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$b \to se^+e^-$ decay beyond next-to-leading order

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Abstract

A more complete calculation of $b \to se^+e^-$ decay is given including the next-to-leading log QCD corrections to the photon magnetic operator O_7. The differential decay rate is found to be slightly enhanced for large invariant mass of the e^+e^- pair, while the integrated width is suppressed comparing to the previous results. The dependence on the renormalization scale is reduced considerably.

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The rare decay $b \to se^+e^-$ is an important channel for the precise study of the standard model. Although some calculations show that there exist large $c\bar{c}$ resonance contributions which interfere with the short distance contributions [1, 2, 3, 4, 5, 6, 7], it is still of great interest to perform a high precision short distance calculation. The QCD corrected coefficients for the operators in $b \to se^+e^-$ are also important for the study of the exclusive processes such as $B \to K(K^*)e^+e^-$ which has been searched for in the past years [8], and $B_s, B_d \to \gamma\ell^+\ell^-$ [9].

The process $b \to se^+e^-$ and its large QCD corrections have already been investigated in many works [10, 11, 12, 13, 14, 15]. Efforts have also been made aiming at the complete next-to-leading order (NLO) corrections [16, 17, 18] whose effects are estimated within 20%. Since there is a large logarithm in the Wilson coefficient of the most important operator O_9, represented by $1/\alpha_s$, the renormalization-group-improved perturbation theory for C_9 has the structure $O(1/\alpha_s) + O(1) + O(\alpha_s) + \ldots$ whereas the corresponding series for the other operators is $O(1) + O(\alpha_s) + \ldots$ [18]. The complete next-to-leading log QCD corrections to $b \to se^+e^-$ decay is hence of the order $O(1)$ [16, 18]. Recently $b \to s\gamma$ decay was calculated by Chetyrkin, Misiak and Münz [19] to the next-to-leading logarithm, which is of the order $O(\alpha_s)$. Since the operator O_7 is important in the region of small invariant mass for the e^+e^- pair, we will try to include the next-to-leading order QCD corrected coefficients $O(\alpha_s)$ for the operator O_7 in the decay $b \to se^+e^-$.

As discussed in ref.[16, 18], this will be part of the next to NLO calculations of the decay $b \to se^+e^-$. Up to now, there is no such order calculations of operator O_9 given in the literature. One may expects that the calculation of only one part of the higher order contribution alone will depend on the renormalization scheme used. Fortunately, as also shown by Buras and Münz [18], the magnetic operator O_7 does not mix with O_9 or O_{10} and has no impact on the coefficients $C_1 - C_6$. So that a complete higher order calculation of O_7 will not result in a scheme dependence of the decay rate.

The effective Hamiltonian appears as

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu). \tag{1}$$

The operator basis consists of 10 operators $O_1 - O_{10}$[12, 16]:

$$\begin{aligned}
O_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}), \\
O_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta}), \\
O_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + ... + (\bar{b}_{L\beta}\gamma_\mu b_{L\beta})], \\
O_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + ... + (\bar{b}_{L\beta}\gamma_\mu b_{L\alpha})], \\
O_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + ... + (\bar{b}_{R\beta}\gamma_\mu b_{R\beta})], \\
O_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + ... + (\bar{b}_{R\beta}\gamma_\mu b_{R\alpha})], \\
O_7 &= \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \\
O_8 &= \frac{g_3}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} X^a b_R G^a_{\mu\nu}, \\
O_9 &= (e^2/16\pi^2)(\bar{s}_L\gamma^\mu b_L)\bar{e}\gamma_\mu e, \\
O_{10} &= (e^2/16\pi^2)(\bar{s}_L\gamma^\mu b_L)\bar{e}\gamma_\mu\gamma_5 e. \qquad (2)
\end{aligned}$$

The coefficients C_i's are calculated by running the renormalization group equations from M_W to the scale $\mu \sim m_b$. For the operator O_9, the coefficient is given to next-to-leading order [16, 18]:

$$C_9(\mu) = P_0 + \frac{C(x) - B(x)}{\sin^2\Theta_W} - 4C(x) - D(x) + P_E E(x), \qquad (3)$$

with

$$\begin{aligned}
P_0 &= \frac{\pi}{\alpha_s(M_W)}(-0.1875 + \sum_{i=1}^{8} p_i \eta^{a_i+1}) + 1.2468 + \sum_{i=1}^{8} \eta^{a_i}[r_i + s_i\eta], \qquad (4) \\
P_E &= 0.1405 + \sum_{i=1}^{8} q_i \eta^{a_i+1}. \qquad (5)
\end{aligned}$$

Here $\eta = \alpha_s(M_W)/\alpha_s(\mu)$, and

$$\begin{aligned}
B(x) &= \frac{x}{4(1-x)} + \frac{x}{4(x-1)^2}\ln x, \qquad (6) \\
C(x) &= \frac{x(x-6)}{8(x-1)} + \frac{x(3x+2)}{8(x-1)^2}\ln x, \qquad (7) \\
D(x) &= \frac{-19x^3 + 25x^2}{36(x-1)^3} + \frac{x^2(5x^2 - 2x - 6)}{18(x-1)^4}\ln x - \frac{4}{9}\ln x, \qquad (8) \\
E(x) &= \frac{x(18 - 11x - x^2)}{12(1-x)^3} + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4}\ln x - \frac{2}{3}\ln x, \qquad (9)
\end{aligned}$$

with $x = m_t^2/M_W^2$. The coefficients a_i, p_i, r_i, s_i, and q_i are

$$
\begin{aligned}
a_i &= (\tfrac{14}{23}, \tfrac{16}{23}, \tfrac{6}{23}, -\tfrac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456),\\
p_i &= (0, 0, -\tfrac{80}{203}, \tfrac{8}{33}, 0.0433, 0.1384, 0.1648, -0.0073),\\
r_i &= (0, 0, 0.8966, -0.1960, -0.2011, 0.1328, -0.0292, -0.1858),\\
s_i &= (0, 0, -0.2009, -0.3579, 0.0490, -0.3616, -0.3554, 0.0072),\\
q_i &= (0, 0, 0, 0, 0.0318, 0.0918, -0.2700, 0.0059).
\end{aligned} \tag{10}
$$

The operator O_{10} does not renormalize under QCD, its coefficient is given by

$$C_{10}(\mu) = \frac{B(x) - C(x)}{\sin^2 \Theta_W}. \tag{11}$$

Previously the operator O_7, which contributes mainly in the region for small invariant mass of the e^+e^- pair, is given to leading order $O(1)$ by [16, 18]:

$$C_7^{(0)eff}(\mu) = \eta^{\frac{16}{23}} C_7^{(0)}(M_W) + \frac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_8^{(0)}(M_W) + \sum_{i=1}^{8} h_i \eta^{a_i}, \tag{12}$$

where

$$C_7^{(0)}(M_W) = \frac{3x^3 - 2x^2}{4(x-1)^4}\ln x + \frac{-8x^3 - 5x^2 + 7x}{24(x-1)^3}, \tag{13}$$

$$C_8^{(0)}(M_W) = \frac{-3x^2}{4(x-1)^4}\ln x + \frac{-x^3 + 5x^2 + 2x}{8(x-1)^3}, \tag{14}$$

$$h_i = \left(\frac{626126}{272277}, -\frac{56281}{51730}, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057\right). \tag{15}$$

Now we try to include the next-to-leading order effects $O(\alpha_s)$, which is the so-called next to NLO in decay $b \to se^+e^-$. The definition of x should be changed to

$$x = \frac{m_t^2}{M_W^2}\left(\frac{\alpha_s(M_W)}{\alpha_s(m_t)}\right)^{\frac{24}{23}}\left(1 - \frac{8}{3}\frac{\alpha_s(m_t)}{\pi}\right). \tag{16}$$

The coefficient C_7 at the scale $\mu \sim m_b$ can be written as:

$$C_7^{eff}(\mu) = C_7^{(0)eff}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_7^{(1)eff}(\mu). \tag{17}$$

The next-to-leading contribution $C_7^{(1)eff}(m_b)$ was calculated in ref.[19] recently:

$$
\begin{aligned}
C_7^{(1)eff}(\mu) = {} & \eta^{\frac{39}{23}} C_7^{(1)eff}(M_W) + \frac{8}{3}\left(\eta^{\frac{37}{23}} - \eta^{\frac{39}{23}}\right) C_8^{(1)eff}(M_W)\\
& + \left(\frac{297664}{14283}\eta^{\frac{16}{23}} - \frac{7164416}{357075}\eta^{\frac{14}{23}} + \frac{256868}{14283}\eta^{\frac{37}{23}} - \frac{6698884}{357075}\eta^{\frac{39}{23}}\right) C_8^{(0)}(M_W)\\
& + \frac{37208}{4761}\left(\eta^{\frac{39}{23}} - \eta^{\frac{16}{23}}\right) C_7^{(0)}(M_W) + \sum_{i=1}^{8}(e_i \eta E(x) + f_i + g_i \eta)\eta^{a_i}.
\end{aligned} \tag{18}
$$

The resulting explicit expressions for $C_7^{(1)eff}(M_W)$ and $C_8^{(1)eff}(M_W)$ in the $\overline{MS}$ scheme read

$$\begin{aligned} C_7^{(1)eff}(M_W) &= \frac{-16x^4-122x^3+80x^2-8x}{9(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{6x^4+46x^3-28x^2}{3(x-1)^5}\ln^2 x \\ &+\frac{-102x^5-588x^4-2262x^3+3244x^2-1364x+208}{81(x-1)^5}\ln x \\ &+\frac{1646x^4+12205x^3-10740x^2+2509x-436}{486(x-1)^4}, \end{aligned} \tag{19}$$

$$\begin{aligned} C_8^{(1)eff}(M_W) &= \frac{-4x^4+40x^3+41x^2+x}{6(x-1)^4}\mathrm{Li}_2\left(1-\frac{1}{x}\right)+\frac{-17x^3-31x^2}{2(x-1)^5}\ln^2 x \\ &+\frac{-210x^5+1086x^4+4893x^3+2857x^2-1994x+280}{216(x-1)^5}\ln x \\ &+\frac{737x^4-14102x^3-28209x^2+610x-508}{1296(x-1)^4}, \end{aligned} \tag{20}$$

and the numbers e_i–g_i are as follows

$$\begin{aligned} e_i &= (\ \tfrac{4661194}{816831},\ -\tfrac{8516}{2217},\ 0,\ 0,\ -1.9043,\ -0.1008,\ 0.1216,\ 0.0183), \\ f_i &= (\ -17.3023,\ 8.5027,\ 4.5508,\ 0.7519,\ 2.0040,\ 0.7476,\ -0.5385,\ 0.0914), \\ g_i &= (\ 14.8088,\ -10.8090,\ -0.8740,\ 0.4218,\ -2.9347,\ 0.3971,\ 0.1600,\ 0.0225). \end{aligned}$$

A complete higher order calculation includes three parts, the matching condition, the renormalization group running at higher order, which we have already done, and also the third step, the higher order calculation of the matrix element. In this calculation, we should replace the formula (17) to the following:

$$C_7^{eff}(\mu)=F^{1/2}\left\{C_7^{(0)eff}(\mu)+\frac{\alpha_s(\mu)}{4\pi}\left[C_7^{(1)eff}(\mu)+\sum_{i=1}^{8}C_i^{(0)}(\mu)\left(r_i+\ell_i ln\frac{m_b}{\mu}\right)\right]\right\}, \tag{21}$$

where the ℓ_i and r_i are given in [20, 19][1]. And

$$F=1-\frac{8}{3}\frac{\alpha_s(m_b)}{\pi}. \tag{22}$$

Now in the decay $b\to se^+e^-$ we have given the calculation of C_9 to the NLO order and C_7 next to NLO order.

[1]The numbers ℓ_i and r_i in the literatures are not complete yet, that makes the final result slightly scheme-dependent [20].

In our numerical analysis, the values of $\alpha_s(\mu)$ in all the above formulae are calculated using the next to leading order expression

$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{v(\mu)}\left[1 - \frac{\beta_1}{\beta_0}\frac{\alpha_s(M_Z)}{4\pi}\frac{\ln v(\mu)}{v(\mu)}\right], \tag{23}$$

where

$$v(\mu) = 1 - \beta_0 \frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right), \tag{24}$$

$\beta_0 = \frac{23}{3}$ and $\beta_1 = \frac{116}{3}$.

In the spectator model, the inclusive decay $\overline{B} \to X_s e^+ e^-$ is dominated by the free quark transition $b \to s e^+ e^-$. The semileptonic decay $\overline{B} \to X_c e \overline{\nu}$ [22] is used to eliminate large uncertainties of m_b^5 in the decay width. The normalized differential decay rate over $\hat{s} = (p_e^+ + p_e^-)^2/m_b^2$ is

$$\begin{aligned} \frac{1}{\Gamma(\overline{B}\to X_c e\overline{\nu})}\frac{d}{d\hat{s}}\Gamma(\overline{B}\to X_s e^+e^-) &= \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{\alpha_{QED}^2(1-\hat{s})^2}{4\pi^2 f(m_c/m_b)\kappa(m_c/m_b)}\left[(1+2\hat{s})\left(|C_9^{eff}|^2 + C_{10}^2\right)\right. \\ &+ \left. 4\left(1+\frac{2}{\hat{s}}\right)|C_7^{eff}|^2 + 12 Re\left(C_7^{eff*}C_9^{eff}\right)\right]. \end{aligned} \tag{25}$$

Here

$$\begin{aligned} C_9^{eff} &= C_9(\mu)\tilde{\eta}(\hat{s}) + g(m_c/m_b,\hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ &-\frac{1}{2}g(1,\hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) - \frac{1}{2}g(0,\hat{s})(C_3 + 3C_4) \\ &+\frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned} \tag{26}$$

where

$$\tilde{\eta}(\hat{s}) = 1 + \frac{\alpha_s(\mu)}{\pi}\omega(\hat{s}), \tag{27}$$

$$\begin{aligned} \omega(\hat{s}) &= -\frac{2}{9}\pi^2 - \frac{4}{3}\mathrm{Li}_2(\hat{s}) - \frac{2}{3}\ln\hat{s}\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) \\ &-\frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln\hat{s} + \frac{5+9\hat{s}-6\hat{s}^2}{6(1-\hat{s})(1+2\hat{s})}. \end{aligned} \tag{28}$$

The functions $g(z,\hat{s})$ arise from the one-loop matrix element of the four-quark operators, and can be written as [12]

$$g(z,\hat{s}) = -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}\beta - \frac{2}{9}(2+\beta)\sqrt{|1-\beta|}\begin{cases} \ln\left|\frac{\sqrt{1-\beta}+1}{\sqrt{1-\beta}-1}\right| - i\pi, & \beta < 1, \\ 2\arctan(1/\sqrt{\beta-1}), & \beta > 1, \end{cases} \tag{29}$$

with $\beta = 4z^2/\hat{s}$. The factor $f(m_c/m_b)$ arises from the non-negligible corrections of m_c/m_b in the semileptonic decay,

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x,$$

and

$$\kappa(z) = 1 - \frac{2\alpha_s(\mu)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-z)^2 + \frac{3}{2}\right]$$

is the QCD correction factor to the semileptonic decay. We take the experimental result $Br(\overline{B} \to X_c e\overline{\nu}) = 10.4\%$ [21], the branching ratio for $\overline{B} \to X_s e^+e^-$ is found.

Taking parameters as $M_W = 80.33\mathrm{GeV}$, $m_t = 175\mathrm{GeV}$, $m_b = 5.0\mathrm{GeV}$, $m_c/m_b = 0.29$, $|V_{tb}V_{ts}^*/V_{cb}| = 0.976$ and the QCD coupling constant $\alpha_s(M_Z) = 0.118$ [21], the values for $C_7^{eff}(\mu)$ (formula (17)) and $C_9(\mu)$ are

$$\begin{aligned} C_7^{eff}(\mu) &= -0.33, \quad C_9(\mu) = 4.45, \quad \text{for } \mu = 2.5\mathrm{GeV}, \\ C_7^{eff}(\mu) &= -0.30, \quad C_9(\mu) = 4.19, \quad \text{for } \mu = 5.0\mathrm{GeV}, \\ C_7^{eff}(\mu) &= -0.28, \quad C_9(\mu) = 3.80, \quad \text{for } \mu = 10\mathrm{GeV}. \end{aligned} \tag{30}$$

The differential decay width is depicted in Fig.1 as a function of $\hat{s}$. In Fig.1 the renormalization scale is taken to be $\mu = m_b$. Since the higher order QCD correction for the operator O_7 suppresses the Wilson coefficients $C_7^{eff}(\mu)$, for very small invariant mass of the e^+e^- pair this decay rate is suppressed due to the $|C_7^{eff}(\mu)|^2$ term in (25). While for the region of large invariant mass of the e^+e^- pair, the $b \to se^+e^-$ differential decay rate is slightly enhanced, due to the cross term $C_7^{eff*}(\mu)C_9^{eff}(\mu)$ in (25). The net effect is to reduce the branching ratio from 8.55×10^{-6} to 8.20×10^{-6} for $\mu = m_b$, and the branching ratio is now 8.66×10^{-6} for $\mu = 2.5\mathrm{GeV}$, or 7.98×10^{-6} for $\mu = 10\mathrm{GeV}$. If we take a lower cut on the the invariant mass spectrum for the electron pair as $\hat{s} = 0.1$, these numbers change into 4.05×10^{-6} for $\mu = 2.5\mathrm{GeV}$, 4.20×10^{-6} for $\mu = 5.0\mathrm{GeV}$, or 4.44×10^{-6} for $\mu = 10\mathrm{GeV}$.

Although the result given here is not quite different from the previous ones, the uncertainty due to the renormalization scale from $\mu = m_b/2$ to $\mu = 2m_b$ is reduced considerably from 17% to 8.5 %. More precise prediction can be achieved only after the next to NLO calculation of operator O_9 is done.

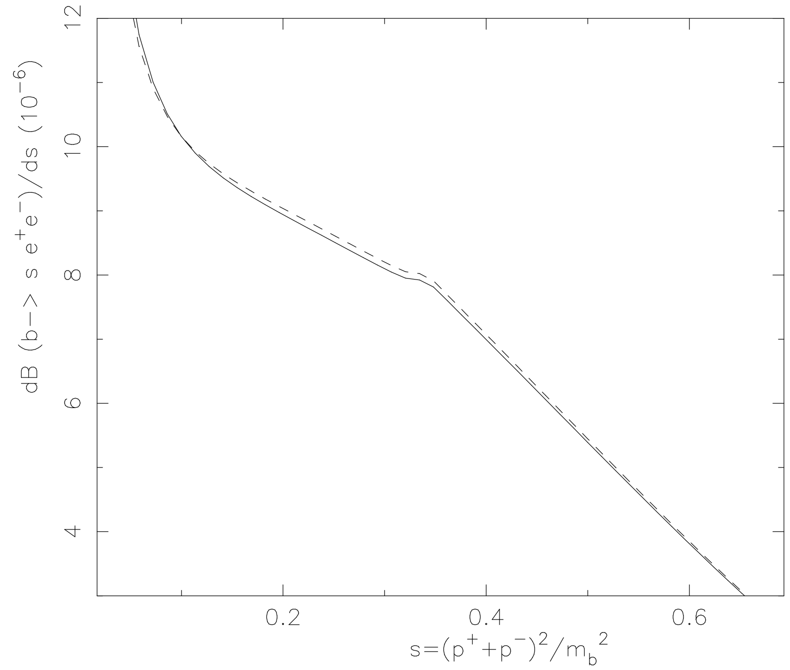

Figure 1: Differential branching ratios of $b \to s e^+ e^-$, as a function of $\hat{s} = (p^+ + p^-)^2/m_b^2$. The solid line denotes the result without higher order QCD corrections to operator O_7, while the dashed one corresponds to result with higher order QCD corrections to operator O_7.

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