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by

J.H. Field

Deutsches Elektronen-Synchrotron DESY, Hamburg

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Abstract

Jet production processes responsible for scale-breaking effects in the F_2 nucleon structure function are analysed in leading log approximation. Resumming soft and virtual gluons to all orders in α_S introduces exponential Sudakov type form factors in all multijet cross sections. Allowing for a confinement threshold in jet production gives a steeper Q^2 dependence between $Q^2 = 10$ and 100 $(\mathrm{GeV}/c)^2$ than that previously predicted. As $Q^2 \to \infty$ the number of gluon jets becomes Poisson distributed.

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The Q^2 evolution of structure functions has been calculated in leading log (LL) approximation in several different ways: by use of the Operator Product Expansion and Renormalisation Group Equations (OPE-RGE) [1,2], solution of coupled integro-differential equations [3], or by summing, to all orders in α_S, certain classes of Feynman diagrams [4, 5, 6]. The corresponding jet production has been calculated to $O(\alpha_S)$ [7, 8] and also to all orders in initial state parton shower models [9, 10, 11] or by use of a 'Jet-Calculus' algorithm [12].

The aim of the work reported here is to derive predictions for jet production, within the LL approximation, directly from the 'classical' QCD structure function evolution prediction [1 - 6] with the minimum number of additional assumptions or parameters. It turns out that this is possible by introducing a suitable jet-resolution parameter:

$$\varepsilon = 1 - z_{MAX} = \hat{s}_{ij}/(Q^2 + \hat{s}_{ij})$$

(where $\hat{s}_{ij}$ is the effective mass of two partons i, j) and resumming the perturbation series in α_S for the evolved quark density $q(x, Q^2)$ to give the all orders contribution to an observable jet production process. The parameter ε is analogous to the y parameter used to define jets in $O(\alpha_S^2)$ calculations of the process $e^+e^- \to q\bar{q}Ng$ (N = 0, 1, 2)[13]. It plays exactly the same rôle as k_{MAX}/E in discussions of radiatively corrected cross-sections in QED. The result found gives, in the Bjorken limit $Q^2 \to \infty$, x = const. or as $\hat{s}_{ij} \to 0$, the QCD analogue of the Bloch-Nordsieck theorem of QED [14.]

Only the non-singlet part of the F_2 structure function of the nucleon in LL approximation is considered here. With the usual definition of quark density functions:

$$F_2(x, Q^2) = \sum_q e_q^2 x \left[q(x, Q^2) + \bar{q}(x, Q^2) \right] \quad (1)$$

the LL result for $q(x, Q^2)$ is the solution of the Bethe-Salpeter equation [4, 5, 6]:

$$q(x, Q^2) = q(x, Q_0^2) + \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{Q^2} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} q\left(\frac{x}{z}, t\right) P_{qq}(z) \quad (2)$$

If q, P, p and p' are the 4 momenta of the virtual photon, target nucleon, interacting quark in the nucleon (assumed on-shell, with vanishing transverse momentum) and the interacting (space-like) quark at the $qq\gamma$ vertex, then the variables in (2) are defined as:

$$x = Q^2/(2Pq) \qquad z = Q^2/(2pq) \qquad t = -(p')^2$$

The choice of t as the argument of α is justified in [4].

Eqn (2) may be solved formally [15], or by iteration, to give:

$$q = q_0 + q_0 \otimes P_{qq} + (q_0 \otimes P_{qq}) \otimes P_{qq} + \ldots$$

where $\otimes$ denotes the double convolution (including the factor $\alpha_s(t)/2\pi$) in (2) and $q_0 = q(x, Q_0^2)$. Taking Mellin moments to decouple the z integrals:

$$q(n, Q^2) \equiv \int_0^1 q(x, Q^2)\, x^{n-1}\, dx$$

and evaluating the multiple t integrals analytically, gives:

$$q(n, Q^2) = q(n, Q_0^2) \sum_{N=0}^{\infty} \frac{\left[b\, P_{qq}(n)\, L\right]^N}{N!} \tag{3}$$

where b = 6/25 for 4 quark flavours and

$$L \equiv \ln\left[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)\right]$$

The non-singlet splitting function P_{qq} contains contributions both from virtual plus soft gluons $P_{qq}^{V,S}$ and from hard gluons P_{qq}^{H}.

Expressions for $P_{qq}^{V,S}$, P_{qq}^{H} at $O(\alpha_s)$ have been derived in various infra-red regularisation schemes and for a fixed value of α_s by Humpert and van Neerven [16]. Factorising the quark mass singularity at scale Q_0^2, and renormalising α_s in the 1-loop approximation with 4 quark flavours:

$$\alpha_s(Q^2) = 12\pi/\left[25 \ln(Q^2/\Lambda^2)\right]$$

gives:

$$P_{qq}^{V,S}(z) = \delta(1-z)\frac{4}{3}\left[2\ln\varepsilon + \frac{3}{2}\right] \tag{4a}$$

$$P_{qq}^{H}(z) = \frac{4}{3}\left[\frac{1+z^2}{1-z}\right] \qquad z < z_{MAX} \tag{4b}$$

$$= 0 \qquad z > z_{MAX}$$

where: $\varepsilon = \hat{s}_0/(Q^2 + \hat{s}_0)$, $z_{MAX} = 1 - \varepsilon$

In (4) only the LL terms (which are independent of the method used to regularise the infra-red divergences), are retained. $\hat{s}$ is the effective mass of the 'current jet' quark and radiated gluon. The gluon is 'soft' if:

$$\hat{s} < \hat{s}_0$$

and is 'hard' if:

$$\hat{s} > \hat{s}_0$$

Taking the Mellin moment of (4b):

$$P_{qq}^{H}(n) = \frac{4}{3}\left[\frac{z^n}{n} - \frac{z^{n+1}}{n+1} - 2\sum_{j=1}^{n}\frac{z^j}{j} - 2\ln(1-z)\right]_0^{z_{MAX}} \tag{5a}$$

If $1 - z_{MAX} \ll 1$ then:

$$P_{qq}^{H}(n) = \frac{4}{3}\left[\frac{1}{n(n+1)} - 2\sum_{j=1}^{n}\frac{1}{j} - 2\ln\varepsilon\right] \tag{5b}$$

Adding $P_{qq}^{V,S}(n)$, $P_{qq}(n)^{H}$ using (4a), (5b) gives:

$$P_{qq}(n) = \frac{4}{3}\left[\frac{3}{2} + \frac{1}{n(n+1)} - 2\sum_{j=1}^{n}\frac{1}{j}\right] = \frac{d_{qq}^{n}}{b} \tag{6}$$

The resolution parameter ε cancels in the sum of $P_{qq}^{V,S}(n)$, $P_{qq}^{H}(n)$ and (3), (6) give the familiar result [1 - 6 ,17]

$$q(n, Q^2) = q(n, Q_0^2)\left[\alpha_s(Q_0^2)/\alpha_s(Q^2)\right]^{d_{qq}^{n}} \tag{7}$$

The step from (3) to (7) via (6) can be made alternatively by the introduction of singular functions in the definition of $P_{qq}(z)$ [3] If this is done, however, the resolution parameter ε, essential to define physically meaningful jet cross sections, disappears from the formalism.

Eqn (2) is a perturbation series in powers of α_s. For real gluon radiation the Nth term corresponds to the radiation of N gluons. Taking into account the virtual and soft contributions at $O(\alpha_s)$ the zeroth order quark density is modified by a 'K-factor':

$$q^{(1)}(n, Q^2)^{2Jet} = q(n, Q_0^2)K^{(1)} = q_0(n)\left[1 + \frac{8}{25}\left(2\ln\varepsilon + \frac{3}{2}\right)L\right] \tag{8}$$

$q^{(1)}$, $K^{(1)}$ indicate that q, K are evaluated up to $O(\alpha_s)$. From (3), (5b) hard gluon radiation gives the contribution:

$$q^{(1)\,3Jet}(n,Q^2) = q_0(n)\frac{8}{25}\left[\frac{1}{n(n+1)} - 2\sum_{j=1}^{n}\frac{1}{j} - 2\ln\varepsilon\right] \qquad (9)$$

Eqns (8, 9) correspond to the observation of 2, 3 jets in the final state when ε is used as a resolution parameter. Since q is positive (it is proportional to F_2 which, in turn, is proportional to the total virtual γ proton scattering cross section) both (8) and (9), which give different observed final states, must be positive. This is always true for (9), but not always for (8). Choosing physically sensible values of Q^2, Q_0^2, $\hat{s}_0$ and Λ (in GeV units):

$$Q^2 = 200,\ Q_0^2 = 2.0,\ \hat{s}_0 = 4.0,\ \Lambda = 0.2$$

gives:

$$K^{(1)} = -0.57$$

so in this case (8) violates unitarity. In the Bjorken limit $Q^2 \to \infty$, n = const. or as $\hat{s}_0 \to 0$ the $O(\alpha_s)$ '2 jet' cross section is:

$$\propto -\frac{16}{25}\ln Q^2 \ln\ln Q^2$$

while the '3 jet' cross section is:

$$\propto \frac{16}{25}\ln Q^2 \ln\ln Q^2$$

No meaningful jet cross section can therefore be defined at high Q^2 if the perturbation series in (3) is truncated at $O(\alpha_s)$.

The K-factor can however be calculated to all orders in α_s by summing to infinity the terms in (2) that contain N powers of $P_{qq}^{V,S}$ and no powers of P_{qq}^{H}. This gives :

$$K^{\infty} = \exp\left[\frac{8}{25}\left(2\ln\varepsilon + \frac{3}{2}\right)L\right] \qquad (10)$$

Similarly summing to infinity over M all terms in (2) that contain M powers of $P_{qq}^{V,S}$ and N powers of P_{qq}^{H} exactly the same K-factor (10) is found also for the N + 2 jet contribution. Unresolved soft and virtual gluons are now included in the multijet contributions to $q(n, Q^2)$ to all orders in α_s in LL approximation.

Taking into account confinement effects, it is however clear that the jet resolution parameter ε (which corresponds to the total effective mass of all partons produced or scattered in the hard quark photon collision) must depend on the jet multiplicity. Roughly speaking, if a '3 jet' event is resolved from '2 jets' for $\hat{s} > \hat{s}_0$ then a '4 jet' event will be distinguishable from a '3 jet' event only for $\hat{s}_0' > \hat{s}_0$, say for $\hat{s}_0' = 2\,\hat{s}_0$. This implies that larger values of ε are needed to define meaningful jet cross sections as N, the number of hard final state gluons, increases.

The definition of a multijet final state is arbitrary. Here a simple 'minimal' definition is used that permits an analytical solution to be given for the 'M-jet' contribution to $q(n, Q^2)$. Choosing:

$$\varepsilon^{(N)} = N\hat{s}_0/(Q^2 + N\hat{s}_0)$$

then (7) may be re-written as a series in 'resolved M-jet' contributions:

$$q(n,Q^2) = q_0(n)\sum_{M=2}^{\infty} K\left(\varepsilon^{(M-1)}\right)\frac{\left\{\left[d_{qq}^{n} - \frac{12}{25} - \frac{16}{25}\ln\varepsilon^{(M-2)}\right]L\right\}^{M-2}}{(M-2)!} \qquad (11)$$

Notice that M here includes the current quark and target diquark jets, so that M = N + 2. Eqn (11) is the main result of this letter.

If $\varepsilon^{(M-1)}$, $\varepsilon^{(M-2)}$ are replaced in (11) by $\varepsilon^{(1)}$ then (11) becomes formally identical to (3). Eqns (3), (11) also have a common asymptotic limit when $\ln Q^2/\hat{s}_0 \gg \ln M$. When $Q^2 \to Q_0^2$ (i.e. $L \to 0$) (3), (11) also give the same $q^{(1)}(n, Q^2)$.

For non asymptotic Q^2 values (11) will predict a somewhat different Q^2 evolution of the moments than (7). This is because the N dependence of ε takes into account threshold effects in jet production related to confinement. The evolution of the n = 1, 4, 8 moments as predicted by (11) for $Q_0^2 = 2\ (\mathrm{GeV}/c)^2$, $\hat{s}_0 = 4\ (\mathrm{GeV}/c)^2$ and $\Lambda = 100$ MeV/c is shown in Fig. 1a, b, c. The 'asymptotic' prediction given by (7) with the same values of Q_0^2, Λ is shown for comparison. It can be seen that from $Q^2 = 10$ to $100\ (\mathrm{GeV}/c)^2$ (11) predicts a somewhat steeper decrease with Q^2 of the higher n moments than (7). The '22 jet' curve E is already asymptotic in M, the number of jets, for $Q^2 < 10^4\ (\mathrm{GeV}/c)^2$, while (7), (11) give essentially identical predictions, for $Q^2 > 100\ (\mathrm{GeV}/c)^2$, for all the moments shown in Fig. 1.

It should be remarked that the curves in Fig. 1 showing the contributions of $\leq$ M jets should not be interpreted directly in terms of observable exclusive jet cross sections. This is because the simple effective mass cuts given by $\varepsilon^{(M)}$ do not take into account final state configurations that are irresolvable because two final state jets are almost parallel. Using a realistic resolution criterion such as [13]:

$$\hat{s}_{ij} > \hat{s}_c \quad i \neq j \quad i, j = 1, 2, \ldots M$$

will increase the relative contributions of smaller jet multiplicities as compared to the curves shown in Fig. 1. Using the analytical expression (11) as a starting point such a re-assignment of events into 'truly observable' jet classes can most conveniently be done by Monte-Carlo methods.

considering the limit $\ln Q^2/\hat{s}_0 >> \ln M$, $\ln n$ of (11) [illegible] $\sim \ln n$ for large n values [1]) the M-jet distribution becomes independent of n. In this limit:

$$\sigma(\gamma p \to X + N g) \simeq \exp(-\bar{N}) \frac{(\bar{N})^N}{N!} \qquad (12)$$

The number of gluon jets obeys a Poisson distribution with mean value:

$$\bar{N} = \frac{16}{25} \ln \frac{Q^2}{\hat{s}_0} \ln\left[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)\right] \qquad (13)$$

For the case N = 0 the RHS of (12) reduces to the square of the asymptotic Sudakov form factor of a quark in QCD as derived by Mueller [18]. From (12), the same Sudakov factor appears also in the N gluon-jet cross section. Coquereaux and De Rafael showed, by solution of a renormalisation group equation, that the same asymptotic form factor occurs in the quark-quark elastic scattering amplitude in LL approximation [19].

The Poisson distribution (12) is just that found by Bloch and Nordsieck [14] for the distribution of N soft photons radiated in potential scattering of an electron. In this case $N \simeq \ln(\omega/\omega_0)$ where ω is the total energy of the N photons and ω_0 is an infra-red cut-off energy. In (12) as $Q^2 \to \infty$ or $\hat{s}_0 \to 0$ the contribution of any finite number of gluons to the multigluon cross section vanishes. In [14] the N photon radiation probability was found to vanish as $\omega_0 \to 0$. The divergences encountered above, in the limit $Q^2 \to \infty$, in the terms in the perturbation series in powers of α_s (3) are analogous to those encountered in the $\omega_0 \to 0$ limit when radiative corrections are evaluated using an expansion in powers of α [14]. The parameter $\hat{s}_0$ defines the total effective energy radiated in gluons (regardless of their number) just as ω in [14] defines the total radiated energy (regardless of the number of photons). The suppression of low multiplicity jet contributions with increasing Q^2 is evident in Fig. 1. This is due to the increased probability of multi-gluon radiation as Q^2 becomes larger.

If the limit $\hat{s}_0 \simeq Q^2$ is considered, i.e. very hard jet production, the K factors all tend to unity and there is no Sudakov suppression. This limit is however of little

experimental interest because of vanishingly small cross sections near to purely kinematical boundaries.

The 3 jet contribution to the RHS of (11):

$$q(n, Q^2)^{3Jet} = q_0(n) K^{\infty}(\varepsilon^{(2)}) \left[d^n_{qq} - \frac{12}{25} - \frac{16}{25} \ln \varepsilon^{(1)} \right] L \qquad (15)$$

may be compared with that given by the prescription of Ref. [7], where the 'Q^2-evolved' quark density is multiplied by the hard scattering process cross section $\sigma(\gamma^* q \to g q)$

$$q(n, Q^2)^{3Jet} = q(n, Q^2) \left[d^n_{qq} - \frac{12}{25} - \frac{16}{25} \ln \varepsilon^{(1)} \right] L \qquad (16)$$

The hard gluon region is defined in the same way in (15) and (16). Comparing (15), (16) it can be seen that replacing the scale invariant quark density $q(n, Q_0^2)$ by the 'evolved' one $q(n, Q^2)$ does not properly take into account soft and virtual gluon interference effects.

Considering the n = 1 moment where $q(1, Q^2) = q(1, Q_0^2)$ (Adler sum rule) (16) gives a divergent contribution $\propto \ln Q^2 \ln\ln Q^2$ as $Q^2 \to \infty$ inconsistent with the total structure function evolution given by (7). On the other hand (15) gives a vanishing contribution in the same limit (see Fig. 1a) consistent with the Bloch-Nordsieck theorem.

In summary, the effects of soft and virtual gluon interference on observed jet cross sections in deep inelastic scattering may be taken into account by multiplying the relevant hard scattering cross section by a factor K^{∞} defined in (10). This is a generalisation of the square of the Sudakov form factor for a quark [18] for the case of a 2 jet cross section. The same or similar factors are expected in all jet production cross sections when soft and virtual gluon radiation effects are properly taken into account. The ansatz of estimating jet production by multiplying a hard scattering cross section by Q^2-evolved quark densities does not predict correctly observable jet cross sections in the case considered here. With the particular choice of jet resolution parameter used in (11) the Q^2 evolution of the high n moments of the nucleon structure function is predicted to be steeper in the range Q^2 = 10 to 100 $(\mathrm{GeV}/c)^2$ than for the asymptotic result (7).

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Figure Caption

Fig. 1 a, b, c: Q^2 evolution of the $n = 1, 4, 8$ moments of F_2 for the nucleon (non singlet part only) between $Q^2 = 2$ and 10^4 $(GeV/c)^2$. A, B, C, D, E show the contributions of 2, $\leqslant$ 3, 4, 5, ≥ 2 jets (as defined in Eqn (11)) to the moments. ASYM is the asymptotic prediction given by Eqn (7). $Q_0^2 = 2$ $(GeV/c)^2$, $\hat{s}_0 = 4$ $(GeV/c^2)^2$, $\Lambda = 0.1$ GeV/c.

References

1. D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633, D9 (1974) 980

2. H. Georgi and H. D. Politzer, Phys. Rev. D9 (1974) 416

3. G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298

4. Yu. L. Dokshitzer, Zh. Eksp. Teor. Fiz. 73 (1977) 1216 Sov. Phys. JETP 46 (1977) 4
Yu. L. Dokshitzer, D. I. Dyakonov and S. I. Troyan, Phys. Rep. 58 (1980) 270

5. C. H. Llewellyn Smith, Acta Phys. Austriaca XIX (1978) 331

6. W. R. Frazer and J. F. Gunion, Phys. Rev. D19 (1979) 2447

7. K. H. Craig and C. H. Llewellyn Smith, Phys. Lett. 72B (1978) 349

8. A. Mendez, Nucl. Phys. B145 (1978) 199

9. R. Odorico, Nucl. Phys. B228 (1983) 381

10. G. C. Fox and S. Wolfram, Nucl. Phys. B168 (1980) 285

11. T. D. Gottschalk, Cal. Tech. Pre-prints CALT-68-1241, 1299, 1364

12. K. Konishi, A. Ukawa and G. Veneziano, Nucl. Phys. B157 (1979) 45

13. F. Gutbrod, G. Kramer and G. Schierholz, Z. Phys. C21 (1984) 235

14. F. Bloch and A. Nordsieck, Phys. Rev. 52 (1937) 54
A. Nordsieck, Phys. Rev. 52 (1937) 59

15. R. K. Ellis et al., Nucl. Phys. B152 (1979) 285

16. B. Humpert and W. L. van Neerven, Nucl. Phys. B184 (1981) 225

17. G. Altarelli, Phys. Rep. 81 (1982) 1

18. A. H. Mueller, Phys. Rev. D20 (1979) 2037

19. R. Coquereaux and E. De Rafael, Phys. Lett. 69B (1977) 181

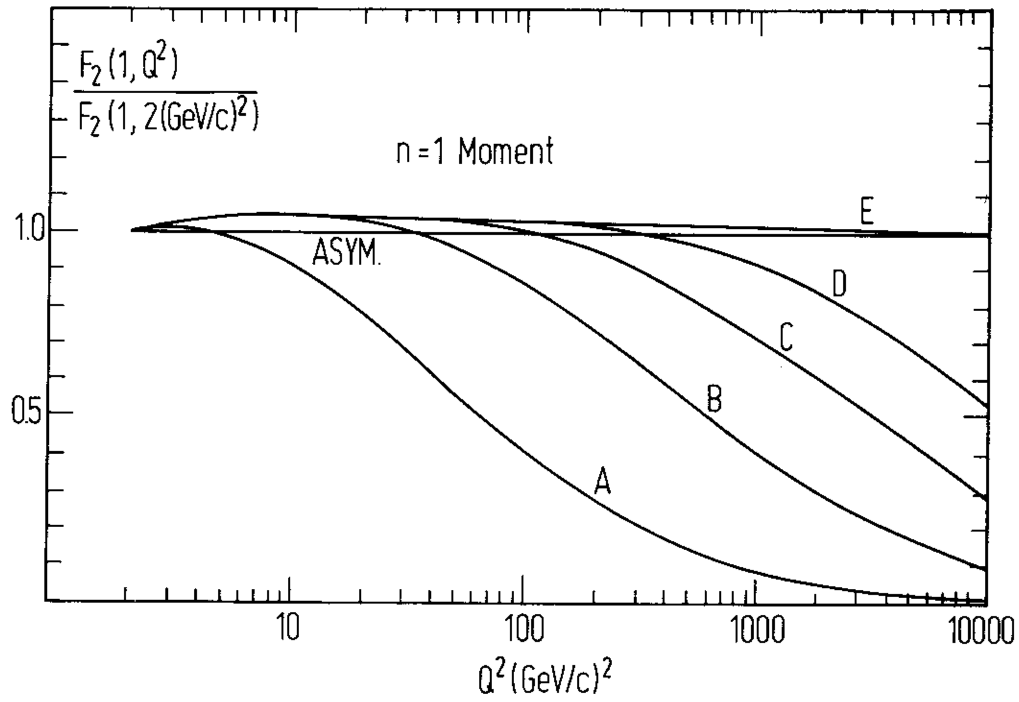
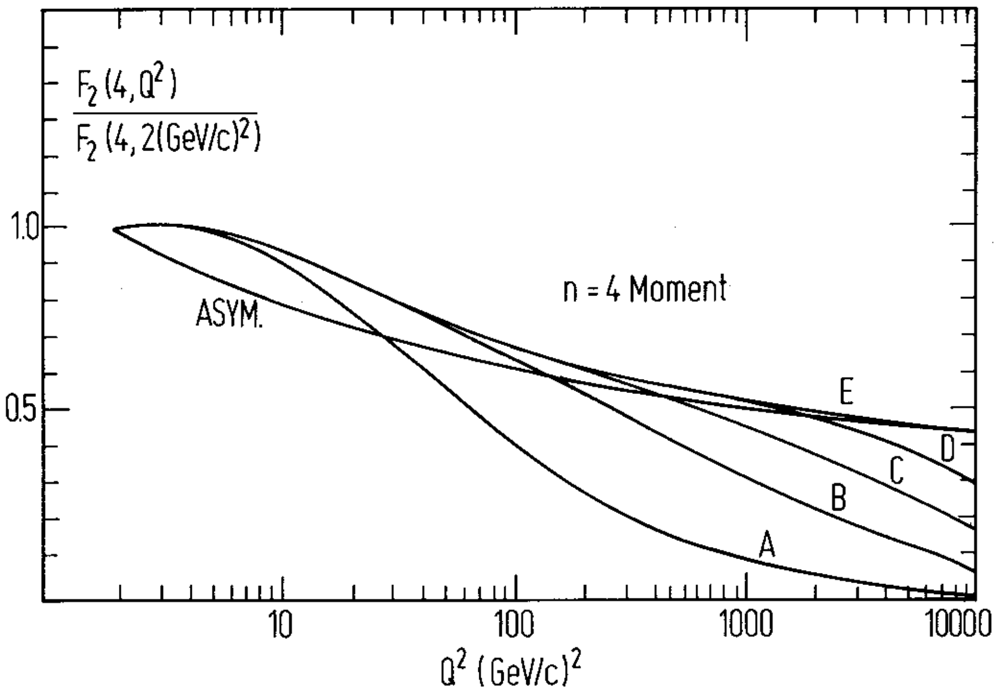
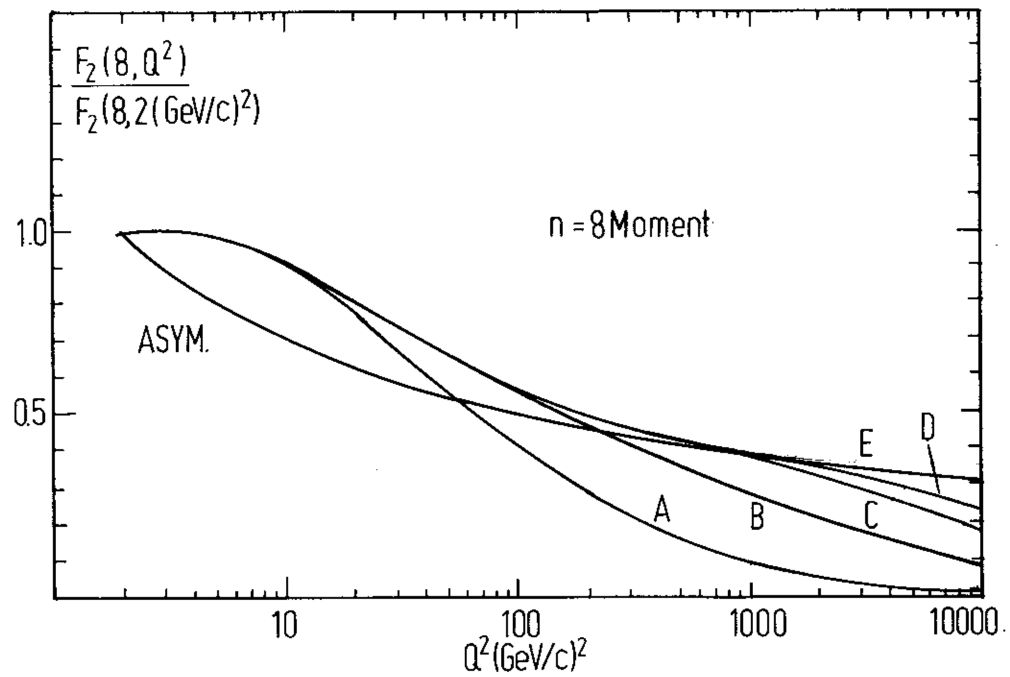


Fig. 1a



$\frac{F_2(4,Q^2)}{F_2(4,2(\text{GeV/c})^2)}$
1.0
0.5
n = 4 Moment
ASYM.
A
B
C
D
E
10
100
1000
10000
Q^2 (GeV/c)^2

Fig.1b



$\frac{F_2(8,Q^2)}{F_2(8,2(\text{GeV/c})^2)}$
1.0
0.5
n = 8 Moment
ASYM.
A
B
C
D
E
10
100
1000
10000
Q^2(GeV/c)^2

Fig.1c