate and output gap disturbances are less strongly considered according to the SBC estimates, wherein a high degree of interest rate smoothing close to unity is observed with $\hat{c}_{3,SBC} = 0.907$. In contrast, the consideration of the past realization in r_t seems to be moderate under ABC with $\hat{c}_{3,ABC} = 0.459$ where, again, this estimate is nonsignificant. The estimated variance in demand and interest rate shocks is roughly two times higher under ABC than SBC ($\sigma^2_{\nu,SBC} = 0.379$ versus $\sigma^2_{\nu,ABC} = 0.656$ and $\sigma^2_{u,SBC} = 0.187$ versus $\sigma^2_{u,ABC} = 0.455$). The opposite seems to be true in the case of a cost–push shock with $\sigma^2_{\eta,SBC} = 0.276$ versus $\sigma^2_{\eta,ABC} = 0.130$.

Overall, these results show that (among other matters) implications for monetary policy strategies differ depending on what method is used to derive estimates. Under ABC, strong and moderate responses to changes in the variables are considered grounded in the Taylor rule. The latter exhibits only a moderate degree of smoothing. In addition, the impact of the household's decision-making regarding consumption (and, hence, the realization of the output gap) is noteworthy. A comparison of the estimates to the ones obtained from SBC point to the direction of less impactful monetary policy limiting itself to interest rate smoothing. We assert that there is evidence of a higher number of nonsignificant parameter estimates under ABC than in SBC – a matter that should be taken into account when interpreting results.

5 Conclusion

We have explored and applied the ABC method combined with the choice of specific moment conditions to estimate a standard macroeconomic model under rational expectations. This approach effectively approximates the likelihood function in NKM, as it has, in general, the potential to circumvent analytically intractable high-dimensional likelihood functions. This estimation procedure is fairly new to this type of research and has been applied by only a few scholars in economics, such as Lux (2023). In a follow-up paper, Lux (2024) points out the lack of parameter identification in simple behavioral macroeconomic models due to a sparse database. We show that this identification issue might be well addressed by augmenting the ABC method with bootstrapping procedures. The results from our Monte Carlo experiments indicate that the augmented ABC performs well in terms of estimation accuracy and asymptotic tendencies. This holds, to some extent, for small samples sizes. The comparison of the estimates derived via the augmented ABC method with those obtained using the standard Bayesian technique uncovered significant differences with noteworthy implications for macroeconomic policy and impact analysis. This remains valid despite numerous parameters being nonsignificant.

Our study also reflects that a relatively large number of structural parameters of DSGE models present identification issues to empirical research because they suffer from substantial variations, especially under relatively small sample sizes. Nevertheless, the ABC method with bootstrapping proposed in this research is still in its experimental stage, and further investigations supported by more data observations are needed. Future research should also be directed to identifying a parsimonious macroeconomic model that exhibits a better approximation of the data generation process to improve predictive accuracy for economic policies.

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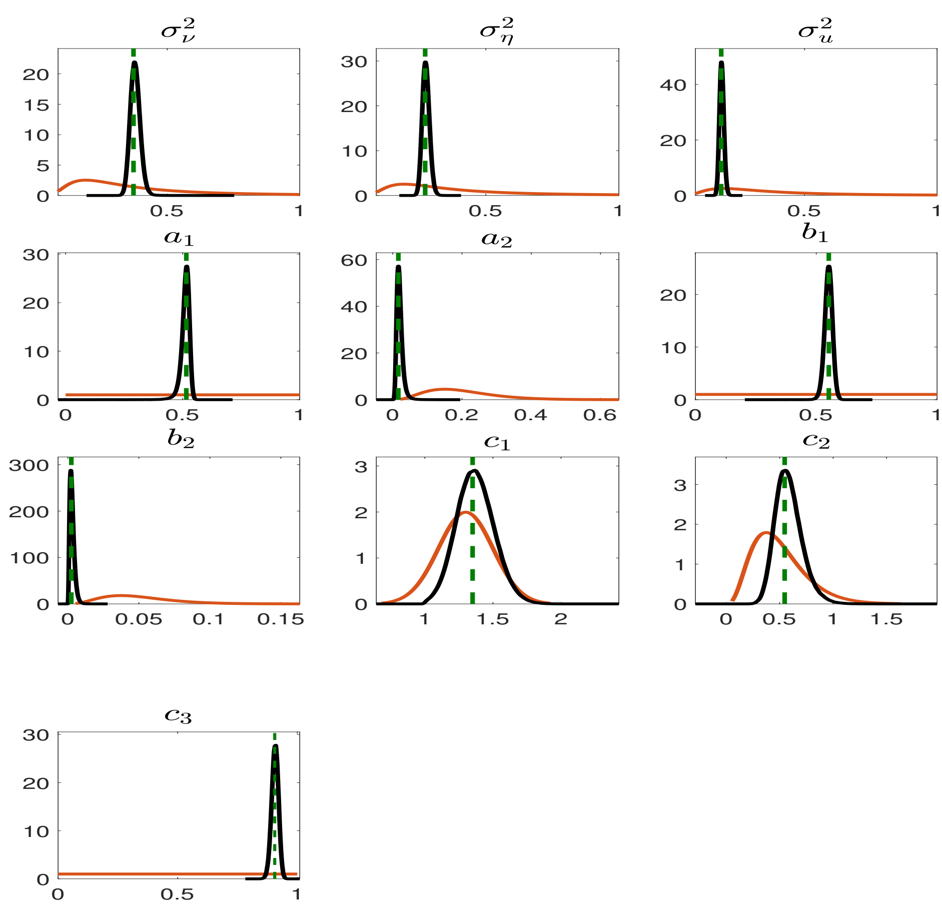
Appendix

A1. Standard Bayesian Estimates: Prior vs. Posterior

Figure A1: Posterior density estimates for the standard Bayesian computation



Note: The dashed (green) line represents the mode of the posterior distribution (black line; generated by the random-walk Metropolis-Hastings procedure). The grey (red) line represents the prior distribution (see Table 1).

A2. Results of Monte Carlo Experiments

Table A1: Monte Carlo Experiments: ABC

	θ_0	$\hat{\theta}$			
		T=100	T=200	T=500	T=1,000
a_1	0.500	0.274	0.271	0.266	0.269
		(0.236)	(0.235)	(0.237)	(0.233)
a_2	0.090	0.360	0.340	0.323	0.324
		(0.284)	(0.257)	(0.237)	(0.237)
b_1	0.580	0.625	0.631	0.621	0.614
		(0.121)	(0.094)	(0.069)	(0.050)
b_2	0.050	0.107	0.097	0.085	0.082
		(0.065)	(0.053)	(0.038)	(0.033)
c_1	1.650	1.699	1.660	1.641	1.645
		(0.134)	(0.096)	(0.071)	(0.069)
c_2	0.375	0.506	0.457	0.422	0.408
		(0.184)	(0.126)	(0.073)	(0.052)
c_3	0.550	0.380	0.360	0.346	0.340
		(0.188)	(0.199)	(0.208)	(0.214)
σ_v^2	0.600	0.683	0.695	0.675	0.675
		(0.143)	(0.141)	(0.098)	(0.093)
σ_η^2	0.300	0.283	0.283	0.280	0.275
		(0.068)	(0.054	(0.044)	(0.042)
σ_u^2	0.400	0.531	0.530	0.517	0.521
		(0.157)	(0.150)	(0.127)	(0.130)
	$\widehat{\text{Sel. Crit.}}$	139.5	169.5	288.0	480.5
	$\widehat{\text{obj.}}$	111.9	135.0	224.3	374.2

Note: The simulation size is set to 100. $(\cdot)$ indicates root mean square error for $\hat{\theta}$.

Table A2: Monte Carlo experiments: ABC with criterion from Bootstrap I

	θ_0	$\hat{\theta}$			
		T=100	T=200	T=500	T=1,000
a_1	0.500	0.344	0.288	0.261	0.271
		(0.181)	(0.218)	(0.242)	(0.235)
a_2	0.090	0.346	0.347	0.319	0.296
		(0.266)	(0.262)	(0.234)	(0.211)
b_1	0.580	0.648	0.645	0.613	0.588
		(0.110)	(0.102)	(0.072)	(0.037)
b_2	0.050	0.102	0.202	0.080	0.066
		(0.057)	(0.056)	(0.034)	(0.018)
c_1	1.650	1.640	1.637	1.649	1.661
		(0.129)	(0.093)	(0.090)	(0.081)
c_2	0.375	0.636	0.509	0.405	0.372
		(0.291)	(0.159)	(0.064)	(0.047)
c_3	0.550	0.383	0.351	0.359	0.386
		(0.179)	(0.205)	(0.199)	(0.173)
σ^2_ν	0.600	0.724	0.728	0.657	0.628
		(0.162)	(0.156)	(0.083)	(0.056)
σ^2_η	0.300	0.302	0.288	0.275	0.275
		(0.045)	(0.046)	(0.050)	(0.046)
σ^2_u	0.400	0.572	0.556	0.488	0.445
		(0.194)	(0.172)	(0.110)	(0.073)
	$\widehat{\text{Sel. Crit.}}$	227.6	214.0	231.9	249.3
	$\widehat{\text{Obj.}}$	169.5	164.2	180.6	196.9

Note: See Table A1.

Table A3: Monte Carlo experiments: ABC with criterion from Bootstrap II

	θ_0	$\hat{\theta}$			
		T=100	T=200	T=500	T=1,000
a_1	0.500	0.295	0.333	0.393	0.447
		(0.286)	(0.237)	(0.162)	(0.090)
a_2	0.090	0.285	0.238	0.174	0.133
		(0.260)	(0.195)	(0.124)	(0.068)
b_1	0.580	0.568	0.575	0.576	0.576
		(0.101)	(0.051)	(0.032)	(0.024)
b_2	0.050	0.072	0.060	0.054	0.054
		(0.040)	(0.021)	(0.012)	(0.008)
c_1	1.650	1.666	1.667	1.650	1.657
		(0.294)	(0.190)	(0.132)	(0.104)
c_2	0.375	0.463	0.392	0.397	0.381
		(0.271)	(0.142)	(0.125)	(0.098)
c_3	0.550	0.526	0.502	0.514	0.527
		(0.118)	(0.094)	(0.066)	(0.045)
σ_v^2	0.600	0.603	0.609	0.587	0.587
		(0.120)	(0.086)	(0.054)	(0.040)
σ_η^2	0.300	0.258	0.272	0.285	0.292
		(0.103)	(0.063)	(0.037)	(0.027)
σ_u^2	0.400	0.314	0.330	0.341	0.329
		(0.162)	(0.117)	(0.094)	(0.093)
	$\widehat{\text{Sel. Crit.}}$	31.6	37.8	43.7	49.8
	$\widehat{\text{Obj.}}$	32.8	34.3	37.3	41.9

Note: See Table A1.