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Hadron Spectra and Spectral Moments in the Decay $B \to X_s \ell^+ \ell^-$ using HQET

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Abstract

We compute the leading order (in α_s) perturbative QCD and power $(1/m_b^2)$ corrections to the hadronic invariant mass and hadron energy spectra in the decay $B \to X_s \ell^+ \ell^-$ in standard model using the heavy quark expansion technique (HQET). Results for the first two hadronic moments $\langle S_H^n \rangle$ and $\langle E_H^n \rangle$, $n = 1, 2$, are presented here working out their sensitivity on the HQET parameters λ_1 and $\bar{\Lambda}$. Data from the forthcoming B facilities can be used to measure the short-distance contribution in $B \to X_s \ell^+ \ell^-$and determine the HQET parameters from the moments $\langle S_H^n \rangle$. This can be combined with the analysis of semileptonic decays $B \to X \ell \nu_\ell$ to determine them precisely.

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The semileptonic inclusive decays $B \to X_s \ell^+ \ell^-$, where $\ell^\pm = e^\pm, \mu^\pm, \tau^\pm$, offer, together with the radiative electromagnetic penguin decay $B \to X_s + \gamma$, presently the most popular testing grounds for the standard model (SM) in the flavor sector. Concentrating on the decay $B \to X_s \ell^+ \ell^-$, we recall that the first theoretical calculations including partial leading order QCD corrections were reported a decade ago in [1–3], emphasizing the sensitivity of the dilepton mass spectrum and decay rates to the top quark mass in the short-distance contribution. Since the pioneering papers [1–3], a lot of theoretical work has been done on the decay $B \to X_s \ell^+ \ell^-$. In particular, the complete leading order perturbative corrections in the QCD coupling constant α_s to the dilepton invariant mass spectrum and so-called forward-backward (FB) asymmetry of the leptons [4], have been calculated in refs. [5,6] and [7], respectively. In addition, leading order power corrections in $1/m_b^2$ to the decay rate, dilepton invariant mass spectrum and the FB asymmetry have been studied [7], using the heavy quark expansion technique (HQET), correcting an earlier derivation of the dilepton spectrum in this decay [8]. The power corrected dilepton mass spectrum and FB asymmetry have also been derived for the massless s-quark case recently in [9], confirming the results in [7]. Corrections of order $1/m_c^2$ to the dilepton mass spectrum away from the resonant regions have also been worked out [10,11].

In this letter, we study the hadron spectra and moments in $B \to X_s \ell^+ \ell^-$ using HQET. The study of the decay $B \to X \ell \nu_\ell$ in this context has received a lot of interest [12–17]. The hadronic invariant mass spectra in $B \to X_s \ell^+ \ell^-$ and $B \to X_u \ell \nu_\ell$ have striking similarities and differences. For example, both of these processes have at the parton level a delta function behavior $d\Gamma/ds_0 \propto \delta(s_0 - m_q^2)$, $q = u, s$, where s_0 is the partonic invariant mass. Thus, the entire invariant mass spectrum away from $s_0 = m_q^2$ is generated perturbatively (by gluon bremsstrahlung) and through B-hadron non-perturbative effects. The latter are already present in $\mathcal{O}(\alpha_s^0)$, as can be seen by the relation between the b quark mass and the B meson mass. In HQET this takes the form $m_B = m_b + \bar{\Lambda} - (\lambda_1 + 3\lambda_2)/2m_b + ...$ The quantities λ_1, λ_2 and $\bar{\Lambda}$ are the HQET parameters [18,19,8]. Keeping, for the sake of simplicity just the $\bar{\Lambda}$ term, the hadronic invariant mass S_H is related to s_0 and the partonic energy E_0 by $S_H = s_0 + 2\bar{\Lambda}E_0 + \bar{\Lambda}^2$. This gives rise to a non-trivial spectrum in the entire region $\bar{\Lambda}^2 < S_H < M_B^2$. Hence, measurements of these spectra would lead to direct information on the QCD dynamics and a better determination of the non-perturbative parameters, such as the HQET parameters $\bar{\Lambda}$ and λ_1. Following this line of argument, the sensitivity of the lepton and hadron energy spectra on these parameters in the decays $B \to X \ell \nu_\ell$ has been studied quantitatively in literature [12,13,16]; Photon energy moments in the decay $B \to X_s + \gamma$ have also been worked out in [20]. Present status of the HQET parameters is reviewed in [21]. There is a fair amount of theoretical dispersion on $\bar{\Lambda}$ and λ_1 and it will be very instructive to get independent and complementary information on these parameters from other B decays.

We report here a calculation of the hadron spectra in the decay $B \to X_s \ell^+ \ell^-$. Leading order (in α_s) perturbative QCD and power ($1/m_b^2$) corrections to the hadronic invariant mass and hadron energy spectra in this decay are computed at the parton level. Including both the $\mathcal{O}(1/m_b^2)$ and $\mathcal{O}(\alpha_s)$ terms generates hadron spectra with contributions of $\mathcal{O}(\bar{\Lambda}/m_B)$, $\mathcal{O}(\alpha_s\bar{\Lambda}/m_B)$, $\mathcal{O}(\lambda_1/m_B^2)$ and $\mathcal{O}(\lambda_2/m_B^2)$. Relegating the detailed hadronic profile to a subsequent publication [22], here the power- and perturbatively corrected hadronic spectral moments $\langle S_H^n\rangle$ and $\langle E_H^n\rangle$ are presented for the first two moments $n = 1, 2$. The former are sensitive to the HQET parameters $\bar{\Lambda}$ and λ_1 and we work out this dependence numerically, showing that these moments would provide an independent determination of the HQET parameters in $B \to X_s \ell^+\ell^-$. The theoretically constrained contours in the $(\bar{\Lambda} - \lambda_1)$ plane in $B \to X_s \ell^+\ell^-$are compared with the corresponding one from an analysis of the power corrected lepton energy spectrum in $B \to X\ell\nu_\ell$ [13]. We argue that a simultaneous analysis of the moments and spectra in $B \to X_s \ell^+\ell^-$and $B \to X\ell\nu_\ell$ will allow to determine the HQET parameters with a high precision.

We start with the definition of the kinematics of the decay at the parton level, $b(p_b) \to s(p_s)(+g(p_g)) + \ell^+(p_+) + \ell^-(p_-)$, where g denotes a gluon from the $O(\alpha_s)$ correction. The corresponding kinematics at the hadron level can be written as: $B(p_B) \to X_s(p_H) + \ell^+(p_+) + \ell^-(p_-)$. We define by q and s the momentum transfer to the lepton pair and the invariant mass of the dilepton system, respectively, $q = p_+ + p_-$, $s = q^2$; the dimensionless variables with a hat are related to the variables with dimension by the scale m_b, the b-quark mass, e.g., $\hat{s} = \frac{s}{m_b^2}$, $\hat{m}_s = \frac{m_s}{m_b}$ etc.. Further, we define a 4-vector v, which denotes the velocity of both the b-quark and the B-meson, $p_b = m_b v$ and $p_B = m_B v$. We shall also need a variable u, which is defined as $u = -(p_b - p_+)^2 + (p_b - p_-)^2$, with the scaled variable $\hat{u} = u/m_b^2$ satisfying the kinematic relation $\hat{u} = 2v \cdot (\hat{p}_+ - \hat{p}_-)$. The hadronic invariant mass is denoted by $S_H \equiv p_H^2$ and E_H denotes the hadron energy in the final state. The corresponding quantities at parton level are the invariant mass s_0 and the scaled parton energy $x_0 \equiv \frac{E_0}{m_b}$; without gluon bremsstrahlung this simplifies to $s_0 = m_s^2$ and x_0 becomes directly related to the dilepton invariant mass, $x_0 = 1/2(1 - \hat{s} + \hat{m}_s^2)$. From momentum conservation, the following equalities hold in the b-quark, equivalently B-meson, rest frame ($v = (1, 0, 0, 0)$):

$$
\begin{aligned}
x_0 &= 1 - v \cdot \hat{q}\,, \quad \hat{s}_0 = 1 - 2v \cdot \hat{q} + \hat{s}\,, \\
E_H &= m_B - v \cdot q\,, \quad S_H = m_B^2 - 2m_B v \cdot q + s\,. \qquad (1)
\end{aligned}
$$

The relation between the kinematic variables of the parton model and the hadronic states is, using the HQET mass relation $m_B = m_b + \bar{\Lambda} - 1/2m_b(\lambda_1 + 3\lambda_2) + \ldots$, given as

$$
\begin{aligned}
E_H &= \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_B} + \left(m_B - \bar{\Lambda} + \frac{\lambda_1 + 3\lambda_2}{2m_B}\right) x_0 + \ldots\,, \\
S_H &= m_s^2 + \bar{\Lambda}^2 + (m_B^2 - 2\bar{\Lambda} m_B + \bar{\Lambda}^2 + \lambda_1 + 3\lambda_2)\,(\hat{s}_0 - \hat{m}_s^2) \\
&\quad + (2\bar{\Lambda} m_B - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2) x_0 + \ldots\,, \qquad (2)
\end{aligned}
$$

where the ellipses denote terms higher order in $1/m_b$. The quantity λ_2 is known precisely from the $B^* - B$ mass difference, with $\lambda_2 \simeq 0.12$ GeV^2. The other two parameters are of interest here.

The effective Hamiltonian governing the decay $B \to X_s \ell^+ \ell^-$, obtained by integrating out the top quark and the $W^\pm$ bosons, is given as

$$\mathcal{H}_{eff}(b \to s + X, \, X = \gamma, \, \ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=1}^{6} C_i(\mu) O_i + C_7(\mu) \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha F^{\mu\nu} \right.$$
$$\left. + C_8(\mu) O_8 + C_9(\mu) \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \ell + C_{10} \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \gamma_5 \ell \right] , \quad (3)$$

where L and R denote chiral projections, $L(R) = 1/2(1 \mp \gamma_5)$, V_{ij} are the CKM matrix elements and the CKM unitarity has been used in factoring out the product $V_{ts}^* V_{tb}$. The operator basis is taken from [7], where also the Four-Fermi operators $O_1, \ldots, O_6$ and the chromo-magnetic operator O_8 can be seen. Note that O_8 does not contribute to the decay $B \to X_s \ell^+ \ell^-$in the approximation which we use here. The $C_i(\mu)$ are the Wilson coefficients, which depend, in general, on the renormalization scale μ, except for C_{10}. Their numerical values are given in Table 1.

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	C_9	C_{10}
-0.240	$+1.103$	$+0.011$	-0.025	$+0.007$	-0.030	-0.311	$+4.153$	-4.546

Table 1: *Values of the Wilson coefficients used in the numerical calculations corresponding to the central values of the parameters given in eq. (15). Here,* $C_7^{eff} \equiv C_7 - C_5/3 - C_6$*, and for* C_9 *we use the NDR scheme.*

With the help of the above expressions, one can express the Dalitz distribution in $B \to X_s \ell^+ \ell^-$as:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{u}\,\mathrm{d}\hat{s}\,\mathrm{d}(v\cdot\hat{q})} = \frac{1}{2\,m_B} \frac{{G_F}^2\,\alpha^2}{2\,\pi^2} \frac{{m_b}^4}{256\,\pi^4} |V_{ts}^* V_{tb}|^2 \, 2\,\mathrm{Im}\left({T^L}_{\mu\nu}\, L^{L^{\mu\nu}} + {T^R}_{\mu\nu}\, L^{R^{\mu\nu}} \right) , \quad (4)$$

where the hadronic and leptonic tensors ${T^{L/R}}_{\mu\nu}$ and $L^{L/R^{\mu\nu}}$ are given in [7]. Using Lorentz decomposition, the tensor $T_{\mu\nu}$ can be expanded in terms of three structure functions T_i,

$$T_{\mu\nu} = -T_1\, g_{\mu\nu} + T_2\, v_\mu\, v_\nu + T_3\, i\epsilon_{\mu\nu\alpha\beta}\, v^\alpha\, \hat{q}^\beta\,, \quad (5)$$

where the ones which do not contribute to the amplitude in the limit of massless leptons have been neglected.

Concerning the $O(\alpha_s)$ corrections to the hadron spectra, we note that only $O_9 = e^2/(16\pi^2)\bar{s}_\alpha\gamma^\mu L b_\alpha \bar{\ell}\gamma_\mu \ell$ is subject to such corrections. These can be obtained by using the existing results in the literature as follows: The vector current O_9 can be decomposed as $V = (V-A)/2 + (V+A)/2$. Note that the $(V-A)$ and $(V+A)$ currents yield the same hadron energy spectrum [23] and there is no interference term present in this spectrum for massless leptons. So, the correction for the vector current case can be taken from the corresponding result for the charged $(V-A)$ case [24,25].

We have calculated the order α_s perturbative QCD correction for the hadronic invariant mass in the range $\hat{m}_s^2 < \hat{s}_0 \leq 1$. Since the decay $b \to s + \ell^+ + \ell^-$ contributes in the parton model only at $\hat{s}_0 = \hat{m}_s^2$, only the bremsstrahlung graphs $b \to s + g + \ell^+ + \ell^-$ contribute in this range. This makes the calculation much simpler than in the full $\hat{s}_0$ range including virtual gluon diagrams. Also for this distribution, the results can be taken from the existing literature. As the starting point, we use the Sudakov exponentiated double differential decay rate $\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}y}$, derived for the decay $B \to X_u \ell \nu_\ell$ in [14], which we have checked, after changing the normalization for $B \to X_s \ell^+ \ell^-$. Defining the kinematic variables (x, y) as $q^2 = x^2 m_b^2$, $v \cdot q = (x + 1/2(1-x)^2 y) m_b$, the Sudakov-improved Dalitz distribution is given by

$$\frac{d^2\mathcal{B}}{dxdy}(B \to X_s \ell^+ \ell^-) = -\mathcal{B}_0 \frac{8}{3} x(1-x^2)^2(1+2x^2) \exp\Big(-\frac{2\alpha_s}{3\pi}\ln^2(1-y)\Big) \times \left\{\frac{4\alpha_s}{3\pi}\frac{\ln(1-y)}{(1-y)}\Big[1 - \frac{2\alpha_s}{3\pi}(G(x) + H(y))\Big] - \frac{2\alpha_s}{3\pi}\frac{dH}{dy}(y)\right\} C_9^2 \,, \tag{6}$$

where the functions $G(x)$ and $H(y)$ can be seen in [14]. The constant $\mathcal{B}_0$ is defined below.

The most significant effect of the bound state is the difference between m_B and m_b which is dominated by $\bar{\Lambda}$. The spectrum $\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}S_H}$ is obtained after changing variables from (x, y) to (q^2, S_H) and performing an integration over q^2. It is valid in the region $m_B \frac{m_B \bar{\Lambda} - \bar{\Lambda}^2 + m_s^2}{m_B - \bar{\Lambda}} < S_H \leq m_B^2$ (or $m_B \bar{\Lambda} \leq S_H \leq m_B^2$, neglecting m_s) which excludes the zeroth order and virtual gluon kinematics ($s_0 = m_s^2$). The hadronic invariant mass spectrum thus found depends rather sensitively on m_b (or equivalently $\bar{\Lambda}$). An analogous analysis for the decay $B \to X_u \ell \nu_\ell$ has been performed in [15].

The hadronic tensor in eq. (5) can be expanded in inverse powers of m_b with the help of the HQET techniques [8,18,19]. The leading term in this expansion, i.e., $\mathcal{O}(m_b^0)$ reproduces the parton model result. In HQET, the next to leading power corrections are parameterized in terms of λ_1 and λ_2. The contributions of the power corrections to the structure functions T_i has been calculated up to (but not including) $O(1/m_b^3)$ and given in [7]. After contracting the hadronic and leptonic tensors and with the help of the kinematic identities given in eq. (1), we can make the dependence on x_0 and $\hat{s}_0$ explicit,

$$T^{L/R}{}_{\mu\nu} L^{L/R^{\mu\nu}} = m_b{}^2 \left\{2(1 - 2x_0 + \hat{s}_0) T_1{}^{L/R} + \left[x_0^2 - \frac{1}{4}\hat{u}^2 - \hat{s}_0\right] T_2{}^{L/R} \mp (1 - 2x_0 + \hat{s}_0)\,\hat{u}\, T_3{}^{L/R}\right\}$$

and with this we are able to derive the double differential power corrected spectrum $\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}x_0\,\mathrm{d}\hat{s}_0}$ for $B \to X_s \ell^+ \ell^-$. Integrating eq. (4) over $\hat{u}$ first, where the variable $\hat{u}$ is bounded by $-2\sqrt{x_0^2 - \hat{s}_0} \leq \hat{u} \leq +2\sqrt{x_0^2 - \hat{s}_0}$, we arrive at the following expression [22]:

$$\frac{\mathrm{d}^2\mathcal{B}}{\mathrm{d}x_0\,\mathrm{d}\hat{s}_0} = -\frac{8}{\pi}\mathcal{B}_0 \mathrm{Im}\sqrt{x_0^2 - \hat{s}_0}\left\{(1 - 2x_0 + \hat{s}_0) T_1(\hat{s}_0, x_0) + \frac{x_0^2 - \hat{s}_0}{3} T_2(\hat{s}_0, x_0)\right\} + \mathcal{O}(\lambda_i \alpha_s) \,. \tag{7}$$

As the structure function T_3 does not contribute to the branching ratio, we did not consider it in our present work. The functions $T_1(\hat{s}_0, x_0)$ and $T_2(\hat{s}_0, x_0)$ have been derived by us after a lengthy calculation and the resulting expressions are too long to be given here. They can be seen together with other details of the calculations in [22].

The branching ratio for $B \to X_s \ell^+ \ell^-$is usually expressed in terms of the measured semileptonic branching ratio $\mathcal{B}_{sl}$ for the decay $B \to X_c \ell \nu_\ell$. This fixes the normalization constant $\mathcal{B}_0$ to be,

$$\mathcal{B}_0 \equiv \mathcal{B}_{sl} \frac{3\,\alpha^2}{16\pi^2} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c)\kappa(\hat{m}_c)}\,, \tag{8}$$

where $f(\hat{m}_c)$ is the phase space factor for $\Gamma(B \to X_c \ell \nu_\ell)$ and the function $\kappa(\hat{m}_c)$ accounts for both the $O(\alpha_s)$ QCD correction to the semileptonic decay width [26] and the leading order $(1/m_b)^2$ power correction [18]. They are given explicitly in [12].

The hadron energy spectrum can now be obtained by integrating over $\hat{s}_0$. The kinematic boundaries are given as: $max(\hat{m}_s^2, -1 + 2x_0 + 4\hat{m}_l^2) \leq \hat{s}_0 \leq x_0^2$, $\hat{m}_s \leq x_0 \leq \frac{1}{2}(1 + \hat{m}_s^2 - 4\hat{m}_l^2)$. Here we keep $\hat{m}_l$ as a regulator wherever it is necessary. Including the leading power corrections, the hadron energy spectrum in the decay $B \to X_s \ell^+ \ell^-$is derived by us and given in [22].

The lowest spectral moments in the decay $B \to X_s \ell^+ \ell^-$at the parton level are worked out by taking into account the two types of corrections discussed earlier, namely the leading power $1/m_b$ and the perturbative $\mathcal{O}(\alpha_s)$ corrections. To that end, we define:

$$\mathcal{M}_{l^+l^-}^{(n,m)} \equiv \frac{1}{\mathcal{B}_0} \int (\hat{s}_0 - \hat{m}_s^2)^n x_0^m \, \frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\hat{s}_0 \mathrm{d}x_0} \, \mathrm{d}\hat{s}_0 \mathrm{d}x_0\,, \tag{9}$$

for integers n and m. These moments are related to the corresponding moments $\langle x_0^m (\hat{s}_0 - \hat{m}_s^2)^n \rangle$ obtained at the parton level by a scaling factor which yields the corrected branching ratio $\mathcal{B} = \mathcal{B}_0 \mathcal{M}_{\ell^+\ell^-}^{(n,m)}$. Thus, $\langle x_0^m (\hat{s}_0 - \hat{m}_s^2)^n \rangle = \frac{\mathcal{B}_0}{\mathcal{B}} \mathcal{M}_{l^+l^-}^{(n,m)}$. We remind that one has to Taylor expand the correction factor $\mathcal{B}_0/\mathcal{B}$ in terms of the $\mathcal{O}(\alpha_s)$ and power corrections. The moments can be expressed as double expansion in $\mathcal{O}(\alpha_s)$ and $1/m_b$ and to the accuracy of our calculations they can be represented in the following form:

$$\mathcal{M}_{l^+l^-}^{(n,m)} = D_0^{(n,m)} + \frac{\alpha_s}{\pi} {C_9}^2 A^{(n,m)} + \hat{\lambda}_1 D_1^{(n,m)} + \hat{\lambda}_2 D_2^{(n,m)}\,, \tag{10}$$

with a further decomposition into pieces from different Wilson coefficients for $i = 0, 1, 2$:

$$D_i^{(n,m)} = \alpha_i^{(n,m)} {C_7^{\rm eff}}^2 + \beta_i^{(n,m)} C_{10}^2 + \gamma_i^{(n,m)} C_7^{\rm eff} + \delta_i^{(n,m)}. \tag{11}$$

The terms $\gamma_i^{(n,m)}$ and $\delta_i^{(n,m)}$ in eq. (11) result from the terms proportional to $Re(C_9^{\rm eff}) C_7^{\rm eff}$ and $|C_9^{\rm eff}|^2$ in eq. (7), respectively. The explicit expressions for $\alpha_i^{(n,m)}, \beta_i^{(n,m)}, \gamma_i^{(n,m)}, \delta_i^{(n,m)}$ are given in [22].

The leading perturbative contributions for the hadronic invariant mass and hadron energy moments can be obtained analytically,

$$\begin{aligned} A^{(0,0)} &= \frac{25 - 4\pi^2}{9}\,, \quad A^{(1,0)} = \frac{91}{675}\,, \quad A^{(2,0)} = \frac{5}{486}\,, \\ A^{(0,1)} &= \frac{1381 - 210\pi^2}{1350}\,, A^{(0,2)} = \frac{2257 - 320\pi^2}{5400}\,. \end{aligned} \tag{12}$$

The zeroth moment $n = m = 0$ is needed for the normalization and we recall that the result for $A^{(0,0)}$ was derived by Cabibbo and Maiani in the context of the $\mathcal{O}(\alpha_s)$ correction to the semileptonic decay rate $B \to X \ell \nu_\ell$ quite some time ago [26]. Likewise, the first mixed moment $A^{(1,1)}$ can be extracted from the results given in [12] for the decay $B \to X \ell \nu_\ell$ after changing the normalization, $A^{(1,1)} = 3/50$. For the lowest order parton model contribution $D_0^{(n,m)}$, we find, in agreement with [12], that the first two hadronic invariant mass moments $\langle \hat{s}_0 - \hat{m}_s^2 \rangle$, $\langle (\hat{s}_0 - \hat{m}_s^2)^2 \rangle$ and the first mixed moment $\langle x_0 (\hat{s}_0 - \hat{m}_s^2) \rangle$ vanish: $D_0^{(n,0)} = 0$, for $n = 1, 2$ and $D_0^{(1,1)} = 0$.

We can eliminate the hidden dependence on the non-perturbative parameters resulting from the b-quark mass in the moments $\mathcal{M}_{l^+l^-}^{(n,m)}$ with the help of the HQET mass relation. As m_s is of order Λ_{QCD}, to be consistent we keep only terms up to order m_s^2/m_b^2 [27]. An additional m_b-dependence is in the mass ratios $\hat{m}_l = \frac{m_l}{m_b}$. With this we obtain the moments for the physical quantities valid up to $\mathcal{O}(\alpha_s/m_B^2, 1/m_B^3)$, where the second equation corresponds to a further use of $m_s = \mathcal{O}(\Lambda_{QCD})$. We get for the first two hadronic invariant mass moments [1]

$$\begin{aligned}
\langle S_H \rangle &= m_s^2 + \bar{\Lambda}^2 + (m_B^2 - 2\bar{\Lambda} m_B) \langle \hat{s}_0 - \hat{m}_s^2 \rangle + (2\bar{\Lambda} m_B - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2) \langle x_0 \rangle \, , \\
\langle S_H^2 \rangle &= m_s^4 + 2\bar{\Lambda}^2 m_s^2 + 2m_s^2 (m_B^2 - 2\bar{\Lambda} m_B) \langle \hat{s}_0 - \hat{m}_s^2 \rangle + 2m_s^2 (2\bar{\Lambda} m_B - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2) \langle x_0 \rangle \\
&\quad + (m_B^4 - 4\bar{\Lambda} m_B^3) \langle (\hat{s}_0 - \hat{m}_s^2)^2 \rangle + 4\bar{\Lambda}^2 m_B^2 \langle x_0^2 \rangle + 4\bar{\Lambda} m_B^3 \langle x_0 (\hat{s}_0 - \hat{m}_s^2) \rangle \, , \\
&= (m_B^4 - 4\bar{\Lambda} m_B^3) \langle (\hat{s}_0 - \hat{m}_s^2)^2 \rangle + 4\bar{\Lambda}^2 m_B^2 \langle x_0^2 \rangle + 4\bar{\Lambda} m_B^3 \langle x_0 (\hat{s}_0 - \hat{m}_s^2) \rangle \, ,
\end{aligned} \tag{13}$$

and for the hadron energy moments:

$$\begin{aligned}
\langle E_H \rangle &= \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_B} + \left(m_B - \bar{\Lambda} + \frac{\lambda_1 + 3\lambda_2}{2m_B} \right) \langle x_0 \rangle \, , \\
\langle E_H^2 \rangle &= \bar{\Lambda}^2 + (2\bar{\Lambda} m_B - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2) \langle x_0 \rangle \\
&\quad + (m_B^2 - 2\bar{\Lambda} m_B + \bar{\Lambda}^2 + \lambda_1 + 3\lambda_2) \langle x_0^2 \rangle \, .
\end{aligned} \tag{14}$$

One sees that there are linear power corrections, $\mathcal{O}(\bar{\Lambda}/m_B)$, present in all these hadronic quantities except $\langle S_H^2 \rangle$ which starts in $\frac{\alpha_s}{\pi} \frac{\bar{\Lambda}}{m_B}$.

Using the expressions for the HQET moments derived by us [22], we present the numerical results for the hadronic moments in $B \to X_s \ell^+ \ell^-$. The parameters used in arriving at the numerical coefficients are given in Table 1 and specified below:

$$\begin{aligned}
m_W &= 80.26 \text{ (GeV)}, \quad m_Z = 91.19 \text{ (GeV)}, \quad \sin^2\theta_W = 0.2325 \, , \\
m_s &= 0.2 \text{ (GeV)}, \quad m_c = 1.4 \text{ (GeV)}, \quad m_b = 4.8 \text{ (GeV)}, \quad m_t = 175 \pm 5 \text{ (GeV)}, \\
\mu &= {m_b}^{+m_b}_{-m_b/2}, \quad \alpha^{-1} = 129, \quad \alpha_s(m_Z) = 0.117 \pm 0.005 \, , \quad \mathcal{B}_{sl} = (10.4 \pm 0.4)\%.
\end{aligned} \tag{15}$$

[1] Our first expression for $\langle S_H^2 \rangle$, eq. (13), does not agree in the coefficient of $\langle \hat{s}_0 - \hat{m}_s^2 \rangle$ with the one given in [12] (their eq. (4.1)). We point out that m_B^2 should have been replaced by m_b^2 in this expression. This has been confirmed by Adam Falk (private communication). Dropping the higher order terms given in their expressions, the hadronic moments in HQET derived here and in [12] agree.

Inserting the expressions for the moments calculated at the partonic level into eqs. (13) and (14), we find the following expressions for the short-distance hadronic moments, valid up to $\mathcal{O}(\alpha_s/m_B^2, 1/m_B^3)$:

$$\begin{aligned}
\langle S_H \rangle &= m_B^2 \Big(\frac{m_s^2}{m_B^2} + 0.093\frac{\alpha_s}{\pi} - 0.069\frac{\bar{\Lambda}}{m_B}\frac{\alpha_s}{\pi} + 0.735\frac{\bar{\Lambda}}{m_B} + 0.243\frac{\bar{\Lambda}^2}{m_B^2} + 0.273\frac{\lambda_1}{m_B^2} - 0.513\frac{\lambda_2}{m_B^2}\Big) , \\
\langle S_H^2 \rangle &= m_B^4 \Big(0.0071\frac{\alpha_s}{\pi} + 0.138\frac{\bar{\Lambda}}{m_B}\frac{\alpha_s}{\pi} + 0.587\frac{\bar{\Lambda}^2}{m_B^2} - 0.196\frac{\lambda_1}{m_B^2}\Big) , \qquad (16) \\
\langle E_H \rangle &= 0.367 m_B \Big(1 + 0.148\frac{\alpha_s}{\pi} - 0.352\frac{\bar{\Lambda}}{m_B}\frac{\alpha_s}{\pi} + 1.691\frac{\bar{\Lambda}}{m_B} + 0.012\frac{\bar{\Lambda}^2}{m_B^2} + 0.024\frac{\lambda_1}{m_B^2} + 1.070\frac{\lambda_2}{m_B^2}\Big) , \\
\langle E_H^2 \rangle &= 0.147 m_B^2 \Big(1 + 0.324\frac{\alpha_s}{\pi} - 0.128\frac{\bar{\Lambda}}{m_B}\frac{\alpha_s}{\pi} + 2.954\frac{\bar{\Lambda}}{m_B} + 2.740\frac{\bar{\Lambda}^2}{m_B^2} - 0.299\frac{\lambda_1}{m_B^2} + 0.162\frac{\lambda_2}{m_B^2}\Big) .
\end{aligned}$$

Concerning the non-perturbative parts related to the $c\bar{c}$ loop in $B \to X_s \ell^+ \ell^-$, it has been suggested in [10] that an $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ expansion in the context of HQET can be carried out to take into account such effects in the invariant mass spectrum away from the resonances. Using the expressions (obtained with $m_s = 0$) for the $1/m_c^2$ amplitude, we have calculated the partonic energy moments $\triangle\langle x_0^n \rangle$, which correct the short-distance result at order λ_2/m_c^2:

$$\triangle\langle x_0^n \rangle \frac{\mathcal{B}}{\mathcal{B}_0} = -\frac{256 C_2 \lambda_2}{27 m_c^2} \int_0^{1/2(1-4\hat{m}_l^2)} dx_0 x_0^{n+2} \mathrm{Re}\left[F(r) \left(C_9^{\mathrm{eff}}(3 - 2x_0) + 2C_7^{\mathrm{eff}} \frac{-3 + 4x_0 + 2x_0^2}{2x_0 - 1} \right) \right] ,$$

where $r = (1 - 2x_0)/4\hat{m}_c^2$ and $F(r)$ is given in [10]. The invariant mass and mixed moments give zero contribution in the order we are working for $m_s = 0$. Thus, the correction to the hadronic mass moments are vanishing, if we further neglect terms proportional to $\frac{\lambda_2}{m_c^2}\bar{\Lambda}$ and $\frac{\lambda_2}{m_c^2}\lambda_i$, with $i = 1, 2$. For the hadron energy moments we obtain numerically

$$\begin{aligned}
\triangle\langle E_H \rangle_{1/m_c^2} &= m_B \triangle\langle x_0 \rangle = -0.007\,\mathrm{GeV} , \\
\triangle\langle E_H^2 \rangle_{1/m_c^2} &= m_B^2 \triangle\langle x_0^2 \rangle = -0.013\,\mathrm{GeV}^2 , \qquad (17)
\end{aligned}$$

leading to a correction of order -0.3% to the short-distance values presented in Table 2.

With the help of the expressions given above, we have calculated numerically the hadronic moments in HQET for the decay $B \to X_s \ell^+ \ell^-$, $\ell = \mu, e$ and have estimated the errors by varying the parameters within their $\pm 1\sigma$ ranges given in eq. (15). They are presented in Table 2 where we have used $\bar{\Lambda} = 0.39\,\mathrm{GeV}$ and $\lambda_1 = -0.2\,\mathrm{GeV}^2$. Further, using $\alpha_s(m_b) = 0.21$ and $\lambda_2 = 0.12\,\mathrm{GeV}^2$, the explicit dependencies of the hadronic moments given in eq. (16) on the HQET parameters λ_1 and $\bar{\Lambda}$ can be worked out.

$$\begin{aligned}
\langle S_H \rangle &= 0.0055 m_B^2 \Big(1 + 132.61\frac{\bar{\Lambda}}{m_B} + 44.14\frac{\bar{\Lambda}^2}{m_B^2} + 49.66\frac{\lambda_1}{m_B^2}\Big) , \\
\langle S_H^2 \rangle &= 0.00048 m_B^4 \Big(1 + 19.41\frac{\bar{\Lambda}}{m_B} + 1223.41\frac{\bar{\Lambda}^2}{m_B^2} - 408.39\frac{\lambda_1}{m_B^2}\Big) . \qquad (18)
\end{aligned}$$

As expected, the dependence of the energy moments $\langle E_H^n \rangle$ on $\bar{\Lambda}$ and λ_1 is very weak and we do not show these here. While interpreting these numbers, one should bear in mind that there are two comparable expansion parameters $\bar{\Lambda}/m_B$ and α_s/π and we have fixed the latter in showing the numbers. The correlations on the HQET parameters λ_1 and $\bar{\Lambda}$ which follow from (assumed) fixed values of the hadronic invariant mass moments $\langle S_H \rangle$ and $\langle S_H^2 \rangle$ are shown in Fig. 1. We have taken the values for the decay $B \to X_s \mu^+ \mu^-$ from Table 2 for the sake of illustration and have also shown the presently irreducible theoretical errors on these moments following from the input parameters m_t, α_s and the scale μ, given in eq. (15). The errors were calculated by varying these parameters in the indicated range, one at a time, and adding the individual errors in quadrature. As the entries in Table 2 are calculated for the best-fit values of λ_1 and $\bar{\Lambda}$ taken from the analysis of Gremm et al. [13] for the electron energy spectrum in $B \to X \ell \nu_\ell$, there is no surprise that these curves meet at this point. This exercise has to be repeated with real data in $B \to X_s \ell^+ \ell^-$ to draw any quantitative conclusions. Using the CLEO cuts on hadronic and dileptonic masses [29], we estimate that $O(200)$ $B \to X_s \ell^+ \ell^- (\ell = e, \mu)$ events will be available per 10^7 $B\bar{B}$ hadrons [22]. So, there will be plenty of $B \to X_s \ell^+ \ell^-$ decays in the forthcoming B facilities to measure the correlation shown in Fig. 1.

The theoretical stability of the moments has to be checked against higher order corrections and the error estimates presented here will have to be improved. The "BLM-enhanced" two-loop corrections [28] proportional to $\alpha_s^2 \beta_0$, where $\beta_0 = 11 - 2n_f/3$ is the first term in the QCD beta function, can be included at the parton level as has been done in other decays [12,30], but not being crucial to our point we have not done this. More importantly, higher order corrections in α_s and $1/m_b^3$ are not included here. While we do not think that the higher orders in α_s will have a significant influence, the second moment $\langle S_H^2 \rangle$ is susceptible to the presence of $1/m_b^3$ corrections as shown for the decay $B \to X \ell \nu_\ell$ [16]. This will considerably enlarge the theoretical error represented by the dashed band for $\langle S_H^2 \rangle$ in Fig. 1. Fortunately, the coefficient of the $\bar{\Lambda}/m_B$ term in $\langle S_H \rangle$ is large. Hence, a good measurement of this moment alone constrains $\bar{\Lambda}$ effectively. Of course, the utility of the hadronic moments calculated above is only in conjunction with the experimental cuts. Since the optimal experimental cuts in $B \to X_s \ell^+ \ell^-$ remain to be defined, we hope to return to this and related issue of doing an improved theoretical error estimate in a future publication. The power corrections presented here in the hadron spectrum and hadronic spectral moments in $B \to X_s \ell^+ \ell^-$ are the first results in this decay.

HQET	$\langle S_H \rangle$	$\langle S_H^2 \rangle$	$\langle E_H \rangle$	$\langle E_H^2 \rangle$
	(GeV^2)	(GeV^4)	(GeV)	(GeV^2)
$\mu^+\mu^-$	1.64 ± 0.06	4.48 ± 0.29	2.21 ± 0.04	5.14 ± 0.16
e^+e^-	1.79 ± 0.07	4.98 ± 0.29	2.41 ± 0.06	6.09 ± 0.29

Table 2: *Hadronic spectral moments for $B \to X_s \mu^+ \mu^-$ and $B \to X_s e^+ e^-$ in HQET with $\bar{\Lambda} = 0.39\,GeV$ and $\lambda_1 = -0.2\,GeV^2$. The quoted error results from varying μ, α_s and the top mass within the ranges given in eq. (15).*

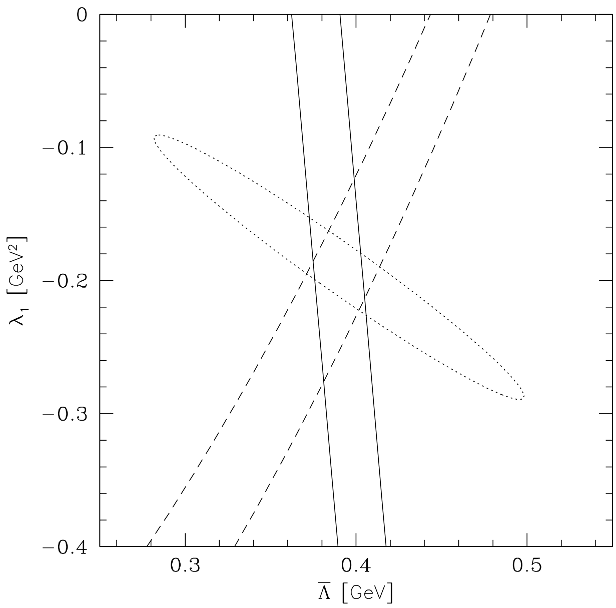

Figure 1: *$\langle S_H \rangle$ (solid bands) and $\langle S_H^2 \rangle$ (dashed bands) correlation in (λ_1-$\bar{\Lambda}$) space for $\langle S_H \rangle = 1.64 \pm 0.06$ GeV^2 and $\langle S_H^2 \rangle = 4.48 \pm 0.29$ GeV^4, corresponding to the values in Table 2. The curves are forced to meet at the point $\lambda_1 = -0.2$ GeV^2 and $\bar{\Lambda} = 0.39$ GeV. The correlation from the analysis of the decay $B \to X \ell \nu_\ell$ from ref. [13] is also shown here (ellipse).*

In summary, we have calculated the $\mathcal{O}(\alpha_s)$ perturbative QCD and leading $\mathcal{O}(1/m_b)$ corrections to the hadron spectra in the decay $B \to X_s \ell^+ \ell^-$, including the Sudakov-improvements in the perturbative part. Hadronic invariant mass spectrum is calculable in HQET over a limited range $S_H > m_B \bar{\Lambda}$ and it depends sensitively on the parameter $\bar{\Lambda}$ (equivalently m_b). These features are qualitatively very similar to the ones found for the hadronic invariant mass spectrum in the decay $B \to X_u \ell \nu_\ell$ [15]. The $1/m_b$-corrections to the parton model hadron energy spectrum in $B \to X_s \ell^+ \ell^-$ are small over most part of this spectrum. However, heavy quark expansion breaks down near the low end-point of this spectrum and near the $c\bar{c}$ threshold. We have calculated the spectral hadronic moments $\langle E_H^n \rangle$ and $\langle S_H^n \rangle$ for $n = 1, 2$ and have worked out their dependence on the HQET parameters $\bar{\Lambda}$ and λ_1. The correlations in $B \to X_s \ell^+ \ell^-$ are shown to be different than the ones in the semileptonic decay $B \to X \ell \nu_\ell$. This allows, in principle, a method to determine them from data in $B \to X_s \ell^+ \ell^-$. We show the kind of constraints following from a Gedanken experiment in $B \to X_s \ell^+ \ell^-$ and the present analysis of data in $B \to X \ell \nu_\ell$ [13] to illustrate this point.

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