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New results for non-perturbative $\mathcal{O}(a)$ improvement in light hadrons *

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We have results from light hadron simulations in quenched QCD at $\beta = 6.0$ and 6.2 using non-perturbatively improved Sheikholeslami–Wohlert fermions in an effort to remove all $\mathcal{O}(a)$ effects. From looking at hadron masses and splittings and the RG-invariant quark masses (where we have one point at $\beta = 5.7$) we suggest this is plausible even with the limited data set. The interpretation of the decay constants appears to be less clear.

1. INTRODUCTION

As part of the QCD Structure Function project, we have been looking at the effect of non-perturbative improvement [1] of Wilson fermions with the Sheikholeslami–Wohlert (SW) term in quenched QCD. In this contribution we describe our principal results. A more detailed description has recently appeared in [2].

The method is now standard. We use the SW term,

$$S_{\mathrm{SW}} = \frac{i}{2}\kappa g c_{\mathrm{SW}} a \sum_x \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \qquad (1)$$

with the coefficients [1] $c_{\mathrm{SW}} = 1.769$ at $\beta = 6.0$ and $c_{\mathrm{SW}} = 1.614$ at $\beta = 6.2$. All the data shown here either uses these values or is unimproved Wilson data.

Apart from improvement of the action, matrix elements and renormalisation require an additional improvement to remove $\mathcal{O}(a)$ effects. Ideally these should be calculated non-perturbatively, but for the time being some of our coefficients have come from tadpole-improved perturbation theory. However, the effectiveness of this procedure is much greater than in the Wilson case, as can be seen in figure 1 which compares the predictions for the critical hopping parameter with the values from the Monte Carlo data.

*Talk presented by P.W. Stephenson

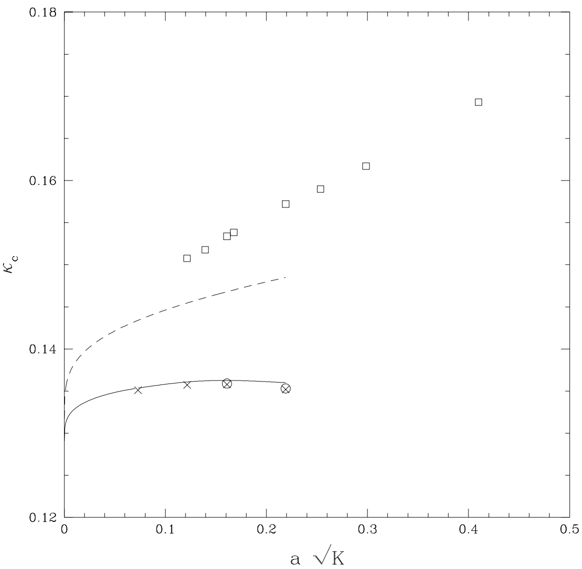

Figure 1. Monte Carlo and tadpole-improved perturbation theory values for κ_c. The dashed line and squares are for Wilson fermions, the solid line, crosses (our data) and circles (ALPHA collaboration data) for improved fermions.

2. LIGHT HADRON MASSES

Our Edinburgh/APE plots confirm that there is a better behaviour as one approaches the chiral limit, however the errors here are too large to draw strong conclusions. New results for large lattices at $\beta = 6.2$ are in production [3]. A useful test of improvement is in the splitting of the pseudoscalar and vector masses. This is not well described by Wilson data. In figure 2, we show the difference in the squares of the vector and pseudoscalar masses for both Wilson and improved data at both beta values against the pseudoscalar mass. The improved data is now consistent with the physical values for both light quark (pion/rho) and strange quark (K/K^*) masses. The string tension has been used to set the scale.

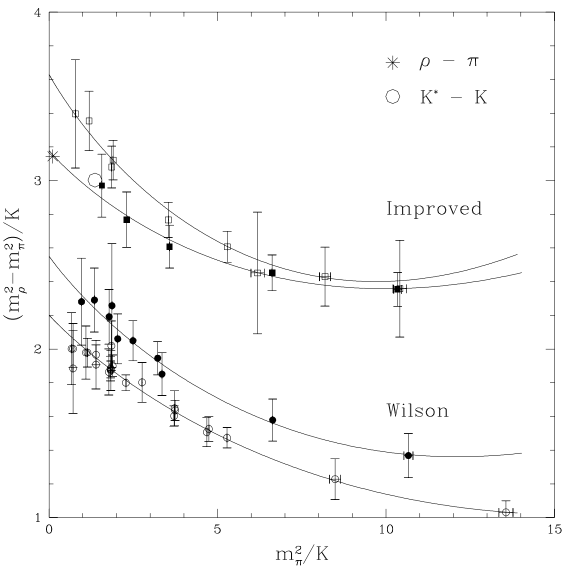

Figure 2. The vector–pseudoscalar mass splitting for Wilson (circles) and improved (squares) fermions. Open symbols are $\beta = 6.0$ and filled symbols $\beta = 6.2$.

Another parameter which has been used is the J parameter [4] which investigates the slope of the graph (assumed linear) rather than the absolute value. Here we see instead no significant improvement towards the physical value of 0.49: at $\beta = 6.0$, in fact, the value has changed with improvement from 0.413(6) to 0.38(2), and at 6.2 there is only an insignificant change in the other direction, from 0.40(3) to 0.42(3). It remains to be seen whether this discrepancy comes from differing discretisation errors in the dependence of the vector and pseudoscalar masses on the quark mass (and can therefore be rescued by performing separate extrapolations to the continuum limit at different quark masses), or whether it is a more fundamental problem with the quenched approximation. Our results in figure 2 provide scant support for suggestions of any intrinsic problems with the masses themselves, even in the chiral limit.

3. QUARK MASSES

We have calculated the light and strange quark masses using two substantially different methods which allows us to check the consistency in the continuum limit.

The first method is the traditional one: the masses are deduced from the bare values contained in κ. A single overall renormalisation factor Z_m is required; this is scale dependent and we pick the common value of 2 GeV as the scale in the $\overline{MS}$ scheme. In the second method, we use the PCAC relation to deduce the sum of the light quark masses from the axial and pseudoscalar currents A and P. Here Z_P carries the scale dependence.

In figure 3 we show the strange quark mass. In addition to our $\beta = 6.0$ and 6.2 results, we show a single point for the standard method at $\beta = 5.7$ using an improvement coefficient $c_{\rm SW} = 2.25$, which is near to and probably slightly above the value required for full $\mathcal{O}(a)$ improvement [6]. We have again relied on tadpole improved perturbation theory for various coefficients.

The quantity displayed is the renormalisation group invariant mass $\hat{m}$; our definition of this is [7] (see reference for values of quantities):

$$m(\mu^2) = \hat{m}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^{\gamma_0/2\beta_0}(1+A_1\frac{\alpha_s(\mu^2)}{\pi}+\cdots) \quad (2)$$

The formula has been used to two loops to produce the results shown. Dividing the invariant masses shown in the graph by 1.65 roughly gives the values normalised at 2.0 GeV.

We use a quadratic extrapolation with no linear term for the improved data, not including the $\beta = 5.7$ result, so the lines are exactly determined. It is clear that the agreement is much better in the improved case, largely due to the movement of the result from the traditional method into line with the others. Maybe this is connected with the improved κ_c behaviour noted above.

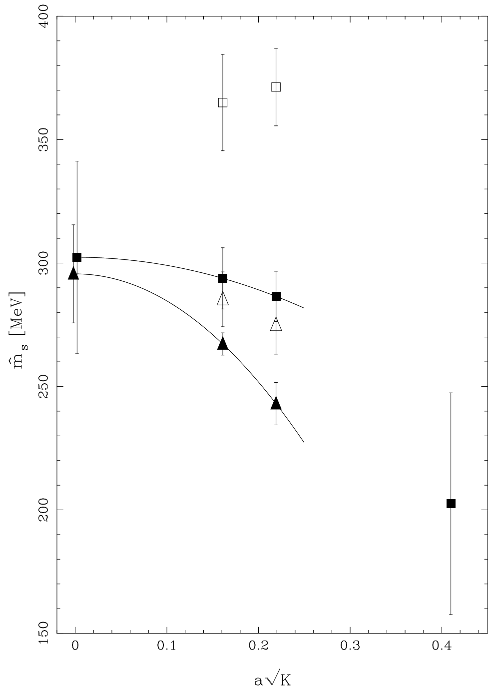

Figure 3. The strange quark mass from the standard method (squares) and the PCAC Ward identity (triangles) for Wilson (open) and improved (filled) quarks.

4. DECAY CONSTANTS

Our decay constants come from observables with a smeared source and a local sink. In the case of the f_π and f_ρ it is less clear that one can validly claim $\mathcal{O}(a)$ errors in the one case and $\mathcal{O}(a^2)$ in the other. We show the values at the strange quark mass for the K and K^* in figure 4. The discretisation errors are larger with the improved action and it is difficult to come to further conclusions about the scaling behaviour, which seems here to be worse than in the chiral limit. We note that the contribution of the improvement terms is also larger in this quark mass region.

5. SUMMARY

Although we have only two values of the coupling, our data from light hadron and quark masses appears to be consistent with the removal

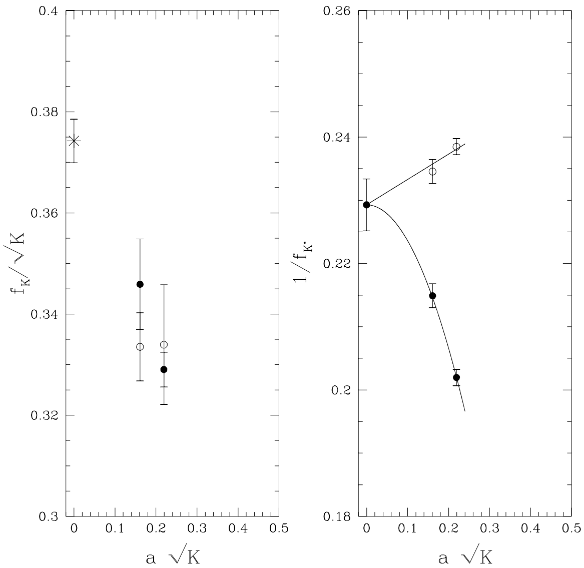

Figure 4. K and K^* decay constants, with Wilson (open) and improved (filled) fermions. The experimental value (star) is shown for the f_K.

of all $\mathcal{O}(a)$ effects by the non-perturbatively improved fermion action. The splitting of the vector and pseudoscalar masses is now in agreement with experiment comfortably within our errors. New larger lattice data at $\beta = 6.2$ [3] will extend this work. This work was supported in part by the Deutsche Forschungsgemeinschaft.

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