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ABSTRACT

The decays $\pi^0 \to \gamma\gamma$, $\rho \to \pi\gamma$, $\omega \to \pi\gamma$, $\omega \to 3\pi$ and $\gamma \to 3\pi$ are studied in the framework of the chiral invariant effective Vector Meson Lagrangian beyond the tree level. The standard Lagrangian is enlarged by including an infinite number of radial excitations which are summed according to the dual model. As a result tree level diagrams are modified by a universal form factor at each vertex containing off-mass-shell mesons, but still respecting chiral anomaly low energy theorems. These vertex corrections bring the tree level predictions into better agreement with experiment. The presence of the $\omega \to 3\pi$ contact term is confirmed but its strength is considerably smaller than at tree level.

There has been a recent revival of the effective Lagrangian approach [1] to soft hadronic physics, motivated in part by the absence of analytical solutions to QCD at low energies. In particular, efforts have been made [2] to construct effective Lagrangians which exhibit the global symmetries of QCD [3], respect chiral anomaly theorems [4], and incorporate successful phenomenological models such as e.g. Vector Meson Dominance (VMD) [5]. An example of such a Lagrangian, to lowest order in the number of fields and derivatives, is given by

$$\begin{aligned}\mathcal{L}_{eff} = {} & g_{\rho\pi\pi}\,\epsilon_{ijk}\,\rho^i_\mu\,\pi^j\,\partial^\mu\pi^k + g_{\omega\rho\pi}\,\delta_{ij}\,\epsilon^{\mu\nu\lambda\sigma}\,\partial_\mu\omega_\nu\,\partial_\lambda\rho^i_\sigma\,\pi^j \\ & + G_\omega\,\epsilon_{ijk}\,\epsilon^{\mu\nu\lambda\sigma}\,\omega_\mu\,\partial_\nu\pi^i\,\partial_\lambda\pi^j\,\partial_\sigma\pi^k \\ & + e\,\frac{M_\rho^2}{f_\rho}\,\rho^{(3)}_\mu A^\mu + e\,\frac{M_\omega^2}{f_\omega}\,\omega_\mu A^\mu + e\,\frac{M_\varphi^2}{f_\varphi}\,\varphi_\mu A^\mu \\ & + \text{radial excitations} + \ldots, \end{aligned} \qquad (1)$$

where additional fields of no interest here have been omitted. The coupling constant G_ω measures the strength of the so called $\omega \to 3\pi$ contact term [6] whose presence may be needed to ensure consistency between this theory and the chiral anomaly low energy theorems controlling the amplitudes for processes such as $\pi^0 \to \gamma\gamma$ and $\gamma \to \pi^+\pi^-\pi^0$. In fact, a confrontation between the chiral anomaly prediction for the ratio of these amplitudes, i.e.

$$\left.\frac{F^{\gamma\to 3\pi}(0,0,\ldots)}{F^{\pi^0\to\gamma\gamma}(0,0,\ldots)}\right|_{C.A.} = \frac{1}{e f_\pi^2} \qquad (2)$$

and that obtained through VMD

$$\left.\frac{F^{\gamma\to 3\pi}(0,0,\ldots)}{F^{\pi^0\to\gamma\gamma}(0,0,\ldots)}\right|_{VMD} = \frac{3}{2}\,\frac{1}{e f_\pi^2}\,, \qquad (3)$$

seems to suggest a quite sizeable contact term [2.c]

$$G_\omega = -\frac{g_{\rho\pi\pi}}{16\pi^2 f_\pi^3} \simeq -46\ \mathrm{GeV}^{-3}. \qquad (4)$$

This result was obtained in VMD after assuming universality [5]: $g_{\rho\pi\pi} = f_\rho$, and the KSFR relation [7]: $g_{\rho\pi\pi} = M_\rho/\sqrt{2}\, f_\pi$. In the framework of chiral invariant VMD at the tree level it is possible to derive a plethora of relations among the various coupling constants, such as e.g. Eq. (4). As elegant as they may look, the ultimate test of these relations has to come from a direct confrontation with experimental data. In doing so, one should not ignore the fact that off-mass-shell mesons couple strongly to their radial excitations and thus induce propagator and/or vertex corrections. Indications that these effects may indeed be important come from various sources, e.g.

$$\left.\frac{g_{\rho\pi\pi}}{f_\rho}\right|_{EXP.} = 1.22\,, \qquad (5)$$

a deviation from the KSFR relation at the 10 % level, the inability of VMD to account properly for the behaviour of the pion form factor [8], some problems of consistency between radiative meson decays and $\omega \to 3\pi$ in VMD [9], etc. With some exceptions (see e.g. [2.d,e,i,j], [10]) these effects have been traditionally neglected in applications of the chiral effective Lagrangian formalism. The standard disclaimer is that predictions are only to be trusted at the 10-20 % level, i.e. at roughly the same level of accuracy as the tree level relations among coupling constants. That a 10 % effect here and another one there may conspire to produce in the end a 100 % correction may be seen by recomputing G_ω as in [2.c] but using instead experimental values for the couplings at each step. The result is now $G_\omega = -(29 \pm 3)$ GeV^{-3}! Other values to be found in the literature include $G_\omega = 0$ [11], which shows the need for a systematic reanalysis of this important issue.

In this note I present an attempt in this direction by studying the related processes: $\pi^0 \to \gamma\gamma$, $\rho \to \pi\gamma$, $\omega \to \pi\gamma$, $\omega \to 3\pi$, and $\gamma \to 3\pi$ in the framework of the effective Lagrangian Eq. (1) beyond the tree level. All of these decays have in common the coupling constant $g_{\omega\rho\pi}$ which may be fixed as usual by requiring consistency with the chiral anomaly theorem in $\pi^0 \to \gamma\gamma$; predictions for the rates of the first three decays above then follow. Requiring consistency with the chiral anomaly theorem in $\gamma \to 3\pi$ fixes in turn the strength of the $\omega \to 3\pi$ contact term G_ω; one may then proceed to predict the $\omega \to 3\pi$ rate as well as the on-mass-shell amplitude for $\gamma \to 3\pi$. The latter can be compared with a recent determination from pion pair production in pion-nucleus collisions [12].

Including explicitly in Eq. (1) an infinite number of vector meson radial excitations with masses and point couplings fixed by the factorizable dual model [8]-[9], it turns out that all tree level diagrams are effectively modified by a form factor at each vertex containing an off-mass-shell meson. To illustrate this with an example let us concentrate on the pion form factor in the zero-width aproximation, i.e.

$$F_\pi(q^2) = \sum_{n=0}^{\infty} \frac{M_{\rho_n}^2}{M_{\rho_n}^2 - q^2} \cdot \frac{g_{\rho_n \pi\pi}}{f_{\rho_n}} . \qquad (6)$$

In the dual model $M_{\rho_n}^2 = M_\rho^2(1+2n)$ and the ratio of coupling constants in Eq. (6) is determined by requiring the form factor to be a ratio of gamma functions. In this way Eq. (6) becomes [8]

$$F_\pi(q^2) = \frac{M_\rho^2}{f_\rho} \frac{g_{\rho\pi\pi}}{M_\rho^2 - q^2} F_\rho(q^2) \qquad (7)$$

where

$$F_\rho(q^2) = \Gamma(\beta - 1) \frac{\Gamma[1 - \alpha'(q^2 - M_\rho^2)]}{\Gamma[\beta - 1 - \alpha'(q^2 - M_\rho^2)]} ,$$

$$\alpha' \equiv 1/2 M_\rho^2 . \qquad (8)$$

The generalization to other fields, e.g. the pionic sector [10] or the baryonic sector [13], is straightforward but it will not be needed here. On the other hand, when more than one particle at a vertex is off the mass shell one simply uses the factorization property of the dual model. In this fashion one is able to dress all tree level vertices appearing in the decays mentioned before with the single form factor $F_\rho(q^2) \simeq F_\omega(q^2) \equiv F_V(q^2)$, where ω-ρ degeneracy is implied. The free parameter β in Eq. (8) has been determined earlier [8] from a chi-squared fit to the pion form factor in the space-like region up to $q^2 \simeq -10$ GeV^2. The result is

$$\beta = 2.3 - 2.4 ; \qquad (9)$$

notice that $\beta = 2$ would correspond to naive (tree level) VMD. In comparison with a few other models, Eq. (7) gives one of the best values of chi-squared. In particular, naive VMD fails to account for the fall-off of the data above $-q^2 \simeq 1$ GeV^2 and misses the ratio (5) by 20 %, although it yields $\langle r_\pi^2 \rangle = 0.39$ fm^2, to be compared with $\langle r_\pi^2 \rangle|_{EXP.} = 0.44 \pm 0.03$ fm^2 [14]. In contrast, Eq. (7) gives $g_{\rho\pi\pi}/f_\rho = 1.20 - 1.22$ and $\langle r_\pi^2 \rangle = 0.44$ fm^2. With β fixed as above the remaining parameters in the effective Lagrangian (1), except for $g_{\omega\rho\pi}$ and G_ω, are known from experiment [15]

$$\begin{aligned} M_\rho &= 770 \text{ MeV}, \qquad M_\omega = 782.6 \text{ MeV} \\ f_\rho &= 5.0 \pm 0.1, \quad f_\omega = 16.3 \pm 0.5, \quad g_{\rho\pi\pi} = 6.09 \pm 0.04. \end{aligned} \tag{10}$$

Deviation of the ω-φ mixing angle from its "ideal" value will be neglected in the sequel; its impact on the present analysis is at the level of the errors to be quoted in the predictions.

Beginning with $\pi^0 \to \gamma\gamma$ and requiring consistency between the chiral anomaly low energy theorem [4]

$$F^{\pi^0\to\gamma\gamma}(0,0,\dots)\Big|_{C.A.} = -\frac{\alpha}{\pi f_\pi} \tag{11}$$

and the expression from the effective Lagrangian

$$F^{\pi^0\to\gamma\gamma}(0,0,\dots) = \frac{8\pi\alpha}{f_\rho f_\omega}\, g_{\omega\rho\pi} |F_V(0)|^2 \tag{12}$$

with $F_V(q^2)$ given by Eq. (8), one obtains

$$g_{\omega\rho\pi} = -(16 \pm 1) \text{ GeV}^{-1}. \tag{13}$$

Requiring the same consistency but with naive VMD ($\beta = 2$) would give instead

$$g_{\omega\rho\pi}\Big|_{VMD} = -(11.1 \pm 0.4) \text{ GeV}^{-1}. \tag{14}$$

The resulting decay rate is given in Table 1. It is of course the same with or without the form factor correction as they have both been normalized to the same value (11). However, there is a difference in the predictions for $\rho \to \pi\gamma$ and $\omega \to \pi\gamma$ (see Table 1). Notice that the form factor correction appears now only once in these amplitudes and it cancels out in the ratio. From the present point of view the agreement between this ratio in VMD and experiment should not be understood as a success of VMD but rather as a result of the fact that the off mass shell correction cancels out. In fact, the individual rates at the tree level do not compare so favorably with the data.

The next two decays, $\omega \to 3\pi$ and $\gamma \to 3\pi$, involve now the contact term, viz.

$$F(\omega\to 3\pi) = 6\left[G_\omega - g_{\rho\pi\pi}\, g_{\omega\rho\pi}\, \frac{|F_V(s)|^2}{M_\rho^2 - s}\right], \tag{15}$$

with $s = M_\omega^2 + \mu_\pi^2 - 2M_\omega E$, such that

$$\Gamma(\omega\to 3\pi) = \frac{M_\omega}{144(2\pi)^3}\int_{E_1}^{E_2} \frac{dE}{\sqrt{s}}\left[(E^2-\mu_\pi^2)(s-4\mu_\pi^2)\right]^{3/2} \times |F(\omega\to 3\pi)|^2, \tag{16}$$

and $E_1 = \mu_\pi$, $E_2 = (M_\omega^2 - 3\mu_\pi^2)/2M_\omega$. For $\gamma \to 3\pi$ one has

$$F^{\gamma\to 3\pi}(q^2, s, t, u) = e\,\frac{M_\omega^2}{f_\omega}\,\frac{|F_V(q^2)|}{M_\omega^2 - q^2}\Big\{ -6\,\mathcal{G}_\omega$$

$$+ 2 g_{\rho\pi\pi}\, g_{\omega\rho\pi}\left[\frac{|F_V(s)|^2}{M_\rho^2 - s} + \frac{|F_V(t)|^2}{M_\rho^2 - t} + \frac{|F_V(u)|^2}{M_\rho^2 - u}\right]\Big\}. \tag{17}$$

Unlike the case of radiative decays these two processes probe now the form factor in the time-like region, i.e. $4\mu_\pi^2 \leq s \lesssim (M_\omega - \mu_\pi)^2$ for $\omega\to 3\pi$, and $s < 10\mu_\pi^2$ for $\gamma\to 3\pi$ [12]. Notice, however, that $F_V(M_\rho^2) = 1$ while the first singularity in $F_V(s)$ lies well above this kinematical region. As a result one finds that unitarity corrections [17] are small for such values of s and thus they will be neglected in the sequel. At this point if one were to set $\mathcal{G}_\omega = 0$ then

$$\Gamma(\omega\to 3\pi)\Big|_{\mathcal{G}_\omega = 0} = \begin{cases} 4.9 \pm 0.4\ \text{MeV}, & \beta = 2\ \ (\text{VMD}) \quad (18a) \\ 6.5 \pm 1.0\ \text{MeV}, & \beta = 2.3 - 2.4 \quad (18b) \end{cases}$$

to be compared with $\Gamma(\omega\to 3\pi)|_{EXP.} = 8.8 \pm 0.3\,\text{MeV}$ [15]. Somewhat higher rates in VMD are often quoted in the literature, but they are just accidental as they involve tree level relations such as e.g. universality or the KSFR relation. Instead, the value in Eq. (18.a) was obtained using Eq. (10), i.e. actual data, and Eq. (14); it should then be understood as the true VMD prediction. Given the fair agreement between (18.b) and experiment does not help to determine the strength of the contact term, which is expected to interfere destructively with the second term in Eq. (15) and thus lower the $\omega\to 3\pi$ rate. However, the issue is rather one of principle, i.e. it is easy to check that with $\mathcal{G}_\omega =$ 0 Eq. (17) would not be consistent with the chiral anomaly theorem [4.e,f]

$$e\,F^{\gamma\to 3\pi}(0, 0, \ldots)\Big|_{C.A.} = -\frac{\alpha}{\pi f_\pi^3}\,. \tag{19}$$

Requiring consistency between Eqs. (17) and (19) determines $\mathcal{G}_\omega$, i.e.

$$\mathcal{G}_\omega = \frac{1}{24\pi^2 f_\pi^3}\,\frac{f_\omega}{|F_V(0)|}\left[1 - 3\,\frac{f_\pi^2}{M_\rho^2}\, g_{\rho\pi\pi}\, f_\rho\, |F_V(0)|\right], \tag{20}$$

or numerically, using experimental data Eq. (10), f_π = 93.2 MeV, and Eqs. (13)-(14),

$$\mathcal{G}_\omega = \begin{cases} -(29 \pm 3)\,\text{GeV}^{-3}, & \beta = 2\ \ (\text{VMD}) \quad (21.a) \\ -(11 \pm 4)\,\text{GeV}^{-3}, & \beta = 2.3 - 2.4 \quad (21.b) \end{cases}$$

Recomputing the $\omega\to 3\pi$ rates with these values of $\mathcal{G}_\omega$ one obtains the results given in Table 1.

Finally, coming to the $\gamma\to\pi^+\pi^-\pi^0$ amplitude its value in the kinematical region: $s \simeq (6\text{-}10)\mu_\pi^2$, $q^2 < 1.3\,\mu_\pi^2$, has been extracted from the differential cross sections for $\pi^- + (A, Z) \to \pi^-\pi^0 + (A, Z)$ [12]. Computing the amplitude through Eq. (17) in the same kinematical region one obtains the results shown in Table 1. Notice that the chiral anomaly low energy theorem gives the slightly smaller value $F(\gamma\to 3\pi) \simeq 9.5\ \text{GeV}^{-3}$, but this corresponds to the full off shell point.

The results displayed in Table 1 serve as conclusions to this work. When experimental values of the coupling constants are systematically used throughout, then tree level VMD predictions are not in such a good agreement with data as is often being claimed. Although off shell extrapolations are by no means unique, I have discussed a convenient and economical framework to estimate vertex corrections which still respects the chiral anomaly low energy theorems. (For other realizations of Extended VMD see e.g. [5.b], [18]. No additional parmeters are introduced as the power fall-off of the vector form factor Eq. (8) is known from a fit to the pion form factor. These vertex corrections bring the tree level predictions into better agreement with experiment. The presence of the $\omega \to 3\pi$ contact term is confirmed but its strength is considerably smaller than at tree level. This is rewarding as it prevents the $\omega \to 3\pi$ rate to become dangerously low.

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TABLE 1

Predictions for the various decay rates. The results in VMD at the tree level were obtained using experimental values for the coupling constants (see text).

DECAY	TREE LEVEL VMD	THIS WORK	EXPERIMENT [15]-[16]
$\Gamma(\pi^0 \to \gamma\gamma)$ (eV)	7.6 ± 0.8	7.6 ± 0.8	7.8 ± 0.4
$\Gamma(\rho^- \to \pi^- \gamma)$ (keV)	58 ± 6	84 ± 11	81 ± 4
$\Gamma(\omega \to \pi\gamma)$ (keV)	655 ± 55	951 ± 105	861 ± 56
$\Gamma(\omega \to \pi\gamma)/\Gamma(\rho^- \to \pi^- \gamma)$	11.2 ± 0.9	11.2 ± 0.9	10.6 ± 0.9
$\Gamma(\omega \to \pi^+\pi^-\pi^0)$ (MeV)	3.5 ± 0.5	6.0 ± 1.0	8.8 ± 0.3
$F^{\gamma \to 3\pi}(q^2, s, t, u)$ (GeV^{-3})	10.9 ± 1.2	13.4 ± 1.6	13.0 ± 0.9

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