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$B \rightarrow X_s \gamma$  in the Standard Model <sup>a</sup>

CHRISTOPH GREUB

*Deutsches Elektronen-Synchrotron, DESY,  
Notkestr. 85, 22607 Hamburg, Germany*

TOBIAS HURTH <sup>b</sup>

*Stanford Linear Accelerator Center,  
Stanford University, Stanford, California 94309, USA  
and*

*Institute for Theoretical Physics, SUNY at Stony Brook,  
Stony Brook, New York 11794-3840, USA*

We give a summary about the various contributions which have to be calculated in order to obtain the next-to-leading logarithmic result for the branching ratio  $BR(B \rightarrow X_s \gamma)$ . Combining all these ingredients, which were obtained by different groups, a complete next-to-leading-logarithmic prediction of the inclusive decay rate was recently presented in the literature. The theoretical uncertainty in the partonic decay rate is now at the 10% level, i.e., less than half of the error in the previous leading-logarithmic result. We also mention the impact of non-perturbative corrections which scale like  $1/m_b^2$  and discuss in some more detail the recently discovered corrections which scale like  $1/m_c^2$ . It turns out that the  $1/m_b^2$ - and the  $1/m_c^2$  terms lead to corrections to the branching ratio  $BR(B \rightarrow X_s \gamma)$  well below the 10% level.

## 1 Introduction

The  $B \rightarrow X_s \gamma$  decay has found increasing attention over the last ten years. It provides an alternative approach in the search for physics beyond the standard model (SM). This decay, like other rare  $B$  meson decays, does not arise at the tree-level in the SM but is induced by one-loop W-exchange diagrams, so

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nonstandard contributions (charged scalar exchanges, SUSY one-loop diagrams etc.) are not suppressed by an extra factor  $\alpha/4\pi$  relative to the standard model amplitude. This high sensitivity for nonstandard contributions implies the possibility for an indirect observation of new physics, a strategy complementary to the direct production of new particles. The  $B \rightarrow X_s \gamma$  decay plays already a very important role in restricting the parameter space of extensions of the SM like the minimal supersymmetric standard model (MSSM)<sup>1,2</sup>. However, even within the SM, the  $B \rightarrow X_s \gamma$  decay is important for constraining the Cabibbo-Kobayashi-Maskawa matrix elements involving the top-quark, in particular  $|V_{ts}|$ . For both reasons, precise experimental and theoretical work on this decay mode is required.

On the theoretical side, the accuracy in the dominating perturbative contribution was recently improved to next-to-leading precision<sup>3,4,5,6,7,8</sup>. The renormalization scale dependence of the previous leading-log result at the  $\pm 25\%$ -level was substantially reduced to  $\pm 6\%$  and the central value was shifted out-side the  $1\sigma$  bound of the CLEO measurement. Furthermore, the analysis of nonperturbative contributions to the  $B \rightarrow X_s \gamma$  decay mode was also recently improved: The inclusive  $B \rightarrow X_s \gamma$  mode is theoretically much cleaner than the corresponding exclusive channels because no specific model is needed to describe the final hadronic state. According to Heavy Quark Effective Theory the class of non-perturbative effects which scales like  $1/m_b^2$  is expected to be well below  $10\%$ <sup>9</sup>. This numerical statement holds also for the recently discovered non-perturbative contributions<sup>10,11,12,13,14</sup> which scale like  $1/m_c^2$ . Thus the inclusive  $B \rightarrow X_s \gamma$  mode is well approximated by the partonic decay rate  $\Gamma(b \rightarrow X_s \gamma)$  which can be analyzed in renormalization group improved perturbation theory.

Before reporting on these theoretical improvements in detail, we summarize the experimental status: The observation of the exclusive  $B \rightarrow K^* \gamma$  mode by CLEO<sup>15</sup> in 1993 was the first evidence for a penguin decay ever. An updated value<sup>16</sup> for the branching ratio is  $BR(B \rightarrow K^* \gamma) = (4.2 \pm 0.8 \pm 0.6) \times 10^{-5}$ . In 1994 the CLEO collaboration measured the inclusive  $B \rightarrow X_s \gamma$  branching ratio to be  $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$  where the first error is statistical and the second is systematic<sup>17</sup>: There are two separate CLEO analyses. The first one measures the inclusive photon spectrum from B-decay near the end point. The second technique constructs the inclusive rate by summing up the possible exclusive final states. The branching ratio stated above is the average of the two measurements, taking into account the correlation between the two techniques.

There is also data from the LEP experiments: While DELPHI<sup>18</sup> in 1996 and L3<sup>19</sup> in 1993 have published the upper bounds  $BR(b \rightarrow s \gamma) < 5.4 \times 10^{-4}$

and  $BR(b \rightarrow s\gamma) < 1.2 \times 10^{-3}$ , respectively, the preliminary measurement  $BR(b \rightarrow s\gamma) = (3.38 \pm 0.74 \pm 0.85) \times 10^{-4}$  by the ALEPH group was reported in the talk by F. Parodi<sup>20</sup> at the 1997 Moriond meeting. A similar number was also quoted by T. Skwarnicki<sup>21</sup> in the heavy flavor meeting held in Santa Barbara in July 1997.

More precise measurements are expected from the upgraded CLEO detector, as well as from the B-factories presently under construction at SLAC and KEK. In view of the expected high luminosity of the B-factories, experimental accuracy of below 10% appears to be in reach.

The rest of the paper is organized as follows: Section 2 is devoted to the partonic (=perturbative) contribution to  $BR(B \rightarrow X_s\gamma)$ . We explain in some detail the various calculational steps leading to the next-to-leading logarithmic result. In section 3 we briefly discuss the impact of the recently discovered non-perturbative corrections which scale like  $1/m_c^2$ .

## 2 Next-to-leading logarithmic corrections for $B \rightarrow X_s\gamma$

It is well-known that the QCD corrections enhance the partonic decay rate  $\Gamma(b \rightarrow s\gamma)$  by more than a factor of two. These QCD effects can be attributed to logarithms of the form  $\alpha_s^n(m_b) \log^m(m_b/M)$ , where  $M = m_t$  or  $M = m_W$  and  $m \leq n$  (with  $n = 0, 1, 2, \dots$ ). In order to get a reasonable result at all, one has to sum at least the leading-log (LL) series ( $m = n$ ). Working to next-to-leading-log (NLL) precision means that one is also resumming all the terms of the form  $\alpha_s(m_b) (\alpha_s^n(m_b) \ln^n(m_b/M))$ .

An appropriate framework to achieve the necessary resummations is an effective low-energy theory, obtained by integrating out the heavy particles which in the SM are the top quark and the  $W$ -boson. The effective Hamiltonian relevant for  $b \rightarrow s\gamma$  and  $b \rightarrow sg$  in the SM and many of its extensions reads

$$H_{eff}(b \rightarrow s\gamma) = -\frac{4G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^8 C_i(\mu) O_i(\mu) \quad , \quad (1)$$

where  $O_i(\mu)$  are the relevant operators,  $C_i(\mu)$  are the corresponding Wilson coefficients, which contain the complete top- and  $W$ - mass dependence, and  $\lambda_t = V_{tb}V_{ts}^*$  with  $V_{ij}$  being the CKM matrix elements<sup>c</sup>. Neglecting operators with dimension  $> 6$  which are suppressed by higher powers of  $1/m_{W/t}$  and using the equations of motion for the operators, one arrives at the following basis of dimension 6 operators<sup>22</sup>

$$O_1 = (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}) (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \quad ,$$

<sup>c</sup>The CKM dependence globally factorizes, because we work in the approximation  $\lambda_u = 0$ .

$$\begin{aligned}
O_2 &= (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\beta} \gamma_\mu c_{L\beta}) , \\
O_7 &= (\epsilon/16\pi^2) \bar{s}_\alpha \sigma^{\mu\nu} (m_b(\mu)R + m_s(\mu)L) b_\alpha F_{\mu\nu} , \\
O_8 &= (g_s/16\pi^2) \bar{s}_\alpha \sigma^{\mu\nu} (m_b(\mu)R + m_s(\mu)L) (\lambda_{\alpha\beta}^A/2) b_\beta G_{\mu\nu}^A . \quad (2)
\end{aligned}$$

Because the Wilson coefficients of the penguin induced four-fermion operators  $O_3, \dots, O_6$  are very small, we do not list them here. In this framework the next-to-leading logarithmic terms  $\alpha_s(m_b)$  ( $\alpha_s^n(m_b) \log^n(m_b/m_{W/t})$ ) in the  $b \rightarrow s\gamma$  amplitude have two sources:

- 1 •** The NLL Wilson coefficients  $C_i(\mu)$  at the scale  $\mu \approx m_b$  contain leading and next-to-leading logarithmic terms in resummed form.
- 2 •** The  $O(\alpha_s)$  corrections to the matrix elements of the operators  $O_i$  yield next-to-leading order terms when multiplied by the (leading logarithmic part of the) Wilson coefficients.

We stress that only the sum of these two sources is *independent of the renormalization scheme*. Let us discuss in some more detail the contributions mentioned in **1** and **2**:

**ad 1 •** From the  $\mu$ -independence of the effective Hamiltonian, one can derive a renormalization group equation (RGE) for the Wilson coefficients  $C_i(\mu)$ :

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} C_j(\mu) \quad , \quad (3)$$

where the  $(8 \times 8)$  matrix  $\gamma$  is the anomalous dimension matrix of the operators  $O_i$ . To solve this first order differential equation one explicitly needs initial conditions  $C_i(\mu_0)$  at some scale  $\mu_0$  as well as the anomalous dimension matrix  $\gamma_{ij}$ .

**1a:** The initial conditions are obtained by matching the effective theory to the full standard model theory at the scale  $\mu_0 = \mu_W$ , where  $\mu_W$  denotes a scale of order  $m_W$  or  $m_t$ . At this scale, the matrix elements of the operators in the effective theory lead to the same logarithms as the full theory calculation. Consequently, the Wilson coefficients  $C_i(\mu_W)$  only pick up small QCD corrections, which can be calculated in fixed-order perturbation theory. In the LL (NLL) program, the matching has to be worked out to order  $\alpha_s^0$  ( $\alpha_s^1$ ) precision. **1b:** Solving the RGE (3) and using the  $C_i(\mu_W)$  of Step **1a** as initial conditions, one performs the evolution of these Wilson coefficients from  $\mu = \mu_W$  down to  $\mu = \mu_b$ , where  $\mu_b$  is of the order of  $m_b$ . As the matrix elements of the operators evaluated at the low scale  $\mu_b$  are free of large logarithms, the latter are contained in resummed form in the Wilson coefficients. For a LL (NLL) calculation, this RGE step has to be performed using the anomalous dimension matrix  $\gamma_{ij}$  up to order  $\alpha_s^1$  ( $\alpha_s^2$ ).

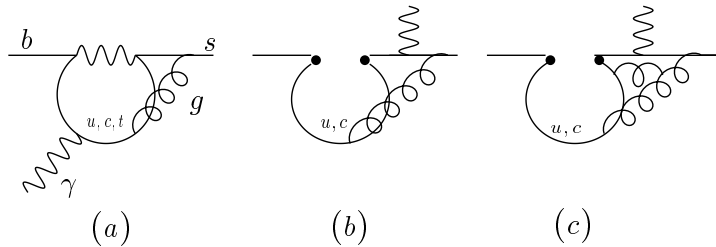


Figure 1: a) Typical diagram (full theory) contributing in the NLL matching calculation. b) Typical diagram contributing to the matrix element of the operator  $O_2$ . c) Typical contribution to the  $O(\alpha_s^2)$  anomalous dimension matrix.

**ad 2 •** The matrix elements of the operators  $\langle s\gamma|O_i(\mu)|b\rangle$  at the scale  $\mu = \mu_b$  have to be calculated to order  $\alpha_s^0$  ( $\alpha_s^1$ ) in the LL (NLL) calculation.

Until recently, only the leading logarithmic (LL) perturbative QCD were known systematically<sup>23</sup>. The error in this approximation was dominated by a large renormalization scale dependence at the  $\pm 25\%$  level. The measurement of the CLEO collaboration<sup>17</sup> overlaps with the estimates based on leading logarithmic calculations (or with some next-to-leading effects partially included) and the experimental and theoretical errors are comparable<sup>3,24,25,26</sup>. However, in view of the expected increase in the experimental precision in the near future, it became clear that a systematic inclusion of the NLL corrections was necessary. This ambitious NLL enterprise was recently completed. All three steps (**1a,1b,2**) involve rather difficult calculations. The most difficult part in Step **1a** is the two-loop (or order  $\alpha_s$ ) matching of the dipole operators  $O_7$  and  $O_8$ . It involves two-loop diagrams both in the full and in the effective theory (see Fig. 1a). This matching calculation was done by Adel and Yao<sup>4</sup> some time ago. As this is a crucial step in the NLL program, Greub and Hurth<sup>8</sup> recently confirmed their findings in a detailed re-calculation, using a somewhat different method. In order to match dimension 6 operators  $O_7$  and  $O_8$ , it is sufficient to extract the terms of order  $m_b \frac{m_b^2}{M^2}$  ( $M = m_W, m_t$ ) from the standard model matrix elements for  $b \rightarrow s\gamma$  and  $b \rightarrow sg$ . Terms

suppressed by additional powers of  $m_b/M$  correspond to higher dimensional operators in the effective theory. In<sup>8</sup> the finite parts of the two-loop diagrams in the SM were calculated by means of the well-known Heavy Mass Expansion (HME) which naturally leads to a systematic expansion of Feynman diagrams in inverse powers of  $M$ . We mention here that the evolution of the Wilson coefficients between  $\mu = m_t$  and  $\mu = m_w$  to LL precision implied an additional contribution of +10% to the leading-log prediction for the decay rate<sup>27</sup>. Most of this contribution is automatically included in the NLL matching at the  $m_W$ -scale<sup>4,8</sup>, because the first term of the LL-sum of<sup>27</sup> is reproduced and higher order terms  $(\alpha_s \log(\frac{m_t}{m_W}))^n (n > 1)$  are rather small. In addition, the NLL matching result includes the first term of the NLL-sum.

Step 2 basically consists of Bremsstrahlung corrections and virtual corrections. While the Bremsstrahlung corrections (together with some virtual corrections needed to cancel infrared singularities) were worked out some time ago by Ali and Greub<sup>3</sup> and have been confirmed and extended by Pott<sup>5</sup>, a complete analysis of the virtual corrections (up to the contributions of the four-fermion operators with very small coefficients) was presented by Greub, Hurth and Wyler<sup>6</sup>. This calculation also involves two-loop diagrams where the full charm quark mass dependence has to be taken into account. A typical diagram is shown in Fig. 1b. By using Mellin-Barnes techniques in the Feynman parameter integrals, the result of these two-loop diagrams was obtained in the form

$$c_0 + \sum_{n=0,1,2,\dots; m=0,1,2,3} c_{nm} \left( \frac{m_c^2}{m_b^2} \right)^n \log^m \frac{m_c^2}{m_b^2} , \quad (4)$$

where the quantities  $c_0$  and  $c_{nm}$  are independent of  $m_c$ . Note, that a finite result is obtained in the limit  $m_c \rightarrow 0$ , as there is no naked logarithm of  $m_c^2/m_b^2$ . This observation is of some importance in the  $b \rightarrow d\gamma$  process, where the  $u$ -quark propagation in the loop is not CKM suppressed. It is, however, even more important that the inclusion of the  $O(\alpha_s)$  matrix elements leads to a drastic reduction of the renormalization scale uncertainty from about  $\pm 25\%$  to about  $\pm 6\%$ . Analytically, the reason is, that the term  $\alpha_s \log(\mu/m_b)$  which dominates the  $\mu$ -dependence of the LL result, is cancelled by a corresponding term appearing in the  $O(\alpha_s)$  matrix element. Finally, the anomalous dimension matrix (at  $O(\alpha_s^2)$ ), Step 1b, has been worked out by Chetyrkin, Misiak and Münz<sup>7</sup>. The calculation of the elements  $\gamma_{i7}$  and  $\gamma_{i8}$  ( $i = 1, \dots, 6$ ) in the  $O(\alpha_s^2)$  anomalous dimension matrix involves a huge number of three loop-diagrams from which the pole parts (in the  $d - 4$  expansion) have to be extracted. For a typical diagram see Fig. 1c. The extraction of the pole parts were simplified by a clever decomposition of the scalar propagator. Moreover, the number

of necessary evanescent operators were reduced by a new choice of a basis of dimension 6 operators. Using the matching result (Step **1a**), these authors obtained the next-to-leading correction to the Wilson coefficient  $C_7(\mu_b)$  which is the only relevant one for the  $b \rightarrow X_s \gamma$  decay rate. Numerically, the LL and the NLL values for  $C_7(\mu_b)$  are rather similar; the NLL corrections to the Wilson coefficient  $C_7(\mu_b)$  lead to a change of the  $b \rightarrow X_s \gamma$  decay rate which does not exceed  $\pm 6\%$ <sup>7</sup>. The new contributions can be split into a part which is due to the order  $\alpha_s$  corrections to the matching (Step 1a) and into a part stemming from the improved anomalous dimension matrix (Step 1b). While individually these two parts are not so small (in the NDR scheme, which was used in<sup>7</sup>), they almost cancel when combined as illustrated in<sup>7</sup>. This shows that all the three different pieces, **1a,1b,2**, are numerically equally important.

Combining the NLL calculations of all the three steps (**1a+b,2**), the first complete theoretical prediction to NLL precision for the  $b \rightarrow X_s + \gamma$  branching ratio was presented in<sup>7</sup>:  $BR(B \rightarrow X_s \gamma) = (3.28 \pm 0.33) \times 10^{-4}$ . The error is due to the  $\pm 6\%$  renormalization scale uncertainty and due to the  $\pm 8\%$  combined uncertainty in the input parameters.

### 3 $1/m_b^2$ and $1/m_c^2$ corrections

Neglecting perturbative QCD corrections and assuming that  $B \rightarrow X_s \gamma$  is due to the operator  $O_7$  only, the calculation of the differential decay rate basically amounts to work out the imaginary part of the forward scattering amplitude  $T(q)$

$$T(q) = i \int d^4x \langle B | T O_7^\dagger(x) O_7(0) | B \rangle \exp(iqx) \quad . \quad (5)$$

Using the operator product expansion for  $T O_7^\dagger(x) O_7(0)$  and Heavy Quark Effective Theory methods, the decay width  $\Gamma(B \rightarrow X_s \gamma)$  reads<sup>9</sup> (modulo higher terms in the  $1/m_b$  expansion)

$$\begin{aligned} \Gamma_{B \rightarrow X_s \gamma}^{(O_7, O_7)} &= \frac{\alpha G_F^2 m_b^5}{32 \pi^4} |V_{tb} V_{ts}|^2 C_7^2(m_b) \left( 1 + \frac{\delta_{rad}^{NP}}{m_b^2} \right) \quad , \\ \delta_{rad}^{NP} &= \frac{1}{2} \lambda_1 - \frac{9}{2} \lambda_2 \quad , \end{aligned} \quad (6)$$

where  $\lambda_1$  and  $\lambda_2$  are the kinetic energy- and the chromomagnetic energy parameters. Using  $\lambda_1 = -0.5 GeV^2$  and  $\lambda_2 = 0.12 GeV^2$ , one gets  $\delta_{rad}^{NP} \simeq -4\%$ . As also the semileptonic decay width gets  $1/m_b^2$  corrections which are negative (see e.g.<sup>28</sup>), these non-perturbative corrections tend to cancel in the branching ratio  $BR(B \rightarrow X_s \gamma)$  and only about 1% remains. This contribution was

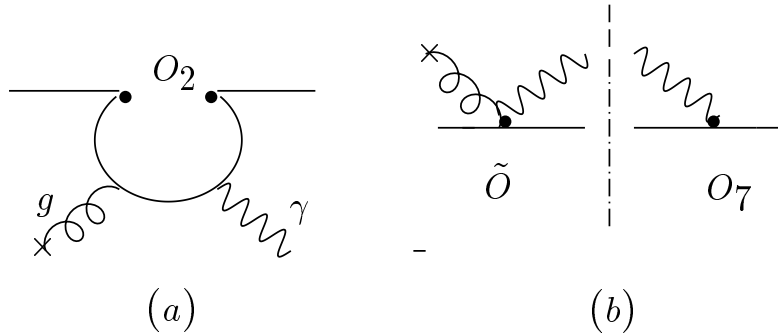


Figure 2: a) Feynman diagram from which the operator  $\tilde{O}$  arises. b) Relevant cut-diagram for the  $(O_2, O_7)$ -interference.

already included in the theoretical NLL prediction presented in section 2 of this article.

Recently, Voloshin<sup>10</sup> considered the non-perturbative effects when including also the operator  $O_2$ . This effect is generated from the diagram in Fig. 2a (and from the one not shown where the gluon and the photon are interchanged);  $g$  is a soft gluon interacting with the charm quarks in the loop. Up to a characteristic Lorentz structure, this loop is given by the integral

$$\int_0^1 dx \int_0^{1-x} dy \frac{xy}{m_c^2 - k_g^2 x(1-x) - 2xyk_g k_\gamma} . \quad (7)$$

As the gluon is soft, i.e.,  $k_g^2, k_g k_\gamma \approx \Lambda^{QCD} m_b/2 \ll m_c^2$ , the integral can be expanded in  $k_g$ . The (formally) leading operator, denoted by  $\tilde{O}$ , is

$$\tilde{O} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* C_2 \frac{e Q_c}{48\pi^2 m_c^2} \bar{s} \gamma_\mu (1 - \gamma_5) g_s G_{\nu\lambda} b \epsilon^{\mu\nu\rho\sigma} \partial^\lambda F_{\rho\sigma} . \quad (8)$$

Working out then the cut diagram shown in Fig. 2b, one obtains the non-perturbative contribution  $\Gamma_{B \rightarrow \bar{X}_s \gamma}^{(\tilde{O}, O_7)}$  to the decay width, which is due to the  $(O_2, O_7)$  interference. Normalizing this contribution by the LL partonic width,



one obtains

$$\frac{\Gamma_{B \rightarrow X_s \gamma}^{(\tilde{O}, O_7)}}{\Gamma_{b \rightarrow s \gamma}^{LL}} = -\frac{1}{9} \frac{C_2}{C_7} \frac{\lambda_2}{m_c^2} \simeq +0.03 \quad . \quad (9)$$

Including this correction with the sign found in <sup>14</sup>, the NLL prediction for the branching ratio becomes  $BR(B \rightarrow X_s \gamma) = (3.38 \pm 0.33) \times 10^{-4}$ .

As the expansion parameter is  $m_b \Lambda_{QCD} / m_c^2 \approx 0.6$  (rather than  $\Lambda_{QCD}^2 / m_c^2$ ), it is not a priori clear whether formally higher order terms in the  $m_c$  expansion are numerically suppressed. More detailed investigations<sup>12,13,14</sup> show that higher order terms are indeed suppressed, because the corresponding expansion coefficients are small.

We mention that the analogous  $1/m_c^2$  effect has been found independently in the exclusive mode  $B \rightarrow K^* \gamma$  in ref. <sup>11</sup>. Numerically, the effect there is also at the few percent level.

## 4 Summary

Collecting all NLL contributions and the small nonperturbative correction which scales with  $m_b^2$ , the final analysis done by Chetyrkin, Misiak and Münz yields  $BR(B \rightarrow X_s \gamma) = (3.38 \pm 0.33) \times 10^{-4}$  when also the +3% shift due to the non-perturbative effects from the  $1/m_c^2$  corrections is included. The theoretical error in the NLL prediction is reduced by a factor of 2 when compared with the LL result. This theoretical value for the branching ratio is in agreement with the CLEO measurement (at the  $2\sigma$ -level) and also with the recent (preliminary) measurement by ALEPH. Clearly, the inclusive  $B \rightarrow X_s + \gamma$  mode will provide an interesting test of the SM and its extensions as soon as more precise experimental data become available.

**Note added:** When finishing this article, we received the new work by Buras, Kwiatkowski and Pott<sup>29</sup>. While these authors fully confirm the matching conditions by<sup>4,8</sup>, their analysis is slightly different, leading to the branching ratio  $BR(B \rightarrow X_s \gamma) = (3.48 \pm 0.31) \times 10^{-4}$ . The shift in the central value is due to systematically discarding next-next-leading order terms, while in the earlier analysis<sup>6,7</sup> some terms of this order were included. Also their estimate for the remaining renormalization scale dependence is somewhat different: The  $\mu$ -uncertainties in the decay width for the radiative decay and the semileptonic decay were treated independently and added in quadrature. In the old analysis<sup>6,7</sup> the scale  $\mu$  was varied simultaneously in both decays. As the semileptonic decay width is an increasing function of  $\mu$  while the radiative decay width is decreasing, a larger  $\mu$ -uncertainty was obtained which *as a more conservative*

estimate we finally prefer. The results are fully compatible after all.

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