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Spectroscopy of doubly charmed baryons: Ξ_{cc}^+ and Ξ_{cc}^{++} .

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Abstract

Using the quark-diquark approximation in the framework of Buchmüller-Tye potential model, we investigate the spectroscopy of doubly charmed baryons: Ξ_{cc}^{++} and Ξ_{cc}^{+} . Our results include the masses, parameters of radial wave functions of states with the different excitations of both diquark and light quark-diquark system. We calculate the values of fine and hyperfine splittings of these levels and discuss some new features, connected to the identity of heavy quarks, in the dynamics of hadronic and radiative transitions between the states of these baryons.



1 Introduction

For experiments at the running and planned high-energy machines such as LHC, B-factories (HERA-B, for example) and Tevatron with high luminosities, there are some hopes and possibilities to observe and study new hadrons, containing two heavy quarks, like the doubly charmed baryons (ccq) (here and throughout this paper, q denotes a light quark u or d).

Looking for such projects, it is important to have reliable theoretical predictions as a guide to the experimental searches for these baryons. In our previous papers we have already studied both the production mechanisms of doubly charmed baryons on the different future facilities [1], [2] and the total inclusive lifetimes of such systems [3]. In this work we would like to address the question on the spectroscopy for these baryons in the framework of potential approach in the Buchmüller-Tye model. The (QQ'q) spectroscopy was also considered in some other models [4, 5, 6, 7].

The baryons with two heavy quarks include the features of both the dynamics in the *D*meson with a fast moving light quark, surrounding a static $\bar{3}$ -color core, and the dynamics of heavy-heavy mesons $(J/\Psi, B_c, \Upsilon)$, with two heavy quarks sensitive to the QCD potential at short distances. So, a rich spectrum is expected. First, there are excitations due to the relative motion of two heavy quarks in the quark-diquark approximation, which we use here. Some comments on the validity of this approximation will be given later. Second, we consider also the excitations of the light quark or combined excitations of both degrees of freedom.

As we will show in this paper, some new features, related with the identity of two heavy quarks, arise in the dynamics of doubly charmed baryons (or doubly beauty baryons). The hadronic or electromagnetic transition between the $2P \rightarrow 1S$ diquark levels is forbidden in the absence of interaction of diquark with the light quark or without taking into account nonperturbative effects. The qualitative picture of this effect will be discussed below, and a rigorous quantitative solution of this problem will be given in our subsequent papers.

This work is organized as follows. Section 2 is devoted to the determination of masses, values of radial wave functions (for S-levels) and their first derivatives (for P-levels) at the origin for both the diquark and the system of light quark-diquark. In Section 3 we evaluate the fine and hyperfine splitting of different levels for the above systems. Section 4 contains some comments on the radiative and hadronic decays of doubly charmed baryons. And, finally, Section 5 draws our conclusion.

2 Mass spectrum of doubly charmed baryons.

Investigating the baryon spectroscopy, one faces a three body problem in the framework of nonrelativistic quantum mechanics. Its reduced hamiltonian has the following expression:

$$H = \frac{p_x^2}{M} + \frac{p_y^2}{M} + v(\mathbf{x}, \mathbf{y}), \qquad (1)$$

where \mathbf{x}, \mathbf{y} are Jacobi variables:

$$\mathbf{x} = \mathbf{r}_2 - \mathbf{r}_1, \tag{2}$$

$$\mathbf{y} = (2\mathbf{r}_3 - \mathbf{r}_1 - \mathbf{r}_2)\sqrt{\frac{m}{2M+m}},\tag{3}$$

where M is the heavy quark mass, and m is the light quark mass.

There are several methods for a solution of three-body Schrödinger equation [6]. The first way is the variational methods, where one expands the wave-function in terms of the eigenstates for a symmetric harmonic oscillator, or gaussians, at different ranges in the **x** and **y** coordinates, or uses the hyperspherical formalism. The other methods are the quark-diquark approximation and Born-Oppenheimer approximation. The former is used in our evaluation of doubly charmed baryon spectrum. The reason is that the ground state of (ccq) consists of a localized (cc) cluster surrounded by the light quark q, with the average distance $\langle r_{cc} \rangle$ much less than $\langle r_{cq} \rangle$. However, when one accounts for the radial or orbital excitations of diquark, the average separation between the heavy quarks increases, and the quark-diquark structure is distroied. So, in this region, our results for the mass spectrum of these baryons are quite crude.

Next, a dramatic simplification of (ccq) dynamics is obtained in the Born-Oppenheimer or adiabatic approximation. Two heavy quarks have much less velocity than that of the light quark. When they move, the light quark wave function readjusts itself almost immediately to the state of minimal energy. Therefore, the calculation can be done in two steps: for any given **x**, one computes the binding energy $\epsilon(\mathbf{x})$, which is, then, used as an effective potential governing the relative motion of heavy quarks. This was done in [4]: first, in the nonrelativistic potential model and, second, in a variant of MIT bag model. From our point of view, this method is the most suitable for the baryon spectroscopy. However, in this work, we will be satisfied with the accuracy, given by the quark-diquark approximation.

In present work, we use the QCD-motivated potential [8] given by Buchmüller and Tye [9], which was justified on the J/Ψ and Υ spectra with the following values of parameters:

$$m_c = 1.486 \ GeV, \quad m_q = 0.385 \ GeV,$$
 (4)

where the light quark mass is obtained by the fitting of theoretical predictions with the Buchmüller-Tye potential for the D-meson mass to its experimental value. Of course the motion of light quark is relativistic inside the D meson, as well as inside the baryons under consideraton and so can not be treated in the framework of nonrelativistic quantum mecanics. But we think, that considering its motion in both cases in the same way, we can obtain good approximation for the values of mass levels of our baryons.

Estimating the mass spectrum of diquark excitations, one has to take into account the factor of 1/2 for the potential due to the fact, that the heavy quarks inside the diquark are in color antitriplet state. At distances ~ 0.6 - 0.8 fm we have an attraction of quarks inside the diquark, so we assume that the shape of potential in this region related with their pairwise interaction is the same as for quark - antiquark interaction in heavy mesons. At larger distances we can not tell anything about it, so our estimates for higher-lying levels is quite rough. But for low-lying energy levels in this system the wave function is already zero in the region of large distances, so that our predictions for them can be trusted. Then, solving the Schrödinger equations for the diquark and light quark excitations, one finds the diquark and Ξ_{cc}^{++} , Ξ_{cc}^{+-} baryons mass spectra and the characteristics of radial wave functions for both the diquark and the system of light quark-diquark $R_d(0)$, $R_l(0)$, $R'_d(0)$ and $R'_l(0)$, shown in Tables 1, 2 and 3.

In calculations we assume, that the threshold mass value of doubly charmed baryons is determined by the hadronic decay into Λ_c -baryon and D-meson, and, hence, it equals 4.26 GeV [10]. The threshold for the stability of diquark is estimated from the following result, stated for a heavy quark-antiquark pair [11]: if a heavy quark and a corresponding antiquark

are separated by a distance greater than 1.4-1.5 fm, then the most energetically favorable and probable configuration results in a production of light quark-antiquark pair, which leads to the fragmentation into a pair of flavored heavy mesons. So, we suppose that the same critial

Diquark state	Mass (GeV)	$\langle r^2 \rangle^{1/2}$ (fm)	Diquark state	Mass~(Gev)	$\langle r^2 \rangle^{1/2}$ (fm)
1S	3.16	0.58	3P	3.66	1.36
2S	3.50	1.12	4P	3.90	1.86
3S	3.76	1.58	3D	3.56	1.13
2P	3.39	0.88	4D	3.80	1.59

Table 1: The (cc)-diquark spectrum: masses and mean-square radii.

distance scale can be used for the colored diquark system, which results in the fragmentation of diquark to the heavy meson and heavy-light diquark.

3 Spin-dependent splitting.

In accordance with the results of refs. [12, 13] and [14], we introduce the additional term to the potential to take into account the spin-orbital and spin-spin interactions, causing the splitting of nL-levels in both the diquark and light quark-diquark system (n is the principal quantum number, L is the orbital momentum). So, it has the form

$$V_{SD}^{(d)}(\mathbf{r}) = \frac{1}{2} \left(\frac{\mathbf{L} \cdot \mathbf{S}}{2m_c^2} \right) \left(-\frac{dV(r)}{rdr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) + \frac{2}{3} \alpha_s \frac{1}{m_c^2} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3} + \frac{4}{3} \alpha_s \frac{1}{3m_c^2} \mathbf{S}_{c1} \cdot \mathbf{S}_{c2} [4\pi \delta(\mathbf{r})] + \frac{1}{3} \alpha_s \frac{1}{m_c^2} (-\frac{1}{4\mathbf{L}^2 - 3} \times [6(\mathbf{L} \cdot \mathbf{S})^2 + 3(\mathbf{L} \cdot \mathbf{S}) - 2\mathbf{L}^2 \mathbf{S}^2]) \frac{1}{r^3},$$
(5)

for the diquark splitting, and

$$V_{SD}^{(l)}(\mathbf{r}) = \frac{1}{2} \left(\frac{\mathbf{L} \cdot \mathbf{S}_{\mathbf{d}}}{2m_{c}^{2}} + \frac{2\mathbf{L} \cdot \mathbf{S}_{\mathbf{l}}}{2m_{l}^{2}} \right) \left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_{s}\frac{1}{r^{3}} \right) + \frac{2}{3}\alpha_{s}\frac{1}{m_{c}m_{l}}\frac{(\mathbf{L} \cdot \mathbf{S}_{\mathbf{d}} + 2\mathbf{L} \cdot \mathbf{S}_{\mathbf{l}})}{r^{3}} + \frac{4}{3}\alpha_{s}\frac{1}{3m_{c}m_{l}}(\mathbf{S}_{d} + \mathbf{L}_{d}) \cdot \mathbf{S}_{l}[4\pi\delta(\mathbf{r})] \qquad (6) + \frac{1}{3}\alpha_{s}\frac{1}{m_{c}m_{l}}(-\frac{1}{4\mathbf{L}^{2} - 3} \times [6(\mathbf{L} \cdot \mathbf{S})^{2} + 3(\mathbf{L} \cdot \mathbf{S}) - 2\mathbf{L}^{2}\mathbf{S}^{2} - 6(\mathbf{L} \cdot \mathbf{S}_{\mathbf{d}})^{2} - 3(\mathbf{L} \cdot \mathbf{S}_{\mathbf{d}}) + 2\mathbf{L}^{2}\mathbf{S}_{\mathbf{d}}^{2}])\frac{1}{r^{3}},$$

n (diquark)	$R_{d(nS)}(0)(R'_{d(nP)}(0))$	n (diquark)	$R_{d(nS)}(0)(R'_{d(nP)}(0))$
1S	0.530	2S	-0.452
2P	0.128	3P	-0.158

Table 2: The characteristics of diquark radial wave functions $R_{d(nS)}(0)$ (in $GeV^{3/2}$), $R'_{d(nP)}(0)$ (in $GeV^{5/2}$).

$n_d(diquark)$ -	Mass~(GeV)	$R_{l(nS)}(0)$	$n_d(diquark)$ -	Mass~(GeV)	$R_{l(nS)}(0)$
$n_l(light \; quark)$		$(R'_{l(nP)}(0))$	$n_l(light \; quark)$		$(R'_{l(nP)}(0))$
1S 1S	3.56	0.499	1S 2P	4.03	0.118
2S 1S	3.90	0.502	2S 2P	4.36	0.119
3S 1S	4.16	0.505	3S 2P	4.62	0.121
2P 1S	3.79	0.501	2P 2P	4.25	0.119
3P 1S	4.06	0.504	3P 2P	4.52	0.119
3D 1S	3.96	0.503	3D 2P	4.42	0.117

Table 3: The mass spectra and characteristics of light quark radial wave functions in the doubly charmed baryons: Ξ_{cc}^{++} and Ξ_{cc}^{+} with the different excitations of diquark, n_d , and light quark-diquark system, n_l .

for the light quark-diquark system, where V(r) is the phenomenological potential (Buchmüller-Tye (BT) potential in our case), S_l and S_d are the light quark and diquark spins, respectively. The first term in both expressions takes into account the relativistic corrections to the potential V(r). The second, third and fourth terms are the relativistic corrections coming from the account for the one gluon exchange between the quarks. α_s is the effective constant of quarkgluon interactions inside the baryons under consideration.

Expression (6) for the additional part of potential, causing the splitting of levels in the light quark-diquark system is obtained from the summing of the pair interactions, like (5), for the light quark with each of the heavy quarks. We include also the correction, connected to the interaction of internal diquark orbital momentum with the light quark spin.

The value of α_s parameter in (5), (6) can be determined in the following way. The splitting of the S-wave heavy quarkonium $(Q_1 \overline{Q}_2)$ is determined by the expression

$$\Delta M(nS) = \frac{8}{9} \alpha_s \frac{1}{m_1 m_2} |R_{nS}(0)|^2, \tag{7}$$

where $R_{nS}(0)$ is the radial wave function of the quarkonium, at the origin. Using the experimental value of 1S-state splitting in the $c\bar{c}$ system [15]

$$\Delta M(1S, c\bar{c}) = 117 \pm 2MeV \tag{8}$$

and the $R_{1S}(0)$ value calculated in the potential model with the BT potential for the $c\bar{c}$ system, one gets the value of $\alpha_s(\Psi)$ coupling constant for the effective Coulomb interaction of heavy quarks.

In the present paper, we take into account the variation of the effective Coulomb coupling constant versus the reduced mass of the system (μ) . In the one-loop approximation at the momentum scale p^2 , the 'running' coupling constant in QCD is determined by the expression

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln\left(p^2/\Lambda_{QCD}^2\right)},\tag{9}$$

where $b = 11 - 2n_f/3$, and $n_f = 3$, when one takes into account the contribution by the virtual light quarks, $p^2 < m_c^2$. We assume that the average kinetic energies of *c*-quarks inside the diquark and the light quark-diquark system (the average kinetic energy weakly depends on the reduced mass of the system) are equal to:

$$\langle T_d \rangle \approx 0.2 \ GeV,$$
 (10)

$$\langle T_l \rangle \approx 0.4 \; GeV,$$
 (11)

so that, using the expression for the kinetic energy,

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2\mu},\tag{12}$$

where μ is the reduced mass of the system, one gets

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln\left(2\langle T \rangle \mu / \Lambda_{QCD}^2\right)},\tag{13}$$

so that $\alpha_s(\Psi) = 0.44$ at $\Lambda_{QCD} \approx 113 \ MeV$.

As one can see from equations (5) and (6), in contrast to the *LS*-coupling in the diquark, there is the *jj*-coupling in the light quark-diquark system, where diquark and light quark have the different masses (here \mathbf{LS}_l is diagonal at the given \mathbf{J}_l momentum, $(\mathbf{J}_l = \mathbf{L} + \mathbf{S}_l, \mathbf{J} = \mathbf{J}_l + \mathbf{\bar{J}})$, \mathbf{J} is the total spin of the system, and $\mathbf{\bar{J}}$ is the total spin of the diquark (as we will see below in the case of interest, $\mathbf{\bar{J}}$ equals to \mathbf{S}_d)).

To calculate the values of level shifts, appearing because of the spin-spin and spin-orbital interactions, one has to average expressions (5), (6) over the wave functions of the corresponding states. Then, because the leading contribution to the spin-orbital splitting of the light quark-diquark system is given by the term $\frac{1}{2} \frac{L \cdot S_l}{2m_l^2} \left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s \frac{1}{r^3}\right)$, we can use the state vectors with the given values of **J** and **J**_l as the first approximation to the eigenvectors of the potential. For the potential terms, which are not diagonal in these states, we can choose another basis of vectors with the given values of **J** and $\mathbf{S} = \mathbf{S}_l + \mathbf{\bar{J}}$

$$|J;J_l\rangle = \sum_{S} (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2S+1)(2J_l+1)} \left\{ \begin{array}{cc} \bar{J} & S_l & S\\ L & J & J_l \end{array} \right\} |J;S\rangle$$
(14)

or \mathbf{J} and \mathbf{J}_d

$$|J;J_l\rangle = \sum_{Jd} (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2J_d+1)(2J_l+1)} \left\{ \begin{array}{cc} \bar{J} & L & J_d \\ S_l & J & J_l \end{array} \right\} |J;Jd\rangle$$
(15)

so that the potential terms of the order of $1/m_c m_l$, $1/m_c^2$ lead, generally speaking, to the mixing of levels with the different J_l values at the given J values. The identity of heavy quarks results in $S_d = 1$ at even L_d and $S_d = 0$ at odd L_d , where L_d is the diquark orbital momentum. Table 3 shows us that we have to take into account only the spin-orbital splitting of 1S2P and 3D1S levels.

In the first case (the splitting of the light quark-diquark system levels $\Delta^{(J)}$ for 1S2P) one has:

$$\Delta^{\left(\frac{5}{2}\right)} = 17.4 \ MeV. \tag{16}$$

The levels with $J = \frac{3}{2}$ (or $\frac{1}{2}$), but with the different J_l , get the mixing. For $J = \frac{3}{2}$, the mixing matrix is equal to

$$\left(\begin{array}{cc} 4.3 & -1.7\\ -1.7 & 7.8 \end{array}\right) MeV \tag{17}$$

with the eigenvectors

$$|1S2P(\frac{3}{2}')\rangle = 0.986|J_l = \frac{3}{2}\rangle + 0.164|J_l = \frac{1}{2}\rangle,$$
(18)
$$|1S2P(\frac{3}{2})\rangle = -0.164|J_l = \frac{3}{2}\rangle + 0.986|J_l = \frac{1}{2}\rangle,$$

and the eigenvalues

$$\lambda'_{1} = 3.6 \ MeV,$$
 (19)
 $\lambda_{1} = 8.5 \ MeV.$

For $J = \frac{1}{2}$, the mixing matrix equals

$$\begin{pmatrix} -3.6 & -55.0 \\ -55.0 & -73.0 \end{pmatrix} MeV$$
 (20)

with the eigenvectors

$$|1S2P(\frac{1}{2}')\rangle = 0.957|J_l = \frac{3}{2}\rangle - 0.291|J_l = \frac{1}{2}\rangle, \qquad (21)$$
$$|1S2P(\frac{1}{2})\rangle = 0.291|J_l = \frac{3}{2}\rangle + 0.957|J_l = \frac{1}{2}\rangle,$$

and the eigenvalues

$$\lambda'_{2} = 26.8 \, MeV,$$
 (22)
 $\lambda_{2} = -103.3 \, MeV.$

In the second case (the splitting of diquark levels $\Delta^{(J_d)}$ for 3D1S) one gets:

$$\Delta^{(3)} = -3.02 \ MeV,$$

$$\Delta^{(2)} = 2.19 \ MeV,$$

$$\Delta^{(1)} = 3.39 \ MeV.$$
(23)

(24)

For the spin-spin interactions inside the diquark, we have only a shift of the energy levels because of the identity of heavy quarks.

The hyperfine splitting for the light quark-diquark system can be computed, making use of the following formula:

$$\Delta_{h.f.}^{(l)} = \frac{2}{9} (S(S+1) - \bar{J}(\bar{J}+1) - \frac{3}{4}) \alpha_s (2\mu T) \frac{1}{m_c m_l} * |R_l(0)|^2,$$
(25)

where $R_l(0)$ is the value of radial wave function in the system of light quark-diquark at the origin, and

$$\Delta_{h.f.}^{(d)} = \frac{1}{9} \alpha_s(2\mu T) \frac{1}{m_c^2} * |R_d(0)|^2, \qquad (26)$$

for the diquark level shifts, where $R_d(0)$ is the radial wave function of diquark at the origin.

For the 1S and 2S-wave states of diquark we have the following values of the shifts (because of the identity of the quarks there are no splittings of these levels)

$$\Delta(1S) = 6.3 MeV,$$

$$\Delta(2S) = 4.6 MeV.$$

The mass spectrum of doubly charmed baryons (Ξ_{cc}^{++}) and Ξ_{cc}^{+} , with the account for the calculated splittings is shown in Fig.1 and Table 4.



Figure 1: The spectrum of doubly charmed baryons: Ξ_{cc}^{++} and Ξ_{cc}^{+} .

4 Transition between diquark levels: $2P \rightarrow 1S - a$ laboratory for the long distance QCD?

Our plans for a future include a detail investigation of hadronic and radiative transitions in the spectrum of doubly charmed baryons. However, here we would like to discuss some new futures in the dynamics of baryons, containing two identical heavy quarks.

So, because of the identity of two heavy quarks, we have a metastable 2P-wave diquark state. This state has the L = 1, S = 0 quantum numbers, and, hence, the transition to the ground state (L = 0, S = 1) would require the simultaneous change of orbital and spin quantum numbers. It is worth to stress that the existence of such state is possible only in the baryons with two heavy quarks, because in ordinary heavy baryons the light diquark is never excited because of extremely large size. We have two possible scenario for the realization of such transition:

1. A three-particle interaction via three-gluon vertex. This interaction breaks down quarkdiquark picture and leads to some new wave functions of these states in the form: $C_1|L = 1, S = 0 \rangle + C_2|L = 0, S = 1 \rangle$, where $|C_2| \ll |C_1|$ for 2P. So, as one can easily see, the account for such interactions makes the radiative M1-transition to the ground state to be possible.

$(n_d(diquark)$ -	Mass (GeV)	$(n_d(diquark)$ -	Mass (GeV)
$n_l(light \; quark)), \; \mathrm{J}^P$		$n_l(light \; quark)), \mathrm{J}^P$	
$(1S \ 1S)1/2^+$	3.478	(3P 1S)1/2 ⁻	3.972
$(1S \ 1S)3/2^+$	3.61	$(3D \ 1S)3/2'^+$	4.007
(2P 1S)1/2 ⁻	3.702	(1S 2P)3/2' ⁻	4.034
$(3D \ 1S)5/2^+$	3.781	(1S 2P)3/2 ⁻	4.039
$(2S \ 1S)1/2^+$	3.812	(1S 2P)5/2 ⁻	4.047
$(3D \ 1S)3/2^+$	3.83	$(3D \ 1S)5/2'^+$	4.05
(2P 1S)3/2 ⁻	3.834	(1S 2P)1/2' ⁻	4.052
$(3D \ 1S)1/2^+$	3.875	$(3S \ 1S)1/2^+$	4.072
(1S 2P)1/2 ⁻	3.927	$(3D \ 1S)7/2^+$	4.089
$(2S \ 1S)3/2^+$	3.944	$(3P \ 1S)3/2^-$	4.104

Table 4: The mass spectra and characteristics of light quark radial wave functions of doubly charmed baryons: Ξ_{cc}^{++} and Ξ_{cc}^{+} with the different excitations of diquark, n_d , and light quark-diquark system, n_l .

2. A nonperturbative transition given by an operator, proportional to $\mu_B \vec{r} \cdot \vec{\nabla} \vec{H}(0) \cdot (\vec{S_1} - \vec{S_2})$, where μ_B is the Bohr magneton, $\vec{H}(\vec{r})$ is the chromomagnetic field and \vec{r} is the distance between two heavy quarks (here we have the same situation as in the case of transition between the orto- and para-hydrogen). This transition goes with the emission of pion. We would like to stress the nonperturbative nature of this transition, because for its realization, the necessary condition is that the chromomagnetic field at the different points is correlated. The absence of such correlation prevents this scenario of transition from realization, because it would require two consequent gluon exchanges with the light quark or quark-gluon sea, wherein the orbital or spin quantum numbers of state would change, what is impossible because of the identity of two heavy quarks.

A detail quantitative investigation of the nature for this transition is a subject of our subsequent papers. It will allow us to gain more information on the nonperturbative dynamics of QCD (in the case of second type transition) and, particularly, on the behaviour of chromomagnetic field at large distances. So, we close this section with the question, posed in its title.

5 Conclusion

In this work we have used the Buchmüller-Tye potential model within the quark-diquark approximation to describe the mass spectrum of doubly charmed baryons: Ξ_{cc}^{++} and Ξ_{cc}^{+} , including the fine and hyperfine splittings of mass levels. We have discussed the uncertainties involved and some possible ways to reduce them: the use of Born-Oppenheimer approximation and the corrected potential for the level splitting. In the previous section of article we have considered a new phenomena, arising in the radiative or hadronic transitions of $2P \rightarrow 1S$ diquark levels. We have commented on some possible scenaria of this transition and lessons, we could learn from a studying the nature of such transitions.

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