

## ON HEAVY QUARKS PHOTOPRODUCTION AND $c \rightarrow D^*$ FRAGMENTATION FUNCTIONS<sup>♣</sup>

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### ABSTRACT

The state of the art of the theoretical calculations for heavy quarks photoproduction is reviewed. The full fixed order next-to-leading order massive calculation and the resummation of large  $\log(p_T/m)$  terms for differential cross sections are described. The implementation of a non-perturbative fragmentation function describing the  $c \rightarrow D^*$  meson transition is also discussed.

### 1. Introduction

Heavy quarks production processes provide a powerful insight into our understanding of Quantum Chromodynamics. The large mass of the heavy quark can make the perturbative calculations reliable, even for total cross sections, by cutting off infrared singularities and by setting a large scale at which the strong coupling can be evaluated and found – possibly – small enough. On the experimental side, the possibility to tag heavy flavoured hadrons by means of microvertex detectors can on the other hand provide accurate measurements.

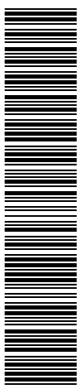
All these potentialities must of course be matched by accurate enough theoretical evaluations of the production cross section. In this talk I shall describe the state of the art of such calculations for heavy quarks photoproduction. I shall first review the next-to-leading order (NLO) QCD evaluations recently presented by Frixione, Mangano, Nason and Ridolfi. These calculations, available for total cross sections, one-particle and two-particles distributions, are now a consolidated result and provide a benchmark for future developments.

Large logarithms appear in the NLO fixed order calculations and potentially make it less reliable in some regimes:  $\log(S/m^2)$  and  $\log(p_T^2/m^2)$  become large when the center of mass energy  $\sqrt{S}$  or the transverse momentum  $p_T$  of the observed quark is much larger than its mass. I shall describe the resummation of  $\log(p_T^2/m^2)$  terms, leaving the high energy resummation to Marcello Ciafaloni's talk<sup>1</sup>.

The perturbative fragmentation function technique used in the resummation of

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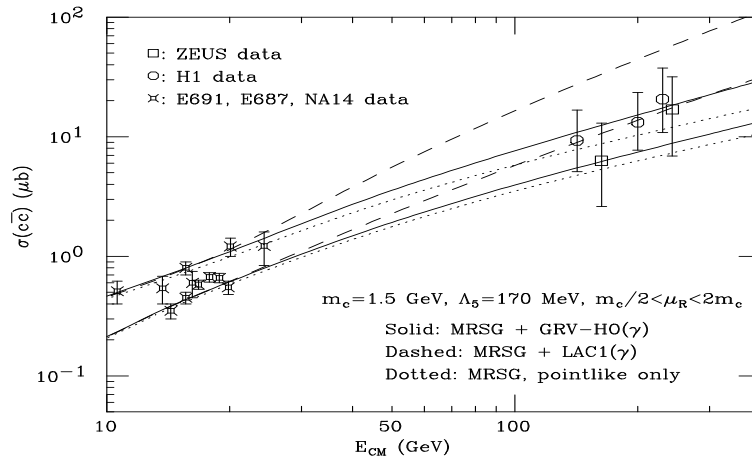


Fig. 1. Total cross section for  $c\bar{c}$  photoproduction<sup>6</sup>.

the large  $\log(p_T^2/m^2)$  terms has a non-perturbative extension which can be used to describe the transition from  $c$  quarks to  $D^*$  mesons. I shall therefore also discuss the determination of these non-perturbative fragmentation functions and their inclusion into the heavy quarks photoproduction calculation, showing a comparison with data from HERA.

## 2. Fixed Order NLO Calculation

Heavy quarks photoproduction at leading order in the strong coupling  $\alpha_s$  looks a very simple process: only the tree level diagram  $\gamma g \rightarrow Q\bar{Q}$  contributes at the partonic level, and the final answer for the total cross section is simple and well behaved, being finite everywhere.

At a deeper thinking, however, problems seem to arise. For instance, one may ask himself why not to include initial state heavy quarks, coming from the hadron and to be scattered by the photon, like  $\gamma Q \rightarrow Qg$ . To include consistently such a diagram is not an easy task, especially if one wants to keep the quark massive. Taking it massless, on the other hand, would not only be a bad approximation but would also produce a divergent total cross section.

A way out of this problem was provided by Collins, Soper and Sterman<sup>2</sup>, who argued that the following factorization formula holds for heavy quarks hadroproduction total cross sections:

$$\sigma(\sqrt{S}, m) = \sum_{ij} \int f_{i/H_1} f_{j/H_2} \hat{\sigma}(ij \rightarrow Q\bar{Q}; \sqrt{S}, m). \quad (1)$$

The sum on the partons runs only on  $i$  and  $j$  being gluons or light quarks, and the heavy quarks are only generated at the perturbative level by gluon splitting. There is therefore no need to try to accommodate them in the colliding hadrons

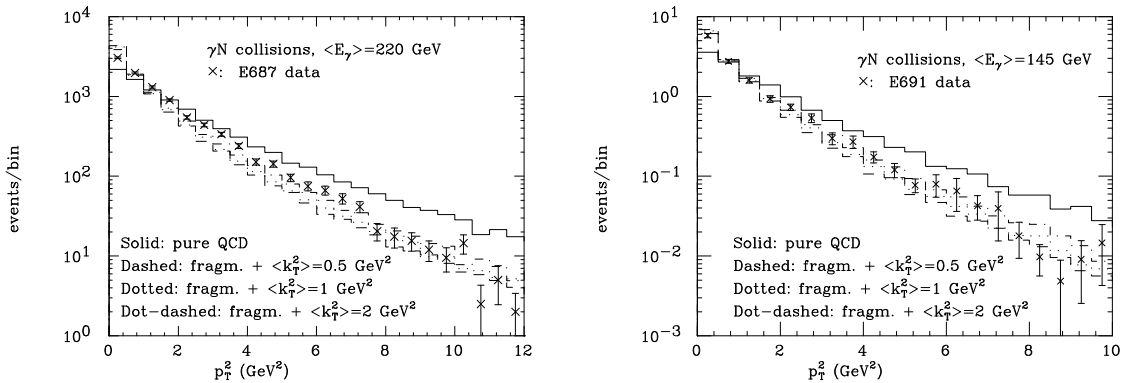


Fig. 2. Differential  $p_T$  distributions for charm production in fixed target experiments<sup>6</sup>.

and the relevant kinematics can be kept exact. Eq. (1) provides the basis for an exact perturbative calculation of heavy quarks production to NLO. For what concerns photoproduction, such a calculation has been first performed by P. Nason and K. Ellis, and subsequently confirmed by J. Smith and W.L. van Neerven<sup>3</sup>.

When going to order  $\alpha_s^2$  in photon-hadron collision, however, a new feature appears. The photon can now couple directly to massless quarks, for instance in processes like  $\gamma q \rightarrow Q\bar{Q}q$ , and in a given region of phase space a collinear singularity will appear. It can be consistently factored out, but this requires the introduction of *photon* parton distribution functions (PDF) which, pretty much like the hadron ones, will describe the probability that before the interaction the photon splits into hadronic components (light quarks or gluons, in this case). Such a behaviour is sometimes called *resolved photon* (as opposed to *direct*). A full NLO calculation for heavy quark photoproduction will therefore also require a NLO calculation for hadroproduction<sup>4</sup>, where one of the PDF's will be the photon's one. A factorization scale  $\mu_\gamma$ , related to the subtraction of the singularity at the photon vertex, will link the two pieces and its dependence on the result will only cancel when both are taken into account.

Frixione, Mangano, Nason and Ridolfi<sup>5</sup> (FMNR) have recently presented Monte Carlo integrators for these two calculations, thereby allowing detailed comparisons with experimental data. A very extensive collection of such comparisons is presented in a recent review<sup>6</sup>, from which we select some plots to be shown here.

A comparison of total cross section experimental results and theoretical predictions for  $c\bar{c}$  photoproduction is shown in fig. 1. Although large uncertainties are present, the comparison suggests agreement between theory and experiment. The new HERA data, at large center of mass energy, can be seen to appear larger than the pointlike (= direct) photon prediction only. This suggests the need for a resolved photon component, but by no means can determine it precisely.

One-particle transverse momentum ( $p_T$ ) distributions are shown in fig. 2. The pure QCD predictions can be seen to be significantly harder than the data. However,

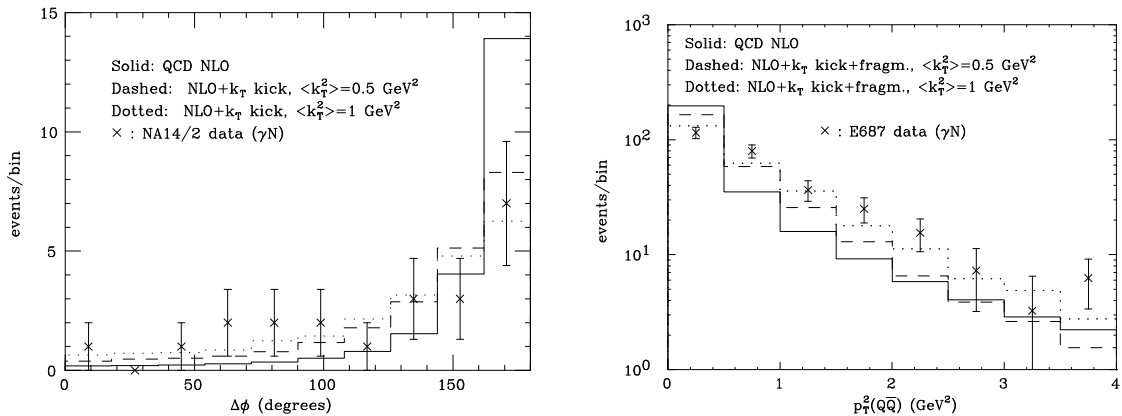


Fig. 3. Two particles correlations in fixed target experiments<sup>6</sup>.

when corrected with two non-perturbative contributions they can be matched to the data. These non-perturbative additions are meant to represent a primordial transverse momentum  $k_T$  of the colliding partons, other than the one already taken into account by the QCD radiative corrections, and the effect of the fragmentation of the produced heavy quark into the observed heavy flavoured hadrons, here described by the so-called Peterson fragmentation function with  $\epsilon = 0.06$ .

Comparisons between data and theory for two-particle correlations, like the azimuthal difference  $\Delta\phi$  or the relative transverse momentum  $p_T(Q\bar{Q})$  of the produced heavy quark pair, are shown in fig. 3. Distributions like these are trivial in leading order QCD, since the  $Q$  and the  $\bar{Q}$  are produced back-to-back. Hence,  $\Delta\phi = \pi$  and  $p_T(Q\bar{Q}) = 0$ . NLO corrections (as well as non-perturbative contributions) can broaden these distributions, and one could think of being able to perform a direct measurement of  $O(\alpha_s^3)$  effects. The plots do however show that non perturbative contributions play a key role in allowing a good description of the data. One can, however, still check that the same inputs allow for a good description of both one- and two-particles distributions, as seems to be the case here.

The overall result of these comparisons can therefore be summarized as follows. Total cross sections seem to be well reproduced by the calculation both at fixed target and HERA regimes, but the huge uncertainties present both on the experimental and the theoretical side do not allow the study of finer details like, for instance, the determination of the resolved component at HERA. For what concerns transverse momentum distributions at fixed target, they can be reproduced after allowing for heavy quark fragmentation effects and for a primordial transverse momentum of the incoming partons of the order of 1 GeV. These same non-perturbative corrections also allow for a description of two-particles correlations, thereby pointing towards a consistent picture.

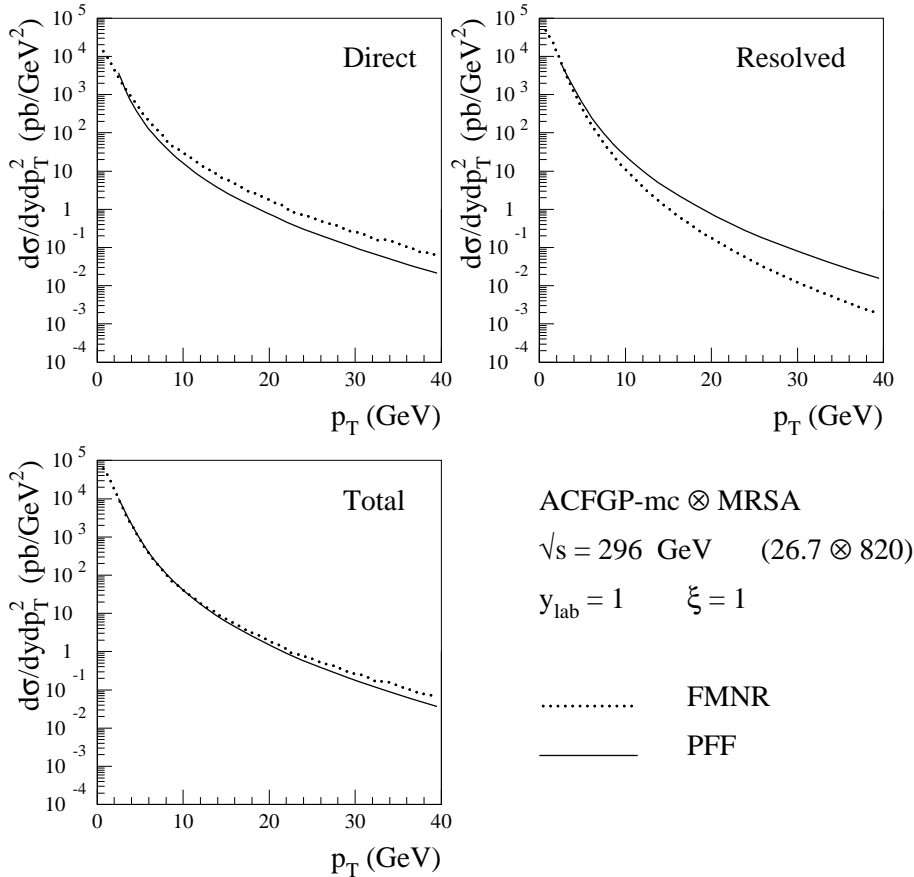


Fig. 4. Comparison between fixed order (FMNR) and resummed (PFF) calculation for charm photoproduction  $p_T$  distribution<sup>7</sup>. It is worth noticing how the two calculations describe differently the (unphysical) resolved and direct components, but agree on their sum (a physical observable).

### 3. Large Transverse Momentum Resummation

Like any perturbative expansion, the NLO calculation for heavy quarks photoproduction is only reliable and accurate as long as the coefficients of the coupling constant remain small. Large terms of the kind  $\log(p_T^2/m^2)$  do however appear in the cross section, and for growing  $p_T$  they will eventually become large enough to spoil the convergence of the series. Such terms need therefore to be resummed to all orders to allow for a sensible phenomenological prediction. Such a resummation has been performed along the following lines<sup>7</sup>.

One observes that in the large- $p_T$  limit ( $p_T \gg m$ ) the only important mass terms are those appearing in the logs, all the others being power suppressed. This means that an alternative description of heavy quark production can be achieved by using *massless* quarks and providing at the same time perturbative distribution and

fragmentation functions also for the heavy quark, describing the logarithmic mass dependence. The factorization formula becomes

$$d\sigma(p_T) = \sum_{ijk} \int F_{i/H_1}(\mu, [m]) F_{j/H_2}(\mu, [m]) d\hat{\sigma}(ij \rightarrow k; p_T, \mu) D_k^Q(\mu, m), \quad (2)$$

with parton indices  $i, j$  and  $k$  also running on  $Q$ , taken massless in  $\hat{\sigma}$ , now an  $\overline{\text{MS}}$  subtracted cross section for light partons production. The dependence on  $m$  of the parton distribution functions  $F_{i/H}$ , shown among square brackets in eq. (2), is only there when  $i$  or  $j$  happens to be the heavy quark  $Q$ .

The key point is that the large mass of the heavy quark allows for the evaluation in perturbative QCD (pQCD) of its distribution and fragmentation functions. Initial state conditions for  $F_{Q/H}(\mu_0 = m)$ <sup>8</sup> and  $D_k^Q(\mu_0 \simeq m)$ <sup>9</sup> can be calculated in pQCD at NLO level in the  $\overline{\text{MS}}$  scheme:

$$F_{Q/H}(x, \mu_0 = m) = 0 \quad (3)$$

$$D_Q^Q(x, \mu_0) = \delta(1-x) + \frac{\alpha_s(\mu_0) C_F}{2\pi} \left[ \frac{1+x^2}{1-x} \left( \log \frac{\mu_0^2}{m^2} - 2 \log(1-x) - 1 \right) \right]_+ \quad (4)$$

$$D_g^Q(x, \mu_0) = \frac{\alpha_s(\mu_0) T_F}{2\pi} (x^2 + (1-x)^2) \log \frac{\mu_0^2}{m^2} \quad (5)$$

$$D_{q,\bar{q},\bar{Q}}^Q(x, \mu_0) = 0 \quad (6)$$

The massive logs will hence appear only through these function, which can then be evolved with the Altarelli-Parisi equations up to the large scale set by  $\mu \simeq p_T$ . This evolution will resum to all orders the large logarithms previously mentioned.

It is important to mention that due to the neglecting of power suppressed mass terms this approach becomes unreliable when  $p_T \simeq m$ . In this region only a case by case comparison with the full NLO massive calculation – here reliable and to be taken as a benchmark – can tell how accurate the resummed result is.

Phenomenological analyses show that the effect of the resummation becomes sizeable only at very large  $p_T$ , say greater than 20 GeV for charm photoproduction. Fig. 4 shows the effect of such a resummation for a fixed photon energy in HERA-like kinematics. The resummed calculation can be seen to match the fixed order one at  $p_T \sim m$ , where resummation effects are not expected to be important, and to behave more softly in the large  $p_T$  region. This particular theoretical refinement should therefore not be phenomenologically overly relevant for present-day HERA physics, data being only available up to  $p_T \simeq 12$  GeV.

#### 4. On the Inclusion of $c \rightarrow D^*$ Fragmentation Effects

When comparing theory with data, one always faces the problem of describing as closely as possible what the experiments do observe. With heavy quarks production the problem lies in the experiments actually seeing the decay products of

heavy flavoured hadrons rather than the heavy quark itself. This is due to the heavy quark strong and non-perturbative binding into a hadron prior to decay. This binding involves the exchange and radiation of low-momentum (order  $\Lambda_{QCD}$ ) gluons, and typically degrades the momentum of the hadron with respect to the one of the original quark. Such a degradation can be described with the help of a non-perturbative fragmentation function (FF) which, lacking the theoretical tools to calculate, can be extract by fitting experimental data.

An often employed parametrization for such a function is the so called Peterson<sup>10</sup> one, which reads

$$D_{np}(z; \epsilon) \sim \frac{1}{z [1 - 1/z - \epsilon/(1 - z)]^2}. \quad (7)$$

The value of  $\epsilon$  is predicted to scale like  $\Lambda_{QCD}^2/m^2$ . For charm to  $D^*$  fragmentation a global analysis<sup>11</sup> based on leading order Montecarlo simulations gives the value  $\epsilon \simeq 0.06$ . This value has so far usually been taken as the reference one, and used for instance together with the NLO fixed order calculation by FMNR in the plots shown in Section 2.

One should however carefully consider how  $\epsilon$  has been extracted from  $e^+e^-$  experimental data. Experiments usually report the energy or momentum fraction ( $x_E$  or  $x_p$ ) of the observed hadron with respect to the beam energy. On the other hand the fraction which appears as the argument of the non-perturbative FF is rather to be taken with respect to the fragmenting quark momentum, usually denoted by  $z$  (see for instance <sup>11</sup> for a discussion on this point). These two fractions are not coincident, due to hard radiation processes which lower the momentum of the quark before it fragments into the hadron. In order to deconvolute these effects one usually runs a Montecarlo simulation of the collision process at hand, including both the perturbative parton showers and the subsequent hadronization of the partons into the observed hadrons. The latter can be parametrized in the Montecarlo by the Peterson fragmentation function, and the value of  $\epsilon$  which best describes the data can be extracted. Clearly this procedure leads to a resulting value for  $\epsilon$  which depends on the details of the description of the perturbative part. Indeed, the showering softens the momentum distribution of the heavy quark, producing an effect qualitatively similar to that of the non-perturbative FF. On the quantitative level, the amount of softening (and hence the value of  $\epsilon$ ) required by the non-perturbative FF to describe the data is related to the amount of softening already performed at the perturbative level. A leading or a next-to-leading description of the showering can therefore produce different values for  $\epsilon$ , whose value is then not a “unique” and “true” one, but rather closely interconnected with the details of the description of the pQCD part of the problem.

In ref. <sup>12</sup> fits to  $D^*$  data taken by the ARGUS and OPAL experiments have been performed with NLO accuracy using a fragmentation description for the heavy quark production like the one described in Section 3, complemented with the inclusion of a

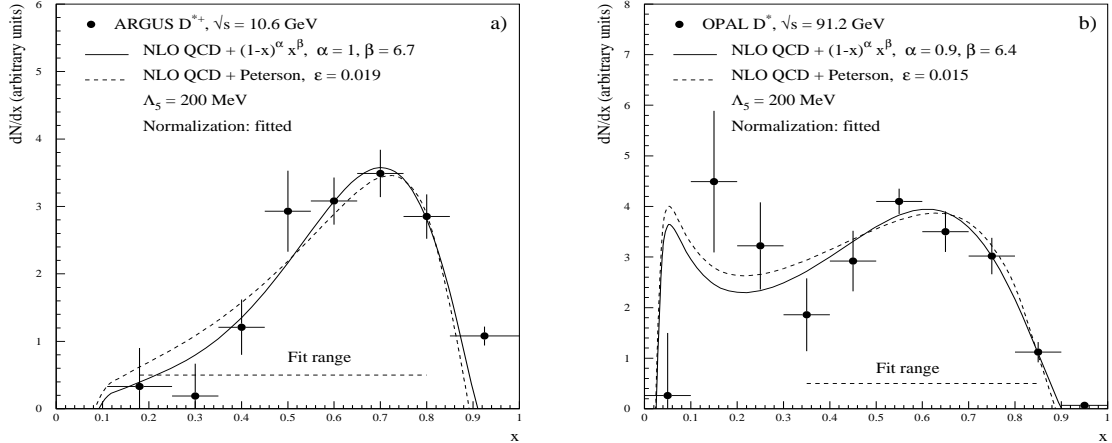


Fig. 5. Distributions of  $D^*$  mesons as measured by the ARGUS and OPAL experiments, together with the theoretical curves<sup>12</sup> fitted to the same data with the  $(1-x)^\alpha x^\beta$  (full line) and the Peterson (dashed line) non-perturbative fragmentation functions.

non-perturbative component via the ansatz

$$D_k^{D^*}(\mu) = D_k^c(\mu) \otimes D_c^{D^*}, \quad (8)$$

represented by the convolution of a perturbatively calculable fragmentation function of the parton  $k$  into the heavy quark  $c$  and the non-perturbative form  $D_c^{D^*}$  describing the  $c \rightarrow D^*$  transition. This non-perturbative form is taken to be scale independent, i.e. all scaling effects are assumed to be described by the Altarelli-Parisi evolution of the perturbative part  $D_k^c(\mu)$ . A similar approach had already been introduced in<sup>9</sup>.

Results for these fits are shown in fig. 5. The value for  $\epsilon$  has been consistently found to be of order 0.02 rather than the customary 0.06 one, resulting instead from fits with leading order evolution. Recalling the previous discussion, this comes to no surprise: next-to-leading order evolution softens more the heavy quark spectrum, and a harder non-perturbative fragmentation function is therefore needed to provide a satisfactory description of the data (see<sup>12</sup> for a full discussion).

Similar fits to  $e^+e^-$  data have also been performed by Binnewies, Kniehl and Kramer<sup>13</sup> (BKK). These authors do instead find, again with NLO evolution, a value for  $\epsilon$  still close to the usual 0.06. This discrepancy, beyond irrelevant nomenclature differences, can be traced back to a discrepancy in the implementation of the factorization scheme. The scheme used in<sup>12</sup>, as originally set up in<sup>9</sup>, is the customary  $\overline{\text{MS}}$  one. Considering for instance the dominant non-singlet component only for simplicity, the  $e^+e^- \rightarrow QX$  momentum distribution  $d\sigma/dx$  can be schematically written as the convolution (= product in Mellin moments space) of a short distance coefficient function, an Altarelli-Parisi evolution kernel  $E(\mu, \mu_0)$ , a perturbative initial state condition for the heavy quark perturbative fragmentation function (PFF) and a fixed non-perturbative FF,

$$d\sigma(\sqrt{S}, m) = \left(1 + \alpha_s(\mu)c(\sqrt{S}, \mu)\right) E(\mu, \mu_0) \left(1 + \alpha_s(\mu_0)d(\mu_0, m)\right) D_{np}, \quad (9)$$



where the perturbative expansions of the coefficient function and the PFF have been explicitly shown. The factorization scale  $\mu$  is taken of the order of the (large) collision energy  $\sqrt{S}$ , and the initial scale  $\mu_0$  is taken of the order of the quark mass  $m$ .

BKK on the other hand, employing a scheme introduced by Kniehl, Kramer and Spira<sup>14</sup> (KKS), write  $d\sigma(\sqrt{S}, m)$  as

$$d\sigma(\sqrt{S}, m) = \left(1 + \alpha_s(\mu)c(\sqrt{S}, \mu) + \alpha_s(\mu)d(\mu_0, m)\right)E(\mu, \mu_0)D_{np}. \quad (10)$$

These two expressions can be seen to differ by  $O(\alpha_s^2)$  terms. However, one of these terms is given by

$$\alpha_s(\mu) - \alpha_s(\mu_0) = -b_0\alpha_s^2 \log \frac{\mu^2}{\mu_0^2} \quad (11)$$

and is, therefore, one of the next-to-leading logarithms (NLL)  $\alpha_s^k \log^{k-1}(\sqrt{S}/m)$  we are resumming. Hence the two calculations differ by a NLL term and cannot possibly *both* implement correctly a resummation at the NLL level.

To better understand the discrepancy, the BKKS scheme can for instance be rewritten in the form (9), with an initial state condition for the PFF containing the large scale  $\mu$  as the argument for  $\alpha_s$  rather than the small one  $\mu_0$ . This choice of a large scale is however in contradiction with the factorization theorem hypotheses, which only allow for small scales in initial conditions, to avoid the appearance of unresummed large logs. Choosing the large  $\mu$  leads at a practical level to the difference being reabsorbed into a different value for the  $\epsilon$  parameter, which happens quite accidentally to be 0.06 rather than 0.02. One can show that, replacing in the BKKS formula (10) the  $\alpha_s(\mu)d(\mu_0, m)$  term with  $\alpha_s(\mu_0)d(\mu_0, m)$  (or alternatively appropriately modifying the NLO splitting vertices in the evolution kernel),  $\epsilon = 0.02$  is once again found from the NLL fits within this scheme too.

On the phenomenological side, and making use of the universality argument, one can now argue that the use of a “harder” Peterson form with  $\epsilon = 0.02$  is probably more suited when combined with a NLO perturbative calculation like the FMNR one which, albeit only at fixed order, contains NLL gluon radiation. Decreasing  $\epsilon$  means increasing the cross section at large  $p_T$ , being the  $p_T$  distribution steeply falling with increasing transverse momentum. This could help reconciling the HERA experimental data<sup>15</sup> with the perturbative NLO calculation, which was shown to underestimate them a little when convoluted with a Peterson with  $\epsilon = 0.06$ : fig. 6 shows, on the left, how the cross section for  $D^*$  photoproduction at HERA increases with decreasing  $\epsilon$  and, on the right, a comparison of the H1 data with the fixed order prediction by FMNR ( $\epsilon = 0.06$ ) and the fragmentation functions one with  $\epsilon = 0.02$ . One should notice that the  $p_T$  values involved are still pretty small: this means that the fixed order calculation is still reliable and the accuracy of the resummed one has to be assessed first by comparing with the former. In this case they are found to be in good agreement, the difference in the plot being mainly given by the different  $\epsilon$  values.

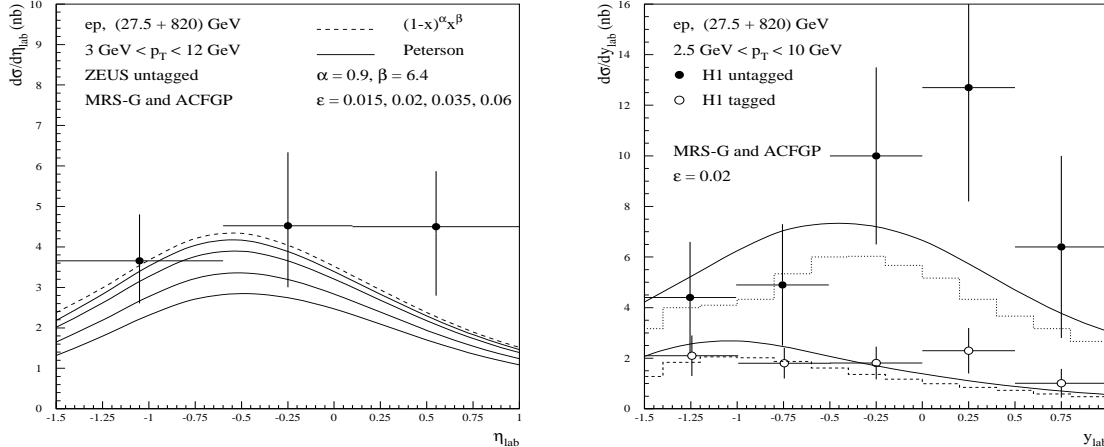


Fig. 6. Effect on the  $D^*$  photoproduction cross section of a decreasing value for  $\epsilon$  (left), and comparison of H1 data with the fixed order calculation by FMNR (histograms,  $\epsilon = 0.06$ ) and the fragmentation function approach (smooth lines,  $\epsilon = 0.02$ ).

Last but not least, it is worth mentioning how, going from LO to NLO analyses, a similar hardening of the non-perturbative fragmentation function is also expected for the  $b$  quark. The corresponding increase of the hadroproduction bottom  $p_T$  distributions<sup>16</sup> would be welcome in the light of the Tevatron data presently overshooting the theoretical predictions by at least 30%.

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