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SCALE INVARIANCE AND SPONTANEOUS SYMMETRY BREAKING

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Abstract

Spontaneous breaking of gauge symmetries is studied in theories with nonlinearly realized scale invariance. The classically sliding vacuum expectation values are fixed through quantum corrections. The anomaly of the dilatation current determines the vacuum energy density as well as the dilaton mass. The coupling of gravity to matter is modified in such a way that the cosmological constant vanishes.

One of the most remarkable properties of the standard model of strong and electroweak interactions is that all masses of elementary particles are tied to the spontaneous breaking of the electroweak gauge group. A direct consequence of this fact is the classical scale invariance of the standard model lagrangian which is only broken by the mass term in the Higgs potential and a possible constant term related to the vacuum energy density.

Theories with mass parameters may still have "hidden", i.e., nonlinearly realized scale invariance [1]. This requires the presence of a Goldstone boson, the dilaton, which couples in a universal way to all mass terms. Motivated by the approximate scale invariance of the standard model we investigate in the following its minimal extension with "hidden" scale invariance, and especially the effect of the dilaton on the spontaneous symmetry breaking of the electroweak gauge group. We will see that quantum corrections to the effective potential [2] play an essential role. They break the classical scale invariance and determine the vacuum energy density as well as the dilaton mass.

In a recent paper Peccei, Solà and Wetterich [3] have considered the possibility that anomalous "hidden" scale invariance may lead to a vanishing cosmological constant. Furthermore they have studied the phenomenology of the pseudo-Goldstone boson of broken scale invariance with a small mass arising from the anomaly. From our investigation of the standard model with "hidden" scale invariance we will obtain the electroweak contribution to the dilaton mass. We will also see that quantum corrections modify the coupling of gravity to matter in such a way that the cosmological constant, which is no longer the vacuum energy density, vanishes.

Let us now consider the scale invariant extension [1] of the scalar sector of the standard model:

$$\mathcal{L} = \frac{1}{2} e^{\frac{2\sigma}{f}} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu \varphi)^\dagger (D^\mu \varphi) - V_0(\varphi, \sigma) \tag{1}$$

where

$$V_0(\varphi,\sigma) = \bar{a}^4 + \bar{\mu}^2 \varphi^\dagger \varphi + \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 \,,$$
$$\bar{a}^4 = a^4 e^{\frac{4\sigma}{f}}, \quad \bar{\mu}^2 = \mu^2 e^{\frac{2\sigma}{f}}, \quad \lambda > 0 \,. \tag{2}$$

Here σ is the dilaton field, f its "decay" constant, φ is the scalar Higgs doublet, $D_\mu = \partial_\mu - \frac{ig}{2} \tau^i W^i_\mu - \frac{ig'}{2} B_\mu$ the $SU(2)_W \times U(1)_Y$ gauge covariant derivative, and $W^i_{\mu\nu}$ and $B_{\mu\nu}$ are the corresponding field strengths. Due to the specific couplings of the Goldstone field σ the action is invariant under dilatations:

$$\begin{aligned}
\delta\sigma &= \delta\alpha \, (f + x^\mu \partial_\mu \sigma) \,, \\
\delta\varphi &= \delta\alpha \, (\varphi + x^\mu \partial_\mu \varphi) \,, \\
\delta W^i_\mu &= \delta\alpha \, (W^i_\mu + x^\nu \partial_\nu W^i_\mu) \,, \\
\delta B_\mu &= \delta\alpha \, (B_\mu + x^\nu \partial_\nu B_\mu) \,,
\end{aligned} \tag{3}$$

which leads to a conserved dilatation current s_μ :

$$\begin{aligned}
\delta I = \delta \int d^4x \, \mathcal{L} &= \delta\alpha \int d^4x \, \partial^\mu s_\mu \,, \\
\partial^\mu s_\mu &= 0 \,.
\end{aligned} \tag{4}$$

The classical equations of motion for the scalar fields read

$$D_\mu D^\mu \varphi + \bar{\mu}^2 \varphi + \lambda (\varphi^\dagger \varphi) \varphi = 0 \,, \tag{5a}$$

$$e^{\frac{2\sigma}{f}} \left(\Box\sigma + \frac{2}{f} \partial_\mu \sigma \partial^\mu \sigma \right) + \frac{2}{f} \bar{\mu}^2 \varphi^\dagger \varphi + \frac{4}{f} \bar{a}^4 = 0 \tag{5b}$$

The existence of non trivial constant solutions σ_0 and φ_0 constrains the allowed parameters λ, μ^2 and a^4. For $\mu^2 \geq 0$ one has $a^4 = 0$. The only stationary point is $\varphi_0 = 0$, σ_0 remains undetermined. As pointed out in ref. [3], for $\mu^2 < 0$ the constant is given by $a^4 = \frac{\mu^4}{2\lambda}$. Now the symmetry breaking vacuum expectation value is undetermined and σ_0 is given by

$$e^{\frac{2\sigma_0}{f}} = - \frac{\lambda \varphi_0^\dagger \varphi_0}{\mu^2} \,. \tag{6}$$

For different relations between a^4, μ^2 and λ ($\mu^4 < 2\lambda a^4$) eq. (5b) yields $\exp(\frac{\sigma_0}{f}) = 0$, i.e., $\bar{a} = \bar{\mu} = 0$. In this case the coupling between the fields σ and φ vanishes and one has the familiar situation of linearly realized scale invariance [2]. The consistency requirements for the couplings a^4, μ^2 and λ imply that the classical energy density vanishes at the stationary points, and the potential takes the special form

$$V_0(\varphi,\sigma) = \frac{\lambda}{2} \left(\frac{\mu^2}{\lambda} e^{\frac{2\sigma}{f}} + \varphi^\dagger \varphi \right)^2 \,. \tag{7}$$

In the quantum theory scale invariance is anomalous [4]. Hence one expects that the special features of the classical theory with "hidden" scale invariance, the sliding of φ_0 and the vanishing of the vacuum energy density, dissappear in the quantum theory. In order to study this point we consider the 1-loop corrections to the effective potential. Since the dilaton interactions are not manifestly renormalizable we treat σ as a classical background field and evaluate the 1-loop contribution to the effective potential with a cutoff $\Lambda = O(f)$. A straightforward calculation yields ($z = \varphi^\dagger \varphi$, cf. ref. [2]):

$$
\begin{aligned}
V_{\Lambda}^{(1)}(\varphi,\sigma) = \frac{1}{(8\pi)^2}\Big\{ & \Lambda^2\left[8\bar{\mu}^2 + (9g^2+3g'^2+12\lambda)z\right] \\
& + \bar{\mu}^4\left(\frac{3}{2}\ln\frac{(\bar{\mu}^2+\lambda z)^2}{\Lambda^4} + \frac{1}{2}\ln\frac{(\bar{\mu}^2+3\lambda z)^2}{\Lambda^4}\right) \\
& + \bar{\mu}^2 z\left(3\lambda\ln\frac{(\bar{\mu}^2+\lambda z)^2}{\Lambda^4} + 3\lambda\ln\frac{(\bar{\mu}^2+3\lambda z)^2}{\Lambda^4}\right) \\
& + \frac{1}{2}z^2\Big(3g^4\ln\frac{g^2 z}{2\Lambda^2} + \frac{3}{2}(g^2+g'^2)^2\ln\frac{(g^2+g'^2)z}{2\Lambda^2} \\
& \qquad + 3\lambda^2\ln\frac{(\bar{\mu}^2+\lambda z)^2}{\Lambda^4} + 9\lambda^2\ln\frac{(\bar{\mu}^2+3\lambda z)^2}{\Lambda^4}\Big) \\
& - \frac{3}{4}\left(g^4+\frac{1}{2}(g^2+g'^2)^2\right)z^2 - \frac{3}{2}(\bar{\mu}^2+\lambda z)^2 - \frac{1}{2}(\bar{\mu}^2+3\lambda z)^2 \\
& + O\left(\frac{1}{\Lambda^2}\right) \Big\} \;.
\end{aligned}
\tag{8}
$$

Obviously the presence of the cutoff Λ breaks scale invariance. Also the renormalized effective potential can not be scale invariant since the renormalization conditions require the choice of a renormalization mass M. There is, however, a class of renormalization conditions which violate scale invariance minimally. They can be defined by the requirement that in the renormalized effective potential all mass parameters have the same coupling to the dilaton field as in the classical lagrangian. This specifies the counter terms

$$
V_c^{(1)} = A\bar{\mu}^2 + B\,\varphi^{+}\varphi + C\bar{\mu}^4 + D\bar{\mu}^2\varphi^{+}\varphi + \frac{1}{2}E(\varphi^{+}\varphi)^2
\tag{9}
$$

up to irrelevant constants for C, D and E. A convenient choice of these constants yields the renormalized 1-loop effective potential $(z = \varphi^{+}\varphi)$:

$$
\begin{aligned}
V(\varphi,\sigma) &= V_0(\varphi,\sigma) + V_{\Lambda}^{(1)}(\varphi,\sigma) + V_c^{(1)}(\varphi,\sigma) \\
&= \bar{a}^4 + \bar{\mu}^2 z + \frac{\lambda}{2}z^2 \\
&\quad + \frac{1}{(8\pi)^2}\Big[\frac{3}{2}(\bar{\mu}^2+\lambda z)^2\left(\ln\frac{(\bar{\mu}^2+\lambda z)^2}{M^4} - 1\right) \\
&\qquad + \frac{1}{2}(\bar{\mu}^2+3\lambda z)^2\left(\ln\frac{(\bar{\mu}^2+3\lambda z)^2}{M^4} - 1\right) \\
&\qquad + \frac{3}{2}g^4 z^2\left(\ln\frac{g^2 z}{2M^2} - \frac{1}{2}\right) \\
&\qquad + \frac{3}{4}(g^2+g'^2)^2 z^2\left(\ln\frac{(g^2+g'^2)z}{2M^2} - \frac{1}{2}\right)\Big] \;,
\end{aligned}
\tag{10}
$$

where a^4 , μ^2 and λ are now renormalized parameters which depend on the renormalization mass M. Their M-dependence can be read off from eq. (10):

$$
M\frac{\partial}{\partial M}a^4(M) = \mu^4(M)\,\gamma_1(\lambda(M)) \,, \qquad \gamma_1(\lambda) = \frac{8}{(8\pi)^2} \;,
\tag{11}
$$

$$
M\frac{\partial}{\partial M}\mu^2(M) = \mu^2(M)\,\gamma_2(\lambda(M)) \,, \qquad \gamma_2(\lambda) = \frac{24\lambda}{(8\pi)^2}
\tag{12}
$$

$$
M\frac{\partial}{\partial M}\lambda(M) = \beta(\lambda(M)) \,, \qquad \beta(\lambda) = \frac{1}{(8\pi)^2}\left(48\lambda^2 + 6g^4 + 3(g^2+g'^2)^2\right) .
\tag{13}
$$

The constraint for a^4 , μ^2 and λ of the classical theory in the case $\mu^2 < 0$ can also be imposed on the renormalized quantities :

$$2\,\lambda(M)\,a^4(M) = \mu^4(M) \quad . \tag{14}$$

Since the scale dependence of λa^4 and μ^4 is different, eq. (14) fixes a renormalization mass M.

It is instructive to compute the change of the effective potential under an infinitesimal scale transformation. One easily finds (cf. eqs. (11)-(13))

$$\begin{aligned}\delta V = \delta\alpha\, \Big[& \partial_\mu (x^\mu V) \\ & + \gamma_1(\lambda)\,\bar\mu^4 + \gamma_2(\lambda)\,\bar\mu^2 \varphi^\dagger\varphi + \frac{\beta(\lambda)}{\lambda}\,\frac{1}{2}(\varphi^\dagger\varphi)^2 \Big] \, .\end{aligned} \tag{15}$$

From eqs. (4) and (15) one obtains, up to derivative terms, for the divergence of the dilatation current (cf. ref. [5])

$$\partial^\mu s_\mu = -\Big(\gamma_1(\lambda)\,\bar\mu^4 + \gamma_2(\lambda)\,\bar\mu^2\varphi^\dagger\varphi + \frac{\beta(\lambda)}{\lambda}\,\frac{1}{2}\,\lambda\,\varphi^\dagger\varphi\Big) \equiv \Delta(\varphi,\sigma) \; . \tag{16}$$

The effective potential (10) yields for stationary points the extremum conditions ($\bar\mu_0 = \mu_0 e^{\sigma_0/f}$, $\bar a_0 = a\, e^{\sigma_0/f}$, $z_0 = \varphi_0^\dagger\varphi_0$) :

$$\begin{aligned}4\bar a_0^4 + 2\bar\mu_0^2 z_0 + \frac{2\bar\mu_0^2}{(8\pi)^2}\Big[& 3(\bar\mu_0^2 + \lambda z_0)\ln\frac{(\bar\mu_0^2+\lambda z_0)^2}{M^4} \\ & + (\bar\mu_0^2 + 3\lambda z_0)\ln\frac{(\bar\mu_0^2+3\lambda z_0)^2}{M^4}\Big] = 0 \; ,\end{aligned} \tag{17a}$$

$$\begin{aligned}\bar\mu_0^2 + \lambda z_0 + \frac{1}{(8\pi)^2}\Big[& 3\lambda(\bar\mu_0^2 + \lambda z_0)\ln\frac{(\bar\mu_0^2+\lambda z_0)^2}{M^4} \\ & + 3\lambda(\bar\mu_0^2 + 3\lambda z_0)\ln\frac{(\bar\mu_0^2+3\lambda z_0)^2}{M^4} \\ & + 3g^4 z_0 \ln\frac{g^2 z_0}{2M^2} + \frac{3}{2}(g^2+g'^2)^2 z_0 \ln\frac{(g^2+g'^2)z_0}{2M^2}\Big] = 0 \; .\end{aligned} \tag{17b}$$

Eqs. (17) have the non-trivial solution

$$|\varphi_0|^2 = \frac{K M^2}{2\lambda(M)}\Big(1 - \frac{4\lambda}{(8\pi)^2}\ln K\Big) \; , \tag{18a}$$

$$e^{\frac{2\sigma_0}{f}} = -\frac{K M^2}{2\mu^2(M)} \; , \tag{18b}$$

$$K = \exp\Big[-\frac{3g^4 \ln\frac{g^2}{4\lambda} + \frac{3}{2}(g^2+g'^2)^2\ln\frac{(g^2+g'^2)}{4\lambda}}{3g^4 + \frac{3}{2}(g^2+g'^2)^2 + 8\lambda^2}\Big] \; , \tag{18c}$$

which determine the vacuum expectation values σ_0 and φ_0 in terms of the renormalization mass M and the couplings λ, g and g'. By a redefinition of the field σ and the parameters f, a, μ , λ one can - even in the presence of the anomaly - absorb a finite vacuum expectation value σ_0 . We assume, without loss of generality that this is already done and work with parameters such that $\sigma_0 = 0$. We note that for $\lambda = O(g^2, g'^2)$ one has $\ln K = O(1)$.

As the quantum corrections break scale invariance the dilaton aquires a mass which is determined by the vacuum expectation value of the anomaly (16). To lowest order in $\frac{v^2}{f^2}$ $(v = (2\sqrt{2}\,G_F)^{-\frac{1}{2}})$ one obtains

$$m_D^2 = -\frac{4\langle\Delta\rangle_0}{f^2} = \frac{8}{(8\pi)^2 f^2}\left(6\,m_W^4 + 3\,m_Z^4 + m_H^4\right) \; , \tag{19}$$

where m_H is the Higgs boson mass, which depends in the usual way on λ, g and g', and satisfies the Weinberg-Linde bound [6]

$$m_H^2 \geq \frac{3\sqrt{2}\,G_F}{16\pi^2}\left(2\,m_W^4 + m_Z^4\right) \; . \tag{20}$$

A charged spin 1/2-fermion with mass m_F adds inside the bracket of (19) the term $-4m_F^4$. The stability of the effective dilaton potential therefore excludes a top-quark mass above the W-mass unless there are further heavy bosons such as a heavy Higgs scalar. Finally, the vacuum energy density is also determined by the vacuum expectation value of the anomaly:

$$\langle V \rangle_0 = \frac{1}{4} \langle \Delta \rangle_0$$
$$= -\frac{1}{2(8\pi)^2} \left(6 m_W^4 + 3 m_Z^4 + m_H^4 \right) . \qquad (21)$$

The "decay" constant f of the dilaton field is the mass scale at which dilatation invariance is spontaneously broken. Hence f is expected to be large, possibly of order the Planck mass M_{Pl}. In this case the dilaton mass lies in the range usually considered for invisible axions. Its couplings to matter are of gravitational strength, as discussed in ref. [3]. The dilaton fermion couplings are proportional to the Higgs boson couplings, $g_{\sigma f\bar{f}} = \frac{v}{f} g_{Hf\bar{f}}$, furthermore the dilaton couples directly to the complete anomalous divergence of the dilatation current. This is analogous to axion couplings [7] and follows, as in the axion case, from a redefinition of fields.

What is the cosmological constant corresponding to the vacuum energy density (21)? Let us express the effective potential in terms of the fields $\phi = e^{-\sigma/f}\varphi$ and σ. From eqs. (10) and (16) one obtains

$$V(\varphi,\sigma) = e^{\frac{4\sigma}{f}} \left(V_0(\phi,0) + V^{(1)}(\phi,0) - \Delta(\phi,0)\frac{\sigma}{f} \right) . \qquad (22)$$

This decomposition shows that (21) results from the minimization with respect to σ irrespective of the detailed dependence of V_0 and $V^{(1)}$ on ϕ.

The coupling of σ to the potential is familiar from the coupling of the metric tensor $g_{\mu\nu}$ to matter. The classical action for the fields ϕ, σ and $g_{\mu\nu}$ reads

$$I = \int d^4x \sqrt{-g}\, e^{\frac{4\sigma}{f}} \Big[\frac{M_{Pl}^2}{16\pi} e^{-\frac{2\sigma}{f}} R + \frac{1}{2} g^{\mu\nu} e^{-\frac{2\sigma}{f}} \partial_\mu\sigma \partial_\nu\sigma$$
$$+ g^{\mu\nu} e^{-\frac{2\sigma}{f}} (\partial_\mu + \tfrac{1}{f}\partial_\mu\sigma)\phi^\dagger (\partial_\nu + \tfrac{1}{f}\partial_\nu\sigma)\phi$$
$$- V_0(\phi,0) \Big] , \qquad (23)$$

where R is the curvature scalar. The action is invariant under global dilatations and local coordinate transformations. Since the metric couples to ϕ only in the form $g_{\mu\nu}\exp(\frac{2\sigma}{f})$, we can read off from eq. (22) the 1-loop effective potential for ϕ,σ and constant, Lorentz-invariant metric $g_{\mu\nu} = C\,\eta_{\mu\nu}$:

$$I^{(1)} = \int d^4x \sqrt{-g}\, e^{\frac{4\sigma}{f}} \left(-V^{(1)}(\phi,0) + \Delta(\phi,0)\left(\frac{\sigma}{f} + \frac{1}{4}\ln\sqrt{-g} \right) \right) . \qquad (24)$$

This is a puzzling result as it seems to imply that, contrary to common belief, the dilatation anomaly also breaks general coordinate invariance!

The anomalous gravitational coupling (24) to the divergence of the dilatation current,

$$I_G^{(1)} = \frac{1}{4}\int d^4x \sqrt{-g}\, s^\mu{}_{;\mu} \ln\sqrt{-g} = -\frac{1}{4}\int d^4x \sqrt{-g}\, s^\mu \Gamma^\nu{}_{\nu\mu} , \qquad (25)$$

is not excluded by previous calculations. It is absent for free scalar fields in curved space [8], and it can also not be obtained from the determination of the ultraviolet counter terms of an interacting theory in a gravitational background field [9], since it arises from a summation of ultraviolet finite contributions to the effective potential. From eq. (24) one cannot conclude that the dilatation anomaly generates a gravitational anomaly, like the one which can arise from fermion loops [10]. In order to study this question one would have to consider energy-momentum conservation which requires the complete effective action and not just the effective potential (24).

What is the effect of the anomalous gravitational coupling (24) on the cosmological constant? The field equations for the metric become

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{8\pi}{M_{Pl}^2} T_{\mu\nu} \quad , \tag{26}$$

with

$$T_{\mu\nu} = \hat{T}_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \Delta(\varphi,\sigma)(\ln\sqrt{g}+1) , \tag{27}$$

$$\hat{T}_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} I_M \quad , \tag{28}$$

where I_M is the matter part of the action without $I_G^{(1)}$ (25). Hence we obtain for the "cosmological constant"

$$\langle T^{\mu}{}_{\mu} \rangle_0 = \langle \hat{T}^{\mu}{}_{\mu} \rangle_0 - \langle \Delta(\varphi,\sigma)(\ln\sqrt{g}+1) \rangle_0 , \tag{29}$$

which vanishes for flat space ($g_{\mu\nu} = \eta_{\mu\nu}$),

$$\langle T^{\mu}{}_{\mu} \rangle = 4 \langle V \rangle_0 - \langle \Delta \rangle_0 = 0 \quad , \tag{30}$$

since the vacuum energy density (21) is given by the anomaly!

This result is surprising. In scale invariant theories, the vacuum energy density is given by the anomaly of the dilatation current and does not vanish. However, the coupling of gravity to matter is modified precisely such that the cosmological constant vanishes! As the effective potential depends only on $\frac{\sigma}{f} + \frac{1}{4}\ln\sqrt{g}$, a stationary point for σ automatically implies that flat space-time is a solution of the coupled, nonlinear system of equations of motion.

The standard model of strong and electroweak interactions offers no explanation as to why the cosmological constant is so small [11]. A remarkable feature of the standard model is its classical scale invariance, which is only broken by a single mass parameter in the Higgs potential. If scale invariance is realized nonlinearly for the complete lagrangian by means of a Goldstone field, the dilaton, the classical vacuum has a flat direction and the cosmological constant vanishes. Quantum corrections fix the scale of spontaneous symmetry breaking without changing the cosmological constant. Reversing the argument, the observed smallness of the cosmological constant suggests the existence of a dilaton, which is expected to have a mass in the invisible axion range [12] and to interact with gravitational strength.

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