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Diffractive J/Ψ Leptoproduction as a Way to Measure the Polarized Gluon Distribution

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Abstract

The cross section of diffractive J/Ψ lepto- (and photo-) production on a polarized target is calculated in the leading log approximation of pQCD assuming a nonrelativistic J/Ψ wave function. In this approximation the spin-spin asymmetry in the small x region is close to $2\Delta G/G$ for the $\gamma^* + p \to J/\Psi + p$ reaction, providing a promising tool to study the spin dependent gluon distribution $\Delta G(x, q^2)$.

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1. The process of diffractive J/Ψ photo- (lepto-) production is very interesting by two reasons. First, it can be calculated within perturbative QCD and, secondly, its cross section is proportional to the gluon density squared. For the unpolarized case the cross section was calculated in the Leading Log Approximation (LLA) [1] assuming a nonrelativistic wave function for the J/Ψ meson . This formula was used succesfully to extract the gluon distribution in the small x region from diffractive J/Ψ production at HERA [2].

Of course, the NLO contributions as well as the relativistic corrections to the J/Ψ wave function should be taken into account; both were discussed in [3]. Fortunately, the corrections cancel each other to a large extent and do in any case not change the x behaviour of the cross section, which is controlled by the x dependence of the gluon density.

In the present paper the same approach will be used to calculate the elastic diffractive J/Ψ electroproduction at small x on a polarized target. The asymmetry turns out to be proportional to the normalized spin dependent gluon density $\Delta G(x,q^2)/G(x,q^2)$. Therefore the elastic diffractive process is presumably a very promising tool to measure (at least to estimate) the value of $\Delta G(x,q^2)$ in the small x region.

2. The Born amplitude of the reaction $\gamma^* + p \to J/\Psi + p$ is described by the sum of the two diagrams in shown fig.1. As the binding energy of the S-wave $c\bar{c}$ system is much smaller than the charm quark mass $m_c = m$ one can follow ref.[4] and write (in no-relativistic approximation) the product of two propagators (k and k' in fig.1) and the J/Ψ vertex (i.e. the J/Ψ-wave function integrated over the relative momenta of c and $\bar{c}$ quark **k**$\simeq$**k'** in the J/Ψ-rest frame) in the form $g\cdot(\hat{k}+m)\gamma_\mu$. The constant g may be expressed in terms of the electronic width Γ^J_{ee} of the $J/\Psi \to e^+e^-$ decay,

$$g^2 \;=\; \frac{3\Gamma^J_{ee} m_J}{64\alpha^2_{e.m.}}, \tag{1}$$

where m_J is the mass of the J/Ψ-meson and $\alpha_{e.m.} = 1/137$ is the electromagnetic coupling constant.

At large energies $s = (q+p)^2 >> |q^2| + m_J^2$ the main contribution comes from the longitudinal polarizations of t-channel gluons (l and $l+Q$ in fig.1), i.e. the spin part of gluon propagator is given by

$$g_{\rho\sigma} \;=\; g^{\perp}_{\rho\sigma} \;+\; \frac{p'_\rho q'_\sigma + q'_\rho p'_\sigma}{(p'q')} \;\simeq\; \frac{p'_\rho q'_\sigma}{(p'q')}$$

with

$$q_\mu = q'_\mu + \frac{q^2}{s}p'_\mu;\; p_\mu = p'_\mu + \frac{m_N^2}{s}q'_\mu \simeq p_\mu;\; s = 2(p'q');\; p'^2 = q'^2 = 0$$

Here m_N is the nucleon mass, p and q are the 4-momenta of the initial proton and photon correspondingly.

However the longitudinal t-channel (Coulomb-like) gluon looses the information about the polarization of the target. Thus at least one gluon must have transverse polarization. Indeed for the longitudinally polarized target with spin vector $s_\mu || p_\mu$ the spin dependent part of the trace in the bottom of the diagram fig.1 (the target loop) looks like

$$B_{\sigma\sigma'} \;=\; \frac{1}{2}\mathrm{Tr}[\hat{p}\gamma_5\gamma_\sigma(\hat{p}+\hat{l})\gamma_{\sigma'}] \;=\; -2i\epsilon^{\sigma\sigma'\alpha\beta}l_\alpha p_\beta \tag{2}$$

where $\epsilon^{\sigma\sigma'\alpha\beta}$ is the antisymmetric tensor and σ (σ') corresponds to the polarization of the left (right) t-channel gluon in fig.1. (Considering the forward amplitude we put the momentum transfer $Q \simeq 0$.)
As p_μ is the longitudinal vector while the $l_\mu \simeq l_{t,\mu}$, two other indices σ, σ' should be the transverse (say, $g_{\rho\sigma} = g^\perp_{\rho\sigma}$) and the longitudinal $g_{\rho'\sigma'} \simeq \frac{p'_{\rho'}q'_{\sigma'}}{(p'q')}$ ones. So the spin dependent part of the matrix element given by the graph in fig.1a takes the form

$$M_a^{(s)} \;=\; i\frac{2}{3}\alpha_s^2\frac{e_c g}{s}4\int\frac{\Phi(l)\mathrm{Tr}[...]_a d^2l_t}{(r^2-m^2)l^4} \tag{3}$$

Here $e_c = \frac{2}{3}\sqrt{\frac{4\pi}{137}}$ is the electric charge of charm quark, α_s is the QCD coupling constant and $\frac{2}{3}$ is the colour coefficient. The function $\Phi(l)$ describes the emission of a gluon pair (l and $l+Q$) by a proton. If l_t is large in comparison with the inverse proton radius R_N ($l >> 1/R_N$) each valence quark emits its own pair of gluons (independently from the other quark-spectators) and then $\Phi(l) \simeq 3$.

$$\mathrm{Tr}[...]_a \;=\; \mathrm{Tr}[\hat{E}_{ph}(\hat{k}+m)\hat{E}_\Psi\hat{p}'(\hat{k}'+\hat{l}+m)\hat{e}^\perp(\hat{r}+m)] -$$

$$\mathrm{Tr}[\hat{E}_{ph}(\hat{k}+m)\hat{E}_\Psi\hat{e}^\perp(\hat{k}'+\hat{l}+m)\hat{p}'(\hat{r}+m)] \;=\; s\cdot m\mathrm{Tr}[\hat{E}_{ph}\hat{E}_\Psi\hat{e}^\perp\hat{l}_t] \;=$$

$$= 4is\cdot ml_t^2(\epsilon_\perp^{\nu\mu}E_{\Psi,\nu}E_{ph,\mu}) \tag{4}$$

where E_{ph} and E_Ψ are the polarization vectors of the photon and the J/Ψ-meson, respectively, and the minus sign in front of the second trace reflects the antisymmetric nature of the $\epsilon^{\sigma\sigma'\alpha\beta}$ tensor in eq.(2); $\hat{p}'$ and $e^{\perp}_{\nu} = i\epsilon^{\nu\mu}_{\perp} l_\mu$ correspond to the gluon polarizations; $\epsilon^{\nu\mu}_{\perp}$ is the two-dimensional antisymmetric tensor acting in the transverse plane.
For the last equality in eq.(4) we have averaged over the direction of $\vec{l}_t$ in the azimuthal plane.

Recall that in our small $x = (|q^2| + m^2)/s$ limit $l_\mu \simeq l_{t,\mu}$ and in the no-relativistic approximation for the forward ($Q_t = 0$) amplitude $k_t = k'_t = 0$, while the longitudinal components $k_z = -k'_z = q_z/2$; $r^2 - m^2 = -(|q^2| + m^2)/2 = -2\bar{q}^2$.

To obtain the contribution of the diagram in fig.1b it is enough to replace the factor $(r^2 - m^2)$ in the denominator of eq.(3) by $(r'^2 - m^2)$ and to put the

$$\mathrm{Tr}[...]_b \ = \ -\mathrm{Tr}[\hat{E}_{ph}(\hat{k} - \hat{l} + m)\hat{p}'(\hat{k} + m)\hat{E}_\Psi \hat{e}^\perp(\hat{r}' + m)] \ +$$
$$\mathrm{Tr}[\hat{E}_{ph}(\hat{k} - \hat{l} + m)\hat{e}^\perp(\hat{k} + m)\hat{E}_\Psi \hat{p}'(\hat{r}' + m)] \ = \ s \cdot m \mathrm{Tr}[\hat{E}_{ph}\hat{E}_\Psi \hat{e}^\perp \hat{l}_t] \qquad (5)$$

instead of $\mathrm{Tr}[...]_a$. $(r' = (k - l - q),\ \ r'^2 - m^2 = -2\bar{q}^2 - 2|l_t|^2)$.

The negative sign in front of the first trace in eq.(5) reflects another colour sign of the antiquark in fig.1b.

3. Note that as the $\mathrm{Tr}[...]_a = \mathrm{Tr}[...]_b \propto l_t^2$ we have the logarithmic dl_t^2/l_t^2 integration in eq.(3) This is nothing else but the first step of the DGLAP evolution of the spin dependent gluon distribution $\Delta G(x, \bar{q}^2)$ with the splitting function [5] $\Delta P_{gq} = 2C_F \frac{1-(1-z)^2}{z} \simeq 4C_F$ for $z << 1$ $(C_F = (N_c^2 - 1)/2N_c = 4/3)$. In order to make the (Born) calculation more realistic we have to include the ladder 'evolution' gluons (shown by the dashed lines in fig.1). This is achieved by the replacements [1]

$$C_F \int \frac{\alpha_s}{\pi} \Phi(l) \frac{dl_t^2}{l_t^2} \ \rightarrow \ \Delta G(x, \bar{q}^2)\ . \qquad (6)$$

Strictly speaking, even at zero transverse momentum $Q_t = 0$ one does not obtain the exact gluon structure function, as a non-zero component of the longitudinal momentum is transferred through the two-gluon ladder. However, in the region of interest, $x << 1$, the value of $|t_{min}| = m_N^2 x^2$ is so small that we may safely put $t \equiv Q^2 = 0$ and identify the ladder coupling to the proton with $\Delta G(x, \bar{q}^2)$ (see [3] for more details). The arguments of the gluon structure function should be $x = (|q^2| + m^2)/s$ and $\bar{q}^2 = (|q^2| + m^2)/4$; the scale $\bar{q}^2$ reflects the fact that the gluons couple to a quark which carries away one half of the initial photon momentum.

Now in the leading $\log(q^2)$ approach the spin dependent part of the amplitude takes the form

$$M^{(s)}_{\mu\nu} \;=\; i16\pi^2\alpha_s \frac{e_c g m}{2\bar{q}^2}\Delta G(x,\bar{q}^2)\cdot 2i\epsilon^{\perp}_{\mu\nu} \tag{7}$$

where we have summed up the contributions of the graphs in fig.1a and fig.1b and taken into account that there are two types of such diagrams (a gluon l can interact with a c-quark k or $\bar{c}$-antiquark k').The indices μ, ν correspond to the initial photon and outgoing J/Ψ-meson, respectively.

4. In the same notation the forward amplitude for the unpolarized case (and a transversely polarized photon) looks like:

$$M^{(u)}_{\mu\nu} \;=\; i16\pi^2\alpha_s \frac{e_c g m}{(2\bar{q}^2)^2} x G(x,\bar{q}^2)\cdot s g^{\perp}_{\mu\nu} \tag{8}$$

In other words we have

$$M_{\mu\nu} \;\propto\; i\left(G(x,\bar{q}^2)g^{\perp}_{\mu\nu} \pm \Delta G(x,\bar{q}^2)\cdot i\epsilon^{\perp}_{\mu\nu}\right) \tag{9}$$

Here the $\pm$ sign reflects the helicity (polarization) of the target proton and the identity $s/2\bar{q}^2 = 2/x$ was used.

In terms of the spin-spin asymmetry $A = A_{LL}$ eq.(9) means that for the elastic diffractive reaction $\gamma^* + p \rightarrow J/\Psi + p$ [1]

$$A \;=\; \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} \;=\; \frac{2\Delta G(x,\bar{q}^2)G(x,\bar{q}^2)}{(G(x,\bar{q}^2))^2 + (\Delta G(x,\bar{q}^2))^2} \;\simeq\; \frac{2\Delta G(x,\bar{q}^2)}{G(x,\bar{q}^2)} \tag{10}$$

The arrows indicate the helicities of the incoming photon and the target nucleon.

It turns out that the asymmetry of diffractive production (in the small x approximation) is close to $A \simeq 2\Delta G/G$. The factor 2, extremly favourable when extracting $\Delta G/G$ from experimental asymmetry data, comes in because

[1] The polarization of the photon emitted by the 100% polarized initial lepton in DIS is $P_{ph} = \frac{1-(1-y)^2}{1+(1+y)^2}$, where y is the lepton momentum fraction carried by the photon in the target proton rest frame.

the cross section of the diffractive process is proportional to the gluon density squared.

It is anticipated that the main part of the corrections (NLO contributions and the relativistic corrections to the J/Ψ wave function) are cancelled when calculating the asymnmetry by eq.(10). Indeed, it was shown in [3] that the corrections do mainly affect the absolute normalization of the cross section (see also [6]). For example, the uncertainty coming from the value of the QCD coupling constant (more exactly from the scale at which α_s is evaluated) do cancel in the ratio eq.(10). Thus the accuracy of the expression (10) for the asymmetry is expected to be even better than that of the unpolarized difractive amplitude (8) [1].

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