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Lifetimes of doubly charmed baryons: Ξ_{cc}^{+} and Ξ_{cc}^{++}.

V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko
State Research Center of Russia "Institute for High Energy Physics"
Protvino, Moscow region, 142284 Russia
Fax: +7-095-2302337
E-mail: kiselev@mx.ihep.su

Abstract

We have performed a detailed investigation of total lifetimes for the Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons in the framework of operator product expansion over the inverse mass of charmed quark, whereas, to estimate matrix elements of operators obtained in OPE, some approximations of nonrelativistic QCD are used. This approach allows one to take into account the corrections to the spectator decays of c-quarks, which reflect the fact, that these quarks are bound, as well as the contributions, connected to the effects of both the Pauli interference for the Ξ_{cc}^{++}-baryon and the weak scattering for the Ξ_{cc}^{+}-baryon. The realization of such program leads to the following estimates for the total lifetimes of doubly charmed baryons: $\tau_{\Xi_{cc}^{++}} = 0.43 \pm 0.1\ ps$ and $\tau_{\Xi_{cc}^{+}} = 0.11 \pm 0.01\ ps$.

1 Introduction

A study on weak decays of doubly charmed baryons is of great interest because of two reasons. The first one is connected to the investigation on the basic properties of weak interactions at the fundamental level, including the precise determination of CKM matrix parameters. The second reason is related to the possibility to explore QCD as it is provided by the systems containing the heavy quarks. In the limit of a large scale given by the heavy quark mass, some aspects in the dynamics of strong interactions become simpler and one gets a possibility to draw definite model-independent predictions. Of course, both these topics appear in the analysis of weak decays for the doubly charmed baryons, whose dynamics is determined by an interplay between the strong and weak interactions. That is why these baryons are the attractive and reasonable subjects for the theoretical and experimental consideration.

The doubly charmed $\Xi_{cc}^{(*)}$-baryon represents an absolutely new type of objects in comparison with the ordinary baryons containing light quarks only. The basic state of such baryon is analogous to a $(\bar{Q}q)$-meson, which contains a single heavy antiquark $\bar{Q}$ and a light quark q. In the doubly heavy baryon the role of heavy antiquark is played by the (cc)-diquark, which is in antitriplet color-state. It has a small size in comparison with the scale of the light quark confinement. Nevertheless the spectrum of (ccq)-system states has to differ essentially from the heavy meson spectra, because the composed (cc)-diquark has a set of the excited states (for example, $2S$ and $2P$) in contrast to the heavy quark. The energy of diquark excitation is twice less than the excitation energy of light quark bound with the diquark. So, the representation on the compact diquark can be straightforwardly connected to the level structure of doubly heavy baryon.

Naive estimates for the lifetimes of doubly charmed baryons were done by the authors slightly early [1]. A simple consideration of quark diagrams shows, that in the decay of Ξ_{cc}^{++}-baryons, the Pauli interference for the decay products of charmed quark and the valent quark in the initial state takes place in an analogous way to the D^+-meson decay. In the decay of Ξ_{cc}^{+}, the exchange by the W-boson between the valence quarks plays an important role like in the decay of D^0. These speculations and the presence of two charmed quarks in the initial state result in the following estimates for the lifetimes:

$$\begin{aligned} \tau(\Xi_{cc}^{++}) &\approx \frac{1}{2}\tau(D^+) \simeq 0.53 \text{ ps}, \\ \tau(\Xi_{cc}^{+}) &\approx \frac{1}{2}\tau(D^0) \simeq 0.21 \text{ ps}. \end{aligned}$$

In this work we discuss the systematic approach to the evaluation of total lifetimes for the doubly charmed baryons on the basis of both the optical theorem for the inclusive decay width and the operator product expansion (OPE) for the transition currents in accordance with the consequent nonrelativistic expansion of hadronic matrix elements derived in OPE. Using OPE at the first step, we exploit the fact, that, due to the presence of heavy quarks in the initial state, the energy release in the decay of both quarks is large enough in comparison with the binding energy in the state. Thus, we can use the expansion over the ratio of these scales. Technically, this step repeats an analogous procedure for the inclusive decays of heavy-light mesons as it was reviewed in [2]. Exploring the nonrelativistic expansion of hadronic matrix elements at the second step, we use the approximation of nonrelativistic QCD [3, 4], which allows one to reduce the evaluation of matrix elements for the full QCD operators, corresponding to the interaction of heavy quarks inside the diquark, to the expansion in powers of $\frac{p_c}{m_c}$, where $p_c = m_c v_c \sim 1\ GeV$ is a typical momentum of the heavy quark inside the baryon. The same procedure for the matrix elements, determined by the strong interaction of heavy quarks with the light quark, leads to the expansion in powers of $\frac{\Lambda_{QCD}}{m_c}$.

This way, taking into account the radiation of hard gluons in these decays, leads to the expansion in powers of α_s, $v_c = \frac{p}{m_c}$ and $\frac{\Lambda_{QCD}}{m_c}$. It is worth to note, that this expansion would be well defined, provided the expansion parameters to be small. In the $c \to su\bar{d}$ transition, the ratio of typical momentum for the heavy quark inside the hadron to the value of energy, released in the decay, is not so small. We would like also to stress the important roles, played by both the Pauli interference and the weak scattering, suppressed as $\frac{1}{m_c^3}$ with respect to the leading spectator contribution, but the former ones are enhanced by a numerical factor, caused by the ratio of two-particle and three-particle phase spaces [5]. Numerical estimates show that the value of these contributions is considerably large, and it is of the order of $40 - 140\%$. These effects take place in the different baryons, Ξ_{cc}^{++} and Ξ_{cc}^{+}, and, thus, they enhance the difference of lifetimes for these baryons. The final result for the total lifetimes of doubly charmed baryons is the following:

$$\begin{aligned} \tau_{\Xi_{cc}^{++}} &= 0.43 \pm 0.1\ ps, \\ \tau_{\Xi_{cc}^{+}} &= 0.11 \pm 0.01\ ps. \end{aligned}$$

2 Operator product expansion

Now let us start the description of our approach for the calculation of total lifetimes for the doubly charmed baryons. The optical theorem, taking into account the integral quark-hadron duality, allows us to relate the total decay width of the heavy quark with the imaginary part of its forward scattering amplitude. This relationship, applied to the $\Xi_{cc}^{(*)}$-baryon total decay width $\Gamma_{\Xi_{cc}^{(*)}}$, can be written down as:

$$\Gamma_{\Xi_{cc}^{(*)}} = \frac{1}{2M_{\Xi_{cc}^{(*)}}} \langle \Xi_{cc}^{(*)} | \mathcal{T} | \Xi_{cc}^{(*)} \rangle, \tag{1}$$

where the $\Xi_{cc}^{(*)}$ state in (1) has the ordinary relativistic normalization, $\langle \Xi_{cc}^{(*)} | \Xi_{cc}^{(*)} \rangle = 2EV$, and the transition operator $\mathcal{T}$ is determined by the expression

$$\mathcal{T} = \Im m \int d^4x\ \{\hat{T} H_{eff}(x) H_{eff}(0)\}, \tag{2}$$

where H_{eff} is the standard effective hamiltonian, describing the low energy interactions of initial quarks with the decays products, so that

$$H_{eff} = \frac{G_F}{2\sqrt{2}} V_{uq_1} V_{cq_1}^* [C_+(\mu) O_+ + C_-(\mu) O_-] + h.c. \tag{3}$$

where

$$O_\pm = [\bar{q}_{1\alpha} \gamma_\nu (1-\gamma_5) c_\beta][\bar{u}_\gamma \gamma^\nu (1-\gamma_5) q_{2\delta}](\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$

and

$$C_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{33-2f}}, \quad C_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{-12}{33-2f}},$$

where f is the number of flavors.

Assuming that the energy release in the heavy quark decay is large, we can perform the operator product expansion for the transition operator $\mathcal{T}$ in (1). In this way we find a series of local operators with increasing dimension over the energy scale, wherein the contributions to $\Gamma_{\Xi_{cc}^{(*)}}$ are suppressed by the increasing inverse powers of the heavy quark masses. This formalism has already been applied to calculate the total decay rates for the hadrons, containing a single heavy quark [2] (for the most early work, having used similar methods, see also [6, 7]). Here we would like to stress that the expansion, applied in this paper, is simultaneously in the powers of the inverse heavy quark mass and the relative velocity of heavy quarks inside the hadron. Thus, the latter points to the difference from the description of both the heavy-light mesons (the expansion in powers of $\frac{\Lambda_{QCD}}{m_c}$) and the heavy-heavy mesons [8] (the expansion in powers of relative velocity of heavy quarks inside the hadron, where one can apply the scaling rules of nonrelativistic QCD [4]).

In this work we will extend this approach to the treatment of baryons, containing two heavy quarks. The operator product expansion applied has the form:

$$\mathcal{T} = C_1(\mu)\bar{c}c + \frac{1}{m_c^2}C_2(\mu)\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c + \frac{1}{m_c^3}O(1). \tag{4}$$

The leading contribution in OPE is determined by the operator $\bar{c}c$, corresponding to the spectator decays of c-quarks. The use of the equation of motion for the heavy quark fields allows one to eliminate some redundant operators, so that no operators of dimension four contribute. There is a single operator of dimension five, $Q_{GQ} = \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$. As we will show below, significant contributions come from the operators of dimension six $Q_{2Q2q} = \bar{Q}\Gamma q\bar{q}\Gamma' Q$, representing the effects of Pauli interference and weak scattering for Ξ_{cc}^{++} and Ξ_{cc}^{+}, correspondingly. Furthermore, there are also other operators of dimension six $Q_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l}Q$ and $Q_{62Q} = \bar{Q}D_\mu G^{\mu\nu}\Gamma_\nu Q$. In what follows, we do not calculate the corresponding coefficient functions for the latter two operators, so that the expansion is certainly complete up to the second order of $\frac{1}{m}$, only.

Further, the different contributions to OPE are given by the following

$$\begin{aligned} \mathcal{T}_{\Xi_{cc}^{++}} &= \mathcal{T}_{35c} + \mathcal{T}_{6,PI}, \\ \mathcal{T}_{\Xi_{cc}^{+}} &= \mathcal{T}_{35c} + \mathcal{T}_{6,WS}, \end{aligned}$$

where the first terms account for the operators of dimension three O_{3Q} and five O_{GQ}, the second terms correspond to the effects of Pauli interference and weak scattering. The explicit formulae for these contributions have the following form:

$$\mathcal{T}_{35c} = 2 * (\Gamma_{c,spec}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2 + K_{0c})P_1 + K_{2c}P_2]O_{Gc}), \tag{5}$$

where $\Gamma_{0c} = \frac{G_F^2 m_c^2}{192\pi^3}$ and $K_{0c} = C_-^2 + 2C_+^2$, $K_{2c} = 2(C_+^2 - C_-^2)$. This expression has been derived in [9] (see also [10]), and it is also discussed in [2]. The phase space factors P_i look like [2, 11]:

$$P_1 = (1 - y)^4, \quad P_2 = (1 - y)^3,$$

where $y = \frac{m_s^2}{m_c^2}$.

$\Gamma_{c,spec}$ denotes the contribution to the total decay width of the free decay for one of the two c-quarks, which is explicitly expressed below.

For the effects of Pauli interference and weak scattering, we find the following formulae:

$$
\begin{aligned}
\mathcal{T}_{PI} &= -\frac{2G_F^2}{4\pi} m_c^2 (1-\frac{m_u}{m_c})^2 \\
&([(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5 c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j)] \\
&[(C_+ + C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_-C_+)] + \\
&[(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)c_i) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5 c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 - 6C_-C_+)),
\end{aligned} \quad (6)
$$

$$
\begin{aligned}
\mathcal{T}_{WS} &= \frac{2G_F^2}{4\pi} p_+^2 (1-z_+)^2 [(C_+^2 + C_-^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2 - C_-^2)) \\
&(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j) + \\
&k^{\frac{1}{2}}(C_+^2 - C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i)],
\end{aligned} \quad (7)
$$

where $p_+ = p_c + p_q$, $p_- = p_c - p_q$ and $z_\pm = \frac{m_c^2}{p_\pm^2}$, $k = \alpha_s(\mu)/\alpha_s(m_c)$.

In the numerical estimates for the evolution of coefficients C_+ and C_-, we have taken into account the threshold effects, connected to the b-quark, as well as the threshold effects, related to the c-quark mass in the Pauli interference and weak scattering.

In expression (5), the scale μ is approximately equal to m_c. For the Pauli interference and weak scattering, this scale was chosen in the way to obtain an agreement between the experimental differences in the lifetimes of Λ_c, Ξ_c^+ and Ξ_c^0-baryons and the theoretical predictions, based on the effects, mentioned above. This problem is discussed below. Anyway, the choice of these scales allows some variations, and a complete answer to this question requires calculations in the next order of perturbative theory.

The contribution of the leading operator $\bar{c}c$ corresponds to the imaginary part of the diagram in Fig. 1, as it stands in expression (4). The coefficient of $\bar{c}c$ can be obtained in the usual way by matching of the Fig. 1 diagram, corresponding to the leading term in expression (4), with the operator $\bar{c}c$. This coefficient is equivalent to the free quark decay rate, and it is known in the next-to-leading logarithmic approximation of QCD [12, 13, 14, 15, 16], including the strange quark mass effects in the final state [16]. To calculate the next-to-leading logarithmic effects, the Wilson coefficients in the effective weak lagrangian are required at the next-to-leading accuracy, and the single gluon exchange corrections to the diagram in Fig.1 must be considered. In our numerical estimates we use the expression for Γ_{spec}, including the next-to-leading order corrections, s-quark mass effects in the final state, but we neglect the Cabibbo-suppressed decay channels for the c-quark. The bulky explicit expression for the spectator c-quark decay is placed in the Appendix.

Similarly, the contribution by O_{GQ} is obtained, when an external gluon line is attached to the inner quark lines in Fig. 1 in all possible ways. The corresponding coefficients are known in the leading logarithmic approximation. Finally, the dimension six operators and their coefficients arise due to those contributions, wherein one of the internal u or $\bar{d}$ quark line is "cut" in the diagram of Fig. 1. The resulting graphs are depicted in Figs. 2 and 3. These contributions correspond to the effects of Pauli interference and weak scattering. We have calculated the expressions for these effects with account for both the s-quark mass in the final state and the logarithmic renormalization of effective electroweak lagrangian at low energies.

Since the simultaneous account for the mass effects and low-energy logarithmic renormalization of such contributions has been performed in this work for the first time, we would like to discuss this question in some details.

The straightforward calculation of diagrams of Figs. 2 and 3 with the account for the s-quark mass yields the following expressions:

$$\begin{aligned}
\mathcal{T}_{PI} &= -\frac{2G_F^2}{4\pi}p_-^2[(\frac{(1-z_-)^3}{12}g^{\alpha\beta}+(\frac{(1-z_-)^3}{2}-\frac{(1-z_-)^3}{3})\frac{p_-^\alpha p_-^\beta}{p_-^2})] \\
&[(C_++C_-)^2(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(q_j\gamma_\beta(1-\gamma_5)q_j)+ \qquad (8)\\
&(5C_+^2-6C_+C_-+C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(q_j\gamma_\beta(1-\gamma_5)q_i)],\\
\mathcal{T}_{WS} &= \frac{2G_F^2m_c^2}{4\pi}p_+^2(1-z_+)^2[(C_+^2+C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(q_j\gamma_\beta(1-\gamma_5)q_j)+\\
&(C_+^2-C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(q_j\gamma_\beta(1-\gamma_5)q_i)]. \qquad (9)
\end{aligned}$$

For p_+ and p_- we use their threshold values:

$$p_+ = p_c(1+\frac{m_q}{m_c}),\quad p_- = p_c(1-\frac{m_q}{m_c}),$$

taking into account that the logarithmic renormalization of effective low-energy lagrangian has the following form [5, 6]:

$$\begin{aligned}
L_{eff,log} &= \frac{G_f^2m_c^2}{2\pi}\{\frac{1}{2}[C_+^2+C_-^2+\frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2-C_-^2)](\bar{c}\Gamma_\mu)(\bar{d}\Gamma^\mu d)+\\
&\frac{1}{2}(C_+^2-C_-^2)k^{\frac{1}{2}}(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma^\mu c)+\frac{1}{3}(C_+^2-C_-^2)k^{\frac{1}{2}}(k^{\frac{-2}{9}}-1)(\bar{c}\Gamma_\mu t^a c)j_\mu^a- \qquad (10)\\
&\frac{1}{8}[(C_++C_-)^2+\frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2+C_-^2-6C_+C_-)](\bar{c}\Gamma_\mu c+\frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c)(\bar{u}\Gamma^\mu u)-\\
&\frac{1}{8}k^{\frac{1}{2}}(5C_+^2+C_-^2-6C_+C_-)(\bar{c}_i\Gamma_\mu c_k+\frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{u}_k\Gamma^\mu u_i)-\\
&\frac{1}{8}[(C_+-C_-)^2+\frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2+C_-^2+6C_+C_-)](\bar{c}\Gamma_\mu c+\frac{2}{3}\bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\Gamma^\mu s)-\\
&\frac{1}{8}k^{\frac{1}{2}}(5C_+^2+C_-^2+6C_+C_-)(\bar{c}_i\Gamma_\mu c_k+\frac{2}{3}\bar{c}_i\gamma_\mu\gamma_5 c_k)(\bar{s}_k\Gamma^\mu s_i)-\\
&\frac{1}{6}k^{\frac{1}{2}}(k^{\frac{-2}{9}}-1)(5C_+^2+C_-^2)(\bar{c}\Gamma_{mu}t^a c+\frac{2}{3}\bar{c}\gamma_\mu\gamma_5 t^a c)j^{a\mu}\},
\end{aligned}$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, $k = (\alpha_s(\mu)/\alpha_s(m_c))$ and $j_\mu^a = \bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d + \bar{s}\gamma_\mu t^a s$ is the color current of light quarks ($t^a = \lambda^a/2$ being the color generators). Having performed the manipulations, we have obtained formulae (6), (7).

Here we would like to make a note, concerning the terms of effective lagrangian, containing the color current of light quarks. In the analysis below, we have omitted these terms, because they contribute in the lagrangian with the strength factor $k^{-\frac{2}{9}}-1$, whose numerical value is equal to 0.054 (see below).

To calculate the contribution of semileptonic modes to the total decay width of $\Xi_{cc}^{(*)}$-baryons (we have taken into account the electron and muon decay modes only) we use the following expressions [10] (see also [16]):

$$\begin{aligned}\Gamma_{sl} &= 4\Gamma_c(\{1-8\rho+8\rho^3-\rho^4-12\rho^2\ln\rho\}+ \\ &E_c\{5-24\rho+24\rho^2-8\rho^3+3\rho^4-12\rho^2\ln\rho\}+ \\ &K_c\{-6+32\rho-24\rho^2-2\rho^4+24\rho^2\ln\rho\}+ \\ &G_c\{-2+16\rho-16\rho^3+2\rho^4+24\rho^2\ln\rho\}),\end{aligned} \tag{11}$$

where $\Gamma_c = |V_{cs}|^2 G_F^2 \frac{m_c^5}{192\pi^3}$, $\rho = \frac{m_s^2}{m_c^2}$. The quantities $E_c = K_c + G_c$, K_c and G_c are given by the expressions:

$$\begin{aligned}K_c &= -\langle \Xi_{cc}^{(*)}(v)|\bar{c}_v \frac{(iD)^2}{2m_c^2} c_v|\Xi_{cc}^{(*)}(v)\rangle, \\ G_c &= \langle \Xi_{cc}^{(*)}(v)|\bar{c}_v \frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_c^2} c_v|\Xi_{cc}^{(*)}(v)\rangle,\end{aligned} \tag{12}$$

where the spinor field c_v in the effective heavy quark theory is defined by the form:

$$c(x) = e^{-im_c v\cdot x}\Big[1+\frac{iD}{2m_c}\Big]c_v(x). \tag{13}$$

Thus, we can see, that the evaluation of total lifetimes for the doubly charmed baryons is reduced to the problem of estimation for the matrix elements of operators, appearing in the above expressions, which is the topic of next section.

3 Evaluation of matrix elements.

Let us calculate the matrix elements for the operators, obtained as the result of OPE for the transitions under consideration. In general, it is a complicated nonperturbative problem, but, as we will see below, in our particular calculation we can get some reliable estimates for the matrix elements of required operators.

Using the equation of motion for the heavy quarks, the local operator $\bar{c}c$ can be expanded in the following series over the powers of $\frac{1}{m_c}$:

$$\langle \Xi_{cc}^{(*)}|\bar{c}c|\Xi_{cc}^{(*)}\rangle_{norm} = 1 - \frac{\langle \Xi_{cc}^{(*)}|\bar{c}[(i\vec{D})^2-(\frac{i}{2}\sigma G)]c|\Xi_{cc}^{(*)}\rangle_{norm}}{2m_c^2} + O(\frac{1}{m_c^3}). \tag{14}$$

Thus, this evaluation can be reduced to the calculation of matrix elements for the following operators:

$$\bar{c}(i\vec{D})^2c, \quad (\frac{i}{2})\bar{c}\sigma Gc, \quad \bar{c}\gamma_\alpha(1-\gamma_5)c\bar{q}\gamma^\alpha(1-\gamma_5)q, \quad \bar{c}\gamma_\alpha\gamma_5 c\bar{q}\gamma^\alpha(1-\gamma_5)q.$$

The first operator corresponds to the time dilation, connected to the motion of heavy quarks inside the hadron, the second is related to the spin interaction of heavy quarks with the chromomagnetic field of light quark and the other heavy quark. Further, the third and fourth operators are the four-quark operators, representing the effects of Pauli interference and weak scattering.

In the system, containing the nonrelativistic heavy quark, the quark-antiquark pairs with the same flavor can be produced with a negligible rate, since the energy greater than

m_Q is required. In this situation, it is useful to integrate out the small components of the heavy-quark spinor field and to present the result in terms of the two component spinor Ψ_Q. Following this approach, we find that all contributions from virtualities greater than μ, where $m_c > \mu > m_c v_c$, can be explicitly taken into account in the perturbative theory. This method is general and analogous to the effective heavy quark theory. So,

$$\begin{aligned}\bar{c}c &= \Psi_c^+\Psi_c - \frac{1}{2m_c^2}\Psi_c^+(i\vec{D})^2\Psi_c + \frac{3}{8m_c^4}\Psi_c^+(i\vec{D})^4\Psi_c - \\ &\quad \frac{1}{2m_c^2}\Psi_c^+ g\vec{\sigma}\vec{B}\Psi_c - \frac{1}{4m_c^3}\Psi_c^+(\vec{D}g\vec{E})\Psi_c + ... \qquad (15)\\ \bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c &= -2\Psi_c^+ g\vec{\sigma}\vec{B}\Psi_c - \frac{1}{m_c}\Psi_c^+(\vec{D}g\vec{E})\Psi_c + ... \qquad (16)\end{aligned}$$

In these expressions we have omitted the term $\Psi_c^+\vec{\sigma}(g\vec{E}\times\vec{D})\Psi_c$, corresponding to the spin-orbital interaction, because it vanishes in the ground states of doubly charmed baryons. By definition, the two-component spinor Ψ_c has the same normalization as Q,

$$\int d^3x\Psi_c^+\Psi_c^+ = \int d^3x Q^+ Q. \qquad (17)$$

Then, with the required accuracy, Ψ_c can be expressed through the big components of spinor Q

$$Q \equiv e^{-imt}\begin{pmatrix}\phi\\ \chi\end{pmatrix} \qquad (18)$$

due to the following formula

$$\Psi_c = \left(1 + \frac{(i\vec{D})^2}{8m_c^2}\right)\phi. \qquad (19)$$

(this can be checked with the use of the equation of motion). Let us note that the covariant derivative should be taken in the adjoint representation, when it acts on the chromoelectric field,

$$(\vec{D}\vec{E}) = (\vec{\partial}T^a - gf^{abc}T^b\vec{A}^c)\vec{E}^a. \qquad (20)$$

Radiative corrections modify the coefficients of the chromomagnetic term $(\vec{\sigma}\vec{B})$ and "Darwin" term in (15). However, in the situation at hand, these effects can be consistently neglected.

Now let us consider the significance of different contributions to the expansions in (15) and (16). Evaluating the contributions of chromomagnetic and "Darwin" terms, we have to take into account the interaction of heavy quark with the light quark as well as the interaction with the other heavy quark. In the first case, the procedure of calculation is analogous to that for the heavy-light mesons. So, the "Darwin" term is suppressed by a factor of $\frac{\Lambda_{QCD}}{m_c}$ in comparison with the chromomagnetic term, and, thus, we neglect its contribution. In the second case, the analysis is analogous to that for the heavy-heavy mesons, so that we can use the scaling rules of nonrelativistic QCD [4]. In this approach, the contributions of different operators can be estimated, using the following relations in Coulomb gauge:

$$\Psi_c \sim (m_c v_c)^{\frac{3}{2}}, \quad \vec{D} \sim m_c v_c, \quad gE \sim m_c^2 v_c^3, \quad gB \sim m_c^2 v_c^4, \quad g \sim v_c^{\frac{1}{2}}.$$

From these scaling rules for the heavy-heavy interaction, we can deduce that the contribution of the "Darwin" term has the same order as that of chromomagnetic term.

Let us now start the calculation of matrix elements with the use of potential models for the bound states of hadrons. While estimating the matrix element value of the kinetic energy, we note, that the heavy quark kinetic energy consists of two parts: the kinetic energy of the heavy quark motion inside the diquark and the kinetic energy, related to the diquark motion inside the hadron. According to the phenomenology of meson potential models, in the range of average distances between the quarks: 0.1 fm $< r <$ 1 fm, the average kinetic energy of quarks is constant and independent of both the quark flavors, constituting meson, and the quantum numbers, describing the excitations of the ground state. Therefore, we determine $T = m_d v_d^2/2 + m_l v_l^2/2$ as the average kinetic energy of diquark and light quark, and $T/2 = m_{c1} v_{c1}^2/2 + m_{c2} v_{c2}^2/2$ as the average kinetic energy of heavy quarks inside the diquark (the coefficient 1/2 takes into account the antisymmetry of color wave function for the diquark). Finally, we have the following expression for the matrix element of the heavy quark kinetic energy:

$$\frac{\langle \Xi_{cc}^{(*)} | \Psi_c^+ (i\vec{D})^2 \Psi_c | \Xi_{cc}^{(*)} \rangle}{2M_{\Xi_{cc}^{(*)}} m_c^2} \simeq v_c^2 \simeq \frac{m_l T}{2m_c^2 + m_c m_l} + \frac{T}{2m_c}. \qquad (21)$$

We use the value $T \simeq 0.4$ GeV, which results in $v_c^2 = 0.146$, where the dominant contribution comes from the motion of heavy quarks inside the diquark.

Now we would like to estimate the matrix element of chromomagnetic operator, corresponding to the interaction of heavy quarks with the chromomagnetic field of the light quark. For this purpose, we will use the following definitions: $O_{mag} = \sum_{i=1}^{2} \frac{g_s}{4m_c} \bar{c}^i \sigma_{\mu\nu} G^{\mu\nu} c^i$ and $O_{mag} \sim \lambda(j(j+1) - s_d(s_d+1) - s_l(s_l+1))$, where s_d is the diquark spin (as was noticed by the authors earlier [1], there is only the vector state of the cc-diquark in the ground state of such baryons), s_l is the light quark spin and j is the total spin of the baryon. Since both c-quarks additively contribute to the total decay width of baryons, we can use the diquark picture and substitute for the sum of c-quark spins the diquark spin. This leads to the parameterization for O_{mag}, as it is given above, and, moreover, it allows us to relate the value of the matrix element for this operator to the mass difference between the excited and ground state of baryons:

$$O_{mag} = -\frac{2}{3}(M^*_{\Xi_{cc}^{(*)}} - M_{\Xi_{cc}^{(*)}}). \qquad (22)$$

The account for the interaction of heavy quarks inside the diquark leads to the following expressions for the chromomagnetic and "Darwin" terms:

$$\frac{\langle \Xi_{cc}^{(*)} | \Psi_c^+ g \vec{\sigma} \cdot \vec{B} \Psi_c | \Xi_{cc}^{(*)} \rangle}{2M_{\Xi_{cc}^{(*)}}} = \frac{2}{9} g^2 \frac{|\Psi(0)|^2}{m_c}, \qquad (23)$$

$$\frac{\langle \Xi_{cc}^{(*)} | \Psi_c^+ (\vec{D} \cdot g\vec{E}) \Psi_c | \Xi_{cc}^{(*)} \rangle}{2M_{\Xi_{cc}^{(*)}}} = \frac{2}{3} g^2 |\Psi(0)|^2. \qquad (24)$$

where $\Psi(0)$ is the diquark wave function at the origin.

Collecting the results given above, we find the matrix elements of operators (15) and (16):

$$\frac{\langle \Xi_{cc}^{(*)} | \bar{c}c | \Xi_{cc}^{(*)} \rangle}{2M_{\Xi_{cc}^{(*)}}} = 1 - \frac{1}{2} v_c^2 - \frac{1}{3} \frac{M_{\Xi_{cc}^{(*)}} - M_{\Xi_{cc}^{(*)}}}{m_c} - \frac{g^2}{9m_c^3} |\Psi(0)|^2 -$$

$$\frac{1}{6m_c^3}g^2|\Psi(0)|^2 + ...$$

$$\approx 1 - 0.074 - 0.004 - 0.003 - 0.005 + \dots \quad (25)$$

We can see that the largest contribution to the decrease of the decay width comes from the time dilation, connected to the motion of heavy quarks inside the baryon. For the matrix element of the operator $\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c$, we get:

$$\frac{\langle\Xi_{cc}^{(*)}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{cc}^{(*)}\rangle}{2M_{\Xi_{cc}^{(*)}}} = -\frac{4}{3}\frac{(M^*_{\Xi_{cc}^{(*)}} - M_{\Xi_{cc}^{(*)}})}{m_c} - \frac{4g^2}{9m_c^3}|\Psi(0)|^2 - \frac{g^2}{3m_c^3}|\Psi(0)|^2. \quad (26)$$

Now let us continue with the calculation of the matrix elements for the four-quark operators, corresponding to the effects of Pauli interference and weak scattering. The straightforward calculation in the framework of nonrelativistic QCD gives

$$(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q) = 2m_c V^{-1}(1 - 4S_c S_q), \quad (27)$$

$$(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q) = -4S_c S_q \cdot 2m_c V^{-1}, \quad (28)$$

where $V^{-1} = |\Psi_1(0)|^2$, and $\Psi_1(0)$ is the light quark wave function at the origin of two c-quarks. We suppose, that $|\Psi_1(0)|$ has the same value as that in the D-meson. So, we find:

$$|\Psi_1(0)|^2 \approx \frac{f_D^2 m_D^2}{12m_c}. \quad (29)$$

Then, again remembering that both c-quarks additively contribute to the total decay width and using the diquark picture, we can substitute for $S_{c_1} + S_{c_2}$ the S_d, where S_d is the diquark spin. Thus, we have

$$\langle\Xi_{cc}^{(*)}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{cc}^{(*)}\rangle = 10m_c \cdot |\Psi_1(0)|^2, \quad (30)$$

$$\langle\Xi_{cc}^{(*)}|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_c c^{(*)}\rangle = 8m_c \cdot |\Psi_1(0)|^2. \quad (31)$$

The color antisymmetry of the baryon wave function results in relations between the matrix elements of operators with the different sums over the color indexes:

$$\langle\Xi_{cc}^{(*)}|(\bar{c}_i T_\mu c_k)(\bar{q}_k\gamma^\mu(1-\gamma_5)q_i|\Xi_{cc}^{(*)}\rangle = -\langle\Xi_{cc}^{(*)}|(\bar{c}T_\mu c)(\bar{q}\gamma^\mu(1-\gamma_5)q|\Xi_{cc}^{(*)}\rangle,$$

where T_μ is any spinor structure. Thus, we completely derive the expressions for the evaluation of the required matrix elements.

4 Numerical estimates

Now we are ready to collect the contributions, described above, and to estimate the total lifetimes of baryons Ξ_{cc}^{++} and Ξ_{cc}^{+}. For the beginning, we list the values of parameters, which we have used in our calculations, and give some comments on their choice.

$$\begin{aligned} m_c &= 1.6\ GeV, & m_s &= 0.45\ GeV, & |V_{cs}| &= 0.9745, \\ M_{\Xi_{cc}^{++}} &= 3.56\ GeV, & M_{\Xi_{cc}^{+}} &= 3.56\ GeV, & M^*_{\Xi_{cc}^{(*)}} - M_{\Xi_{cc}^{(*)}} &= 0.1\ GeV, \\ T &= 0.4\ GeV, & |\Psi(0)| &= 0.17\ GeV^{\frac{3}{2}}, & m_l &= 0.30\ GeV. \end{aligned}$$

For the parameters $M_{\Xi_{cc}^{++}}$, $M_{\Xi_{cc}^{+}}$ and $M^*_{\Xi_{cc}^{(*)}} - M_{\Xi_{cc}^{(*)}}$ we use the mean values, given in the literature. Their evaluation has been also performed by the authors in the potential model for the doubly charmed baryons with the Buchmüller-Tye potential, and also in papers [17, 18, 19, 20]. For f_D we use the value, given in [6, 21] and for T we take it from [22]. The mass m_c corresponds to the pole mass of the c-quark. For its determination we have used a fit of theoretical predictions for the lifetimes and semileptonic width of the D^0-meson from the experimental data. This choice of c-quark mass seems effectively to include unknown contributions of higher orders in perturbative QCD to the total decay width of baryons under consideration.

The renormalization scale μ is chosen in the following way: $\mu_1 = m_c$ in the estimate of Wilson coefficients C for the effective four-fermion weak lagrangian with the c-quarks at low energies and $\mu_2 = 1.2\ GeV$ for the Pauli interference and weak scattering (k-factor). The latter value of renormalization scale has been obtained from the fit of theoretical predictions for the lifetimes differences of baryons Λ_c, Ξ_c^+, Ξ_c^0 over the experimental data. Here we would like to note, that the theoretical approximations used in [5] include the effect of logarithmic renormalization and do not take into account the mass effects, related to the s-quark in the final state. For the corresponding contributions to the decay widths of baryons with the different quark content we have:

$$\begin{aligned} \triangle\Gamma_{nl}(\Lambda_c) &= c_d\langle O_d\rangle_{\Lambda_c} + c_u\langle O_u\rangle_{\Lambda_c}, \\ \triangle\Gamma_{nl}(\Xi_c^+) &= c_s\langle O_s\rangle_{\Xi_c^+} + c_u\langle O_u\rangle_{\Xi_c^+}, \\ \triangle\Gamma_{nl}(\Xi_c^0) &= c_d\langle O_d\rangle_{\Xi_c^0} + c_s\langle O_s\rangle_{\Xi_c^0}, \end{aligned} \tag{32}$$

where $\langle O_q\rangle_{X_c} = \langle X_c|O_q|X_c\rangle$, $O_q = (\bar{c}\gamma_\mu c)(\bar{q}\gamma^\mu q)$ and $q = u, d, s$. The coefficients $c_q(\mu)$ are equal to:

$$\begin{aligned} c_d &= \frac{G_f^2 m_c^2}{4\pi}[C_+^2 + C_-^2 + \frac{1}{3}(4k^{\frac{1}{2}} - 1)(C_-^2 - C_+^2)], \\ c_u &= -\frac{G_f^2 m_c^2}{16\pi}[(C_+ + C_-)^2 + \frac{1}{3}(1 - 4k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_+C_-)], \\ c_s &= -\frac{G_f^2 m_c^2}{16\pi}[(C_+ - C_-)^2 + \frac{1}{3}(1 - 4k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_+C_-)]. \end{aligned} \tag{33}$$

We use the spin averaged value of the D-meson mass for the estimation of the effective light quark mass m_l as it stands below:

$$m_D = m_c + m_l + \frac{T \cdot m_l}{m_c + m_l} \approx 1.98\ \ GeV. \tag{34}$$

The s-quark mass can be written down as:

$$m_s = m_l + 0.15\, GeV. \tag{35}$$

As we have already mentioned, the spectator decay width of c-quark $\Gamma_{c,spec}$ is known in the next-to-leading order of perturbative QCD [12, 13, 14, 15, 16]. The most complete calculation, including the mass effects, connected to the s-quark in the final state, is given in [16]. In the present work we have used the latter result for the calculation of the spectator contribution to the total decay width of doubly charmed baryons. In the calculation of the semileptonic decay width, we neglect the electron and muon masses in the final state. Moreover, we neglect the τ-lepton mode.

Now, let us proceed with the numerical analysis of contributions by the different decay modes into the total decay width. In table 1 we have listed the results for the fixed values of parameters, described above. From this table one can see the significance of effects caused by both the Pauli interference and the weak scattering in the decays of doubly charmed baryons. The Pauli interference gives the negative correction about 36% for the Ξ_{cc}^{++}-baryons, and the weak scattering increases the total width by 144% for Ξ_{cc}^{+}. As it has been already noted in the Introduction, these effects take place differently in the baryons, and, thus, they enhance the difference of lifetimes for these hadrons.

It is worth here to recall that the lifetime difference of D^+ and D^0-mesons is generally explained by the Pauli interference of c-quark decay products with the antiquark in the initial state, while in the current consideration, we see the dominant contribution of weak scattering. This could not be surprising, because under a more detailed consideration we will find, that the formula for the Pauli interference operator for the D-meson coincides with that for the weak scattering in the case of baryons, containing, at least, a single c-quark.

Finally, collecting the different contributions for the total lifetimes of doubly charmed baryons, we obtain the following values:

$$\tau_{\Xi_{cc}^{++}} = 0.43\, ps, \quad \tau_{\Xi_{cc}^{+}} = 0.12\, ps.$$

Rather broad variations of both the c-quark mass in the range of $1.6 - 1.65\ \ GeV$ and the mass difference for the strange and ordinary light quarks in (35) in the range of $0.15 - 0.2\ \ GeV$, lead to the uncertainties in the lifetimes: $\delta\tau_{\Xi_{cc}^{++}} = \pm 0.1\ \ ps$, $\delta\tau_{\Xi_{cc}^{+}} = \pm 0.01\ \ ps$.

5 Conclusion

In this work we have performed a detailed investigation on the lifetimes of doubly charmed baryons Ξ_{cc}^{++}, Ξ_{cc}^{+} on the basis of the operator product expansion for the transition currents. For the first time, we have presented the formulae with the simultaneous account for both the mass effects and low-energy logarithmic renormalization for the contributions to the total decay width of baryons, containing heavy quarks, as it is caused by the effects of Pauli interference and weak scattering. The usage of the diquark picture has allowed us to evaluate the matrix elements of operators derived. Further, we have discussed the procedure to choose the values of parameters for the total lifetimes of these baryons. The obtained results show the significant role of both the Pauli interference and the weak scattering.

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Appendix

In this appendix we present the explicit formulae [16] for the spectator decay of the c-quark in next-to-leading order of the perturbation theory with the account for the mass effects, related to the s-quark in the final state.

The coefficients C_+ and C_- in the effective lagrangian with the account for the next-to-leading order in perturbative QCD acquire the additional multiplicative factors:

$$F_\pm(\mu) = 1 + \frac{\alpha_s(m_W) - \alpha_s(\mu)}{4\pi} \frac{\gamma_\pm^{(0)}}{2\beta_0} \left(\frac{\gamma_\pm^{(1)}}{\gamma_\pm^{(0)}} - \frac{\beta_1}{\beta_0} \right) + \frac{\alpha_s(m_W)}{4\pi} B_\pm,$$

where $\gamma_\pm^{(i)}$ is the coefficients of anomalous dimensions for the operators $O_\pm$:

$$\gamma_\pm = \gamma_\pm^{(0)} \frac{\alpha_s}{4\pi} + \gamma_\pm^{(1)} \left(\frac{\alpha_s}{4\pi}\right)^2 + O(\alpha_s^3),$$

with

$$\gamma_+^{(0)} = 4, \quad \gamma_-^{(0)} = -8, \quad \gamma_+^{(1)} = -7 + \frac{4}{9} n_f, \quad \gamma_-^{(1)} = -14 - \frac{8}{9} n_f,$$

in the naive dimensional regularization (NDR) with the anticommutating γ_5, and n_f is a number of flavours taken into account. β_i is the initial two coefficients of QCD β-function,

$$\beta = -g_s\{\beta_0 \frac{\alpha_s}{4\pi} + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + O(\alpha_s^3)\},$$
$$\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta_1 = 102 - \frac{38}{3} n_f.$$

The coefficients $B_\pm$ are written down in accordance to the requirement of agreement between the effective lagrangian, evaluated at the scale $\mu = m_W$, and the Standard Model one up to terms of the order of $\alpha_s^2(m_W)$:

$$B_\pm = \pm B \frac{N_c \mp 1}{2N_c},$$

where $N_c = 3$ is the number of colors. In the NDR scheme for B, we find $B = 11$.

Using the effective lagrangian in the next-to-leading order of perturbative QCD and calculating the one-gluon corrections, we get the following expression for the spectator c-quark decay:

$$\begin{aligned}
\Gamma(c \to s u \bar{d}) \;=\; & \Gamma_0 [2C_+^2(\mu) + C_-^2(\mu) + \frac{\alpha_s(m_W) - \alpha_s(\mu)}{2\pi} \{2C_+^2(\mu) R_+ + C_-^2(\mu) R_-\} + \\
& \frac{\alpha_s(\mu)}{2\pi} \{2C_+^2(\mu) B_+ + C_-^2(\mu) B_-\} + \\
& \frac{3}{4} \{C_+(\mu) + C_-(\mu)\}^2 \frac{2}{3} \frac{\alpha_s(\mu)}{\pi} \{G_a + G_b\} + \\
& \frac{3}{4} \{C_+(\mu) - C_-(\mu)\}^2 \frac{2}{3} \frac{\alpha_s(\mu)}{\pi} \{G_c + G_d\} + \\
& \frac{1}{2} \{C_+^2(\mu) - C_+^2(\mu)\} \frac{2}{3} \frac{\alpha_s(\mu)}{\pi} \{G_a + G_b + Ge\}],
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\Gamma_0 &= \frac{G_f^2 m_c^5}{192\pi^3} |V_{cs}|^2 f_1(m_s^2/m_c^2), \\
f_1(a) &= 1 - 8a + 8a^3 - a^4 - 12a^2 \ln a,
\end{aligned}$$

and

$$R_\pm = B_\pm + \frac{\gamma_\pm^{(0)}}{2\beta_0} \left(\frac{\gamma_\pm^{(1)}}{\gamma_\pm^{(0)}} - \frac{\beta_1}{\beta_0} \right).$$

For G_a, G_b, G_c, G_d and G_e we have found:

$$
\begin{aligned}
(G_a+G_b)f_1(a) &= \frac{31}{4}-\pi^2-a[80-\ln a]+32a^{3/2}\pi^2 \\
&\quad -a^2[273+16\pi^2-18\ln a+36\ln^2 a]+32a^{5/2}\pi^2 \\
&\quad -\frac{8}{9}a^3[118-57\ln a]+O(a^{7/2}), \qquad (37)\\
(G_c+G_d)f_1(a) &= \frac{31}{4}-\pi^2-8a[10-\pi^2+3\ln a]-a^2[117-24\pi^2+ \\
&\quad (30-8\pi)\ln a+36\ln^2 a]-\frac{4}{3}a^3[79+2\pi^2 \\
&\quad -62\ln a+6\ln^2 a]+O(a^4), \qquad (38)\\
(G_a+G_b+G_e+B)f_1(a) &= (6\ln\frac{m_c^2}{\mu^2}+11)f_1(a)-\frac{51}{4}-\pi^2+8a[21-\pi^2-3\ln a] \\
&\quad +32a^{3/2}\pi^2-a^2[111+40\pi^2-258\ln a+36\ln^2 a] \\
&\quad +32a^{5/2}\pi^2-\frac{4}{9}a^3[305+18\pi^2+30\ln a-54\ln^2 a] \\
&\quad +O(a^{7/2}). \qquad (39)
\end{aligned}
$$

These approximations can be used in the range of a values: $a < 0.15$, where $a = (\frac{m_s}{m_c})^2$, which, indeed, takes place in the calculations under consideration.

Mode or decay mechanism	Width, ps^{-1}	Contribution in % (Ξ_{cc}^{++})	Contribution in % (Ξ_{cc}^{+})
$c_{spec} \to sdu$	2.894	124	32
$c \to se^+\nu$	0.380	16	4
PI	-1.317	-56	–
WS	5.254	–	59
$\Gamma_{\Xi_{cc}^{++}}$	2.337	100	–
$\Gamma_{\Xi_{cc}^{+}}$	8.909	–	100

Table 1: The contributions of different modes to the total decay width of doubly charmed baryons.

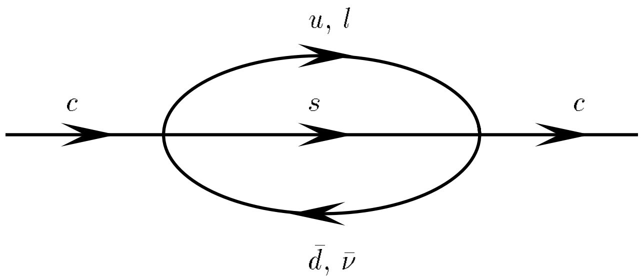

Figure 1: The spectator contribution to the total decay width of doubly charmed baryons.

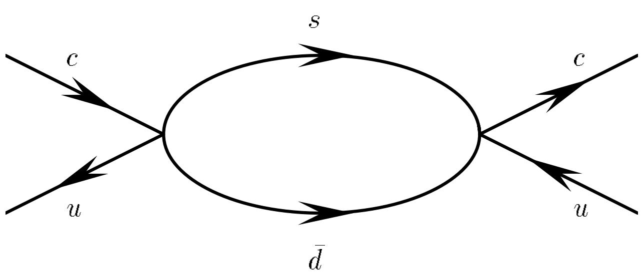

Figure 2: The Pauli interference of c-quark decay products with the valence quark in the initial state for the Ξ_{cc}^{++}-baryon.

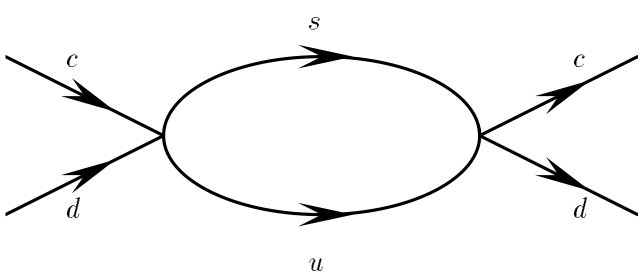

Figure 3: The weak scattering of the valence quarks in the initial state for the Ξ_{cc}^{+}-baryon.