DTP/98/60
DESY 98-156

DTP/98/60
DESY 98-156
hep-ph/9901420
January 1999

Υ photoproduction at HERA compared to estimates of perturbative QCD

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Abstract

We estimate the cross section for $\gamma p \to \Upsilon p$ by two independent methods. First, by studying the corrections to the naive leading-order QCD formula and, second, by using parton-hadron duality. The estimates are in good agreement with each other and with the recent measurements of the cross section at HERA.

The elastic photoproduction of Υ mesons has recently been measured for the first time both by the ZEUS [1] and H1 [2] collaborations. The observed cross section $B\sigma \equiv \sigma(\gamma p \to \Upsilon p)\ B(\Upsilon \to \mu^+\mu^-)$ is found to be rather large

$$B\sigma(\Upsilon) \ = \ 13.3 \ \pm \ 6.0 \, ^{+\,2.7}_{-\,2.3} \, \text{pb} \quad \text{at} \quad \langle W \rangle \ = \ 120 \ \text{GeV} \qquad \text{(ZEUS [1])}, \tag{1}$$

$$B\sigma(\Upsilon) \ = \ 16.0 \ \pm \ 7.5 \ \pm \ 4.0 \, \text{pb} \quad \text{at} \quad \langle W \rangle \ = \ 160 \ \text{GeV} \qquad \text{(H1 (preliminary) [2])} \tag{2}$$

where Υ stands for the sum over the $\Upsilon(1S), \Upsilon(2S)$ and $\Upsilon(3S)$ states and where $\langle W \rangle$ is the average photon-proton centre-of-mass energy. In both presentations it was noted that the measurements are higher than the predictions of various QCD models by about two standard deviations.

In this paper we estimate elastic Υ photoproduction from perturbative QCD by two independent methods. Both calculations are found to give cross sections which agree well with the data, and so we conclude that there is no disagreement with QCD. The first method is to study the main corrections to the naive QCD formula [3] for heavy vector meson photoproduction. The second method makes use of parton-hadron duality similar to [4], where ρ electroproduction at HERA was described successfully.

Method I

The first method is based on the leading-order expression for the photoproduction of a heavy vector meson of mass M_V [3]

$$\left.\frac{d\sigma}{dt}\,(\gamma p \to V p)\right|_{t=0} \ = \ \frac{\alpha_S^2 \Gamma_{ee}^V}{3\alpha M_V^5}\, 16\pi^3 \left[x g\left(x, \frac{M_V^2}{4}\right)\right]^2 \tag{3}$$

where Γ_{ee}^V is the partial width of the $V \to ee$ decay, α_S is the QCD coupling, $\alpha = 1/137$ is the QED coupling and $g(x,\mu^2)$ is the gluon density measured at $x = M_V^2/W^2$ and the scale $\mu = m_Q \simeq M_V/2$. We discuss below four types of corrections to this naive formula.

(a) Relativistic corrections

A non-relativistic wave function for the heavy vector meson was used to derive (3), so first we quantify the size of the relativistic corrections. For J/ψ photoproduction these corrections have been the subject of debate [5, 6, 7]. Since the velocity of the charm quarks can be sizeable in the J/ψ meson $\left(\langle v^2 \rangle \simeq \frac{1}{4}\right)$ the kinematical correction coming from the quark propagators suppresses the J/ψ cross section by a factor of about 2 according to Ref. [6] or up to a factor of 10 in [7]. However it was shown in [6] that to a good approximation this correction is nullified in the cross section formula (3) since it has been written in terms of the J/ψ mass, instead of the current quark mass ($2m_c < M_\psi$) which should have been used in perturbative QCD. So it turns out that there are practically no $\langle v^2 \rangle$ corrections to the leading order result (3). This result is

consistent with a more detailed study by Hoodbhoy [8]. Strictly speaking to estimate the $\langle v^2 \rangle$ corrections we should also take into account the contributions coming from more complicated components of the vector meson wave function which contain one or more gluons besides the $q\bar{q}$ pair. These components were included in a self-consistent way by Hoodbhoy [8], who showed that the total $O(v^2)$ relativistic correction (including corrections to the quark propagator and extra gluons in the J/ψ wave function) to the naive cross section formula (3) amount to at most only 7% for J/ψ photoproduction. The correction should be even smaller for Υ production. Therefore we neglect these relativistic effects below.

(b) **Real part**

Only the imaginary part of the $\gamma p \to Vp$ amplitude is expressed in terms of $xg(x,\mu^2)$ in (3). To restore the real part we may use the approximation

$$\mathrm{Re}\, A/\mathrm{Im}\, A \;\simeq\; \pi\lambda/2 \tag{4}$$

where

$$\lambda \;=\; \frac{\partial \log A}{\partial \log s} \;=\; \frac{1}{xg(x,\mu^2)}\,\frac{\partial(xg(x,\mu^2))}{\partial \ln(1/x)}. \tag{5}$$

This approximation is valid for small λ, say $\lambda < \frac{1}{2}$, for the even signature amplitude A. The real part correction factor

$$C_b \;=\; \left(1 \;+\; \frac{\pi^2\lambda^2}{4}\right) \;=\; 1.43(1.36) \text{ or } 1.54(1.46) \tag{6}$$

for $\langle W \rangle = 120(160)$ GeV and scale $\mu^2 = 25$ GeV^2, according to whether the MRS(R2) [9] or the GRV [10] gluon distribution is used.

(c) **The effect of off-diagonal partons**

At leading order the elastic photoproduction of vector mesons is mediated by two-gluon exchange. Strictly speaking the amplitude is proportional to the off- diagonal gluon distribution $x'g(x,x')$, where the momentum fraction x' carried by the "second" gluon is much smaller than the fraction $x \simeq M_V^2/W^2$ carried by the "first" gluon. The effect, $R = x'g(x,x')/xg(x)$, was estimated in Ref. [11]. The correction is almost negligible for ρ production, giving about 10% enhancement of the J/ψ amplitude, but is much more important for Υ photoproduction since the scale $\mu^2 \simeq M_\Upsilon^2$ and the value of $x = M_\Upsilon^2/W^2$ are much larger. In fact the correction factor to the cross section formula (3) for Υ production is [11]

$$C_c \;=\; R^2 \;\simeq\; (1.4)^2 \;\simeq\; 2. \tag{7}$$

(d) **NLO corrections**

Finally we have NLO corrections which come from an explicit integration over the loop which corresponds to the convolution of the exchanged gluons with the vector meson wave

function. The integral is not of a pure logarithmic form, which was assumed in deriving the leading order result (3). The typical range of integration corresponds to the scale $\mu^2 = \frac{1}{4}M_V^2$ and the non-logarithmic contribution is well approximated by a NLO correction with a coefficient of about $\frac{1}{2}$ [6]. That is the correction can be written as

$$C_d \simeq \left(1 + \alpha_S\left(\tfrac{1}{4}M_V^2\right)/2\right) \simeq 1.2 \tag{8}$$

for Υ photoproduction.

We are now in the position to estimate the cross section for elastic Υ photoproduction. If we assume that the elastic Υ differential cross section, $d\sigma/dt \sim \exp(bt)$, has a slope $b = 4$ GeV^{-2}, as for elastic J/ψ photoproduction, then the leading order formula gives

$$B\sigma(\Upsilon_{1S})|_{\rm LO} \simeq 1.7\,{\rm pb} \tag{9}$$

at $\langle W\rangle = 120$ GeV. Taking into account the corrections $C_{b,c,d}$ we obtain

$$B\sigma(\Upsilon_{1S}) \simeq 6\,{\rm pb}. \tag{10}$$

However the cross sections measured by the ZEUS and H1 collaborations, (1) and (2), did not select the pure $\Upsilon(1S)$ state, but rather the data were integrated over an interval of the $\mu^+\mu^-$ mass which includes at least the $1S$, $2S$ and $3S$ resonances. We estimate the combined contributions of the $2S$ and $3S$ states to be 40% of the $\Upsilon(1S)$ cross section, in agreement with the measurement of the CDF collaboration [12]. Thus multiplying (10) by a factor of 1.4 we estimate

$$B\sigma(\Upsilon) = 8.4\,{\rm pb}, \tag{11}$$

which is compatible with the ZEUS measurement given in (1). Going from $\langle W\rangle = 120$ GeV to 160 GeV, the cross section is predicted to increase by a factor of 1.6, due to the larger gluon density at smaller x, and so the prediction is in agreement with the preliminary H1 measurement quoted in (2).

Method II

The second method that we use to estimate the elastic photoproduction of vector mesons from perturbative QCD is based on parton-hadron duality. The procedure [4] is to calculate the amplitude for open $q\bar{q}$ production, then to project the amplitude onto the $J^P = 1^-$ $q\bar{q}$ state and finally to integrate the cross section over an appropriate interval ΔM of the mass of the $q\bar{q}$ pair which includes the resonance peak. As there are almost no other possibilities for hadronization at $M_{q\bar{q}} \simeq M_V$ we should obtain a reasonable estimate of the cross section for vector meson production. This framework was found [4] to describe successfully the energy and Q^2 dependence of ρ-meson electroproduction ($\gamma^* p \to \rho p$), both for longitudinally and transversely polarized ρ mesons, including the Q^2 dependence of the σ_L/σ_T ratio.

The computer code used here to estimate the $\gamma p \to \Upsilon p$ cross section takes into account all the corrections mentioned above, with the exception of the enhancement due to the use of

off-diagonal gluons. (The enhancement is negligible for ρ production.) We therefore include the extra factor $C_c \simeq 2$ of (7). As the recent measurements (1) and (2) cover a rather large $\mu^+\mu^-$ mass interval[1] ($\Delta M \sim 2$ GeV) over the Υ resonances a description based on parton-hadron duality might even be more appropriate to describe the data.

In practice we have to extend our code to include the mass m of the heavy b quark. First we restore the mass term $m\delta_{\lambda\lambda'}$ in the expression for the $\gamma \to q\bar{q}$ matrix element, see Eq. (32) of Ref. [13]. Here we will use the notation $\lambda = i = +, -$ and $\lambda' = i' = +, -$ for the helicities of the quark and antiquark. The cross section is then given by (see also [13])

$$\left.\frac{d^2\sigma^T}{dM^2dt}\right|_{t=0} = \frac{\pi^2 e_q^2 \alpha}{3(Q^2+M^2)^2} \int 2dz \sum_{i,i'} |B_{ii'}|^2 \tag{12}$$

where the helicity amplitudes are (for photon helicity +1)

$$B_{++} = \frac{mI_L}{2\sqrt{z(1-z)}}, \qquad B_{--} = 0 \tag{13}$$

and

$$B_{+-} = \frac{-zk_TI_T}{\sqrt{z(1-z)}}, \qquad B_{-+} = \frac{(1-z)k_TI_T}{\sqrt{z(1-z)}}. \tag{14}$$

The variable z is the momentum fraction carried by the quark and k_T is its transverse momentum. The integrals I_L, I_T are defined in [13]. The second modification due to the mass of the quark is associated with the fact that the $\gamma \to q\bar{q}$ helicity amplitudes $B_{ii'}$ are defined in the proton rest frame (pRF), while the projection onto the $J^P = 1^-$ state was done using the quark helicities in the $q\bar{q}$ rest frame ($q\bar{q}$RF). Unfortunately helicity is not a good quantum number for a heavy quark. It can be changed by a Lorentz boost. So we have to compute the helicity amplitudes $A_{jj'}$ in the $q\bar{q}$RF in terms of $B_{ii'}$ in the pRF. We have

$$A_{jj'} = \sum_{i,i'} c_{ij}\, c_{j'i'}\, B_{ii'} \tag{15}$$

where we will calculate the coefficients (c_{ij} for the quark and $c_{j'i'}$ for the antiquark) via the polarized quark density matrix ρ. For a quark with 4-momentum k_μ and polarisation vector a_μ (satisfying $a^2 = -1$ and $a.k = 0$) we have [14]

$$\rho = \frac{(\not{k} + m)}{2}\,\frac{(1+\gamma_5\not{a})}{2}, \tag{16}$$

where $(1+\gamma_5\not{a})/2$ projects onto the state with polarisation vector a_μ. For the states with helicities $j = \pm$ in the $q\bar{q}$RF these vectors take the form

$$a_\mu^j = (a_0; \boldsymbol{a}_T, a_z) = (a_0; \boldsymbol{a}) = \pm(k; k_0\boldsymbol{k}/k)/m \tag{17}$$

[1]Note that in the parton-hadron duality approach ΔM is related to the formation time rather than to the width of the resonances. The $\gamma p \to \Upsilon p$ data are integrated over an interval larger than the formation time but which includes at least the first three Υ states and possibly some small contribution from the $b\bar{b}$ continuum.

where $k = |\boldsymbol{k}|$. Let b^i_μ be the analogous polarisation vectors describing helicities $i = \pm$ in the pRF. After the boost from the pRF to the $q\bar{q}$RF these vectors are given by

$$b^i_\mu \;=\; \pm\left(\frac{M}{2m} \;-\; \frac{m}{zM};\; \frac{\boldsymbol{k}_T}{m},\; \frac{M}{m}\left(z-\tfrac{1}{2}\right) \;+\; \frac{m}{zM}\right) \tag{18}$$

where M is the mass of the $q\bar{q}$ pair, $z = \frac{1}{2} + k\cos\theta/M$ and θ is the quark decay angle in the $q\bar{q}$RF.

Now that we have all the polarisation vectors in the $q\bar{q}$RF we can calculate the product of the two different projectors[2]

$$\tfrac{1}{4}\mathrm{Tr}\left[(1+\gamma_5\not{a}^j)(1+\gamma_5\not{b}^i)\right] \;=\; 1-(a.b) \;=\; 2c^2_{ij}. \tag{19}$$

Thus we have

$$c_{++} \;=\; c_{--} \;=\; c_{+-} \;=\; -c_{-+} \;=\; \sqrt{\frac{1-(a.b)}{2}}, \tag{20}$$

where c_{-+} is chosen to be negative to ensure the orthogonality of the b^i states when expressed in the a^j basis. Note that if the quark mass $m \to 0$ then

$$a^\pm_\mu \;\to\; b^\pm_\mu \;=\; \pm\, k_\mu/m \tag{21}$$

leading to the unit matrix $c_{ij} = \delta_{ij}$.

The procedure is therefore to decompose the original amplitude $B_{ii'}$ in the pRF in terms of the amplitudes $A_{jj'}$ with helicities $j, j' = +, -$ in the $q\bar{q}$RF, as in (15). The resulting helicity amplitudes are projected onto the $J^P_m = 1^-_m$ $q\bar{q}$ states so as to obtain the production amplitudes in spin states $|1,m\rangle$ with $m = 0, \pm 1$

$$T_m \;=\; \sum_{j,j'} \sqrt{\frac{3}{2}} \int_{-1}^{1} d\cos\theta\; A_{jj'}\; d^1_{1m}(\theta)\; \delta_{m,(j-j')}. \tag{22}$$

The cross section of diffractive $J^P = 1^-$ $q\bar{q}$ pair production is then obtained by summing over the $m = 0, \pm 1$ amplitudes squared in an analogous way to that described in Ref. [4].

The expressions for $I_{L,T}$ of (13) and (14) were written in [13] for the (dominant) imaginary part of the amplitude. They are integrals over the gluon transverse momentum ℓ_T with weight w corresponding to the $q\bar{q}$-loop

$$\mathrm{Im}I_{L,T} \;=\; \int d\ell^2_T\; f(x,\ell^2_T)\; w_{L,T}(\ell^2_T, Q^2, M^2 \cdots) \tag{23}$$

where f is the unintegrated gluon distribution

$$f(x,\ell^2_T) \;=\; \frac{\partial x g(x,\ell^2_T)}{\partial \ell^2_T}. \tag{24}$$

[2] We omit the projector $\frac{1}{2}(\not{k}+m)$ of (16) so that the trace includes both quark and antiquark components. Hence the factor 2 in $2c^2_{ij}$ on the right hand side of (19).

The real part of the amplitude has been included using the same approximation as in Eqs. (4, 5), by replacing $f(x,\ell_T^2)$ with the derivative $(\pi/2)\partial f(x,\ell_T^2)/\partial \ln(1/x)$, that is we have computed

$$\mathrm{Re}I_{L,T} \;=\; \frac{\pi}{2}\int d\ell_T^2\,\frac{\partial f(x,\ell_T^2)}{\partial \ln 1/x}\,w_{L,T}(\ell_T^2,Q^2,M^2,\ldots). \tag{25}$$

Finally the NLO corrections (analogous to C_d of (8)) are approximated in the code by use of a K-factor, see [4, 13]. As in [13] (and (8)) the scale in the coupling $\alpha_S(\mu^2)$ was chosen to be $\mu^2 = M^2/4$.

To compare with the ZEUS data for $\gamma p \to \Upsilon p$ we integrate over the mass interval 8.9–10.9 GeV. We take the mass of the b quark to be $m_b = 4.6$ GeV and, as in Ref. [4], we use the MRS(R2) set of partons [9]. In this way we find

$$\sigma(\gamma p \to \textstyle\sum \Upsilon p) \;\simeq\; 620\ \mathrm{pb} \tag{26}$$

at $\langle W\rangle = 120$ GeV. This should be compared with the value 635 ± 310 pb obtained from the ZEUS measurement. The "ZEUS" value is calculated from (1) by assuming the ratios

$$B\sigma(\Upsilon_{1S}) \;:\; B\sigma(\Upsilon_{2S}) \;:\; B\sigma(\Upsilon_{3S}) \;\simeq\; 0.7 \;:\; 0.15 \;:\; 0.15, \tag{27}$$

which are consistent with the expectations of (3) and with the CDF data [12], and using the known leptonic branching ratios of the Υ states. What is the uncertainty in our cross section estimate (26) due to the choice of m_b? The PDG [15] give $4.1 < m_b < 4.4$ GeV for the running mass evaluated at $\mu = m_b$ in the $\overline{\mathrm{MS}}$ scheme, which corresponds to the range $4.5 < m_b < 4.8$ GeV for the pole mass. The latter mass is relevant for our perturbative calculations. If we were to use $m_b = 4.4$ or 4.8 GeV then the cross section (26) would become 900 or 380 pb respectively.

Fig. 1 shows the ZEUS and H1 data, together with our QCD estimates obtained from the two independent methods. The comparison is given for the $\gamma p \to \Upsilon(1S)p$ cross section as a function of the photon-proton centre-of-mass energy W. To obtain the $\Upsilon(1S)$ cross section for method II, in which we integrate over the mass interval 8.9–10.9 GeV, we divide the $\gamma p \to (\sum \Upsilon)p$ result by a factor 1.7. This factor is obtained using (27). For completeness we also show in Fig. 1 the prediction from the naive leading order formula (3) of method I as well as the result of method II without the corrections from the real part of the amplitudes. We note that the $\gamma p \to \Upsilon p$ process is particularly sensitive to off-diagonal gluon effects. These effects give an enhancement of about a factor of 2, which appears to be required by the data.

To estimate the ratio of Υ to J/ψ photoproduction we calculate both cross sections using all the corrections listed for method I. For $\langle W\rangle = 120$ GeV we find

$$\sigma(\Upsilon_{1S})/\sigma(J/\psi) \;\simeq\; 1/400 \tag{28}$$

where the MRS(R2) gluon has been used. This is near the lower bound observed by ZEUS [1]. To be specific we underestimate Υ production and overestimate[3] J/ψ production. We note

[3]Note that in Ref. [6] absorptive corrections were included and that a better description of J/ψ photoproduction was obtained. Here absorptive corrections were not discussed as they should be small for Υ photoproduction.

that if, for example, the steeper GRV gluon was used the ratio would be 1/1500. Clearly, as expected, the ratio is very sensitive to the x and scale dependence of the gluon in the region of very small x and low scale that is sampled by J/ψ production. In the case of Υ production we sample $x \sim 0.01$ and $Q^2 \sim 25$ GeV^2 where the predictions are much more stable with respect to different parametrizations of the gluon.

We conclude that the recent measurements of elastic Υ photoproduction are in good agreement with the expectations of perturbative QCD. Of course when more precise data become available the effects that are estimated here — relativistic corrections, NLO contributions, off-diagonal parton effects and the contribution of the real part of the amplitude — should be calculated in detail. The elastic photoproduction of J/ψ and Υ at HERA will be a good laboratory to probe these effects.

Acknowledgements

M.G.R. thanks the Royal Society, INTAS (95–311) and the Russian Fund of Fundamental Research (98 02 17629) for support.

Note added

While writing this note we received a paper on the same subject by Frankfurt, McDermott and Strikman [16]. Their procedure is comparable to our method I and also yields a cross section in agreement with the HERA data. However although our final values are similar, there are large, physically significant, differences in the various component factors. Unlike the present work, they do not make use of the result of Hoodbhoy [8] and so have a large suppression from the relativistic corrections. In [16] this suppression is compensated by a larger off-diagonal effect (2.6 as compared to 2), a larger real part and an enhancement due to re-scaling (giving an effective Q^2 of about 40 as compared to 25 GeV^2).

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Figure Caption

Fig. 1 The $\gamma p \to \Upsilon(1S)p$ cross section as a function of the γp centre-of-mass energy W. The data points are the ZEUS [1] and preliminary H1 [2] measurements. The horizontal lines indicate the range of W sampled by the measurements, the vertical lines the systematic and statistical errors added linearly. The continuous curves are obtained from the two QCD calculations (model I and II) described in the text. The dotted curve corresponds to the naive leading-order formula (3), the dashed curve shows our prediction for method II if we neglect the corrections from the real part of the amplitudes.