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Stability of a Colliding Beam in a Linear Collider

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Abstract

Plasma fluid instabilities of a beam collision in a linear collider are studied. A dispersion relation for the instability of arbitrary order is derived from both the fluid equation and the kinetic equation. The analysis shows that for a single collision these instabilities are negligible even for a large disruption parameter (e.g., ~ 10). However, in the multiple-bunch collision case, the cumulative effect of small growth of instability may reduce the luminosity in spite of the small disruption parameter for each bunch.

1 Introduction

When two beams collide in a linear collider, the intense electromagnetic fields of each beam can deflect the particle trajectories of the other beam, causing a mutual pinch of the two beams[1,2]. If the beams are sufficiently dense, particles would execute plasma oscillations during beam passage where plasma instabilities may occur. The simulation results, however, show that the growth of instabilities is negligible for the disruption parameter D less than 32. The theoretical study has been done by Fawley and Lee[3] for the transverse dipole instability (kink instability), which occurs when the centers of mass of the two beams do not coincide. Fawley and Lee show that for a small disruption parameter and a small transverse offset, this instability does not cause significant reduction of luminosity. This growth of instability is limited by two natures of a beam collision: the short duration of the interaction and the single collision. The second nature, however, will be broken for the proposed scheme of collision to use trains of closely-spaced bunchlets instead of single pair of $e^+ - e^-$ bunches[4]. In this multiple-bunch collision case, an instability would be allowed to grow successively at each collision, and may reduce the luminosity in spite of a small disruption parameter for each bunch.

The purpose of this paper is a theoretical investigation of the transverse (plasma) fluid instabilities due to beam-beam interaction. We consider two idealized models for beam streams. The first model is a so-called "two-string model"[5] in which beams are treated as fluids and are represented by two elliptical cylinders with a uniform charge distribution across the cross section. Fawley and Lee use this model. The dispersion relation for the dipole instability is derived in Sec.2. The second model is a "ribbon beam model"[6] in which beams have a flat distribution like a ribbon in the real space, and have a uniform distribution in phase space. The dispersion relation for instability of any order is derived starting with the kinetic equation for a Vlasov plasma. This is done in Sec.3. Comparison of the two dispersion relation shows that the result does not depend on the model very much. In both models, the evolution of the beam cross section during the interaction is not taken into account. The condition for this assumption is discussed in Appendices A and B in terms of the strong-weak beam picture. In the final section, we briefly discuss the cumulative growth of instability in the multiple-bunch collision case.

2 Kink instability

Suppose that two elliptically cylindrical beams moving towards one another collide with a relative vertical displacement Y in their centers of mass. See Fig.1. This displacement induces surface charges which generate an electromagnetic field inside the beams. Each beam is assumed to have the same number of particles N per unit length of the beam, with each beam having opposite charges. The equations of motion for the two beams follow from the two relativistic fluid equations:

$$m\gamma N(\frac{\partial v_y}{\partial t} + v\frac{\partial v_y}{\partial z}) = -NF_y, \quad (1)$$

$$m\gamma N(\frac{\partial w_y}{\partial t} - v\frac{\partial w_y}{\partial z}) = NF_y, \quad (2)$$

where v_y and w_y are the vertical velocities of the electron and the positron beams, respectively, m is the mass of a particle, γ is the Lorentz factor, v ($\simeq c$, the velocity of light) is the longitudinal velocity of beams, and F_y is the Lorentz force produced by the surface charge. We study cold beams, i.e., zero emittance beams, so that the term of plasma pressure vanishes from the fluid equations. If we assume that the beams have a uniform charge distribution across the cross section, and that no redistribution of charges occurs during the collision, the Lorentz force is given by[7]

$$F_y = \frac{8Ne^2}{4\pi\epsilon_0}\frac{Y}{r_y(r_x + r_y)}, \quad (3)$$

where e is the elementary charge, ϵ_0 is the dielectric constant of vacuum, and r_x and r_y are the semi axes of the beams in the x and y directions. In terms of the displacements, y_1 and y_2, of the center of mass of the electron and positron beams from the beam axis, Eqs.(1) and (2) are written as

$$(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z})^2 y_1 = -\frac{8Nr_e c^2}{r_y(r_x + r_y)\gamma}(y_1 - y_2), \quad (4)$$

$$(\frac{\partial}{\partial t} - v\frac{\partial}{\partial z})^2 y_2 = -\frac{8Nr_e c^2}{r_y(r_x + r_y)\gamma}(y_2 - y_1), \quad (5)$$

where r_e ($= \frac{e^2}{4\pi\epsilon_0 mc^2}$) is the classical electron radius.

We consider the space-time variation of y_1 and y_2 of the form $\exp(ikz - i\omega t)$. Then Eqs.(4) and (5) become algebraic equations, and from the condition for a non-trivial solution, one derives the dispersion relation

$$1 = \frac{\omega_y^2}{(\omega + vk)^2} + \frac{\omega_y^2}{(\omega - vk)^2}, \quad (6)$$

where

$$\omega_y = \sqrt{\frac{8Nr_e c^2}{r_y(r_x + r_y)\gamma}} \tag{7}$$

is the relativistic plasma frequency. The solutions for this quartic equation are given by

$$\omega^2 = [(kv)^2 + \omega_y^2] \pm \sqrt{4\omega_y^2(kv)^2 + \omega_y^4}. \tag{8}$$

If the second term in RHS is larger than the first term in the square bracket, then ω can be pure imaginary. The criterion for the onset of instabilities is, therefore,

$$|k| < \sqrt{2}\frac{\omega_y}{v}, \tag{9}$$

and the maximum growth rate is obtained when

$$k_{max} = \pm\frac{\sqrt{3}}{2}\frac{\omega_y}{v}, \tag{10}$$

and its value is

$$\omega_{max} = \frac{i}{2}\omega_y. \tag{11}$$

It follows that the initial displacement may have grown in the optimal condition by a factor of

$$g_{max} = \exp(\frac{\omega_y l}{4v}) \tag{12}$$

after the interaction time $t = l/2v$ where l is the length of the bunch.

It should be mentioned here that the exponent in Eq.(12) is half that of Fawley and Lee, apart from the phase mixing damping term which is neglected in the present analysis. The reason is that their introduction of the longitudinal bunch coordinates takes the interaction time as l/v, not $l/2v$. Since the correct growth factor is the square root of theirs, the kink instability in fact is quite less severe than their estimation.

For Gaussian beams, using $D \cong 10(\frac{\omega_y l}{2\pi c})^2$[1], the growth factor (12) is expressed in terms of the disruption parameter by

$$g_{max} \approx \exp(\frac{\sqrt{D}}{2}). \tag{13}$$

The condition $g_{max}Y_0 < \sigma_y$ where σ_y is the vertical rms beam size sets a tolerance on the initial displacement Y_0 of the beams as

$$\frac{Y_0}{\sigma_y} < \exp(-\frac{\sqrt{D}}{2}). \tag{14}$$

For example, for $D = 1$, the fractional displacement is allowed up to 60%.

In the above model, the evolution of the beam envelope due to the pinch effect is not taken into account. In fact, as shown in Appendix A with the strong-weak picture, the beam envelope r_p ($p = x, y$) can schrink by an optimal factor of $\frac{\omega_p}{c}\beta_p^*$ where β_p^* is the beta function at the collision point. The simplification of the steady envelope is justified in the regime where this factor is around unity.

3 Kinetic theory

For the systematic investigation of the instability of arbitrary order, we use the Vlasov technique[8] in which the particle distribution around the equilibrium distribution is solved in a self-consistent manner. However, for simplicity, we restrict ourselves to the one-dimensional case in which the beam motion occurs only in one degree of freedom (vertical direction hereafter). A suitable beam profile for this purpose is the ribbon beam in which the beam is assumed to have a flat distribution, and the vertical force does not depend on the horizontal position. The cross section of the beam is shown in Fig.2.

The equation of motion for a particle in beam 1 is

$$\frac{d^2y}{dt^2} = -\frac{2\pi N r_e c^2}{w\gamma}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} SIGN(y-y_2)\Lambda_2(y_2,\dot{y_2},z,t)dy_2 d\dot{y_2}, \tag{15}$$

where w is the horizontal half width of the ribbon beam at the collision point, Λ_i is the distribution function of the beam i normalized such as

$$\int\int \Lambda_i dy d\dot{y} = 1, \tag{16}$$

and the function $SIGN$ means the sign of the argument. We have the same equation for a particle in beam 2 with the index 1 replaced with 2. We assume that Λ_i can be decomposed into the common equilibrium distribution Λ_0 which does not change during the collision and a small nonequilibrium component $\Delta\Lambda_i$:

$$\Lambda_i = \Lambda_0(y,\dot{y}) + \Delta\Lambda_i(y,\dot{y},z,t). \tag{17}$$

Here, we neglect the dependences of the distribution Λ_0 on z and t for mathematical simplicity. The criterion for this assumption is shown in Appendix B. Then the equation of motion (15) can be written as

$$\frac{d^2y}{dt^2} = -F(y) - \Delta F(y,z,t), \tag{18}$$

where F and ΔF come from Λ_0 and $\Delta\Lambda_2$, respectively. The incoherent motion of particle is determined by $F(y)$, and ΔF gives the coherent perturbation on it. Moreover, we assume that the equilibrium part of the force is nearly linear in y. The effect of the small nonlinearity is approximated by the amplitude dependence of the oscillation frequency. It follows that the incoherent motion of a particle is expressed in terms of action-angle variables I, ψ by

$$y = \sqrt{\frac{2I}{\omega_y}}\cos\psi, \tag{19}$$

$$\dot{y} = -\sqrt{2I\omega_y}\sin\psi, \tag{20}$$

where $\omega_y = \omega_y(I)$. The amplitude dependence of ω_y is derived later.

The linearized Vlasov equations with respect to the perturbation terms are

$$\frac{\partial\Delta\Lambda_1}{\partial t} + v\frac{\partial\Delta\Lambda_1}{\partial z} + \psi'\frac{\partial\Delta\Lambda_1}{\partial\psi} + I'\frac{\partial\Lambda_0}{\partial I} = 0, \tag{21}$$

$$\frac{\partial\Delta\Lambda_2}{\partial t} - v\frac{\partial\Delta\Lambda_2}{\partial z} + \psi'\frac{\partial\Delta\Lambda_2}{\partial\psi} + I'\frac{\partial\Lambda_0}{\partial I} = 0. \tag{22}$$

If we substitute ψ' and I' given by Hamilton's equations, Eqs.(21) and (22) become

$$\begin{aligned} \frac{\partial \Delta\Lambda_{1,2}}{\partial t} &\pm v\frac{\partial \Delta\Lambda_{1,2}}{\partial z} + \omega_y(I)\frac{\partial \Delta\Lambda_{1,2}}{\partial \psi} - \frac{2\pi N r_e c^2}{w\gamma}\sqrt{\frac{2I}{\omega_y}}\sin\psi \\ &\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} SIGN(y-\xi)\Delta\Lambda_{1,2}(\xi,\dot{\xi},z,t)d\xi d\dot{\xi}\frac{d\Lambda_0}{dI} = 0, \end{aligned} \quad (23)$$

where we have replaced the partial derivative of Λ_0 by the total derivative, using the fact that the unperturbed Hamiltonian is a function only of I. Although Λ_0 should be decided so as to satisfy the Vlasov equation for Λ_0, we here use a water-bag model of Λ_0: [1]

$$\Lambda_0 = \begin{cases} \dfrac{1}{2\pi I_0} & \text{for } I \le I_0, \\ 0 & \text{for } I > I_0. \end{cases} \quad (24)$$

It is obvious at a glance that the solution of Eq.(23) has the form

$$\Delta\Lambda_1 = \delta(I-I_0)\sum_{m=-\infty}^{\infty} f_m e^{im\psi}e^{ikz-i\omega t}, \quad (25)$$

$$\Delta\Lambda_2 = \delta(I-I_0)\sum_{n=-\infty}^{\infty} g_n e^{in\psi}e^{ikz-i\omega t}. \quad (26)$$

Substituting Eqs.(25) and (26) into Eq.(23), multiplying it by $e^{-im\psi}$ or $e^{-in\psi}$, and integrating both sides over I and ψ, we obtain

$$(-i\omega + ivk + im\omega_y(I_0))f_m - \frac{\omega_{y0}}{8\pi}\sum_{n=-\infty}^{\infty} M_{mn}g_n = 0, \quad (27)$$

$$(-i\omega - ivk + in\omega_y(I_0))g_n - \frac{\omega_{y0}}{8\pi}\sum_{m=-\infty}^{\infty} M_{nm}f_m = 0, \quad (28)$$

where

$$\omega_{y0}^2 = \frac{8Nr_ec^2}{w\gamma h} \quad (29)$$

with the half height of the beam $h = \sqrt{2I_0/\omega_y}$, and we have defined the matrix $\mathbf{M}$ whose elements are given by

$$M_{mn} = \begin{cases} \dfrac{-32im}{[(n+m)^2-1][(n-m)^2-1]} & \text{for } n+m = \text{even}, \\ 0 & \text{for } n+m = \text{odd}. \end{cases} \quad (30)$$

Defining new coefficients $f_m^c = f_m + f_{-m}$ and $g_n^c = g_n + g_{-n}$, then the properties of the matrix $\mathbf{M}$:

$$\begin{aligned} M_{-m,n} &= -M_{m,n}, \\ M_{m,-n} &= M_{m,n} \end{aligned} \quad (31)$$

[1] Some disadvantages arising from the use of water-bag model are listed up in Ref.9

lead to the combined form of the m^{th} and $(-m)^{th}$ modes

$$f_m^c = \frac{i\omega_{y0}}{4\pi} \frac{m\omega_y(I_0)}{[(\omega - vk)^2 - (m\omega_y(I_0))^2]} \sum_{n=1}^{\infty} M_{mn} g_n^c, \tag{32}$$

$$g_n^c = \frac{i\omega_{y0}}{4\pi} \frac{n\omega_y(I_0)}{[(\omega + vk)^2 - (n\omega_y(I_0))^2]} \sum_{m=1}^{\infty} M_{nm} f_m^c. \tag{33}$$

The condition for a non-trivial solution for f_m^c and g_n^c yields the dispersion relation

$$det \begin{pmatrix} \mathbf{I} & -\mathbf{M}^- \\ -\mathbf{M}^+ & \mathbf{I} \end{pmatrix} = 0, \tag{34}$$

where $\mathbf{I}$ is the unit matrix, and $\mathbf{M}^\pm$ are the matrices with elements $(m, n \geq 1)$

$$M_{mn}^\pm = \frac{i\omega_{y0}}{4\pi} \frac{m\omega_y(I_0)}{[(\omega \pm vk)^2 - (m\omega_y(I_0))^2]} M_{mn}. \tag{35}$$

At last, we give the unperturbed Hamiltonian calculated from the equilibrium distribution function (24)

$$H_0 = \frac{8}{9\pi} I_0 \omega_{y0} [(7 + \frac{I}{I_0}) E(\sqrt{\frac{I}{I_0}}) - 4(1 - \frac{I}{I_0}) K(\sqrt{\frac{I}{I_0}})], \tag{36}$$

where $K(x)$ and $E(x)$ are the complete elliptic integrals of the first and the second kinds, respectively. The dynamic tune is

$$\omega_y(I) = \frac{\partial H_0}{\partial I} = \frac{4}{3\pi} \omega_{y0} [(1 + \frac{I}{I_0}) E(\sqrt{\frac{I}{I_0}}) + (1 - \frac{I}{I_0}) K(\sqrt{\frac{I}{I_0}})], \tag{37}$$

from which we find that ω_{y0} is the plasma frequency at the zero amplitude, and

$$\omega_y(I_0) = \frac{8}{3\pi} \omega_{y0}. \tag{38}$$

Since the system is determined only by a single frequency at the edge of the distribution, $\omega_y(I_0)$ will be written merely as ω_y hereafter for simpler notation.

The structure of the matrix $\mathbf{M}$ suggests the following two things;

1. There is no coupling between modes of different polarity: e.g., the dipole mode cannot couple to the quadrupole mode.

2. The coupling between different modes with the same polarity decreases sharply as the distance from the principal diagonal increases.

Accordingly, if we disregard all the non-diagonal terms of the matrix $\mathbf{M}$, the dispersion relation becomes

$$1 = \frac{m^2\omega_y^2}{(\omega + vk)^2} + \frac{m^2\omega_y^2}{(\omega - vk)^2} - \frac{m^4\omega_y^4}{(\omega^2 - v^2k^2)^2}(1 - \frac{\alpha_m^2}{m^2}) \tag{39}$$

with

$$\alpha_m = \frac{3}{32} |M_{mm}|, \tag{40}$$

which has a form similar to Eq.(6). The condition for the onset of instabilities has, hence, a form similar to Eq.(9):

$$m\frac{\omega_y}{v}\sqrt{(1-\frac{\alpha_m}{m})} < |k| < m\frac{\omega_y}{v}\sqrt{(1+\frac{\alpha_m}{m})}, \tag{41}$$

and an instability with the maximum growth rate

$$\omega_{max} = \frac{i}{2}\omega_y\alpha_m \tag{42}$$

occurs at the wave number

$$k_{max} = \pm m\frac{\omega_y}{v}\sqrt{1-\frac{\alpha_m^2}{4m^2}}. \tag{43}$$

Substituting the actual values of $\alpha_m(\alpha_1 = 1, \alpha_2 = \frac{2}{5}, \ldots)$ into Eq.(42), we find for a dipole instability ($m = 1$)

$$\omega_{max} = \frac{i}{2}\omega_y \tag{44}$$

which agrees with ω_{max} obtained in Sec. 2 with the two-string model, and for a quadrupole instability ($m = 2$)

$$\omega_{max} = \frac{i}{5}\omega_y. \tag{45}$$

Let us examine the limit on D determined by the quadrupole instability for a Gaussian beam. If one requires that the maximum beam size should not be larger than twice the initial size at any rate, one gets

$$D < [5\log(\delta/\sigma_y)]^2, \tag{46}$$

where δ is the quadrupole displacement of the beam envelope from the equilibrium one. For example, for $\delta/\sigma_y = 37\%$, D must be less than 25.

4 Discussion

The growth of (plasma) fluid instability of a beam collision is negligible for the disruption parameter of the order of 10. This is, as pointed out somewhere[1], due to the fact that the two beams pass through one another before plasma instabilities grow up amply, which takes several plasma oscillations of particles. However, if another beam comes in to collide, the instabilities may have another chance to build up their amplitudes. Since the force exerted on the beam is the restoring force, the coherent motion of the beam traversing the sequence of incoming beams and drift spaces is the pseudo harmonic oscillation with a growing amplitude. It is readily seen that the oscillation amplitude of instability can be enhanced after b collisions by a factor of g^b in the optimal condition for the growth where g is the growth factor for a single collision. This means that even if the disruption parameter is only 0.1 for each bunch, the amplitude of dipole instability may reach to the same level after 18 collisions as for the disruption limit $D \approx 32$ of a single collision. Besides, since the luminosity enhancement due to the pinch effect is hardly expected for such a small D, the luminosity may drop off significantly as a bunch collides with one after another bunch of an incoming bunch-train.

The scaling of the growth factor with the collision number b depends on the case. Under the condition that the luminosity per bunch-train and the density of particles per bunch are kept constant[2], the parameters scale as

$$\begin{aligned} N &\propto 1/b, \\ l &\propto 1/b, \\ D &\propto 1/b, \end{aligned}$$

with the result that the growth factor becomes independent of b. Therefore, if the tolerance of the initial perturbation is satisfied for D of $b = 1$, we can assure that the instabilities are negligible.

In the end, it should be mentioned that the growth factor for a Gaussian beam may be smaller than the estimation owing to Landau damping due to a spread in the plasma frequency.

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Appendix A: Evolution of envelope

In this appendix, we consider the evolution of the envelope of an elliptically cylindrical beam which travels through an incoming beam with the same cross section. We take a strong-weak picture, i.e., the incoming beam (the strong beam) is not perturbed by the beam of concern (the weak beam). The equation of motion for a particle in a weak beam is

$$\frac{d^2y}{dz^2} = -k_y^2 y, \tag{47}$$

where

$$k_y = \frac{\omega_y}{c} \tag{48}$$

is the plasma wave number. The solution can be expressed in the well-known form:

$$y = a\sqrt{\beta(z)}\exp(i\int_0^z \frac{dz}{\beta}), \tag{49}$$

where β must satisfy

$$2\beta\beta'' - \beta'^2 + 4\beta^2 k_y^2 = 4. \tag{50}$$

It is readily seen that the problem is identical to the propagation of the Twiss parameters in a transport line. Instead of an external guide field, the incoming beam here plays the role

[2]This holds almost for Montague's multiple-bunchlet collision scheme.

of a dynamic focussing quadrupole. Provided that $\beta' = 0$ at the collision point, the Twiss parameters are given by

$$\beta = \frac{1}{k_y^2\beta_y^*}(k_y^2\beta_y^{*^2}\cos^2\phi + \sin^2\phi), \tag{51}$$

$$\alpha = \frac{\sin 2\phi}{2k_y\beta_y^*}(k_y^2\beta_y^{*^2} - 1), \tag{52}$$

$$\gamma = \frac{1}{\beta_y^*}(k_y^2\beta_y^{*^2}\sin^2\phi + \cos^2\phi), \tag{53}$$

where β_y^* is the beta function at the collision point, and

$$\phi = k_y(z - z_0). \tag{54}$$

The point z_0 where the particle of concern collides with the head of the incoming beam depends on the position of particle in the beam. If we take the coordinate such that the head of beam arrives at $z = 0$ at $t = 0$, then $z_0 = (z - vt)/2$ where v is the velocity of the beam.

The beam size oscillates according to $\sqrt{\beta}$. If $k_y^2\beta_y^{*^2}$ is larger than 1, the beam is at first compressed, i.e., pinched. If $l > \frac{\pi}{k_y} = \frac{\lambda_y}{2}$, the beam size schrinks at maximum at $z - z_0 = \frac{\lambda_y}{4}$ by a factor of $k_y^2\beta_y^{*^2}$, and then expands.

Appendix B: Pinch condition

The pinch condition can be generalized to any axisymetric beam[10]. In the steady state, the plasma pressure p of the beam must be balanced to the pinch force of the opposite beam:

$$\gamma\frac{dp}{dr} = nE_r - j_z B_\theta, \tag{55}$$

where n and j_z are the local charge density and the local current density of the beam at r, respectively, and E_r and B_θ are the electromagnetic fields created by the opposite beam. Since $nE_r = -j_z B_\theta$, and $B_\theta = \frac{\mu_0}{2\pi r}I(r)$ where $I(r) = \int_0^r 2\pi r j_z(r)dr$, Eq.(55) is written as

$$2\pi r\gamma\frac{dp}{dr} = -2\mu_0 I(r) j_z(r), \tag{56}$$

where μ_0 is the magnetic permeability. The plasma pressure is connected to the temperature T and the plasma density n by

$$p = nkT, \tag{57}$$

where k is the Boltzmann constant. In general, T as well as n may be a function of r. However if one neglects this r-dependence, and integrates the both sides of Eq.(56), one obtains Benett's pinch condition

$$\gamma NkT_{av} = \frac{\mu_0}{4\pi}I_0^2, \tag{58}$$

or, using $I_0 = Nev$,

$$Nr_e = \frac{kT_{av}\gamma}{mv^2}, \tag{59}$$

where T_{av} is the average temperature of the beam, N is the number of particles per unit length, and I_0 is the total current. The result, Eq.(59), is independent of the form of the charge distribution. If N is greater than the value given by Eq.(59), the beam is pinched.

For example, for a cylindrical beam with an uniform charge distribution over the cross section, one has

$$kT_{av} = \frac{\epsilon}{3\beta_y^*} mv^2 \tag{60}$$

with the emittance ϵ, which leads to

$$Nr_e = \frac{\epsilon\gamma}{3\beta_y^*}. \tag{61}$$

In fact, this pinch condition agrees with $k_y^2 \beta_y^{*2} = 1$ obtained in Appendix A, except that the factor 3 in Eq.(61) is replaced by 4. This is because the temperature has actually the following r-dependence

$$kT = \frac{mv^2}{2\beta_y^*}(\epsilon - \frac{r^2}{\beta_y^*}). \tag{62}$$

If one does the derivation again starting from Eq.(56) with the explicit r-dependence, one obtains exactly the same condition.

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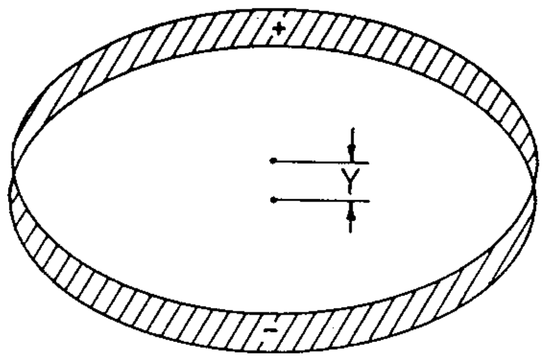

Figure 1: Collision of two beams with a relative displacement Y of their centers of mass.

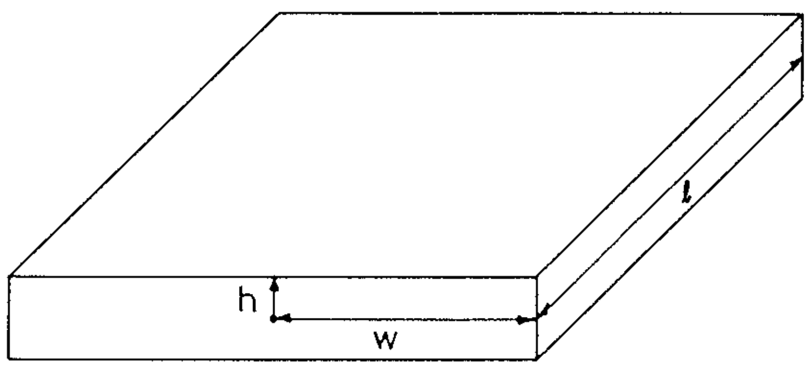

Figure 2: A ribbon beam.