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QCD – Instantons in $e^{\pm}P$ Scattering[1]

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Abstract. We review recent theoretical results from our systematic study of QCD-instanton contributions to $e^{\pm}P$ processes, involving a hard momentum scale $\mathcal{Q}$. The main issues are: the absence of IR divergencies due to a dynamical suppression of instantons with large size $\rho > 1/\mathcal{Q}$, the reliable calculability of instanton-induced amplitudes and our inclusive framework to systematically calculate properties of the I-induced multi-parton final state.

INTRODUCTION

Instantons [1] are well known to represent topology changing tunnelling transitions in non-abelian gauge theories. These transitions induce processes which are *forbidden* in perturbation theory, but have to exist in general [2] due to Adler-Bell-Jackiw anomalies. Correspondingly, these processes imply a violation of certain fermionic quantum numbers, notably, $B+L$ in the electroweak gauge theory and chirality (Q_5) in (massless) QCD.

An experimental discovery of such a novel, non-perturbative manifestation of non-abelian gauge theories would clearly be of basic significance.

The interest in instanton (I)-induced processes during recent years has been revived by the observation [3] that the strong exponential suppression, $\propto \exp(-4\pi/\alpha)$, of the corresponding tunnelling rates *at low energies* may be overcome at *high energies*, mainly due to multi-gauge boson emission in addition to the minimally required fermionic final state. Since $\alpha_s \gg \alpha_W$, QCD-instantons are certainly much less suppressed than electro-weak ones. A pioneering and encouraging theoretical estimate of the size of the QCD-instanton induced contribution to the gluon structure functions in deep-inelastic scattering (DIS) was recently presented in Ref. [4].

A systematic phenomenological and theoretical study is under way [5–11], which clearly indicates that deep-inelastic $e^{\pm}P$ scattering at HERA now offers

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a unique window to experimentally detect QCD-instanton induced processes through their characteristic *final-state signature*. While our phenomenological approach and the ongoing experimental searches for QCD instantons were reviewed in WG III [12,13], we shall focus here on our recent theoretical results.

They clearly indicate that $e^{\pm}P$ scattering, involving a hard momentum scale $\mathcal{Q}$, plays a distinguished rôle for studying manifestations of QCD-instantons. This mostly refers to the DIS regime, but possibly also to hard photoproduction, where $\mathcal{Q}$ denotes the large transverse momentum of a jet.

In this talk, we shall concentrate on the following important issues:

We set up the relevant I-induced amplitudes for DIS at the parton level in leading semi-classical approximation and outline why they are well-defined and calculable for small $\alpha_s(\mathcal{Q})$. We concentrate on the crucial feature that the generic IR divergencies from integrating over the I-size ρ are absent in DIS, since the hard momentum scale $\mathcal{Q}$ provides a dynamical cutoff, $\rho \lesssim \mathcal{O}(1/\mathcal{Q})$. As an example, the cross section for the simplest I-induced process is explicitly evaluated and compared to the corresponding contribution from perturbative QCD. Finally, we briefly sketch our inclusive framework to systematically calculate properties of the I-induced multi-parton final state. It accounts for the exponentiation of produced gluons including final-state tree-graph corrections.

2. INSTANTON-INDUCED PROCESSES IN LEADING SEMI-CLASSICAL APPROXIMATION

The main I-induced contribution to deep-inelastic $e^{\pm}P$ scattering comes from the processes,

$$\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_{\mathrm{g}}\, \mathrm{g},$$

which correspond to $\triangle Q_5 = 2\, n_f$ and thus vanish to any order of conventional perturbation theory in (massless) QCD.

The evaluation of the corresponding amplitudes involves the following steps:

We start with the basic building blocks in Euclidean configuration space (see e.g. Ref. [9]), the classical instanton gauge field $A_\mu^{(I)}(x;\rho,U)$, the quark zero modes $\kappa^{(I)}(x;\rho,U)$, $\overline{\phi}^{(I)}(x;\rho,U)$ and the (non-zero mode) quark propagators in the I-background $S^{(I)}(x,y;\rho,U), \overline{S}^{(I)}(x,y;\rho,U)$. The classical fields and quark propagators depend on collective coordinates, the I-size ρ and the colour orientation matrices U.

"Instanton-perturbation theory" is generated by expanding the (Euclidean) path integral for the relevant Green's functions about the classical instanton solution for small α_s. Next, Fourier-transforms (FT) to momentum space with respect to external lines are performed and the external legs are LSZ amputated. Finally, the result is analytically continued to Minkowski space.

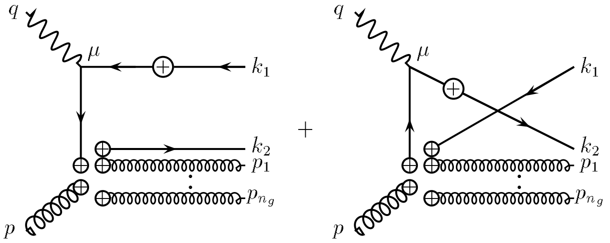


FIGURE 1. I-induced contribution to DIS for $n_f = 1$ in leading semi-classical approximation.

After performing the FT's with respect to the external lines, the leading-order amplitude in Euclidean space takes the following form [9] for the simplified case of one flavour ($n_f = 1$) (see Fig. 1),

$$\begin{aligned}\mathcal{T}_\mu^{(I)\,a\,a_1\ldots a_{n_g}} &= -\mathrm{i}\,e_q\,d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]\int dU \int_0^\infty \frac{d\rho}{\rho^5}\,(\rho\,\mu_r)^{\beta_0}\;\times\\ &\lim_{p^2\to 0} p^2\,\mathrm{tr}\left[\lambda^a\,\epsilon_g(p)\cdot A^{(I)}(p;\rho,U)\right]\prod_{i=1}^{n_g}\lim_{p_i^2\to 0} p_i^2\,\mathrm{tr}\left[\lambda^{a_i}\,\epsilon_g^*(p_i)\cdot A^{(I)}(-p_i;\rho,U)\right]\;\times\\ &\chi_R^\dagger(k_2)\left[\lim_{k_2^2\to 0}(\mathrm{i}k_2)\,\kappa^{(I)}(-k_2;\rho,U)\,\lim_{k_1^2\to 0}\mathcal{V}_\mu(q,-k_1;\rho,U)\right. \qquad (1)\\ &\left.+\lim_{k_2^2\to 0}\mathcal{V}_\mu^c(q,-k_2;\rho,U)\,\lim_{k_1^2\to 0}\overline{\phi}^{(I)}(-k_1;\rho,U)\,(-\mathrm{i}\,\overline{k}_1)\right]\chi_L(k_1)\,,\end{aligned}$$

The instanton-density [2] with renormalization scale μ_r and 1st coefficient $\beta_0 = 11 - 2/3\,n_f$ of the perturbative QCD beta-function,

$$d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right](\rho\,\mu_r)^{\beta_0}\;;\; d \text{ being a known constant,} \qquad (2)$$

is 1-loop renormalization group (RG) invariant.

First of all, note the strong IR divergence (large ρ), if the ρ-integral in Eq. (1) were performed with just the I-density. Hence, convergence may only come from further ρ dependence inherent in the matrix element. In particular, any possible cut-off for large ρ in terms of the inverse hard scale $1/\mathcal{Q}$, must be hidden in the FT'ed photon-fermion "vertices", $\mathcal{V}_\mu(q,-k_1;\rho,U)$ and $\mathcal{V}_\mu^c(q,-k_2;\rho,U)$ (c.f. Figs. 1, 2).

FT and LSZ-amputation of the instanton gauge field $A_\mu^{(I)}$ and quark zero modes κ and $\overline{\phi}$ is straightforward [3]. They only contribute *positive* powers of ρ. On the other hand, the LSZ-amputation of the current quark in the FT'ed photon-fermion vertices, $\mathcal{V}_\mu(q,-k_1;\rho,U)$ and $\mathcal{V}_\mu^c(q,-k_2;\rho,U)$, is quite non-trivial, since they involve a two-fold FT of the complicated quark propagators

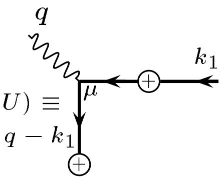
$\mathcal{V}_\mu(q,-k_1;\rho,U) \equiv$  $\equiv \int d^4x\, \mathrm{e}^{-\mathrm{i}q\cdot x}\left[\overline{\phi}^{(I)}(x;\rho,U)\,\overline{\sigma}_\mu\, S^{(I)}(x,-k_1;\rho,U)(-\mathrm{i}\overline{k}_1)\right]$

FIGURE 2. The photon-fermion vertex, $\mathcal{V}_\mu(q,-k_1;\rho,U)$, generating the *dynamical cut-off* $\rho \lesssim \mathcal{O}(1/\mathcal{Q})$.

in the I-background (see Fig. 2). After a long and tedious calculation, we find [9],

$$\lim_{k_1^2\to 0}\mathcal{V}_\mu(q,-k_1;\rho,U) = 2\pi\mathrm{i}\rho^{3/2}\left[\epsilon\sigma_\mu\overline{V}(q,k_1;\rho)\,U^\dagger\right] \tag{3}$$

$$\lim_{k_2^2\to 0}\mathcal{V}^c_\mu(q,-k_2;\rho,U) = 2\pi\mathrm{i}\rho^{3/2}\left[U\,V(q,k_2;\rho)\overline{\sigma}_\mu\epsilon\right], \tag{4}$$

where with the shorthand $q' \equiv q-k$,

$$V(q,k;\rho) = \frac{k}{2q\cdot k}\rho\sqrt{q^2}\,K_1\left(\rho\sqrt{q^2}\right) + \left[\frac{q'}{q'^2} - \frac{k}{2q\cdot k}\right]\rho\sqrt{q'^2}\,K_1\left(\rho\sqrt{q'^2}\right). \tag{5}$$

Obviously, the integration over the instanton size ρ in the amplitudes $\mathcal{T}^{(I)}_\mu$, Eq. (1), is finite due to the exponential decrease of the "form factors" in $V(q,k;\rho)$,

$$\mathcal{Q}\rho K_1(\mathcal{Q}\rho) \overset{\mathcal{Q}\rho\to\infty}{\to} \sqrt{\frac{\pi}{2}}\sqrt{\mathcal{Q}\rho}\,\exp\left[-\mathcal{Q}\rho\right]. \tag{6}$$

The ρ-integral may even be performed analytically, after inserting the various LSZ amputated FT's into Eq. (1). After continuation to the DIS-regime of Minkowski space, the (effective) hard scale,

$$\mathcal{Q} \equiv \min\left\{Q \equiv \sqrt{-q^2}\,,\ \sqrt{-(q-k_1)^2}\,,\ \sqrt{-(q-k_2)^2}\right\} \geq 0\,, \tag{7}$$

then provides a *dynamical IR cutoff for the instanton size,* $\rho \lesssim \mathcal{O}(1/\mathcal{Q})$.

As a highly non-trivial check of our calculations, electromagnetic current conservation, $q^\mu \mathcal{T}_\mu^{(I)\,a\,a_1\ldots a_{n_g}} = 0$, is manifestly satisfied.

Like in perturbative QCD, the leading-order I-induced amplitudes are well-behaved as long as we avoid the collinear singularities, arising when the internal quark virtualities $t \equiv -(q-k_1)^2$ or $u \equiv -(q-k_2)^2 \to 0$ vanish in Eq. (5) (c.f. Fig. 1). Hence high Q^2 processes of moderate multiplicity, where *the current quark* is produced at a *fixed angle* relative to the photon, are reliably calculable in instanton perturbation theory.

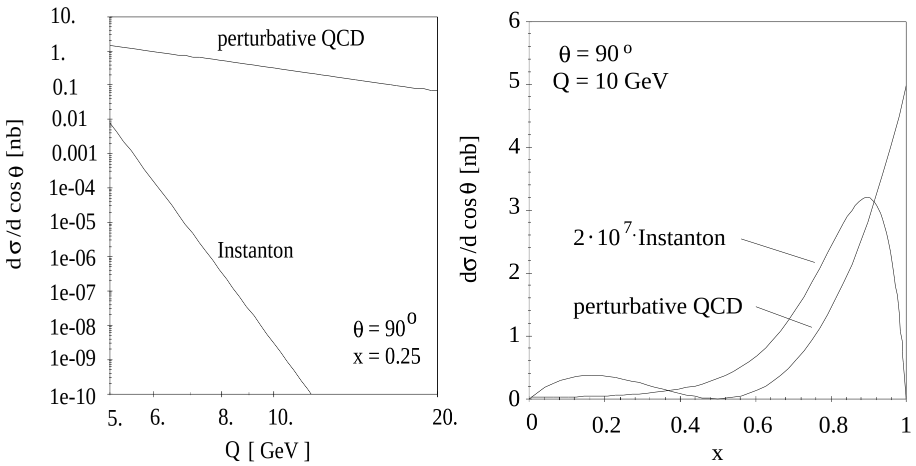


FIGURE 3. I-induced fixed-angle scattering for $\mathcal{Q} \gtrsim 5$ GeV is well under control. ($e_q = 2/3$, $\Lambda = 234$ MeV, $\mu_r = Q$).

Altogether, we may conclude that deep-inelastic scattering is very well suited for studying manifestations of QCD-instantons!

Example: We have worked out explicitly the cross section for the simplest process [9] for $n_f = 1$ without final-state gluons, $\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_R$ ($\triangle Q_5 = 2$). Although being only a *small* fraction of the total I-induced contribution, it may be considered as a calculable *lower bound*. Moreover, it contains all essential features of the dominant multi-gluon process. The residual dependence on the renormalization scale μ_r is very weak if the 2-loop RG-invariant I-density is used [14]. In Figs. 3, we display a comparison with the cross sections for the appropriate chirality-conserving process within leading-order perturbative QCD, $\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_L$ ($\triangle Q_5 = 0$).

From the requirement that the average instanton size $\langle\rho\rangle$ contributing for a given virtuality $\mathcal{Q}$, should be small enough to neglect higher-order corrections of I-perturbation theory, a lower limit on the hard scale $\mathcal{Q}$ can be easily obtained [9],

$$\langle\rho\rangle \lesssim \frac{1}{500\ \mathrm{MeV}} < \frac{1}{\Lambda} \Rightarrow \mathcal{Q} \gtrsim 5\ \mathrm{GeV}. \tag{8}$$

3. INCLUSIVE APPROACH TO THE MULTI-PARTICLE FINAL STATE

A crucial theoretical task is to find the best framework allowing to make contact with experiment (HERA), without upsetting the validity of I-perturbation theory. Experimentally, the best bet [5] is to hunt for I-"footprints" in the multi-particle final state rather than in totally inclusive observables like $F_2(x_{\mathrm{Bj}}, Q^2)$. Unlike the configuration space approach of

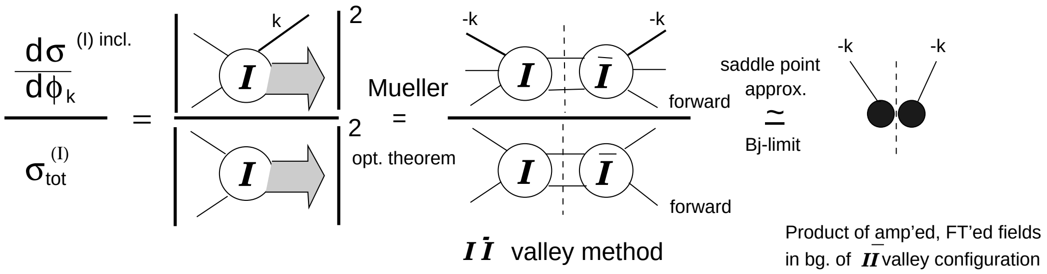


FIGURE 4. Application of the Mueller optical theorem [17], the $I\overline{I}$-valley method [15,16] and saddle-point integration over the collective coordinates to estimate the normalized, I-induced 1-parton *inclusive* q^*g subprocess cross section; $\mathrm{d}\phi_k$ denotes the Lorentz-invariant phase-space element.

Ref. [4], our momentum-space picture allows to keep control over the various I-approximations also at small $x_{\rm Bj}$ through *kinematical cuts* on (reconstructed) *final-state momentum variables*!

A "brute-force" possibility to calculate the desired I-induced cross sections for a *multi-parton final state* consists simply in squaring our I-induced amplitudes from Sect. 2, performing the necessary phase-space integrations and summing over unobserved partons. For the dominating final states with many gluons this procedure becomes increasingly tedious and also inaccurate.

Let us sketch here a second more elegant and presumably also more accurate approach [11], which includes an implicit summation over the *exponentiating* leading-order gluon emission as well as gluonic final-state tree-graph corrections according to the $I\overline{I}$-valley method [15,16]:

First of all, it turns out [5,7,10] that $\mathrm{d}\sigma^{(I)}\,(\gamma^* + g \Rightarrow 2n_f\, q + n_g\, g)$ factorizes into a calculable "splitting" function associated with the $\gamma^* q q^*$-vertex (c. f. Fig. 1) and cross sections for the I-induced *subprocess* $q^*g \to (2n_f - 1)\, q + n_g\, g$, on which we shall concentrate from now on.

Next, we remember that any given final state may be equivalently described in terms of $\sigma_{\rm tot}$ and the set of 1,2,... parton *inclusive* cross sections. The latter, in turn, may be evaluated via the so-called "Mueller optical theorems" [17], expressing $n = 1, 2\ldots$ particle *inclusive* cross sections as appropriate discontinuities of $2 + n \to 2 + n$ forward elastic amplitudes in generalization of the usual optical theorem (c. f. Fig. 4). Although no rigorous proof exists, much of the Regge inspired multi-particle phenomenology of the 70's rested on the validity of these Mueller optical theorems. The I-induced $1, 2\ldots$ parton inclusive cross sections can now be evaluated in complete analogy to existing calculations [16] of $\sigma_{\rm tot}^{(I)}$ by means of the $I\overline{I}$ valley method [15,16] applied to the $2 \to 2$ forward amplitude in the $I\overline{I}$ background. By normalizing the inclusive cross sections to $\sigma_{\rm tot}^{(I)}$, common, poorly known pre-exponential factors largely cancel, such that quite stable and accurate results are obtained (c. f. Fig. 4).

Let us give some examples. From calculating the (normalized) 1-parton inclusive cross section along these lines, we obtain (after phase-space integration) the average gluon and quark multiplicity $\langle n_{q+g}\rangle \propto 1/\alpha_s$, as well as the *isotropy* of I-induced parton production and the transverse energy flow vs. pseudo-rapidity $\frac{d\langle E_T\rangle}{d\eta_I} = \frac{E_I^{\rm tot}}{\langle n\rangle}\frac{1}{\cosh\ \eta_I}$, both in the c.m.s. of the I-subprocess. From the (normalized) 2-parton inclusive cross section, we obtain (after phase-space integration) in the Bjorken limit $\langle n^2\rangle - \langle n\rangle^2 \approx 0$, implying a *Poisson* distribution for the I-induced exclusive n-parton cross sections. Furthermore, we gain information on various momentum correlations of the produced partons.

All along, there are most non-trivial consistency conditions in form of *energy/momentum/charge* sum rules [18], like e. g.

$$\sum_{\rm q,g}\int d\phi_k\ k_\mu\ \frac{1}{\sigma_{\rm tot}^{(I)}}\frac{d\sigma^{(I)\,{\rm incl.}}}{d\phi_k}\ =\ (p+q')_\mu, \tag{9}$$

which turn out to be satisfied in the Bjorken limit.

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