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PERTURBATIVE QCD- AND POWER-CORRECTED HADRON SPECTRA AND SPECTRAL MOMENTS IN THE DECAY $B \to X_s \ell^+ \ell^-$

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Leading order (in α_s) perturbative QCD and power $(1/m_b^2)$ corrections to the hadronic invariant mass and hadron energy spectra in the decay $B \to X_s \ell^+ \ell^-$ are reviewed in the standard model using the heavy quark expansion technique (HQET). In particular, the first two hadronic moments $\langle S_H^n \rangle$ and $\langle E_H^n \rangle$, $n = 1, 2$, are presented working out their sensitivity on the HQET parameters λ_1 and $\bar{\Lambda}$. Data from the forthcoming B facilities can be used to measure the short-distance contribution in $B \to X_s \ell^+ \ell^-$ and determine the HQET parameters from the moments $\langle S_H^n \rangle$. This could be combined with complementary constraints from the decay $B \to X \ell \nu_\ell$ to determine these parameters precisely.

1 Introduction

The semileptonic inclusive decays $B \to X \ell^+ \ell^-$, where $\ell^\pm = e^\pm, \mu^\pm, \tau^\pm$ and X represents a system of light hadronic states, offer, together with the radiative electromagnetic penguin decays $B \to X + \gamma$, presently the most popular testing grounds for the standard model (SM) in the flavour sector. In this contribution, we summarize the main steps in the derivation of the hadron spectra and hadron spectral moments in $B \to X_s \ell^+ \ell^-$using perturbative QCD and the heavy quark expansion technique HQET [1,2,3], published recently by us [4,5]. This work, which incorporates the leading order (in α_s) perturbative QCD and power $(1/m_b^2)$ corrections to the hadronic spectra, complements the derivation of the dilepton invariant mass spectrum and the forward-backward asymmetry of the charged lepton [6], calculated in the HQET framework some time ago by us in collaboration with T. Morozumi and L. Handoko [7]. (See, also Buchalla et al. [8].) Both the hadron and dilepton spectra are needed to distinguish the signal ($B \to X_s \ell^+ \ell^-$) from the background processes and in estimating the effects of the experimental selection criterion. We shall concentrate here on the short-distance contribution which can be extracted from data with the help of judicious cuts, such as those employed recently by the CLEO collaboration [9]. The residual effects from the resonant (long-distance) contributions have been studied in these distributions elsewhere [7,10], to which we refer for details and references to the earlier work.

We also underline the theoretical interest in measuring the first few hadronic spectral moments $\langle S_H^n \rangle$ and $\langle E_H^n \rangle$ $(n = 1, 2)$. The former are sensitive to the HQET parameters $\bar{\Lambda}$ and λ_1; we work out this dependence numerically and argue that a combined analysis of the moments and spectra in $B \to X_s \ell^+ \ell^-$and $B \to X \ell \nu_\ell$ will allow to determine the HQET parameters with a high precision. Since these parameters are endemic to a large class of phenomena in B decays, their precise knowledge is of great advantage in reducing the theoretical errors in the determination of the CKM matrix elements V_{td}, V_{ts}, V_{cb} and V_{ub}.

2 Kinematics and HQET Relations

We start with the definition of the kinematics of the decay at the parton level, $b(p_b) \to s(p_s)(+g(p_g)) + \ell^+(p_+) + \ell^-(p_-)$, where g denotes a gluon from the $O(\alpha_s)$ correction. The corresponding kinematics at the hadron level can be written as: $B(p_B) \to X_s(p_H) + \ell^+(p_+) + \ell^-(p_-)$. We define by q the momentum transfer to the lepton pair $q = p_+ + p_-$ and $s \equiv q^2$ is the invariant dilepton mass squared. We shall also need the variable u defined as $u \equiv -(p_b - p_+)^2 + (p_b - p_-)^2$. The hadronic invariant mass and the hadron energy in the final state is denoted by S_H and E_H, respectively; corresponding quantities at parton level are the invariant mass s_0 and the scaled parton energy $x_0 \equiv \frac{E_0}{m_b}$. From energy-momentum conservation, the following equalities hold in the b-quark, equivalently B-meson, rest frame ($v = (1, 0, 0, 0)$):

$$\begin{aligned} x_0 &= 1 - v \cdot \hat{q} \,, \quad \hat{s}_0 = 1 - 2v \cdot \hat{q} + \hat{s} \,, \\ E_H &= m_B - v \cdot q \,, \quad S_H = m_B^2 - 2m_B v \cdot q + s \quad , \end{aligned} \tag{1}$$

where dimensionless variables with a hat are scaled by the b-quark mass, e.g., $\hat{s} = \frac{s}{m_b^2}$, $\hat{m}_s = \frac{m_s}{m_b}$ etc. Here, the 4-vector v denotes the velocity of both the b-quark and the B-meson, $p_b = m_b v$ and $p_B = m_B v$.

The relation between the B-meson and b-quark mass is given by the HQET mass relation $m_B = m_b + \bar{\Lambda} - 1/2m_b(\lambda_1 + 3\lambda_2) + \ldots$, where the ellipses denote terms higher order in $1/m_b$. The quantity λ_2 is known precisely from the $B^* - B$ mass difference, with $\lambda_2 \simeq 0.12$ GeV^2. The other two parameters are considerably uncertain at present [11,12] and are of interest here.

The hadronic variables E_H and S_H can be expressed in terms of the partonic variables x_0 and $\hat{s}_0$ by the following relations

$$E_H = \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_B} + \left(m_B - \bar{\Lambda} + \frac{\lambda_1 + 3\lambda_2}{2m_B}\right) x_0 + \dots ,$$
$$S_H = m_s^2 + \bar{\Lambda}^2 + (m_B^2 - 2\bar{\Lambda} m_B + \bar{\Lambda}^2 + \lambda_1 + 3\lambda_2)(\hat{s}_0 - \hat{m}_s^2) + (2\bar{\Lambda} m_B - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2) x_0 + \dots .$$

The dominant non-perturbative effect on the hadron spectra is essentially determined by the binding energy $\bar{\Lambda} = m_B - m_b + ...$, in terms of which one has the following transformation:

$$E_0 \to E_H = \bar{\Lambda} + E_0 + \dots , \quad s_0 \to S_H = s_0 + 2\bar{\Lambda} E_0 + \bar{\Lambda}^2 + \dots . \tag{2}$$

Thus, changing the variables of integration $(s_0, E_0) \to (s_H, E_0)$ and integrating over E_0 in the range $\sqrt{S_H} - \bar{\Lambda} < E_0 < 1/2m_B(S_H - 2\bar{\Lambda} m_B^2 + m_B^2)$, one gets an invariant hadron mass spectrum $d\Gamma/dS_H$ in the kinematic range $\bar{\Lambda}^2 < S_H < m_B^2$. In particular, already for the partonic decay $b \to s\ell^+\ell^-$ with $m_s = 0$, and hence $s_0 = 0$, one gets a non-trivial distribution in S_H for $\bar{\Lambda}^2 < S_H < \bar{\Lambda} m_B$. The kinematic boundary of the distribution $d\Gamma/dS_H$ is extended by the bremsstrahlung process $b \to s + g + \ell^+\ell^-$, where now $\bar{\Lambda} m_B < S_H < m_B^2$ (with $m_s = 0$). The $\mathcal{O}(\alpha_s)$ contribution leads to a double logarithmic (but integrable) singularity at $S_H = \bar{\Lambda} m_B$. Perturbation theory is valid for $\Delta^2 < S_H < m_B^2$, with $\Delta^2 > \bar{\Lambda} m_B$.

3 Matrix Element for $B \to X_s \ell^+ \ell^-$ in the Effective Hamiltonian Approach

The effective Hamiltonian governing the decay $B \to X_s \ell^+ \ell^-$ is given as [7]:

$$\mathcal{H}_{eff}(b \to s) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \tag{3}$$
$$\left[\sum_{i=1}^{6} C_i(\mu) O_i + C_7(\mu) \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha F^{\mu\nu} + C_9(\mu) \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \ell + C_{10} \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \gamma_5 \ell \right] ,$$

where G_F is the Fermi coupling constant, $L(R) = 1/2(1 \mp \gamma_5)$, and C_i are the Wilson coefficients. Note that the chromo-magnetic operator does not contribute to the decay $B \to X_s \ell^+ \ell^-$ in the approximation which we use here.

The matrix element for the decay $B \to X_s \ell^+ \ell^-$ can be factorized into a leptonic and a hadronic part as

$$\mathcal{M}(B \to X_s \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left(\Gamma^L{}_\mu L^{L\mu} + \Gamma^R{}_\mu L^{R\mu}\right) , \tag{4}$$

with

$$L^{L/R}{}_\mu \equiv \bar{\ell} \gamma_\mu L(R) \ell , \tag{5}$$

$$\Gamma^{L/R}{}_\mu \equiv \bar{s} \left[R \gamma_\mu \left(C_9^{\rm eff}(\hat{s}) \mp C_{10} + 2C_7^{\rm eff} \frac{\hat{\not{q}}}{\hat{s}} \right) + 2\hat{m}_s C_7^{\rm eff} \gamma_\mu \frac{\hat{\not{q}}}{\hat{s}} L \right] b , \tag{6}$$

with $C_7^{\rm eff} \equiv C_7 - C_5/3 - C_6$. The effective Wilson coefficient $C_9^{\rm eff}(\hat{s})$ receives contributions from various pieces. The resonant $c\bar{c}$ states also contribute to $C_9^{\rm eff}(\hat{s})$; hence the contribution given below is just the perturbative part:

$$C_9^{\rm eff}(\hat{s}) = C_9 \eta(\hat{s}) + Y(\hat{s}) . \tag{7}$$

Here $\eta(\hat{s})$ and $Y(\hat{s})$ represent, respectively, the $\mathcal{O}(\alpha_s)$ correction [13] and the one loop matrix element of the Four-Fermi operators [14,15].

With the help of the above expressions, the differential decay width becomes on using $p_\pm = (E_\pm, \boldsymbol{p}_\pm)$,

$$\mathrm{d}\Gamma = \frac{1}{2m_B} \frac{G_F{}^2 \alpha^2}{2\pi^2} |V_{ts}^* V_{tb}|^2 \frac{\mathrm{d}^3 \boldsymbol{p}_+}{(2\pi)^3 2E_+} \frac{\mathrm{d}^3 \boldsymbol{p}_-}{(2\pi)^3 2E_-} \times \left(W^L{}_{\mu\nu} L^{L\mu\nu} + W^R{}_{\mu\nu} L^{R\mu\nu} \right) , \tag{8}$$

where $W_{\mu\nu}^{L,R}$ and $L_{\mu\nu}^{L,R}$ are the hadronic and leptonic tensors, respectively, and can be seen in the literature [7]. The hadronic tensor $W_{\mu\nu}^{L/R}$ is related to the discontinuity in the forward scattering amplitude, denoted by $T_{\mu\nu}^{L/R}$, through the relation $W_{\mu\nu}^{L/R} = 2\,\mathrm{Im}\,T_{\mu\nu}^{L/R}$. Transforming the integration variables to $\hat{s}$, $\hat{u}$ and $v \cdot \hat{q}$, one can express the triple differential distribution in $B \to X_s \ell^+ \ell^-$ as:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{u}\, \mathrm{d}\hat{s}\, \mathrm{d}(v \cdot \hat{q})} = \frac{1}{2\, m_B} \frac{G_F{}^2 \alpha^2}{2\, \pi^2} \frac{m_b{}^4}{256\, \pi^4} |V_{ts}^* V_{tb}|^2 \times 2\, \mathrm{Im} \left(T^L{}_{\mu\nu} L^{L\mu\nu} + T^R{}_{\mu\nu} L^{R\mu\nu} \right) . \tag{9}$$

Using Lorentz decomposition, the tensor $T_{\mu\nu}$ can be expanded in terms of three structure functions T_i,

$$T_{\mu\nu}^{L/R} = -T_1^{L/R} g_{\mu\nu} + T_2^{L/R} v_\mu v_\nu + T_3^{L/R} i\epsilon_{\mu\nu\alpha\beta} v^\alpha \hat{q}^\beta , \tag{10}$$

where the ones which do not contribute to the amplitude for massless leptons have been neglected.

4 Hadron Spectra in $B \to X_s \ell^+ \ell^-$

We discuss first the perturbative $O(\alpha_s)$ corrections recalling that only the matrix element of the operator $O_9 \equiv e^2/(16\pi^2) \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \ell$ is subject to such corrections. The corrected hadron energy spectrum in $B \to$

$X_s\ell^+\ell^-$can be obtained by using the existing results in the literature on the decay $B \to X\ell\nu_\ell$ by decomposing the vector current in O_9 as $V = (V-A)/2 + (V+A)/2$. The $(V-A)$ and $(V+A)$ currents yield the same hadron energy spectrum [16] and there is no interference term present in this spectrum for massless leptons. So, the correction for the vector current case can be taken from the corresponding result for the charged $(V-A)$ case[13].

The $\mathcal{O}(\alpha_s)$ perturbative QCD correction for the hadronic invariant mass is discussed next. As already mentioned, the decay $b \to s + \ell^+ + \ell^-$ yields a delta function at $\hat{s}_0 = \hat{m}_s^2$ and hence only the bremsstrahlung diagrams $b \to s+g+\ell^+ +\ell^-$ contribute in the range $\hat{m}_s^2 < \hat{s}_0 \leq 1$. The resulting distribution $d\mathcal{B}(B \to X_s\ell^+\ell^-)/ds_0$ in the parton model in the $\mathcal{O}(\alpha_s)$ approximation and the Sudakov exponentiated form can be seen in our paper [5]. We remark that the Sudakov exponentiated double differential distribution for the decay $B \to X_u\ell\nu_\ell$ has been derived by Greub and Rey [17], which we have checked and used after changing the normalization for $B \to X_s\ell^+\ell^-$. The hadronic invariant mass spectrum $d\mathcal{B}(B \to X_s\ell^+\ell^-)/dS_H$, shown in Fig. 1 depends rather sensitively on m_b (or equivalently $\bar{\Lambda}$). An analogous analysis for the decay $B \to X_u\ell\nu_\ell$ has been performed earlier, with very similar qualitative results [18].

Next, we discuss the power corrections to the hadronic spectra. The structure functions $T_i^{L/R}$ in the hadronic tensor in Eq. (10) can be expanded in inverse powers of m_b with the help of the HQET techniques[1,2,3]. The leading term in this expansion, i.e., $\mathcal{O}(m_b^0)$, reproduces the parton model result[14,15]. In HQET, the next to leading power corrections are parameterized in terms of λ_1 and λ_2. After contracting the hadronic and leptonic tensors and with the help of the kinematic identities given in Eq. (1), we can make the dependence on x_0 and $\hat{s}_0$ explicit,

$$T^{L/R}{}_{\mu\nu}\, L^{L/R\mu\nu} = {m_b}^2 \Big\{ 2(1-2x_0+\hat{s}_0){T_1}^{L/R} + \left[x_0^2 - \frac{1}{4}\hat{u}^2 - \hat{s}_0 \right] {T_2}^{L/R} \mp (1-2x_0+\hat{s}_0)\hat{u}\, {T_3}^{L/R} \Big\} . \tag{11}$$

By integrating Eq. (9) over $\hat{u}$, the double differential power corrected spectrum can be expressed as [5]:

$$\frac{\mathrm{d}^2\mathcal{B}}{\mathrm{d}x_0\,\mathrm{d}\hat{s}_0} = -\frac{8}{\pi}\mathcal{B}_0 \mathrm{Im}\sqrt{x_0^2-\hat{s}_0}\,\Big\{(1-2x_0+\hat{s}_0)T_1(\hat{s}_0,x_0) + \frac{x_0^2-\hat{s}_0}{3}T_2(\hat{s}_0,x_0)\Big\} + \mathcal{O}(\lambda_i\alpha_s) . \tag{12}$$

The structure function T_3 does not contribute to the double differential distribution and we do not consider it any further. The functions $T_1(\hat{s}_0,x_0)$ and $T_2(\hat{s}_0,x_0)$, together with other details of the calculations, have been given by us elsewhere [5].

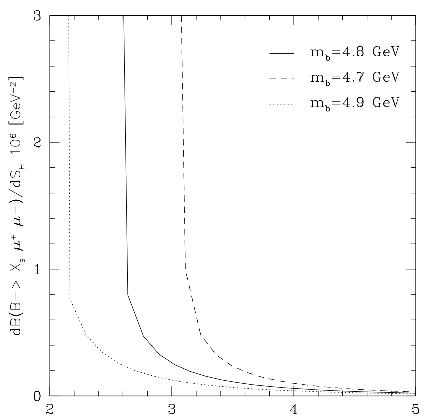

Figure 1: The differential branching ratio $d\mathcal{B}(B \to X_s\ell^+\ell^-)/dS_H$ in the hadronic invariant mass, S_H, shown for three values of m_b in the range where only bremsstrahlung diagrams contribute.

The branching ratio for $B \to X_s\ell^+\ell^-$is usually expressed in terms of the measured semileptonic branching ratio $\mathcal{B}_{sl}$ for the decay $B \to X_c\ell\nu_\ell$. This fixes the normalization constant $\mathcal{B}_0$ to be,

$$\mathcal{B}_0 \equiv \mathcal{B}_{sl} \frac{3\,\alpha^2}{16\pi^2}\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{1}{f(\hat{m}_c)\kappa(\hat{m}_c)} , \tag{13}$$

where $f(\hat{m}_c)$ is the phase space factor for $\Gamma(B \to X_c\ell\nu_\ell)$ and $\kappa(\hat{m}_c)$ accounts for both the $O(\alpha_s)$ QCD correction to the semileptonic decay width [19] and the leading order $(1/m_b)^2$ power correction[1]. The hadron energy spectrum can now be obtained by integrating over $\hat{s}_0$ with the kinematic boundaries: $max(\hat{m}_s^2, -1+2x_0+4\hat{m}_l^2) \leq \hat{s}_0 \leq x_0^2$, $\hat{m}_s \leq x_0 \leq \frac{1}{2}(1+\hat{m}_s^2 - 4\hat{m}_l^2)$. The hadron energy spectrum $d\mathcal{B}(B \to X_s\ell^+\ell^-)/dE_0$ in the parton model (dotted line) and including leading power corrections (solid line) are shown in Fig. 2. For $m_b/2 < E_0 < m_b$ the two distributions coincide. Note that the $1/m_b^2$-expansion breaks down near the lower end-point of the hadron energy spectrum and at the $c\bar{c}$ threshold. Hence, only suitably averaged spectra are useful for comparison with experiments in these regions. Apart from these regions, the HQET and parton model spectra are remarkably close to each other.

5 Hadron Spectral Moments in $B \to X_s\ell^+\ell^-$

The lowest spectral moments in the decay $B \to X_s\ell^+\ell^-$at the parton level are worked out by taking into account the two types of corrections discussed earlier, namely the leading power $1/m_b$ and the perturbative

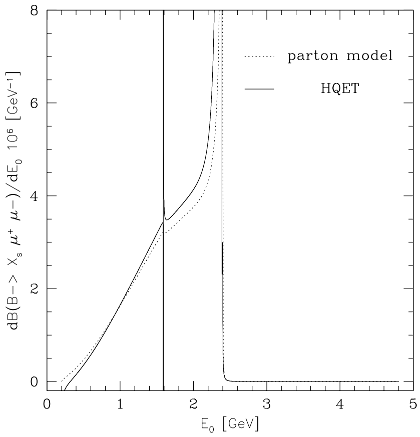

Figure 2: Hadron energy spectrum $d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dE_0$ in the parton model (dotted line) and including leading power corrections (solid line). For $m_b/2 < E_0 < m_b$ the two distributions coincide.

$\mathcal{O}(\alpha_s)$ corrections. To that end, we define the moments for integers n and m:

$$\mathcal{M}_{l^+l^-}^{(n,m)} \equiv \frac{1}{\mathcal{B}_0} \int (\hat{s}_0 - \hat{m}_s^2)^n x_0^m \, \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\hat{s}_0 \mathrm{d}x_0} \, \mathrm{d}\hat{s}_0 \mathrm{d}x_0 \,, \quad (14)$$

which obey $\langle x_0^m (\hat{s}_0 - \hat{m}_s^2)^n \rangle = \frac{\mathcal{B}_0}{\mathcal{B}} \mathcal{M}_{l^+l^-}^{(n,m)}$. These moments can be expanded as a double Taylor series in α_s and $1/m_b$:

$$\mathcal{M}_{l^+l^-}^{(n,m)} = D_0^{(n,m)} + \frac{\alpha_s}{\pi} {C_9}^2 A^{(n,m)} + \hat{\lambda}_1 D_1^{(n,m)} + \hat{\lambda}_2 D_2^{(n,m)} \,, \quad (15)$$

with a further decomposition of $D_i^{(n,m)}$, $i = 0, 1, 2$, into pieces from different Wilson coefficients:

$$D_i^{(n,m)} = \alpha_i^{(n,m)} {C_7^{\mathrm{eff}}}^2 + \beta_i^{(n,m)} C_{10}^2 + \gamma_i^{(n,m)} C_7^{\mathrm{eff}} + \delta_i^{(n,m)} . \quad (16)$$

The terms $\gamma_i^{(n,m)}$ and $\delta_i^{(n,m)}$ in Eq. (16) result from the terms proportional to $Re(C_9^{\mathrm{eff}}) C_7^{\mathrm{eff}}$ and $|C_9^{\mathrm{eff}}|^2$ in Eq. (12), respectively. The explicit expressions for $\alpha_i^{(n,m)}, \beta_i^{(n,m)}, \gamma_i^{(n,m)}, \delta_i^{(n,m)}$ are given in our paper [5].

The leading perturbative contributions for the hadronic invariant mass and hadron energy moments can be obtained analytically,

$$A^{(0,0)} = \frac{25 - 4\pi^2}{9} \,, \quad A^{(1,0)} = \frac{91}{675} \,, \quad A^{(2,0)} = \frac{5}{486} \,,$$
$$A^{(0,1)} = \frac{1381 - 210\pi^2}{1350} \,, A^{(0,2)} = \frac{2257 - 320\pi^2}{5400} \,. \quad (17)$$

The zeroth moment $n = m = 0$ is needed for the normalization; the result for $A^{(0,0)}$ was first derived by Cabibbo and Maiani [19]. Likewise, the first mixed moment $A^{(1,1)}$ can be extracted from the results for the decay $B \to X \ell \nu_\ell$ [20] after changing the normalization, $A^{(1,1)} = 3/50$. For the lowest order parton model contribution $D_0^{(n,m)}$, we find, in agreement with [20], that the first two hadronic invariant mass moments $\langle \hat{s}_0 - \hat{m}_s^2 \rangle$, $\langle (\hat{s}_0 - \hat{m}_s^2)^2 \rangle$ and the first mixed moment $\langle x_0 (\hat{s}_0 - \hat{m}_s^2) \rangle$ vanish: $D_0^{(n,0)} = 0$, for $n = 1, 2$ and $D_0^{(1,1)} = 0$.

Using the expressions for the HQET moments derived by us [5], we present the numerical results for the hadronic moments in $B \to X_s \ell^+ \ell^-$. The parameters used are : $m_s = 0.2$ GeV$, m_c = 1.4$ GeV$, m_b = 4.8$ GeV$, m_t = 175 \pm 5$ GeV$, \mu = {m_b}^{+m_b}_{-m_b/2}, \alpha_s(m_Z) = 0.117 \pm 0.005$, $\alpha^{-1} = 129$. We find for the short-distance hadronic moments, valid up to $\mathcal{O}(\alpha_s/m_B^2, 1/m_B^3)$:

$$\begin{aligned}
\langle S_H \rangle &= m_B^2 \Big(\frac{m_s^2}{m_B^2} + 0.093 \frac{\alpha_s}{\pi} - 0.069 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} \\
&\quad + 0.735 \frac{\bar{\Lambda}}{m_B} + 0.243 \frac{\bar{\Lambda}^2}{m_B^2} + 0.273 \frac{\lambda_1}{m_B^2} - 0.513 \frac{\lambda_2}{m_B^2} \Big) \,, \\
\langle S_H^2 \rangle &= m_B^4 \Big(0.0071 \frac{\alpha_s}{\pi} + 0.138 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} \\
&\quad + 0.587 \frac{\bar{\Lambda}^2}{m_B^2} - 0.196 \frac{\lambda_1}{m_B^2} \Big) \,, \qquad (18) \\
\langle E_H \rangle &= 0.367 m_B \Big(1 + 0.148 \frac{\alpha_s}{\pi} - 0.352 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} + 1.691 \frac{\bar{\Lambda}}{m_B} \\
&\quad + 0.012 \frac{\bar{\Lambda}^2}{m_B^2} + 0.024 \frac{\lambda_1}{m_B^2} + 1.070 \frac{\lambda_2}{m_B^2} \Big) \,, \\
\langle E_H^2 \rangle &= 0.147 m_B^2 \Big(1 + 0.324 \frac{\alpha_s}{\pi} - 0.128 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} + 2.954 \frac{\bar{\Lambda}}{m_B} \\
&\quad + 2.740 \frac{\bar{\Lambda}^2}{m_B^2} - 0.299 \frac{\lambda_1}{m_B^2} + 0.162 \frac{\lambda_2}{m_B^2} \Big) \,,
\end{aligned}$$

where the numbers shown correspond to the central values of the parameters.

The dependence of the hadronic moments given in Eq. (18) on the HQET parameters λ_1 and $\bar{\Lambda}$ has been worked out numerically. In doing this, the theoretical errors on these moments following from the errors on the input parameters m_t, α_s and the scale μ have been estimated by varying these parameters in the indicated $\pm 1\sigma$ ranges, one at a time, and adding the individual errors in quadrature. The correlations on the HQET parameters λ_1 and $\bar{\Lambda}$ which follow from (assumed) fixed values of the hadronic invariant mass moments $\langle S_H \rangle$ and $\langle S_H^2 \rangle$ (calculated using $\bar{\Lambda} = 0.39\,\mathrm{GeV}$, $\lambda_1 = -0.2\,\mathrm{GeV}^2$ and $\lambda_2 = 0.12\,\mathrm{GeV}^2$) are shown in Fig. 3 (for the decay $B \to X_s \mu^+ \mu^-$). The (λ_1-$\bar{\Lambda}$) correlation from the analysis of Gremm et al. [11] for the electron energy spectrum in $B \to X \ell \nu_\ell$ is shown as an ellipse in this figure. With the measurements of $\langle S_H \rangle$ and $\langle S_H^2 \rangle$ in the

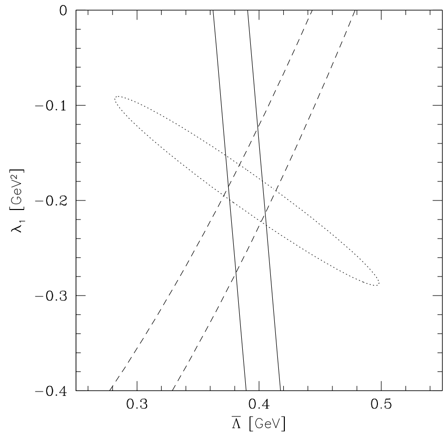

Figure 3: $\langle S_H \rangle$ (solid bands) and $\langle S_H^2 \rangle$ (dashed bands) correlation in (λ_1-$\bar{\Lambda}$) space for the decay $B \to X_s \ell^+ \ell^-$. The correlation from the analysis of the decay $B \to X \ell \nu_\ell$ by Gremm et al. [11] is shown as an ellipse.

decay $B \to X_s \ell^+ \ell^-$, one has to solve the experimental numbers on the l.h.s. of Eq. (18) for λ_1 and $\bar{\Lambda}$. It is, however, clear that the constraints from the decays $B \to X_s \ell^+ \ell^-$ and $B \to X \ell \nu_\ell$ are complementry. Using the CLEO cuts on hadronic and dileptonic masses [9], we estimate that $O(200)$ $B \to X_s \ell^+ \ell^- (\ell = e, \mu)$ events will be available per 10^7 $B\bar{B}$ hadrons [5]. So, there will be plenty of $B \to X_s \ell^+ \ell^-$decays in the forthcoming B facilities to measure the correlation shown in Fig. 3.

Of course, the utility of the hadronic moments calculated above is only in conjunction with the experimental cuts which could effectively remove the resonant (long-distance) contributions. The optimal experimental cuts in $B \to X_s \ell^+ \ell^-$remain to be defined, but for the cuts used by the CLEO collaboration we have studied the effects in the HQET-like Fermi motion (FM) model [21]. We find that the hadronic moments in the HQET and FM model are very similar and CLEO-type cuts remove the bulk of the $c\bar{c}$ resonant contributions [5].

In summary, we have calculated the dominant contributions to the hadron spectra and spectral moments in $B \to X_s \ell^+ \ell^-$ including contributions up to terms of $\mathcal{O}(\alpha_s/m_B^2, 1/m_B^3)$. We have presented the results on the spectral hadronic moments $\langle E_H^n \rangle$ and $\langle S_H^n \rangle$ for $n = 1, 2$ and have worked out their dependence on the HQET parameters $\bar{\Lambda}$ and λ_1. The correlations in $B \to X_s \ell^+ \ell^-$are shown to be different than the ones in the semileptonic decay $B \to X \ell \nu_\ell$. This complementarity allows, in principle, a powerful method to determine them precisely from data on $B \to X \ell \nu_\ell$ and $B \to X_s \ell^+ \ell^-$in forthcoming high luminosity B facilities.

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