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NON-PERTURBATIVE STUDY OF QUANTUM FIELD THEORIES WITH SCALAR FIELDS

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Non-perturbative study of quantum field theories with scalar fields*

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Abstract

Some recent non-perturbative results in quantum field theories with scalar fields are reviewed.

1 Looking for critical points

Recently there is an increasing interest in non-perturbative studies of the standard electroweak theory. The main motivation is to achieve a better understanding of the Higgs sector and of the Higgs mechanism for mass generation.

The simplest prototype model of the Higgs sector is the "standard Higgs model" which is an SU(2) gauge field theory interacting with a scalar SU(2) doublet. The continuum Lagrangean depends on the gauge field-strength matrix $F(x)_{\mu\nu}$ and the complex scalar doublet field $\phi(x)$:

$$\mathcal{L} = \frac{1}{2} Tr \left(F(x)_{\mu\nu} F(x)_{\mu\nu} \right) + (D_\mu \phi(x)^\dagger)(D_\mu \phi(x)) + \lambda \left(\phi(x)^\dagger \phi(x) - \frac{v^2}{2} \right)^2 \tag{1}$$

The lattice action is also becoming standard by now. Using the $2\otimes 2$ matrix field $\varphi \equiv (\tilde{\phi}\;\phi)$ we have:

$$S = \beta \sum_P \left(1 - \frac{1}{2} Tr\, U_P \right) + \sum_x \left[\varphi_x^+ \varphi_x + \lambda (\varphi_x^+ \varphi_x - 1)^2 \right] - \kappa \sum_{(x\mu)} Tr \left(\varphi_{x+\hat{\mu}}^+ U(x,\mu) \varphi_x \right) \tag{2}$$

The first piece is the Wilson lattice action for the gauge field (a sum over plaquettes P). The gauge variable is $U(x,\mu) \in SU(2)$ defined on the links $(x, x+\hat{\mu})$. The bare parameters in the lattice action are: $\beta \equiv 4/g^2$ for the gauge coupling, λ for the scalar quartic self-coupling and the hopping parameter κ which is the bare mass parameter for the scalar field.

The first question one has to investigate in a non-perturbative framework is the phase diagram in the space of bare parameters (Fig. 1). The phase transition surface between the confinement-like and Higgs-like regions was localized in extensive numerical Monte Carlo

*Lecture given at the EPS Conference on High Energy Physics, Uppsala, June 1987 calculations (for a partial list of references see [1]). The phase transition is most probably of first order everywhere on the surface, except for the boundary at $\beta = \infty$ and at the opposite edge at small β. The first order signal is, however, weakening for large β and/or λ, therefore a change to second order along some "tricritical line" [2] cannot be excluded from the numerical data alone. One has to keep in mind, however, that a second order phase transition line in some $\lambda = const.$ plane almost inevitably implies the existence of a non-trivial continuum limit. (See the discussion in the next Section, in particular the CPCP's with constant renormalized gauge coupling g_r in Fig. 3.) For small λ and large β the first order nature of the phase transition is also supported by perturbation theory [3].

The confinement-Higgs phase transition was also investigated as a function of the temperature [4]. The "symmetry restoration" transition [3] from the low temperature Higgs phase to the high temperature phase without spontaneous symmetry breaking is manifested on the lattice by a small upwards shift of the phase transition surface on asymmetric ($L^3 \cdot T^{-1}$) lattices. The small shift was, indeed, observed but the calculations are not yet good enough to give a quantitative value of the phase transition temperature. For the study of the phase transition certain "order parameters" are useful, which are able to distinguish between confinement and Higgs mechanism in a theory with matter fields [5].

It follows from the investigation of the phase transition surface that the up to now well established critical points, where a continuum limit can in principle be defined, are either on the critical line at $\beta = \infty$ (see line CR in Fig. 1) or on the opposite edge of the phase transition surface at small β. Critical points, however, quite often define only a trivial theory, therefore the search for the fixed points of the renormalization group (RG) transformations has a great importance. By determining the critical exponents it is also possible to obtain information about the properties of the continuum limit at a particular fixed point. A systematic study of RG fixed points was carried out recently in several models with scalar fields (see e. g. [6]). In the standard SU(2) Higgs model the results are consistent with trivial continuum limits everywhere, but the analysis is not yet conclusive with respect to the fixed point at small β. This question of a non-trivial fixed point in the interiour of the bare parameter space would be particularly interesting in the SU(2)$\otimes$U(1) Higgs model, because of the possible non-trivial interplay between the two non-asymptotically free couplings [7]. The numerical calculations in the SU(2)$\otimes$U(1) Higgs model are not much more complicated than in the SU(2) case [8]. There is, however, one more parameter and a third phase with free (non-confined) charges.

Simple models with scalar fields are also well suited for trying out new methods in the numerical Monte Carlo calculations. A recent example is the study of finite volume effects in the 4-dimensional Ising model, which is a limiting case (with infinite bare quartic coupling) of the 1-component ϕ^4 model [9]. Due to the simplicity of the model a good statistics ($10^6 - 10^7$ sweeps per point) can be collected with moderate computing resources. This made possible to check the theory of finite volume effects developed by Lüscher [10]. From the difference of the lowest 2-particle energy level minus twice the 1-particle mass it was possible to determine the scattering length (see Fig. 2). The application of this method to the calculation of the W-W scattering lenght in SU(2) Higgs models is straightforward.

2 Is the continuum limit of the standard model trivial?

Since the continuum limit of the pure ϕ^4 model is known to be trivial (see [11] and references therein), the question naturally arises whether the introduction of the SU(2) gauge coupling can prevent triviality? The answer is most probably no [12]! According to the weak gauge coupling expansion [13] the "curves of constant physics" with non-vanishing interaction have an endpoint at $\lambda = \infty$ for a finite maximum cut-off [14]. In other words, in the cut-off dependent "effective" theory there is an upper limit for the renormalized quartic coupling (λ_r) which goes to zero for infinite cut-off. Since the Higgs- to W-mass ratio m_H/m_W is a monotonous function of λ_r, there is also a cut-off dependent upper limit for m_H/m_W. The results of an approximate RG transformation scheme [15] and direct numerical studies in the standard SU(2) Higgs model [16,17] show that if the Higgs-mass in lattice unit is of order 1, the upper limit is

$$\frac{m_H}{m_W} \leq 9 - 10 \tag{3}$$

The main difficulty in the numerical Monte Carlo calculations is to avoid the effects of the finite lattice size, due to the large mass ratio. However, on the basis of the finite volume study in the 4-dimensional Ising model [9] it seems plausible that the estimate in Eq. (3) is reasonably safe.

Why is the continuum limit of the standard Higgs model trivial? One could imagine, as it was speculated in Ref. [18,19], that a λ-independent continuum limit exists at the $\beta = \infty$ critical line. In this case the quartic coupling would be a function of the gauge coupling (the bare quartic coupling would be "irrelevant" in the sens of renormalization group). From the dynamical point of view, the non-zero quartic self-coupling would be a consequence of the residual gauge interaction between the SU(2)-colour singlet physical states. In order to see why this does not happen let us consider (in the Higgs phase) the behaviour of the renormalized couplings λ_r and g_r^2 near the phase transition surface for fixed bare couplings λ, g^2. As a parameter ("reference quantity") let us take $\tau \equiv \log \mu_W^{-1}$ ($\mu_W \equiv am_W$ is the W-mass in lattice units). The functions $\lambda_r(\tau)$, $g_r^2(\tau)$ obey the differential equations

$$\begin{aligned} \frac{d\lambda_r(\tau)}{d\tau} &= -\beta_\lambda(\lambda_r, g_r^2, \tau) \to -\frac{1}{16\pi^2}\left(96\lambda_r^2 - 9\lambda_r g_r^2 + \frac{9}{32}g_r^4\right) \\ \frac{dg_r^2(\tau)}{d\tau} &= -\beta_{g^2}(\lambda_r, g_r^2, \tau) \to \frac{1}{16\pi^2}\frac{43}{3}g_r^4 \end{aligned} \tag{4}$$

Here the limiting universal form of the Callan-Symanzik β-functions for small renormalized couplings is also given. The consequence of Eq. (4) is that λ_r decreases and becomes negative for large τ. At some point the Higgs-phase becomes unstable and a first order phase transition to the confining phase occurs. In order to see why this implies the triviality of the continuum limit let us consider the "curves of partially constant physics" (CPCP's) [14] defined by fixing the value of the renormalized gauge coupling squared g_r^2 in a plane with fixed bare quartic coupling λ. The solution of Eq. (4), with reasonable initial values at $\tau = 0$, implies that the CPCP's, instead of going to the critical point at $\beta = \infty$, have an endpoint on the phase transition surface for some finite cut-off (Fig. 3). The decrease of λ_r resulting in the first order phase transition could only be stopped if there was an infrared fixed point (IRFP) in the renormalization group equations. Let us also note that the triviality of the pure ϕ^4 model has a different origin: there is an IRFP, but it is at vanishing interaction $\lambda_r = \lambda_{r*} = 0$. This implies the triviality of ϕ^4 in the limit of going to the critical line for fixed bare λ [11].

The situation is different if there is a non-trivial IRFP (for non-vanishing values of the renormalized couplings). Such a non-trivial IRFP is known to occur in the standard model in the case of strong Yukawa couplings (for a partial list of references see [20]). One possibility is that there are only three fermion generations with a very heavy top quark: $m_t \simeq 240\ GeV$. However, such a heavy top quark seems to be in conflict with the 1-loop radiatively corrected relation between low energy parameters and the measured vector boson masses (see e. g. [21]). Therefore let us consider the other possibility, namely a fourth generation heavy quark doublet. The 1-loop RG equations for the Yukawa coupling of the heavy quarks G and the scalar quartic coupling λ are [20]:

$$16\pi^2 \frac{dG}{dt} = 12G^3 - 8g_3^2 G - \frac{9}{4} g_2^2 G$$

$$16\pi^2 \frac{d\lambda}{dt} = 96\lambda^2 - 9g_2^2\lambda + \frac{9}{32} g_2^4 + 48G^2\lambda - 12G^4 \tag{5}$$

Here g_3^2 and g_2^2 are the gauge coupling squared for SU(3) and SU(2), respectively. Small contributions, like small Yukawa couplings, the U(1)-coupling etc., are neglected here for simplicity. The parameters of the IRFP are:

$$G_* = \frac{2}{3} g_3^2 + \frac{3}{16} g_2^2$$

$$\lambda_* = -\frac{1}{4} G_*^2 + \frac{3}{64} g_2^2 + \sqrt{\left(\frac{1}{4} G_*^2 - \frac{3}{64} g_2^2\right)^2 + \frac{1}{8} G_*^4 - \frac{3}{1024} g_2^4} \tag{6}$$

The corresponding heavy quark- and Higgs-mass is:

$$m_{Q*} = m_W \frac{2G_*}{g_2} \simeq 230 GeV, \qquad m_{H*} = m_W \frac{\sqrt{32\lambda_*}}{g_2} \simeq 300 GeV \tag{7}$$

Here the mean values of the gauge couplings $g_{2,3}^2$ in the 100 GeV range were taken: $g_2^2 \simeq 0.44$, $g_3^2 \simeq 1.25$. The remarkable property of the IRFP is, that the values of the Yukawa and quartic couplings are not free parameters any more, but are given by the gauge couplings.

The existence of a non-trivial IRFP in the RG equations is a necessary condition for the existence of the non-trivial continuum limit of the standard model. The question whether it is also sufficient has to be decided in future detailed non-perturbative studies. In particular, the problem of the chiral SU(2)⊗U(1) gauge couplings ("chiral fermions") on the lattice has to be clearified. At present one can only give general arguments. In general, one has to require that there must exist at least one critical point at large bare quartic and Yukawa couplings (in the region of attraction of the IRFP), where both the heavy quark and Higgs boson masses tend to zero in lattice units: am_Q, $am_H \to 0$. This seems plausible, because the chiral symmetry for $m_Q = 0$ is expected to induce a second order critical line (like in QCD) and in a strongly interacting situation the Higgs boson state can be dynamically produced as a fermion-antifermion bound state. Also the existence of the "skyrmion-like" solutions in the pure bosonic system [22] suggests that in a quasi-continuum situation the boson and fermion degrees of freedom live always together. The simultaneous vanishing of am_Q and am_H in the fermion-scalar theory without gauge couplings is also supported by the IRFP in the ratio of the quartic to Yukawa coupling squared (see the discussion in the next Section). In any case, the existence of the non-trivial IRFP implies the possibility of strong interaction for very high cut-off's. Therefore, even if the continuum limit would turn out trivial, the non-trivial IRFP would be important from the pragmatic point of view. Quite generally, the IRFP structure of a theory is of decisive importance for its large cut-off behaviour. The rôle of a non-trivial IRFP can, of course, be studied in simple prototype models like the one discussed at the end of the next Section.

The IRFP values of the Yukawa and quartic couplings are relatively large, therefore the reliability of perturbation theory has to be questionned. If one can trust the above estimates, the IRFP corresponds to a semi-strongly interacting situation: the quark mass is roughly equal to half of its unitarity bound value, the Higgs mass to a third of it [23,24]. It cannot, however, be rouled out that the IRFP for the fourth fermion generation is in reality at still stronger couplings. In this case strong non-perturbative effects could play a rôle implying exotic dynamical phenomena like difermions, colour non-singlet fermion-antifermion composites etc. (see, for instance, [25]). In any case, a detailed non-perturbative study of models with strong Yukawa and scalar quartic couplings seems to be important.

3 The σ-model on the lattice

A popular prototype model with Yukawa coupling is the σ-model [26]. Its non-perturbative study is important because

- it describes a degenerate heavy fermion doublet in the electro-weak model (neglecting SU(2)$\otimes$U(1) gauge couplings);
- it is a model for QCD at low energy, therefore one can investigate in the σ-model, for instance, the critical behaviour at vanishing quark mass, the thermodynamics in the hadronic phase, skyrmions on the lattice etc.;
- the σ-model can also serve as a simple testing ground for numerical simulation methods of fermions.

The lattice action of the σ-model in O(4) notations ($R = 0,1,2,3$) with Wilson fermions ($r > 0$) is:

$$S = \sum_x \Big\{ \phi_{Rx}\phi_{Rx} + \lambda(\phi_{Rx}\phi_{Rx} - 1)^2 - \kappa \sum_\mu \phi_{Rx+\hat\mu}\phi_{Rx}$$

$$+F(\bar\psi_x\psi_x) - \phi_{Rx}(\bar\psi_x\Gamma_R\psi_x) - K\sum_\mu(\bar\psi_{x+\hat\mu}[r+\gamma_\mu]\psi_x)\Big\} \quad (8)$$

Here the first line is the O(4)-symmetric ϕ^4 action, with automatic summation over R. F is the bare inverse Yukawa coupling and K is the hopping parameter for the fermion doublet. The 8$\otimes$8 matrices Γ_R, ($R = 0,\ldots,3$) are defined as $\Gamma_R = (1, -i\gamma_5\tau_r)$ with the isospin Pauli matrices $\tau_{1,2,3}$. For the qualitative understanding of strong quartic couplings it is enough to consider the limit $\lambda \to \infty$, because the behaviour at $\lambda = o(1)$ is very similar [19]. In the same way, for strong Yukawa couplings one can consider $F = 0$ (infinite bare Yukawa coupling). In this model with "maximal couplings" the scalar field has unit length: $\phi_{Rx}\phi_{Rx} = 1$, and the lattice action is

$$S = -\sum_x \left\{ \phi_{Rx}(\bar{\psi}_x \Gamma_R \psi_x) + \kappa \sum_\mu \phi_{Rx+\hat{\mu}}\phi_{Rx} + K \sum_\mu (\bar{\psi}_{x+\hat{\mu}}[r+\gamma_\mu]\psi_x) \right\} \tag{9}$$

The elementary scalar field is removed from this model by taking the limit $\kappa = 0$, which corresponds to an infinitely large bare scalar mass. In this case a pure fermionic formulation is possible:

$$S^\psi_{eff} = -K \sum_{x,\mu} (\bar{\psi}_{x+\hat{\mu}}[r+\gamma_\mu]\psi_x) - \sum_x \sum_{n=1}^{\infty} \frac{C_{2n}}{(2n)!} \left[\frac{1}{4} (\bar{\psi}_x \Gamma_R \psi_x)(\bar{\psi}_x \Gamma_R \psi_x) \right]^n \tag{10}$$

The integer constants C_{2n} are given by an O(4) integral. The first few of them are: $C_2 = 1,\ C_4 = -1,\ C_6 = 5,\ C_8 = -56, \ldots$. In this fermionic formulation the scalar state is described by the fermion bilinear $(\bar{\psi}_x \Gamma_R \psi_x)$. A bosonic formulation equivalent to Eq. (10) is:

$$S^\phi_{eff} = Tr \sum_{n=1}^{\infty} \frac{(-K)^n}{2n} [M_1(\phi) + K M_2]^n \tag{11}$$

with $\bar{\phi}_R \equiv (\phi_0, -\phi_r)$ and

$$M_1(\phi)_{xy} = \sum_\mu \delta_{x+\hat{\mu},y} \Gamma_R [(\phi_{Rx} + \bar{\phi}_{Ry}) r + (\phi_{Rx} + \phi_{Ry})\gamma_\mu]$$

$$M_{2,xy} = \sum_{\mu\nu} \delta_{x+\hat{\mu}+\hat{\nu},y} (r+\gamma_\mu)(r+\gamma_\nu) \tag{12}$$

A powerful non-perturbative method for the solution of the lattice σ-model is the hopping parameter expansion in κ and/or K. A double expansion at $\kappa = K = 0$ can be performed by a variant of the linked cluster expansion [27]. It is also possible to use the numerical hopping parameter expansion method developed in QCD [28], also for $\kappa \neq 0$. As a warming up, one can consider a simplified model with 1-component scalar field ϕ_x and either scalar $\phi_x(\bar{\psi}_x\psi_x)$ or pseudoscalar $\phi_x(\bar{\psi}_x\gamma_5\psi_x)$ Yukawa coupling [29].

For studying the consequences of a non-trivial IRFP, a simple prototype model is QCD with a very heavy ($\sim 100 GeV$) degenerate quark doublet interacting with an elementary complex scalar doublet. The lattice action for this model can be obtained from Eq. (8) by adding the SU(3)-colour gauge action and changing the fermion hopping piece:

$$\ldots - K \sum_\mu \left(\bar{\psi}_{x+\hat{\mu}}[r+\gamma_\mu] U_c(x,\mu) \psi_x \right) + \ldots \tag{13}$$

Here $U_c(x,\mu)$ is the SU(3) colour link variable. The numerical Monte Carlo calculations in this model are not much more difficult than in QCD. The scalar field and the couplings to it are, in fact, a minor complication.

The pure fermion-scalar model (without gauge couplings) has an IRFP at vanishing couplings. This can be seen in the RG equations for a quark doublet interacting with a scalar doublet (see Eq. (5)):

$$16\pi^2 \frac{dG}{dt} = 12G^3; \qquad 16\pi^2 \frac{d\lambda}{dt} = 96\lambda^2 + 48G^2\lambda - 12G^4 \tag{14}$$

These equations determine the cut-off dependence of the renormalized couplings in the vicinity of a critical point with $G \to G_r$; $\lambda \to \lambda_r$ and $t \to -\tau$. Since the IRFP is at $\lambda_{r*} = G_{r*} = 0$, the continuum limit is most probably trivial. For the ratio of couplings $y(\tau) \equiv \lambda_r/G_r^2$ we have at large τ:

$$16\pi^2 \frac{dy}{d\tau} = -12G_r^2(8y^2 + 2y - 1) \tag{15}$$

This equation has an IRFP at $y_* = \frac{1}{4}$, therefore at large cut-off's one always has $\lambda_r \simeq \frac{1}{4}G_r^2$. In a spontaneously broken case, like in the standard model, this implies $m_H/m_Q^2 \simeq 2$ (see Eq. (7)).

Let us note in this respect that an IRFP for the ratios of couplings has some relation to the "coupling parameter reduction" considered by Zimmermann [30]. It is interesting to consider the IRFP for coupling ratios in supersymmetric models, like e. q. the Wess-Zumino model of a Majorana fermion and a scalar-pseudoscalar pair [31]. As it can be easily seen from the 1-loop RG equations, the SUSY relation $\lambda_r = G_r^2$ corresponds to an IRFP of the ratio $y_* = 1$. This is another aspect of the importance of the IRFP for the large cut-off behaviour: even if the bare couplings are not supersymmetric, for large cut-off's the renormalized couplings are automatically driven to the IRFP corresponding to SUSY. The requirement of the existence of an IRFP for the coupling ratios is, however, more general than supersymmetry: for instance, the model considered in Eq. (15) is not supersymmetric.

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Figure captions

Fig. 1. The phase diagram of the standard SU(2) Higgs model. Below the phase transition surface there is the confining phase (C), above it the Higgs phase (H). The point G at ($\beta = \infty, \kappa = \frac{1}{8}, \lambda = 0$) is the Gaussian fixed point where perturbation theory is defined. The line CR is the critical line of pure ϕ^4 at $\beta = \infty$.

Fig. 2. The difference of the 2-particle mass minus twice the 1-particle mass as a function of the spatial extension L of the $L^3 \cdot 24$ lattice at $\kappa = 0.074$ in the 4-dimensional Ising model. The line is Lüscher's formula [10] for a scattering length $a_0 = -0.96$.

Fig. 3. The "curves of partially constant physics" (CPCP's) with the renormalized gauge coupling g_r fixed in a plane with constant bare quartic coupling λ. The line PT is the location of the phase transition between confining- and Higgs-phase. The other boundary of the region where CPCP's are drawn is the line (UM) with W-mass equal to unity: $am_W = 1$.

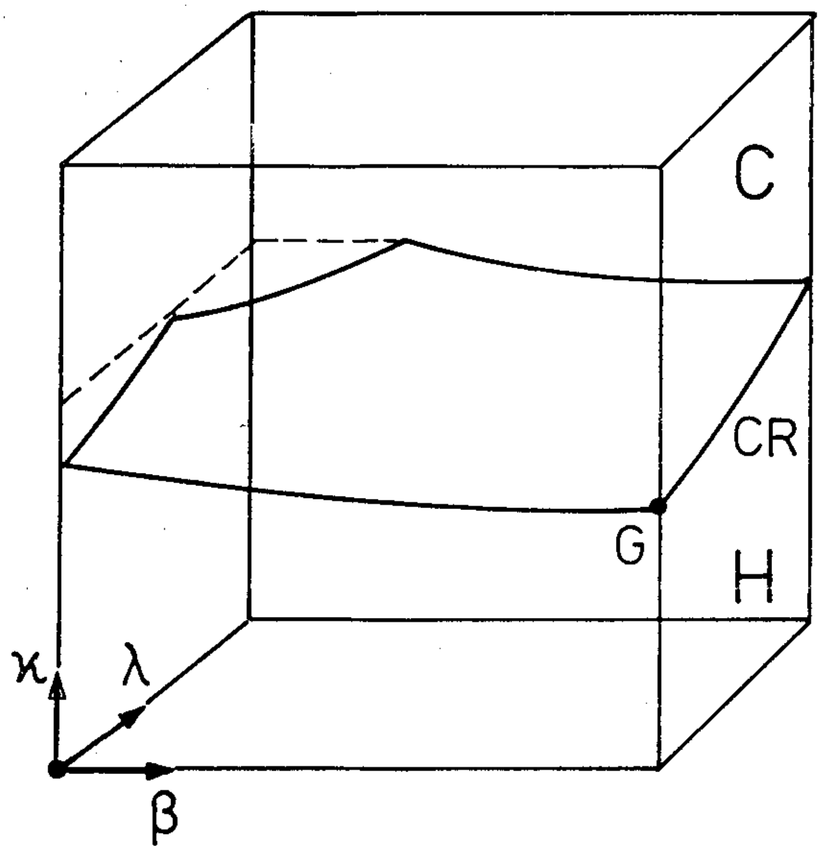

Fig.1

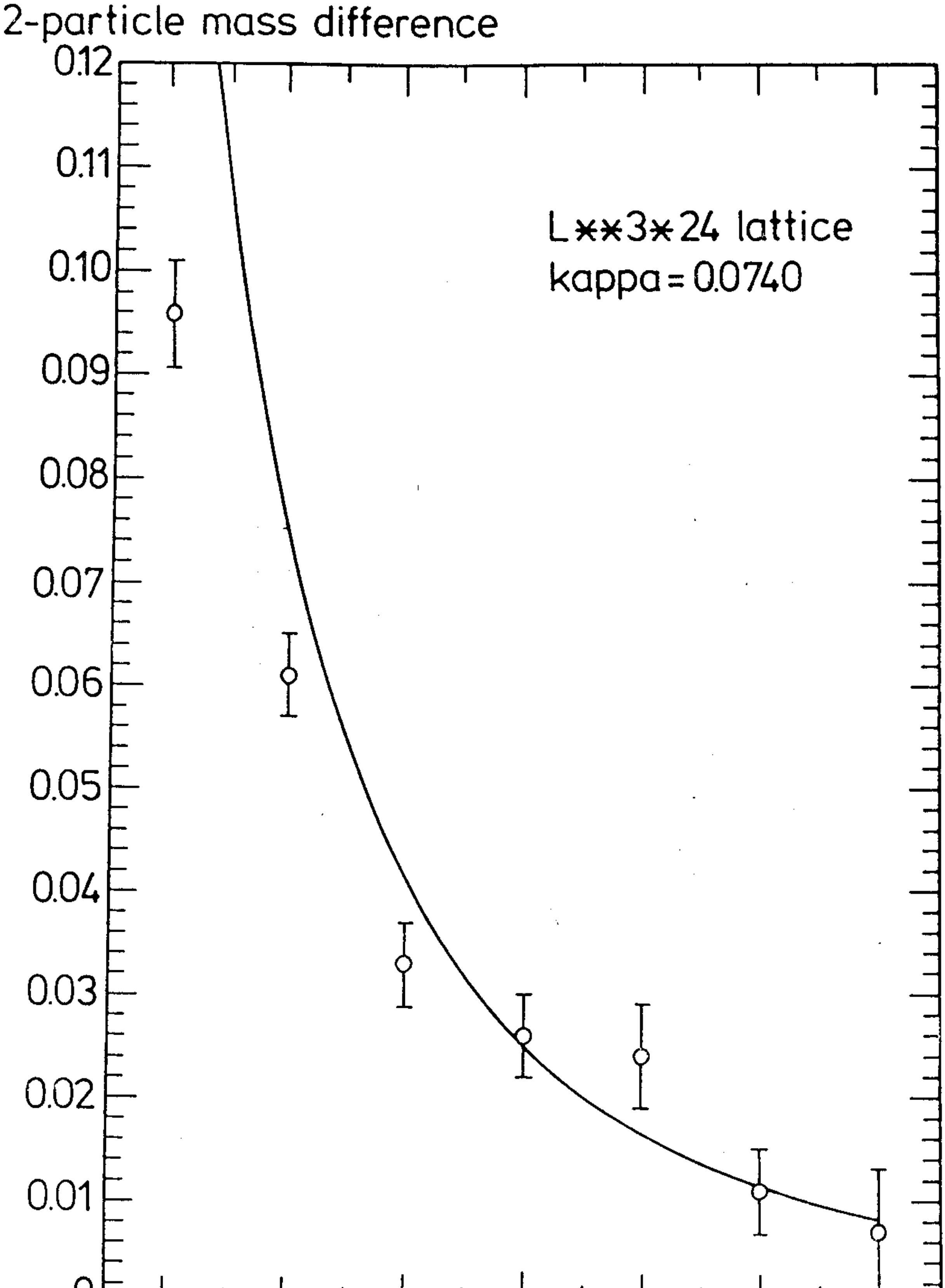
2-particle mass difference
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Fig. 2

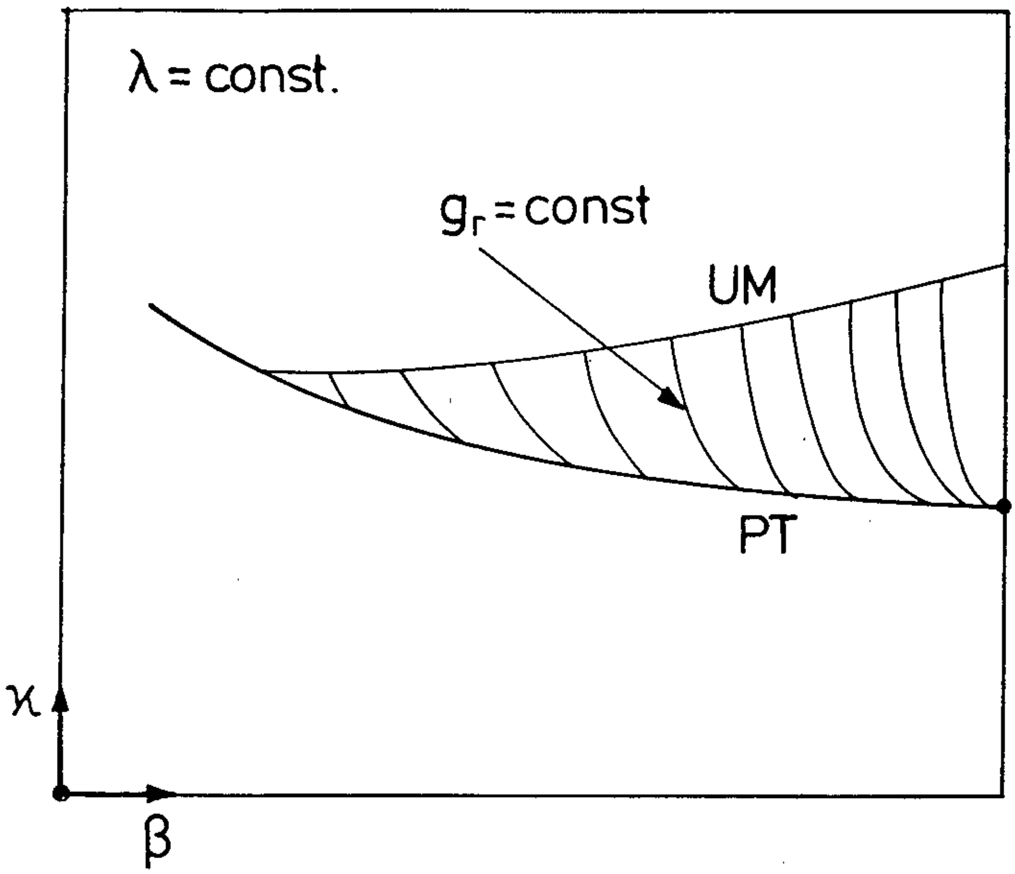

Fig. 3