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Resummation of ϕ^4 free energy up to an arbitrary order

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Abstract

The consistency condition, which guarantees a well organized small-coupling asymptotic expansion for the thermodynamics of massless ϕ^4 -theory, is generalized to any desired order of the perturbative treatment. Based on a strong conjecture about forbidden two-particle reducible diagrams, this condition is derived in terms of functions of four-momentum in place of the common toy mass in previous treatments. It has the form of a set of gap equations and marks the position in the space of these functions at which the free energy is extremal.

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1. Introduction

Thermal field theory [1, 2, 3] must take care of infrared singularities which infect a naive diagrammatic expansion. But the way out is well known [4], namely a reorganization of the perturbation series by introducing some toy mass term to be added and subtracted in the Lagrangian, one term becoming part of the bare Lagrangian and the other being treated as perturbation.

At finite temperature, the convenience of first choosing a suitable effective free Lagrangian became apparent in studies of spontaneously broken symmetry [5, 6] and was then encouraged by the success of the Braaten–Pisarski resummation [7] in understanding the quark–gluon excitations. In particular, to treat hot ϕ^4 theory, the toy mass was given the value of the dynamically generated thermal mass [4, 8]. This choice was seen to work well up to three-loop order [9, 10], while in gauge theories the static limit (Matsubara frequency $P_0 = 0$) of the dynamical mass was found to be sufficient [10, 11, 12]. Even ϕ^4 thermodynamics can be formulated using the zero-mode propagator [13]. The asymptotic expansion for the ϕ^4 pressure is now known up to the g^5 term and well confirmed by calculations [14] using the dimensional reduction method. The proposal to use a gap equation for "an even better choice" of the toy mass can be found in the book of Le Bellac [2] (§ 4.1 there), and was recently discussed in relation with the large- N limit and numerics [15].

There remain questions particularly about the toy mass, as to which choice (given an accuracy of treatment) is required according to which principle. There might be an underlying structure the so far used mass versions are only special cases thereof. In this note we are in search of this general structure. Massless ϕ^4 theory turns out to be sufficiently simple for studying the free energy up to an arbitrary high order of interest.

The idea, basic to this paper, came into mind while re-examining the ϕ^4 part of the paper of Arnold and Zhai [10], whose analysis extends to g^4 . Leaving the toy mass m variable, and before evaluating thermal sum-integrals, their result for the pressure $-F/V$ reads

$$-\frac{1}{V}F = a + \frac{1}{2}m^2b - \frac{1}{8}g^2b^2 + \frac{1}{4}\left(m^2 - \frac{1}{2}g^2b\right)^2 c + \text{const}_m, \quad (1.1)$$

where $b = \sum G_0$ and $c = \sum G_0^2$ are functions of m , as also is a with the property $\partial_m a = -mb$. In our notations (see §2) $G_0 = 1/(m^2 - P^2)$. What one could learn from (1.1), first time and at low order, is as follows. The m value at which the pressure becomes minimal ($m = g\sqrt{b/2}$ from $\partial_m F = 0$) precisely equals the position where the unwanted c term vanishes. The suppression of the c term is in fact "necessary to get a well-behaved expansion in g " [10], because otherwise g^3 terms would arise from perturbative g^4 diagrams. Note that $c = -\frac{1}{2m}\partial_m b$, which becomes $\sim 1/g$ through evaluation. In short, the extremum condition for the free energy agrees with the consistency condition — at

least in the case (1.1) at hand.

The requirement for generalization of this agreement makes the plan for the present paper. We shall search for the extremum condition of the free energy, developed up to a given order $g^{2\lambda}$ of perturbation expansion. Then, we read this condition tentatively as the consistency condition. The latter is then shown to remove all unwanted diagrams, whose two or more lines at the same inner momentum would violate a well-organized asymptotic expansion (see (5.7) below). There might be no other mechanism ruining the expansion. But this, admittedly, we are only able to state as (strong) conjecture.

At first glance, the Lagrangian we shall work with,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\phi(Y\phi) - \frac{g^2}{4!}\phi^4 + \frac{1}{2}\phi(\bar{Y}\phi) \quad , \quad (1.2)$$

is nothing but the usual one. Just Y appears in place of the squared toy mass m^2 . Soon, however, we shall generalize Y to be an even function of four momentum (see (2.1) below). Let the bar over the second Y in (1.2) be a reminder that it is a part of the interaction (imagine $\bar{Y} \propto g^2$, but even higher powers will be included in § 2). Of course, \bar{Y} must be set equal to Y at an appropriate stage of analysis in order to reinstall the original theory. The original theory is massless. At a second glance one could miss counter terms in (1.2). However, our analysis will be mainly diagrammatic, hence making renormalization details more suitable for working in afterwards.

The toy function Y , being part of \mathcal{L}_0 in the above Lagrangian, is nothing but a variable self-energy in the bare propagator $G_0(P) = 1/(Y(P) - P^2)$. Note that this generalization opens the door for using functional methods. So, let us state here also the most useful technical detail, which is the relation

$$2G_0^2(P) \delta_{G_0(P)} \ln(Z) = 2\beta \delta_{Y(P)} F \Big|_{\bar{Y} \text{ held fixed}} = G(P) \quad , \quad (1.3)$$

where Z is the partition function, $F = -T\ln(Z)$, $T = 1/\beta$ the temperature and G the exact propagator to the Lagrangian (1.2). Any wisdom on the free energy corresponds to one on the Greens function and vice versa. We learned about this connection from the text book of Kapusta [1] (equation (3.24) there, or (4.2.5) in [3]). But it goes back to the sixties [16], at least.

Section 2 collects our notations and a few diagrammatic details. In Section 3 the extremum of F is found for a general order $g^{2\lambda}$ at which the perturbation series is truncated. Then, in Section 4, the extremum condition is shown to remove all unwanted, i.e. g -order-reducing diagrams. Simple diagrammatic rules are given which determine the resummed series. The way it reproduces the known results is detailed in Section 5, followed by conclusions in Section 6. Some hints for generating diagrams and combinatoric factors are found in the Appendix.

2. The perturbation series

There are "red" g 's in the Lagrangian (1.2), those in the interaction part (last two terms), and "blue" ones in the bare Lagrangian. The latter remain subordinate parameters up to the step where the functions Y and \bar{Y} are identified. Then they modify the perturbation expansion through the evaluation of sum-integrals. The order n of a diagram (if it carries the prefactor g^{2n}) is determined by red g 's. We work with the Matsubara contour and Minkowski metrics $(+---)$. Four-momenta P have the components $(i2\pi nT, \mathbf{p})$, and $x = (-i\tau, \mathbf{x})$.

Mass terms are made momentum dependent conveniently in Fourier space. Especially, the object $(Y\phi)$ in (1.2) reads

$$(Y\phi) = \int_{x'}^{\beta} \sum_P e^{-iPx} e^{iPx'} Y(P) \phi(x') = \sum_P e^{-iPx} Y(P) \tilde{\phi}(P) . \quad (2.1)$$

Variable "masses" of the above type are familiar (in the more recent thermal field theory) from the formulation of effective actions [17, 18, 19] and have been studied recently in another context [20]. Our notations are in essence those of [20]. The following four symbols, the first two being used in (2.1) above, may need explanation ,

$$\int_x^{\beta} \equiv \int_0^{\beta} d\tau \int d^3r , \quad \sum_P \equiv \frac{1}{\beta V} \sum_n \sum_{\mathbf{p}} , \quad |_P \equiv \beta V \delta_{\tilde{\gamma}(P)} , \quad [P] \equiv \beta V \delta_{n,0} \delta_{\mathbf{p},0} , \quad (2.2)$$

while our Fourier convention might be obvious from (2.1). The harmony between the above four definitions becomes apparent in the following relations :

$$\int_x^{\beta} e^{iPx} = [P] , \quad \sum_K [K - P] = 1 , \quad |_P \sum_K \tilde{\phi}(K) \tilde{j}(K) = \tilde{\phi}(P) . \quad (2.3)$$

Analyzing a diagram, there is always one thermal Kronecker of zero argument left, giving $[0] = \beta V$. The rules for going to continuous three-momenta are $\sum_{\mathbf{p}} \rightarrow V \int d^3p / (2\pi)^3$ and $V \delta_{\mathbf{p},0} \rightarrow (2\pi)^3 \delta(\mathbf{p})$.

Rewriting the partition function $Z = \text{Tr}(e^{-\beta H})$ into the functional integral language (and removing multiple vacuum-to-vacuum amplitudes) is a standard procedure [1], resulting in

$$Z = Z_0 \cdot Z_{\text{int}} , \quad \ln(Z_{\text{int}}) = \left[\left(e^{\mathcal{Q}} - 1 \right) W_0 \right]_{j=0, \text{connected}} . \quad (2.4)$$

We have reasons to supply (2.4) with details. The operator \mathcal{Q} in (2.4) is a sum of two :

$$\mathcal{Q} = \mathcal{Q}_Y + \mathcal{Q}_g \quad \text{with} \quad \mathcal{Q}_Y = \frac{1}{2} \sum_P \bar{Y}(P) |_P |_{-P} \quad (2.5)$$

$$\text{and} \quad \mathcal{Q}_g = -\frac{g^2}{4!} \sum_{P_1 P_2 P_3 P_4} [P_1 + P_2 + P_3 + P_4] |_{P_1} |_{P_2} |_{P_3} |_{P_4} . \quad (2.6)$$

They act on the functional

$$W_0 = e^{\frac{1}{2} \sum_P \tilde{j}(-P) G_0(P) \tilde{j}(P)} \quad , \quad G_0(P) = \frac{1}{Y(P) - P^2} \quad . \quad (2.7)$$

Finally, the bare partition function Z_0 is given by

$$Z_0 = \mathcal{N} \int \mathcal{D}\tilde{\phi} e^{\frac{1}{2} \sum_P \tilde{\phi}(-P) \frac{-1}{G_0(P)} \tilde{\phi}(P)} \quad (2.8)$$

with the functional measure \mathcal{N} to be determined such that, with $Y \rightarrow 0$, Z_0 turns into the partition function of blackbody radiation with only one "polarization" possible [20], see also (4.15) below. Note that, with (2.4), everything about diagrams of any order is cast into one line. Some general properties of the perturbation series, as will be seen, are better distilled from this line than by studying a variety of diagrams.

As a first step into the diagrammatic analysis let us define the self-energy Π through Dyson's equation $G = G_0 + G_0 [\bar{Y} - \Pi] G$. Thus, the exact propagator may be written as

$$G(P) = \frac{1}{Y(P) - P^2 - [\bar{Y}(P) - \Pi(P)]} = G_0 + G_0^2 [] + G_0^3 []^2 + \dots \quad (2.9)$$

Equating $Y = \bar{Y}$ shows that Π is the true (exact) self-energy of the theory to be studied. On the other hand, $\Pi - \bar{Y}$ could be named the "perturbative" self-energy.

We shall have to specify the n -th order term of G . Red g 's do occur squared only. Let an index n (as well as the term "n-th order") refer to the prefactor g^{2n} (red g 's). Then, from (2.9), if $\Pi \sim \bar{Y} \sim g^2$, we would have $G_n = G_0^{n+1} [\bar{Y} - \Pi]^n$. But this is not true, since there are higher orders in Π . This suggests including higher orders in \bar{Y} and Y , too:

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 + \dots \quad , \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \dots \quad (2.10)$$

The Lagrangian (1.2), $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$, is now fully specified.

The n -th order contribution $f_n = -\beta F_n = \ln(Z_{\text{int}})_n$ to the free energy is obtained from the following diagrammatic rules (formulated as near to [1] as possible):

1. Draw all connected diagrams of n -th order. Lines may carry crosses of order m .
2. Determine the combinatoric factor for each diagram.
3. Label the lines with momenta, conserved at vertices, and associate a G_0 with each. (2.11)
4. There is a factor $-g^2/4!$ at each four vertex, and a factor $\frac{1}{2}\bar{Y}_m$ at a cross of order m .
5. Sum over momenta with the symbol (2.2), and put a factor $[0] = \beta V$ in front of all this.

The weak point is in rule 2. How to determine combinatoric factors? We answer this question in Appendix A. Let figure 1 illustrate what the first two of the rules (2.11) bring about.

$$\begin{aligned}
f_3 = -\beta F_3 = & 2 \cdot 144 \text{ (two circles)} + 3 \cdot 144 \text{ (three circles)} + 2 \cdot 144 \text{ (two circles with a dot)} + 4 \cdot 144 \text{ (two circles with a dot)} \\
& + 144 \text{ (two circles with a dot)}_{(1)} + 144 \text{ (three circles with a dot)}_{(1)} + 96 \text{ (two circles with a dot)}_{(1)} + 12 \text{ (two circles with a dot)}_{(1)} \\
& + 24 \text{ (two circles with a dot)}_{(1)} + \frac{4}{3} \text{ (two circles with a dot)}_{(1)} + 12 \text{ (two circles with a dot)}_{(2)} + 2 \text{ (two circles with a dot)}_{(1)} + \text{ (two circles with a dot)}_{(3)}
\end{aligned}$$

Figure 1: All third order contributions (g^6) to the free energy including their combinatoric factors. The perturbative order of mass term insertions is indicated in parentheses. The only two-particle irreducible diagrams are the first and the last one.

3. Varying F with respect to Y

After \bar{Y} is identified with Y , this function drops out in the Lagrangian. Hence, it can not survive in the exact free energy: $\delta_Y F^{\text{exact}} = 0$. If, however, the perturbation series is truncated at some order λ (retaining $g^{2\lambda}$, neglecting $g^{2\lambda+2}$, red g 's), the perturbative object $F^{(\lambda)}$ does depend on Y , even after $Y = \bar{Y}$ has been worked in (let $F^{(\lambda)}$ be defined as the result of this setting). Note the mystery. When varying $F^{(\lambda)}$ with respect to $Y_1 = \bar{Y}_1, \dots, Y_\lambda = \bar{Y}_\lambda$, we do something that has no counterpart on the exact side. In particular, the Y 's are *not* parameters measuring the departure from equilibrium. So, the free energy must not exhibit its minimum property. In fact, varying toy parameters, it may have a maximum [21], as it happens with the low-order example (1.1), indeed.

There is a nice exercise which checks the above statement as well as the setup of § 2: verify the non- Y -dependence of the exact F by explicit calculation. With \mathcal{E} an operator that sets $\bar{Y} = Y$ ("the equalizer") we may do so by

$$\delta_{Y(P)} \mathcal{E} F = \mathcal{E} (\delta_Y F + \delta_{\bar{Y}} F) = \mathcal{E} \left[\frac{T}{2} G - T \mathcal{E} \left[(\delta_{\bar{Y}} \mathcal{Q}_Y) e^{\mathcal{Q} W_0} \right]_{j=0, \text{ connected}} \right]. \quad (3.1)$$

The first term to the right is obtained from (1.3). For the second term, we used (2.4) and the fact that the only dependence on \bar{Y} is through \mathcal{Q}_Y . Now, from (2.5), we realize that $\delta_{\bar{Y}} \mathcal{Q}_Y = \frac{T}{2V} \lfloor_P \lfloor_{-P}$. The two rod operators (together with T/V) generate the full propagator, hence things end up with $G - G = 0$, indeed. The toy functions Y really only reorganize a series without changing its meaning.

To study the truncated free energy $F^{(\lambda)}$ it is convenient to define one more operator. Let \mathcal{P}_λ be a projector which, when applied to a linear combination of red g^2 powers, suppresses all terms $\propto g^{2m}$ with $m > \lambda$. Using \mathcal{P}_λ we may write $F^{(\lambda)} = \mathcal{E} \mathcal{P}_\lambda F$. The extremum of $F^{(\lambda)}$ has to be determined from the equations $\delta_{Y_n(P)} F^{(\lambda)} = 0$ with $n = 1, 2, \dots, \lambda$. By applying these functional derivatives to $\mathcal{E} \mathcal{P}_\lambda F$ one could distinguish the following four steps. First, the derivatives may be interchanged with \mathcal{E} , $\delta_{Y_n(P)} \mathcal{E} = \mathcal{E} (\delta_{Y_n(P)} + \delta_{\bar{Y}_n(P)})$,

as in (3.1). Second, while δ_{Y_n} simply commutes with \mathcal{P}_λ , one realizes for the other derivative, that

$$\delta_{\overline{Y}_n(P)} \mathcal{P}_\lambda = \mathcal{P}_{\lambda-n} \delta_{\overline{Y}_n(P)} \quad . \quad (3.2)$$

The so far reached intermediate result is $\delta_{Y_n(P)} F^{(\lambda)} = \mathcal{E} \mathcal{P}_\lambda \delta_{Y_n(P)} F + \mathcal{E} \mathcal{P}_{\lambda-n} \delta_{\overline{Y}_n(P)} F$. Third, with view to (2.10), we note that $\delta_{Y_n} F = \delta_Y F$ and $\delta_{\overline{Y}_n} F = \delta_{\overline{Y}} F$. But now, for the fourth step, we may simply refer to (1.3) and (3.1), to replace $\delta_{Y(P)} F = TG(P)/2$ and $\delta_{\overline{Y}(P)} F = -TG(P)/2$, respectively. Obviously, the two projectors form a difference:

$$\delta_{Y_n(P)} F^{(\lambda)} = \frac{T}{2} \mathcal{E} (\mathcal{P}_\lambda - \mathcal{P}_{\lambda-n}) G(P) \stackrel{!}{=} 0 \quad . \quad (3.3)$$

Choosing $n = 1, n = 2$, and so on, it becomes obvious that the above condition may be equivalently stated as $\mathcal{E} G_\lambda = \dots = \mathcal{E} G_1 = 0$. With a view to (2.9), this condition reads

$$\begin{aligned} 0 &= G_0^2 (Y_1 - \mathcal{E} \Pi_1) \quad , \\ 0 &= G_0^2 (Y_2 - \mathcal{E} \Pi_2) + G_0^3 (Y_1 - \mathcal{E} \Pi_1)^2 \quad , \\ 0 &= G_0^2 (Y_3 - \mathcal{E} \Pi_3) + 2 G_0^3 (Y_1 - \mathcal{E} \Pi_1) (Y_2 - \mathcal{E} \Pi_2) + G_0^4 (Y_1 - \mathcal{E} \Pi_1)^3 \quad , \end{aligned} \quad (3.4)$$

and so forth. Herewith we arrive at the main result of this section. In the space of the functions $Y_1(P), \dots, Y_\lambda(P)$, the truncated free energy $F^{(\lambda)}$ becomes extremal at the "position"

$$Y_n = \mathcal{E} \Pi_n \quad , \quad n = 1, 2, \dots, \lambda \quad . \quad (3.5)$$

The extremum condition has obtained a formally simple form. It is a multiple condition in the sense that one of the λ equations can be used to simplify another (see (4.10) below). Here we only remember that the self-energy functions Π_n are, of course, to be determined diagrammatically as shown in figure 2. They contain Y through G_0 and \overline{Y}_m ($m < n$) through mass insertions. Remember that only the trivial crosses on outer lines are absent by definition, cf. (2.9). This explains the need for the equalizer \mathcal{E} in (3.5).

$$\Pi_2 = -96 \text{---}\bigcirc\text{---} - 144 \text{---}\bigcirc\bigcirc\text{---} - 24 \text{---}\overset{(1)}{\bigcirc}\text{---} \longrightarrow -96 \text{---}\bigcirc\text{---} = \Pi_2^{2\text{PI}}$$

Figure 2: The self-energy diagrams of second order (g^4), and the two-particle irreducible part left over after posing the consistency condition, as explained in subsection 4.2.

4. Consequences of the consistency condition

While in the preceding section we varied $F^{(\lambda)}$ with respect to Y , we are now interested in the value of $F^{(\lambda)}$ at extremum, given by (3.5). As anticipated in the section head, we

shall learn shortly, that (3.5) is also the right (and only right) consistency condition, i.e. the one that removes two-particle reducible diagrams, thereby much reducing the number of terms in the perturbation series.

Irrespective of presence or absence of external legs, a diagram is called two-particle reducible (2PR) if it decomposes into two pieces by cutting two different lines. Otherwise it is a two-particle irreducible (2PI) or skeleton diagram. All diagrams in figure 1 are 2PR, except the first and the last one, which are 2PI. The decomposition is complete, of course:

$$-\beta F_n = [\ln(Z_{\text{int}})]_n = f_n = f_n^{2\text{PI}} + f_n^{2\text{PR}} \quad (4.1)$$

A contribution to f_n still contains Y_m and \bar{Y}_m as distinguished objects. The quantity of interest, instead, is $\mathcal{E}f_n$ to be taken at its extremum. The operation of imposing the condition (3.5) may be performed in two steps. We may set $\bar{Y}_m = \bar{H}_m$ first, and specify the extremum-”position” for Y_m afterwards. To be specific, we need a last special operator \mathcal{C} (”the cross converter”). \mathcal{C} replaces \bar{Y}_n by \bar{H}_n , then \bar{Y}_{n-1} by \bar{H}_{n-1} , ..., and finally \bar{Y}_1 by $\bar{H}_1 = -12 \text{ } \underline{\text{O}}$. Note that, in this ordering, also \bar{Y} insertions (i.e. crosses) are replaced, which occur in the \bar{H} functions. Diagrammatically, \mathcal{C} converts *all* crosses into normal (cross-free) self-energy insertions. To summarize:

$$[\mathcal{E} f_n]_{\text{at extremum}} = [\mathcal{C} f_n]_{\text{at extremum}} \quad (4.2)$$

There is no need for an equalizer \mathcal{E} to the right of (4.2), because \mathcal{C} had no \bar{Y}_m left over. Note that the second subscript ”at extremum” only applies to functions Y contained in G_0 -lines. In the following we shall concentrate on $\mathcal{C}f_n$, while leaving the Y -specification aside as some trivial last step. To study $\mathcal{C}f_n$, all non-trivial information can be extracted from the functional relation (1.3). As G_0 carries no red g , this relation holds true in each order separately,

$$2G_0^2(P) \delta_{G_0(P)} f_n = G_n(P) \quad (4.3)$$

The further analysis is now decomposed into three parts with the respective subsection head announcing the result.

4.1 $\mathcal{C} f_n^{2\text{PR}} = 0$

To the left of (4.3), f_n is a sum of diagrams, and the number of G_0 lines in a definite diagram may be smaller than $2n$ due to higher-order crosses contained. The variational derivative δ_{G_0} , in turn, produces one term for each G_0 line. A fixed $G_0(P)$ may occur several times at the same momentum argument, q times, say. If $q \geq 2$, it is called part of a ”dressed line” (or of a ” q -cycle” [3]), but if $q = 1$ we call it a ”bare line”. In general, G_0^2 times the differentiation with respect to a definite G_0 (being part of a q -cycle) will produce something with the prefactor $G_0^{q+1}(P)$. Clearly, we may collect all terms with

the same power $q + 1$ to the left of (4.3). On the right hand side of (4.3), this grouping is already made explicit by (2.9). Each group may be identified on both sides. We learn, that (4.3) may be given a second index q , i.e. it is valid separately for a given power of outer G_0 's too. Here we only need the separation into $q = 1$ and $q \geq 2$. This amounts to splitting δ_{G_0} into $\delta_{G_0} = \delta^{\text{bare}} + \delta^{\text{dressed}}$, where the first derivative becomes aware of bare lines only, and the second notices only G_0 's being part of dressed lines. Thus :

$$2 \delta^{\text{bare}} f_n = G_0^{-2} G_{n, q=1} = \bar{Y}_n - \Pi_n , \quad (4.4)$$

$$2 \delta^{\text{dressed}} f_n^{2\text{PR}} = G_0^{-2} G_{n, q \geq 2} = G_0 \sum_{m=1}^{n-1} []_m []_{n-m} + \dots + G_0^{n-1} []_1^n \quad (4.5)$$

with $[]_n$ shorthand for $[\bar{Y}_n - \Pi_n]$. (4.5) needs $n \geq 2$ (and reduction to the last term if $n = 2$). Note that there are no dressed lines in $f_n^{2\text{PI}}$ by definition, but there are bare lines in both parts, $f_n^{2\text{PR}}$ and $f_n^{2\text{PI}}$. The reader (if not the journal) might color all dressed lines in figure 1.

The functional derivative in (4.5) does not change the ratio in which \bar{Y}_m and Π_m occur in a dressed line. But to the right of (4.5) this ratio is the $[\bar{Y}_m - \Pi_m]$ combination. Hence, this combination is just a property of dressed lines. This fact is well illustrated by the figure 1: the 2PR contributions shown may be combined such that the only remaining insertions are $[\bar{Y}_m - \Pi_m]$. To be specific, there remain three terms: $f_3^{2\text{PR}} = -3 \bullet \circ \circ \bullet + \frac{1}{6} \bullet \circ \bullet + \frac{1}{2} \bullet \blacksquare$ with $\bullet \equiv []_1$ and $\blacksquare \equiv []_2$. To check (4.5) in this case, note that there is a dressed line also in $[]_2$, see figure 2.

Application of \mathcal{C} makes $[\bar{Y}_m - \Pi_m]$ to vanish. Any diagram in $f_n^{2\text{PR}}$ contains at least one dressed line. Thus, $\mathcal{C} f_n^{2\text{PR}} = 0$, and we have reached our main conclusion first, namely that all 2PR diagrams disappear under the \mathcal{C} operation, i.e. already under the first step in posing the consistency condition. The conclusion may be reversed. The operation \mathcal{C} is in fact the only one making $f_n^{2\text{PR}}$ to vanish (but let us avoid stating all arguments in reverse order).

$$\underline{4.2} \quad \mathcal{C} \Pi_n = \Pi_n^{2\text{PI}}$$

We return to (4.4). Its right hand side vanishes under the \mathcal{C} operation. Consequently, using (4.1), we have

$$\mathcal{C} \delta^{\text{bare}} f_n^{2\text{PI}} = -\mathcal{C} \delta^{\text{bare}} f_n^{2\text{PR}} = 0 , \quad (4.6)$$

where the vanishing of the right hand side follows immediately from the arguments in the preceding subsection: all dressed lines survive under δ^{bare} , and there is at least one in each $f_n^{2\text{PR}}$ diagram. Note that \mathcal{C} and δ^{bare} commute, if applied to $f_n^{2\text{PR}}$. But they do not, when applied to $f_n^{2\text{PI}}$.

To learn from (4.6), we first observe that, among the $f_n^{2\text{PI}}$ diagrams, there is always the blank circle with a cross of order n (the last one in figure 1). From the rules (2.11)

we have

$$f_n^{\times} \equiv \bigcirc^{\times(n)} = \beta V \sum_P \frac{1}{2} \bar{Y}_n(P) G_0(P) \quad , \quad 2\mathcal{C} \delta^{\text{bare}} f_n^{\times} = \mathcal{C} \bar{Y}_n = \mathcal{C} \Pi_n \quad . \quad (4.7)$$

Now, using (4.7), we may rewrite (4.6) as

$$\mathcal{C} \Pi_n = -2 \delta^{\text{bare}} f_n'^{2\text{PI}} \quad \text{with} \quad f_n'^{2\text{PI}} \equiv f_n^{2\text{PI}} - f_n^{\times} \quad . \quad (4.8)$$

Note that there are neither dressed lines nor crosses in $f_n'^{2\text{PI}}$ diagrams, by definition. The same happens with $\delta^{\text{bare}} f_n'^{2\text{PI}}$. Therefore we dropped one \mathcal{C} in (4.8). $\delta^{\text{bare}} f_n'^{2\text{PI}}$ is made up of diagrams having two ends. If cutting two inner lines of such a diagram, it remains connected. So, it contributes to $\Pi_n^{2\text{PI}}$. Moreover, these contributions form $\Pi_n^{2\text{PI}}$ itself, because through the above reformulations we only removed diagrams. This argument makes (4.8) to become

$$\mathcal{C} \Pi_n = \Pi_n^{2\text{PI}} \quad , \quad n = 1, 2, \dots, \lambda \quad . \quad (4.9)$$

For the simplest example see figure 2. For $n = 1$, admittedly, (4.9) is trivial, since $\Pi_1^{2\text{PI}} = \mathcal{C} \Pi_1 = \Pi_1 = -12 \bigcirc$. The result (4.9) is very welcome for a final simplification of the consistency condition (3.5):

$$Y_1 = \Pi_1 [Y] \quad , \quad Y_2 = \Pi_2^{2\text{PI}} [Y] \quad , \quad \dots \quad , \quad Y_\lambda = \Pi_\lambda^{2\text{PI}} [Y] \quad , \quad (4.10)$$

where the functional dependence on $Y = Y_1 + \dots + Y_\lambda$ is hidden in the bare G_0 lines. Note that (4.10) resulted from an iterative use of the multiple condition (3.5), thereby minimizing the number of sum-integrals involved. Of course, (4.10) may be written as a single equation, $Y = \sum_{n=1}^{\lambda} \Pi_n^{2\text{PI}} [Y]$. But note, that only with a finite number λ of terms, it makes sense to be a generalized gap equation.

$$\underline{4.3} \quad \mathcal{C} f_n^{2\text{PI}} = (1 - 2n) f_n'^{2\text{PI}}$$

We now concentrate on the n -th order diagrams remaining after all the above reductions. To start with, we remember (4.1), $\mathcal{C} f_n^{2\text{PR}} = 0$ and the definition in (4.8) to get

$$\mathcal{C} f_n = \mathcal{C} f_n^{2\text{PI}} = \mathcal{C} f_n^{\times} + f_n'^{2\text{PI}} \quad (4.11)$$

with $f_n'^{2\text{PI}}$ to be obtained from the rules (2.11) by omitting all dressed and/or crossed diagrams. Further such contributions will arise, if (by \mathcal{C}) the n -th order cross in f_n^{\times} is replaced by $\Pi_n^{2\text{PI}}$. So, there could be a relation between the two terms to the right in (4.11). This happens indeed, and is easily established by combining the equations (4.7), (4.8) and (4.9):

$$\mathcal{C} f_n^{\times} = \beta V \sum_P \frac{1}{2} G_0(P) \Pi_n^{2\text{PI}}(P) = -\beta V \sum_P G_0(P) \delta_{G_0(P)} f_n'^{2\text{PI}} \quad . \quad (4.12)$$

Remember that δ_{G_0} removes a sum together with the factor βV . But precisely these details are restored on the above right hand side. The operator $\beta V \sum G_0 \delta_{G_0}$ just counts lines. There are $2n$ lines in each diagram of $f_n'^{2\text{PI}}$, and that is it: $\mathcal{C}f_n^x = -2n f_n'^{2\text{PI}}$. To summarize, the element $\mathcal{C}f_n$, which is needed on the right hand side of (4.2), is given by $\mathcal{C}f_n = \mathcal{C}f_n^{2\text{PI}} = (1 - 2n)f_n'^{2\text{PI}}$, as announced.

The free energy, consistently resummed up to order λ , is thus obtained as

$$\left[-\beta F^{(\lambda)} \right]_{\text{at extremum}} = \left[f_0 + \sum_{n=1}^{\lambda} (1 - 2n) f_n'^{2\text{PI}} \right]_{Y=\sum_{n=1}^{\lambda} \Pi_n^{2\text{PI}}[Y]} \quad (4.13)$$

or, equivalently, from the following set of diagrammatic rules :

1. Drop all remarks on crosses in the rules (2.11), i.e. return to the rules for ϕ^4 theory without toy mass.
2. Draw 2PI diagrams only. Come to a decision for the truncation: $n \leq \lambda$. (4.14)
3. Multiply each combinatoric factor of an n -th order diagrams with $(1 - 2n)$.
4. To specify the function $Y = Y_1 + Y_2 + \dots + Y_\lambda$ in G_0 , solve the gap equations (4.10).

The result (4.14) is agreeably simple. As figure 3 shows, the number of diagrams has reduced so much, that the remainder can be presented up to g^{10} with ease. The combinatoric factor for pearl rings, given in the caption, persists to higher orders and can be proven by induction. Once (4.13), (4.14) are reached, the details on cross insertions may be viewed as some less important aspect of the derivation. To avoid possible misunderstandings (taken up in the next two paragraphs), let us emphasize the generality of the result and recall its meaning. The ingoing question (for the correct decomposition of the Lagrangian whose truncated perturbation series gives a well-organized asymptotic expansion) is answered by the gap equation, i.e. by the subscript to the right of (4.13). But (4.13) itself merely states the result of working with the solution $Y(Q)$.

$$\begin{aligned} -\beta F^{(5)} = & \frac{1}{2} \bigcirc - 1 \cdot 3 \bigcirc\bigcirc - 3 \cdot 12 \bigcirc\bigcirc\bigcirc - 5 \cdot 2 \cdot 12^2 \bigcirc\bigcirc\bigcirc - 7 \cdot \frac{3}{2} \cdot 12^3 \bigcirc\bigcirc\bigcirc\bigcirc \\ & - 7 \cdot 6 \cdot 12^3 \bigcirc\bigcirc\bigcirc\bigcirc - 9 \cdot \frac{6}{5} \cdot 12^4 \bigcirc\bigcirc\bigcirc\bigcirc - 9 \cdot 12 \cdot 12^4 \bigcirc\bigcirc\bigcirc\bigcirc \\ & - 9 \cdot 24 \cdot 12^4 \bigcirc\bigcirc\bigcirc\bigcirc - 9 \cdot \frac{16}{5} \cdot 12^4 \bigcirc\bigcirc\bigcirc\bigcirc \end{aligned}$$

Figure 3: The contributions to the resummed free energy up to 5-th order (g^{10}). The combinatoric factors are written as products with $(1 - 2n)$ the first factor of each. For rings of pearls (4., 5. and 7. diagram) the combinatoric factors follow the rule $(1 - 2n) 12^n / (2n)$. The last diagram is non-planar. The first term is $-\beta F_0$, and its factor denotes the number of blackbody radiations at $Y \rightarrow 0$, cf. (4.15).

By (4.13) one could be strongly remembered to structures observed by Cornwall, Jackiw and Tomboulis [22] in their study of the effective potential. Gap equations mark

stationary points and reduce the effective action diagrams to skeletons with lines being improved propagators [22, 23, 18]. It may happen that (4.13) can be derived along these lines as well (though the diagrams shown there are of rather low order). We have not followed up this possibility. But we are able to perform a near-by other test next.

From even earlier days [21, 16, 24, 3] one knows of the possibility of *exactly* summing self-energies into the lines of skeleton diagrams. But remember that, in the absence of truncation, the terms consistency and gap equation make no sense, and that the exact free energy was realized in (3.1) to be a merely uninteresting limiting case. Nevertheless, we are now invited, to test (4.13) by taking the limit $\lambda \rightarrow \infty$. The condition (4.10) becomes $Y = \sum_{n=1}^{\infty} \Pi_n^{2\text{PI}}[Y]$ and has the solution $Y = \Pi$, with $\Pi \equiv \sum_{n=1}^{\infty} \Pi_n [\bar{Y} = Y = 0]$ the exact self-energy, cf. (2.9). Next, concerning the $-2n$ part of the third rule, we step back to f_n^x : $-\beta F^{(\infty)} = f_0 + \sum_{n=1}^{\infty} f_n'^{2\text{PI}} + \sum_{n=1}^{\infty} \mathcal{C}f_n^x$. Using (4.12), we may perform the second sum. Note that $Y = \Pi$ makes $G_0(P) = 1/(\Pi(P) - P^2)$ to be the exact propagator of the original (Y -free) theory. In presenting f_0 we care about the correct behavior $f_0 \rightarrow VT^3\pi^2/90$ at vanishing coupling. Then, from Appendix A of [20], we are led to introduce the function $r(P) \equiv T^2 e^{\beta|\mathbf{p}|} \delta_{n,0} - P_0^2$ (remember $P_0^2 = -T^2(2\pi n)^2$). Herewith, the present exercise ends up with

$$-\frac{1}{V} F^{(\infty)} = \frac{1}{2} \sum_P \ln \left(\frac{r(P)}{\Pi(P) - P^2} \right) + \frac{1}{2} \sum_P \frac{\Pi(P)}{\Pi(P) - P^2} + \frac{1}{\beta V} \sum_{n=1}^{\infty} f_n'^{2\text{PI}}, \quad (4.15)$$

which (for $r = 1$) is equation (22.16) in [16], or (4.2.10) in [3]. We have thus rederived the old exact statement, indeed.

5. The first few terms

and the one-loop gap equation. Although looking quite different, our resummed free energy must precisely reproduce the asymptotic expansion for the ϕ^4 pressure, as far as known. We shall be content here to demonstrate this reproduction. For this task, we may concentrate on $F^{(2)}$, given by the first three diagrams of figure 3, as well as on the gap equation (4.10) for $\lambda = 2$.

Under renormalization several quantities change their meaning. The Lagrangian (1.2) becomes the renormalized one and g turns to be the running coupling. Among the counter terms, only $Z_2 = 1 + 3g^2/(32\pi^2\varepsilon) + O(g^4)$ is of relevance [10, 12], where ε refers to dimensional regularization: $\sum_P \rightarrow T \sum_n \mu^{2\varepsilon} (2\pi)^{2\varepsilon-3} \int d^{3-2\varepsilon}p$. The ϕ^4 -interaction in (1.2) modifies by $g^2 \rightarrow \mu^{2\varepsilon} Z_2 g^2$. The corresponding g^4 term must be included in the second diagram of figure 3 and in the first of the conditions (4.10). The second condition (4.10), may be simplified immediately: $Y_2(Q) = -\frac{1}{6}g^4 \sum_{P,K} G_0(P)G_0(K)G_0(P+K+Q)$ with the crude approximation $Y \approx g^2 T^2/24$ sufficient in G_0 .

The first condition (4.10) needs more care. Since Y_1 does not depend on the outer momentum Q (a fact special to ϕ^4 theory), we may write $Y_1 \equiv m^2 + \delta$, i.e.

$$m^2 + \delta = \frac{1}{2}g^2 \sum_P \frac{1}{m^2 - P^2 + \delta + Y_2} + \frac{1}{2}g^2(Z_2 - 1) \sum_P \frac{1}{m^2 - P^2 + \delta + Y_2} , \quad (5.1)$$

and determine δ such that the leading part of (5.1), the one with $G_{00} \equiv 1/(m^2 - P^2)$ under the first sum, turns into the one-loop gap equation

$$m^2 = \frac{1}{2}g^2 \sum_P \frac{1}{m^2 - P^2} . \quad (5.2)$$

Expanding (5.1) one obtains $\delta = \frac{1}{2}g^2(Z_2 - 1) \sum_P G_{00} - \frac{1}{2}g^2 \sum_P G_{00}^2 (\delta + Y_2) + O(g^7)$.

For the pressure $-F^{(2)}/V \equiv p$ we have three contributions, $p = p_0 + p_1 + p_2$, corresponding to the first three diagrams of figure 3: $p_0 = \frac{1}{2} \sum_P \ln(r G_0)$, cf. (4.15), $p_1 = \frac{1}{8} Z_2 g^2 (\sum_P G_0)^2$ and $p_2 = -\frac{1}{16} g^4 I_{\text{ball}}$ (overall factors $\mu^{-2\varepsilon}$ are omitted for brevity). As with (5.1), we might collect those leading terms of $p_0 + p_1$, called p_m , which contain G_{00} and require a nontrivial solution of (5.2) for m . Hence, $p = p_m + p_{\text{rest}}$. After a bit of calculation we obtain

$$p_m = \frac{1}{2} \sum_P \ln(r G_{00}) + \frac{g^2}{8} (\sum_P G_{00})^2 , \quad p_{\text{rest}} = -\frac{g^2(Z_2 - 1)}{8} (\sum_P G_{00})^2 + \frac{g^4}{48} I_{\text{ball}} \quad (5.3)$$

with $I_{\text{ball}} = \sum_{Q,P,K} G_0(Q)G_0(P)G_0(K)G_0(Q+P+K)$ [10]. Two terms proportional to $(\delta + Y_2)$ have canceled in p_{rest} , because p is at its minimum. Again, being content with $\leq g^5$, the functions G_0 in p_{rest} may be supplied with the lowest order value of m^2 . Note that, with p_{rest} , two of the five contributions of ref. [10] are obtained, namely those of fourth order in red g 's. Hence, the remaining three diagrams, which are \bigcirc , $\bigcirc\bigcirc$ and $\bigcirc\otimes$, might be anyhow hidden in p_m . But note that, in these three diagrams, cross and lines carry the Arnold–Zhai value m_A^2 , which is given by (5.2) at zero mass to the right. We are thus led to expand p_m around m_A^2 :

$$p_m = [p_m]_A - \frac{1}{2}(m^2 - m_A^2) \left[1 - \frac{1}{2}g^2 \partial_{m_A^2} \sum_P G_{00 A} \right] \sum_P G_{00 A} + O(g^5) , \quad (5.4)$$

where $G_{00 A} \equiv \sum_P 1/(m_A^2 - P^2)$ and, at this point, we relax retaining g^5 terms. They need no additional diagram [13]. Now, the gap equation (5.2), if expanded around m_A^2 ,

$$(m^2 - m_A^2) \left[1 - \frac{1}{2}g^2 \partial_{m_A^2} \sum_P G_{00 A} \right] = \frac{1}{2}g^2 \sum_P G_{00 A} - m_A^2 , \quad (5.5)$$

is seen to be of direct use in (5.4). Moreover, it gives p_m the desired form to exhibit the three diagram contributions in search. In total, the terms obtained combine to the well known expression (in our language)

$$p = \frac{1}{2} \sum_P \ln(r G_{00 A}) - \frac{1}{8} Z_2 g^2 \left(\sum_P G_{00 A} \right)^2 + \frac{1}{2} m_A^2 \sum_P G_{00 A} + \frac{1}{48} g^4 I_{\text{ball}} \quad (5.6)$$

for the pressure up to three-loop order. At these low orders, and already on an algebraic level, the resummed theory has reduced to the traditional setup, indeed.

It might have been remarked, that our low order limit has led to the Arnold and Zhai version [10] of the asymptotic expansion, but not to that of Parwani and Singh [13], whose mass term includes $\delta_{P_0,0}$. But the latter version only amounts to a regrouping of terms having the same order of magnitude [6]. Note that the absence of "forbidden" g^3 terms in equation (12) of [13] is still due to a suitable choice of the toy mass prefactor.

Through all of the preceding sections (starting with the example (1.1)), it was taken for granted, that (a) 2PR diagrams reduce the g order, hence being forbidden in a systematic asymptotic expansion, and that (b) there is no other mechanism producing g^{-1} factors. Concerning statement (a), consider, without loss of generality, a diagram with only one dressed line, a q -cycle with $q = k + 1$ and $k \geq 1$. With $m^2 \sim g^2$ a constant part of Y and $\partial_m = 2m \partial_{m^2}$ we may write

$$\sum_P G_0^{k+1}(P)h(P) = \sum_P \frac{h(P)}{k!} (-\partial_{m^2})^k G_0(P) = \frac{1}{k!} \left(-\frac{1}{2m} \bar{\partial}_m \right)^k \sum_P h(P) G_0(P) \quad , \quad (5.7)$$

where $h(P)$ is a $(k + 1)$ -fold product of 2PI self-energy functions (or cross insertions) and $\bar{\partial}$ is not allowed to act on the m 's in $h(P)$. Even under this restriction, all experience with evaluated skeleton diagrams $\sum h(P)G_0(P)$ [10, 13] shows, that they have a term $\propto m$ in its asymptotic expansion. But this is sufficient to reduce its order through $(-\frac{1}{m}\partial_m)^k m = -(2k - 3)!! m^{1-2k} \sim g^{-2k+1}$. The statement (b) is somewhat delicate as we have no proof for. But it is hard to realize, that the g order could be reduced anyhow else than by (5.7). Statement (b) is the strong conjecture, this paper rests on — and ends up with.

6. Conclusions

To summarize, the small-coupling asymptotic expansion for the ϕ^4 thermodynamics is supplied with a general consistency condition. The latter is derived by requiring the free energy to be extremal, but then shown to guarantee the systematics of the asymptotic expansion. By the corresponding resummation, the pressure is given by simple diagrammatic rules. But the self-energy in their "bare" line propagators need to be self-consistently determined by solving a generalized gap equation. Former treatments are demonstrated to be low-order special cases of this scheme.

Most probably, the observed structure has its counterpart for gauge theories as well, at least with regard to the functional methods used in this paper. The hot Yang-Mills system (pure gluon plasma) is under present study.

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Appendix

Here we comment on the generation of diagrams while retaining the information on combinatoric factors. Diagrams derive from (2.4) by first expanding the exponential. Imagine n operators \mathcal{Q} applied to W_0 . Reduce \mathcal{Q}_Y , (2.5), and \mathcal{Q}_g , (2.6), to the rod operators, because the other details (Σ , weight, Kronecker) are preserved by the rules (2.11). Now move W_0 to the left by commuting it with each $|$, $| W_0 = W_0 (| + j)$, where j is shorthand for $G_0 \tilde{j}$ (but G_0 is preserved by the rules). At the very left W_0 may be omitted due to the $j = 0$ prescription in (2.4). Let ∂ be a rod-derivative, which is not allowed to act on a j on the same vertex. These inner differentiations may be made explicit, instead. After all this, \mathcal{Q} has converted to

$$\mathbf{D} \equiv 6 \alpha + 12 \alpha^\partial + 6 \alpha_{\partial}^\partial + \times + 4 \times^\partial + 6 \times_{\partial}^\partial + 4 \times_{\partial}^{\partial} + \frac{\partial}{\partial} \times_{\partial}^{\partial} + \dagger + 2 \dagger^\partial + \dagger_{\partial}^\partial . \quad (\text{A.1})$$

Unspecified line ends carry j . An unspecified cross represents the sum $\sum_{k=1}^{\infty}$ over k -th order crosses. Two terms, 3∞ and $\textcircled{\times}$, are omitted in (A.1) since they certainly lead to disconnected diagrams. Now, form $(1/n!) \mathbf{D} \dots \mathbf{D} 1$, drop further disconnected pieces and set $j = 0$, finally, i.e. omit diagrams with empty ends. Having obtained a definite diagram, its combinatoric factor may be checked, of course, against $-(4!)^v / (2mS)$ with $v =$ number of vertices, $m =$ multiplicity of a line cut up, $S^{-1} =$ symmetry factor of the self-energy diagram arisen.

Up to $n = 3$, the above \mathbf{D} operations may be well performed by hand (figure 1). But let Miss MAPLE continue to higher orders (figure 3). The program must be given some memory for *which* two vertices have been joined by a line. The corresponding crucial program-line reads `w:=proc(b,a,x); b.a*diff(x,a); end; .`

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