

The Longitudinal Heavy Quark Structure Function $F_L^{Q\bar{Q}}$ in the Region $Q^2 \gg m^2$ at $O(\alpha_s^3)$

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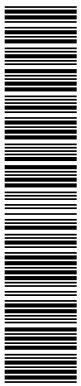
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Abstract

The logarithmic and constant contributions to the Wilson coefficient of the longitudinal heavy quark structure function to $O(\alpha_s^3)$ are calculated using mass factorization techniques in Mellin space. The small x behaviour of the Wilson coefficient is determined. Numerical illustrations are presented.

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1 Introduction

Deeply inelastic electron–nucleon scattering at large momentum transfer provides one of the cleanest possibilities to test the predictions of Quantum Chromodynamics (QCD). In the case of pure photon exchange the structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ describe the scattering cross section. While the former structure function is well measured in a wide kinematic region [1], $F_L(x, Q^2)$ was mainly measured in fixed target experiments [2] and determined in the high y region at HERA [3] using an extrapolation method. Future detailed measurements of the longitudinal structure function $F_L(x, Q^2)$ at HERA are still to be performed [4]. At leading order in the coupling constant the gluon distribution $g(x, Q^2)$ does not contribute to the structure function $F_2(x, Q^2)$ directly, but only to its derivative, which weakens the sensitivity. In the region of smaller values of x the structure function $F_L(x, Q^2) = F_2(x, Q^2) - 2xF_1(x, Q^2)$, however, is dominated by the gluon contribution. Therefore, this structure function may yield essential constraints on $g(x, Q^2)$. In lowest order in the coupling constant (α_s^0) and vanishing target–mass effects, the twist–2 contributions to the structure functions F_2 and F_L obey the Callan–Gross [5] relation

$$F_2(x, Q^2) = 2xF_1(x, Q^2), \quad F_L(x, Q^2) \equiv 0 . \quad (1)$$

$F_L(x, Q^2)$ receives leading order contributions due to target mass effects [6]. The Callan–Gross relation is further broken by QCD corrections. The corresponding Wilson coefficients for massless quarks were calculated in leading (LO) [7], next-to-leading (NLO) [8–10], and next-to-next-to-leading order (NNLO) [11–13]. Since the leading order coefficient functions are polynomial, scheme–invariant quantities one may construct a simple mapping of $F_L^{\text{LO}}(x, Q^2)$ to $g^{\text{LO}}(x, Q^2)$ taking the quark distributions from the F_2 measurement [14]. The leading small x terms for the coefficient functions of F_L have been derived in [15] and agree with the known fixed order results (NLO, NNLO) [8, 9, 12, 13]. The gluonic contribution to $F_L(x, Q^2)$ was calculated using the k_\perp representation in leading order [16], which turns out to be numerical very close to the NLO result [8]. The numerical impact of the small x resummation [15] on F_L was studied in [17]. Similar to the small x resummation for the splitting functions, formally sub–leading terms lead to comparable but widely compensating effects, as seen comparing the magnitude of these terms for fixed orders in the coupling constant. This behaviour was later observed also in [13]. To draw firm conclusions on the effect of these resummations, several sub–leading series of terms have to be known. Higher twist contributions to $F_L(x, Q^2)$, partly under model assumptions, were considered in [18].

Since the longitudinal structure functions $F_L(x, Q^2)$ contains rather large heavy flavor contributions in the small x region [19], a consistent analysis has to account for these effects, which were calculated in leading [20] and next-to-leading order [21, 22].² The NLO corrections [21] could not be performed in analytic form completely. This is also expected for even higher orders, due to the complexity of the phase space integrals. However, complete analytic results may be derived in the asymptotic region $Q^2 \gg m^2$ calculating all contributions but the power suppressed terms $(m^2/Q^2)^k$, [24, 25].³

In the present paper we use the method of Ref. [24] to derive the heavy quark Wilson coefficients for $F_L^{Q\bar{Q}}(x, Q^2)$ to $O(\alpha_s^3)$ in the region $Q^2 \gg m^2$. In Section 2 we give a brief outline of the method. The Wilson coefficients are derived in Section 3. Their small x behaviour

²Fast Mellin–space expressions for these Wilson coefficients were given in [23].

³For related work for other processes, see [26].

is discussed in Section 4. In Section 5 numerical are presented and Section 6 contains the conclusions. Some useful relations are summarized in the Appendix.

2 The Method

In the twist–2 approximation the nucleon structure functions $F_i(x, Q^2)$ are described as Mellin convolutions between the parton densities $f_j(x, \mu^2)$ and the Wilson coefficients $C_i^j(x, Q^2/\mu^2)$

$$F_i(x, Q^2) = \sum_j C_i^j \left(x, \frac{Q^2}{\mu^2} \right) \otimes f_j(x, \mu^2) \quad (2)$$

to all orders in perturbation theory due to the factorization theorem. Here μ^2 denotes the factorization scale and the Mellin convolution is given by the integral

$$[A \otimes B](x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2) . \quad (3)$$

Since the distributions f_j refer to **massless** partons, the heavy flavor effects are contained in the Wilson coefficients only. We are interested in the massive contributions in the region $Q^2 \gg m^2$. These are the non–power corrections in m^2/Q^2 , i.e. all logarithmic contributions and the constant term. We apply the collinear parton model, i.e. the parton 4–momentum is $p = zP$, with P the nucleon momentum. The massive Wilson coefficients itself can be viewed as a quasi cross section in pV^* –scattering, where V^* denotes the exchanged virtual vector boson. In the limit $Q^2 \gg m^2$ the massive Wilson coefficients $H_{2,L,i}^{S,NS}(Q^2/m^2, m^2/\mu^2, x)$, likewise the case for the structure functions (2), factorize

$$H_{2,L,i}^{S,NS} \left(\frac{Q^2}{m^2}, \frac{m^2}{\mu^2}, x \right) = C_{2,L,k}^{S,NS} \left(\frac{Q^2}{\mu^2}, x \right) \otimes A_{k,i}^{S,NS} \left(\frac{m^2}{\mu^2}, x \right) \quad (4)$$

into Wilson coefficients $C_{L,k}^{S,NS}(Q^2/\mu^2, x)$ accounting for light flavors only and massive operator matrix elements $A_{k,i}^{S,NS}(m^2/\mu^2, x)$. The latter take a similar role as the parton densities in (2), but are perturbatively calculable. The factorization (4) is a consequence of the renormalization group and the fact that we restrict the investigation to non–power corrections to $H_{L,i}^{S,NS}(Q^2/m^2, m^2/\mu^2, x)$. The operator matrix elements $A_{k,i}^{S,NS}$ obey the expansion

$$A_{k,i}^{S,NS} \left(\frac{m^2}{\mu^2} \right) = \langle i | O_k^{S,NS} | i \rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{S,NS,(l)}, \quad i = q, g \quad (5)$$

of the twist–2 quark singlet and non–singlet operators $O_k^{S,NS}$ between **partonic** states $|i\rangle$, which are related by collinear factorization to the initial–state nucleon states $|N\rangle$ and $a_s = \alpha_s(\mu^2)/(4\pi)$ denotes the strong coupling constant. The operator matrix elements are process–independent quantities. The process dependence of $H_{L,i}^{S,NS}$ is described by the associated coefficient functions

$$C_{L,k} \left(\frac{Q^2}{\mu^2} \right) = \sum_{l=l_0}^{\infty} a_s^l C_{L,k}^{(l)} \left(\frac{Q^2}{\mu^2} \right), \quad k = NS, S, g . \quad (6)$$

The $\overline{\text{MS}}$ coefficient functions, in the massless limit, corresponding to the heavy quarks only, are denoted by

$$\hat{C}_{L,k} \left(\frac{Q^2}{\mu^2} \right) = C_{L,k} \left(\frac{Q^2}{\mu^2}, N_L + N_H \right) - C_{L,k} \left(\frac{Q^2}{\mu^2}, N_L \right), \quad (7)$$

where N_H, N_L are the number of heavy and light flavors, respectively. In the following we will consider the case of a single heavy quark, i.e. $N_H = 1$. The formalism is easily generalized to more than one heavy quark species. The heavy flavor Wilson coefficient is obtained as the expansion of the product of (5,6) to the respective order in a_s .

3 The Wilson Coefficients in the Region $Q^2 \gg M^2$

In the limit of vanishing nucleon mass effects, cf. [6], the longitudinal structure function emerges only at $O(a_s)$ due to the Callan–Gross relation [5]. The leading order contribution is purely gluonic [7]. At $O(a_s^2)$ $F_L^{Q\bar{Q}}(x, Q^2)$ receives also quarkonic contributions.

To $O(a_s^3)$ the heavy quark Wilson coefficients $H_{L,g(q)}^{S,PS,NS}$ read :

$$H_{L,g}^S \left(\frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) = a_s \hat{C}_{L,g}^{(1)} \left(\frac{Q^2}{\mu^2} \right) + a_s^2 \left[A_{Q,g}^{(1)} \left(\frac{\mu^2}{m^2} \right) \otimes C_{L,q}^{(1)} \left(\frac{Q^2}{\mu^2} \right) + \hat{C}_{L,g}^{(2)} \left(\frac{Q^2}{\mu^2} \right) \right] \\ + a_s^3 \left[A_{Q,g}^{(2)} \left(\frac{\mu^2}{m^2} \right) \otimes C_{L,q}^{(1)} \left(\frac{Q^2}{\mu^2} \right) + A_{Q,g}^{(1)} \left(\frac{\mu^2}{m^2} \right) \otimes C_{L,q}^{(2)} \left(\frac{Q^2}{\mu^2} \right) + \hat{C}_{L,g}^{(3)} \left(\frac{Q^2}{\mu^2} \right) \right] \quad (8)$$

$$H_{L,q}^{PS} \left(\frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) = a_s^2 \hat{C}_{L,q}^{PS,(2)} \left(\frac{Q^2}{\mu^2} \right) + a_s^3 \left[A_{Qq}^{PS,(2)} \left(\frac{\mu^2}{m^2} \right) \otimes C_{L,q}^{(1)} \left(\frac{Q^2}{\mu^2} \right) + \hat{C}_{L,q}^{PS,(3)} \left(\frac{Q^2}{\mu^2} \right) \right] \quad (9)$$

$$H_{L,q}^{NS} \left(\frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) = a_s^2 \hat{C}_{L,q}^{NS,(2)} \left(\frac{Q^2}{\mu^2} \right) + a_s^3 \left[A_{qq,Q}^{NS,(2)} \left(\frac{\mu^2}{m^2} \right) \otimes C_{L,q}^{(1)} \left(\frac{Q^2}{\mu^2} \right) + \hat{C}_{L,q}^{NS,(3)} \left(\frac{Q^2}{\mu^2} \right) \right], \quad (10)$$

where

$$C_{L,q}^{(2)} = C_{L,q}^{NS} + C_{L,q}^{PS} \quad (11)$$

and

$$H_{L,q}^S = H_{L,q}^{NS} + H_{L,q}^{PS}. \quad (12)$$

$C_{L,i}^{(k)}(Q^2/\mu^2)$ are the scale dependent Wilson coefficients in the $\overline{\text{MS}}$ scheme with $C_{L,i}^{(k)}(Q^2/\mu^2) = c_{L,i}^{(k)}$ for $Q^2 = \mu^2$ given in [7–13].

The operator matrix elements were derived in [24] and read :

$$A_{Qg}^{(1)} = -\frac{1}{2} \hat{P}_{qg}^{(0)} \ln \left(\frac{m^2}{\mu^2} \right) + a_{Qg}^{(1)} \quad (13)$$

$$A_{Qg}^{(2)} = \frac{1}{8} \left\{ \hat{P}_{qg}^{(0)} \otimes [P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0] \right\} \ln^2 \left(\frac{m^2}{\mu^2} \right) \\ - \frac{1}{2} \left\{ \hat{P}_{qg}^{(1)} + a_{Qg}^{(1)} [P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0] \right\} \ln \left(\frac{m^2}{\mu^2} \right) \\ + \bar{a}_{Qg}^{(1)} [P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0] + a_{Qg}^{(2)} \quad (14)$$

$$A_{Qq}^{PS,(2)} = -\frac{1}{8} \hat{P}_{qg}^{(0)} \otimes P_{gq}^{(0)} \ln^2 \left(\frac{m^2}{\mu^2} \right) - \frac{1}{2} [\hat{P}_{qq}^{PS,(1)} - a_{Qg}^{(1)} P_{gq}^{(0)}] \ln \left(\frac{m^2}{\mu^2} \right) \\ + a_{Qq}^{PS,(2)} + \bar{a}_{Qg}^{(1)} \otimes P_{gq}^{(0)} \quad (15)$$

$$A_{qq,Q}^{NS,(2)} = -\frac{\beta_{0,Q}}{4} P_{qq}^{(0)} \ln^2 \left(\frac{m^2}{\mu^2} \right) - \frac{1}{2} \hat{P}_{qq}^{NS,(1)} \ln \left(\frac{m^2}{\mu^2} \right) + a_{qq,Q}^{NS,(2)} + \frac{1}{4} \beta_{0,Q} \zeta_2 P_{qq}^0, \quad (16)$$

with

$$\hat{f} = f(N_F + 1) - f(N_F) . \quad (17)$$

For later fast numerical representations we express the above functions $f_i(x)$ in Mellin space,

$$\mathbf{M}[f_i(x)](N) = \int_0^1 dx x^{N-1} f_i(x) \quad (18)$$

at (even) integers N and arrange for analytic continuation to complex values of N starting from these values.

The splitting functions are

$$P_{qq}^{(0)}(N) = 4C_F \left[-2S_1(N-1) + \frac{(N-1)(3N+2)}{2N(N+1)} \right] \quad (19)$$

$$P_{gg}^{(0)}(N) = 8T_R N_F \frac{N^2 + N + 2}{N(N+1)(N+2)} \quad (20)$$

$$P_{gg}^{(0)}(N) = 8C_A \left[-S_1(N-1) - \frac{N^3 - 3N - 4}{(N-1)N(N+1)(N+2)} \right] + 2\beta_0 \quad (21)$$

$$P_{gq}^{(0)}(N) = 4C_F \frac{N^2 + N + 2}{(N-1)N(N+1)} \quad (22)$$

$$\hat{P}_{qq}^{\text{PS},(1)}(N) = 16C_F T_R \frac{5N^5 + 32N^4 + 49N^3 + 38N^2 + 28N + 8}{(N-1)N^3(N+1)^3(N+2)^2} \quad (23)$$

$$P_{qq,Q}^{\text{NS},(1)}(N) = \hat{P}_{qq}^{\text{NS},(1)} = C_F T_R \left\{ \frac{160}{9} S_1(N-1) - \frac{32}{3} S_2(N-1) - \frac{4(N-1)(3N+2)(N^2-11N-6)}{9N^2(N+1)^2} \right\} \quad (24)$$

$$\begin{aligned} \hat{P}_{gg}^{(1)}(N) = & 8C_F T_R \left\{ 2 \frac{N^2 + N + 2}{N(N+1)(N+2)} [S_1^2(N) - S_2(N)] - \frac{4}{N^2} S_1(N) \right. \\ & \left. + \frac{5N^6 + 15N^5 + 36N^4 + 51N^3 + 25N^2 + 8N + 4}{N^3(N+1)^3(N+2)} \right\} \\ & + 16C_A T_R \left\{ -\frac{N^2 + N + 2}{N(N+1)(N+2)} [S_1^2(N) + S_2(N) - \zeta_2 - 2\beta'(N+1)] \right. \\ & \left. + 4 \frac{2N+3}{(N+1)^2(N+2)^2} S_1(N) + \frac{P_1(N)}{(N-1)N^3(N+1)^3(N+2)^3} \right\} , \end{aligned} \quad (25)$$

where

$$P_1(N) = N^9 + 6N^8 + 15N^7 + 25N^6 + 36N^5 + 85N^4 + 128N^3 + 104N^2 + 64N + 16 . \quad (26)$$

The expansion coefficient of the β -function for the case of light and heavy (Q) flavors read

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R N_f , \quad (27)$$

$$\beta_{0,Q} = -\frac{4}{3} T_R . \quad (28)$$

The Mellin transforms lead to harmonic sums. Their analytic continuation for single harmonic sums is given by

$$S_1(N-1) = \psi(N) + \gamma_E \quad (29)$$

$$S_k(N-1) = \frac{(-1)^{k-1}}{(k-1)!} \psi^{(k-1)}(N) + \zeta_k, \quad k \geq 2 \quad (30)$$

$$S_{-1}(N-1) = (-1)^{N-1} \beta(N) - \ln(2) \quad (31)$$

$$S_{-k}(N-1) = (-1)^{k+N} \beta^{(k)}(N) - \left(1 - \frac{1}{2^{k-1}}\right) \zeta_k, \quad k \geq 2. \quad (32)$$

Here $\psi(z) = d \ln[\Gamma(z)]/dz$, γ_E denotes the Mascheroni–Euler number, ζ_k the values of Riemann's ζ -function for integer $k \geq 2$ and

$$\beta(z) = \frac{1}{2} \left[\psi\left(\frac{1+z}{2}\right) - \psi\left(\frac{z}{2}\right) \right]. \quad (33)$$

Multiply nested harmonic sums are reduced to Mellin transforms of basic functions [27–29] for which the analytic continuation to complex values of N is performed [30,31].

The functions emerging in the scale independent contributions of the operator matrix elements are

$$a_{Q_g}^{(1)}(N) = 0 \quad (34)$$

$$\bar{a}_{Q_g}^{(1)}(N) = -\frac{1}{8} \zeta_2 \hat{P}_{gg}^{(0)}(N) \quad (35)$$

$$\begin{aligned} a_{Q_g}^{(2)}(N) = & 4C_F T_R \left\{ \frac{N^2 + N + 2}{N(N+1)(N+2)} \left[-\frac{1}{3} S_1^3(N-1) + \frac{4}{3} S_3(N-1) \right. \right. \\ & \left. \left. - S_1(N-1) S_2(N-1) - 2\zeta_2 S_1(N-1) \right] + \frac{2}{N(N+1)} S_1^2(N-1) \right. \\ & + \frac{N^4 + 16N^3 + 15N^2 - 8N - 4}{N^2(N+1)^2(N+2)} S_2(N-1) \\ & + \frac{3N^4 + 2N^3 + 3N^2 - 4N - 4}{2N^2(N+1)^2(N+2)} \zeta_2 \\ & \left. + \frac{N^4 - N^3 - 16N^2 + 2N + 4}{N^2(N+1)^2(N+2)} S_1(N-1) + \frac{P_2(N)}{2N^4(N+1)^4(N+2)} \right\} \\ & + 4C_A T_R \left\{ \frac{N^2 + N + 2}{N(N+1)(N+2)} \left[4\mathbf{M} \left[\frac{\text{Li}_2(x)}{1+x} \right] (N) + \frac{1}{3} S_1^3(N) + 3S_2(N) S_1(N) \right. \right. \\ & \left. \left. + \frac{8}{3} S_3(N) + \beta''(N+1) - 4\beta'(N+1) S_1(N) - 4\beta(N+1) \zeta_2 + \zeta_3 \right] \right. \\ & - \frac{N^3 + 8N^2 + 11N + 2}{N(N+1)^2(N+2)^2} S_1^2(N) - 2 \frac{N^4 - 2N^3 + 5N^2 + 2N + 2}{(N-1)N^2(N+1)^2(N+2)} \zeta_2 \\ & \left. - \frac{7N^5 + 21N^4 + 13N^3 + 21N^2 + 18N + 16}{(N-1)N^2(N+1)^2(N+2)^2} S_2(N) \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{N^6 + 8N^5 + 23N^4 + 54N^3 + 94N^2 + 72N + 8}{N(N+1)^3(N+2)^3} S_1(N) \\
& - 4 \left. \frac{(N^2 - N - 4)}{(N+1)^2(N+2)^2} \beta'(N+1) + \frac{P_3(N)}{(N-1)N^4(N+1)^4(N+2)^4} \right\} \quad (36)
\end{aligned}$$

$$\begin{aligned}
a_{Qq}^{\text{PS},(2)}(N) = C_{FT_R} \left\{ -8 \frac{N^4 + 2N^3 + 5N^2 + 4N + 4}{(N-1)N^2(N+1)^2(N+2)} S_2(N-1) \right. \\
\left. - 4 \frac{(N^2 + N + 2)^2}{(N-1)N^2(N+1)^2(N+2)} \zeta_2 + \frac{4P_4(N)}{(N-1)N^4(N+1)^4(N+2)^3} \right\} \quad (37)
\end{aligned}$$

$$\begin{aligned}
a_{qq,Q}^{\text{NS},(2)}(N) = C_{FT_R} \left\{ - \left(\frac{224}{27} + \frac{8}{3} \zeta_2 \right) S_1(N-1) + \frac{40}{9} S_2(N-1) - \frac{8}{3} S_3(N-1) \right. \\
+ \frac{2(3N+2)(N-1)}{3N(N+1)} \zeta_2 \\
\left. + \frac{(N-1)(219N^5 + 428N^4 + 517N^3 + 512N^2 + 312N + 72)}{54N^3(N+1)^3} \right\}, \quad (38)
\end{aligned}$$

where

$$P_2(N) = 12N^8 + 54N^7 + 136N^6 + 218N^5 + 221N^4 + 110N^3 - 3N^2 - 24N - 4 \quad (39)$$

$$\begin{aligned}
P_3(N) = 2N^{12} + 20N^{11} + 86N^{10} + 192N^9 + 199N^8 - N^7 - 297N^6 - 495N^5 - 514N^4 \\
- 488N^3 - 416N^2 - 176N - 32 \quad (40)
\end{aligned}$$

$$\begin{aligned}
P_4(N) = N^{10} + 8N^9 + 27N^8 + 33N^7 - 71N^6 - 275N^5 - 403N^4 - 448N^3 \\
- 408N^2 - 208N - 48. \quad (41)
\end{aligned}$$

Note that in the above expressions the equality

$$\mathbf{M} \left[\frac{\text{Li}_2(x)}{1+x} \right] (N) - \zeta_2 \beta(N) = (-1)^N \left[S_{-2,1}(N-1) + \frac{5}{8} \zeta_3 \right] \quad (42)$$

can be applied. Therefore, the operator matrix elements depend on one non-trivial basic function [29,32] only and no sum with index $\{-1\}$ contributes. Concerning the non-trivial harmonic sums emerging the operator matrix elements are of the same complexity as the 2-loop anomalous dimensions.

The longitudinal structure function $F_L(x, Q^2)$ consists of the light and heavy flavor contributions

$$\begin{aligned}
F_L(x, Q^2) &= F_L^{\text{light}}(x, Q^2) + F_L^{\text{heavy}}(x, Q^2) \\
&= C_L^{\text{NS}} \left(x, a_s, \frac{Q^2}{\mu^2} \right) \otimes q_{\text{NS}}(x, \mu^2) + C_L^{\text{S}} \left(x, a_s, \frac{Q^2}{\mu^2} \right) \otimes q_{\text{S}}(x, \mu^2) \\
&\quad + C_L^g \left(x, a_s, \frac{Q^2}{\mu^2} \right) \otimes g(x, \mu^2). \quad (43)
\end{aligned}$$

We choose $Q^2 = \mu^2$ as uniform factorization scale. The Wilson coefficients in (43) are of the form

$$C_L^{(i)} \left(x, a_s, \frac{Q^2}{\mu^2} \right) = C_L^{(i),\text{light}}(x, a_s) + H_L^{(i)} \left(x, a_s, \frac{Q^2}{m^2} \right), \quad i = \text{S, NS, } g. \quad (44)$$

The heavy quark contributions are given by

$$\begin{aligned}
H_{L,g}^S \left(x, a_s, \frac{Q^2}{m^2} \right) &= a_s \hat{c}_{L,g}^{(1)} + a_s^2 \left[\frac{1}{2} \hat{P}_{gg}^{(0)} c_{L,q}^{(1)} \ln \left(\frac{Q^2}{m^2} \right) + \hat{c}_{L,g}^{(2)} \right] \\
&+ a_s^3 \left\{ \left[\frac{1}{8} \hat{P}_{gg}^{(0)} [P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0] \ln^2 \left(\frac{Q^2}{m^2} \right) + \frac{1}{2} \hat{P}_{gg}^{(1)} \ln \left(\frac{Q^2}{m^2} \right) \right. \right. \\
&\quad \left. \left. + a_{Qg}^{(2)} + \bar{a}_{Qg}^{(1)} [P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0] \right] c_{L,q}^{(1)} + \frac{1}{2} \hat{P}_{gg}^{(0)} \ln \left(\frac{Q^2}{m^2} \right) c_{L,q}^{(2)} + \hat{c}_{L,g}^{(3)} \right\} \quad (45)
\end{aligned}$$

$$\begin{aligned}
H_{L,q}^{\text{PS}} \left(x, a_s, \frac{Q^2}{m^2} \right) &= a_s^2 \hat{c}_{L,q}^{\text{PS},(2)} + a_s^3 \left\{ \left[-\frac{1}{8} \hat{P}_{gg}^{(0)} P_{gg}^{(0)} \ln^2 \left(\frac{Q^2}{m^2} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \hat{P}_{qq}^{\text{PS},(1)} \ln \left(\frac{Q^2}{m^2} \right) + a_{Qq}^{\text{PS},(2)} - \bar{a}_{Qg}^{(1)} P_{gg}^{(0)} \right] c_{L,q}^{(1)} + \hat{c}_{L,q}^{\text{PS},(3)} \right\} \quad (46)
\end{aligned}$$

$$\begin{aligned}
H_{L,q}^{\text{NS}} \left(x, a_s, \frac{Q^2}{m^2} \right) &= a_s^2 \left[-\beta_{0,Q} c_{L,q}^{(1)} \ln \left(\frac{Q^2}{m^2} \right) + \hat{c}_{L,q}^{\text{NS},(2)} \right] \\
&+ a_s^3 \left\{ \left[-\frac{1}{4} \beta_{0,Q} P_{qq}^{(0)} \ln^2 \left(\frac{Q^2}{m^2} \right) - \frac{1}{2} \hat{P}_{qq}^{\text{NS},(1)} \ln \left(\frac{Q^2}{m^2} \right) + a_{qq,Q}^{\text{NS},(2)} + \frac{1}{4} \beta_{0,Q} \zeta_2 P_{qq}^{(0)} \right] \right. \\
&\quad \left. \times c_{L,q}^{(1)} + \hat{c}_{L,q}^{\text{NS},(3)} \right\}. \quad (47)
\end{aligned}$$

The Wilson coefficients for heavy quark production consist of terms $\propto (m^2/Q^2)^k$, $k > 0$, $k \in \mathbf{N}$ and the logarithmic and constant contributions $\propto \ln^l(Q^2/m^2)$, $l \geq 0$ for on-shell massive quarks. The latter terms do not vanish in the limit $m^2 \rightarrow 0$ and can be calculated solving the renormalization group equations for $F_i^{Q\bar{Q}}(x, Q^2)$.

4 The Small- x Limit

In the small x limit the heavy quark Wilson coefficient at $O(a_s)$, $H_{L,g}^{(1)}(x, Q^2/m^2; \mu^2/m^2)$, vanishes with x since its leading pole is situated at $N = -1$. One expects the following leading and next-to-leading small- x behaviour

$$H_L^S(x) \propto a_s^2 \frac{d_1^{(1)}}{x} + \sum_{k=2}^{\infty} a_s^{k+1} \left[d_k^{(1)} \frac{\ln^{k-1}(x)}{x} + d_k^{(2)} \frac{\ln^{k-2}(x)}{x} + \dots \right]. \quad (48)$$

In $O(a_s^2)$ the leading [24, 33] small- x terms for $\mu^2 = Q^2$ are

$$d_{1,i}^{(1)} = -32C_i T_R \frac{1}{9}, \quad (49)$$

with $i = A, F$ for the gluonic and pure singlet contribution, respectively. As seen in Eqs. (8, 9, 13, 14, 15), these terms stem from the small x behaviour of $P_{gg}^{(0)}$ and $\hat{P}_{gg}^{(0)}$, (21,22), and from $c_{L,g}^{(2)}, c_{L,q}^{(2)\text{PS}}$. The terms $\propto T_R$ for both contributions scale by the color factors, C_A , resp. C_F . $H_{L,g}^S$ contains an additional term at $O(a_s^3)$, $\propto T_R^2 \ln(Q^2/m^2)$. $H_{L,q}^{(2),\text{NS}}$ does not contain a term $\propto 1/x$ but is less singular for small values of x . The corresponding coefficients are obtained in finding

the contributions $\propto 1/(N-1)$ in (45,46). Most of the respective functions were given above. We further note that $c_{L,q}^{(1)}(N=1) = 2C_F$ and $c_{L,q}^{(2)}(N \rightarrow 1) \propto -(32/9)C_F T_R N_f/(N-1)$, [9,27].

In $O(a_s^3)$ the leading small x contributions to $H_{L,g(q)}^{S,(3)}(x, Q^2/m^2; \mu^2/m^2)$ result from the small x terms in $\hat{c}_{L,g(q)}^{(3)}(x)$ only and are proportional to that of the light flavor contributions [13]. The remaining heavy flavor corrections are less singular. The leading terms $\propto 1/x$ are

$$d_{2,i}^{(1)} = \frac{128}{3} C_A C_i T_R \left[-\frac{34}{9} + \zeta_2 \right] \quad (50)$$

$$\begin{aligned} d_{2,A}^{(2)} = & -32 C_A C_F T_R \left[\frac{1}{3} \ln^2 \left(\frac{Q^2}{m^2} \right) - \frac{10}{9} \ln \left(\frac{Q^2}{m^2} \right) + \frac{28}{27} \right] \\ & - \frac{256}{27} C_F T_R^2 (2N_F + 1) \ln \left(\frac{Q^2}{m^2} \right) \\ & + \frac{32}{3} C_A^2 T_R \left[-\frac{2756}{27} + \frac{65}{3} \zeta_2 + 20 \zeta_3 \right] + \frac{64}{3} C_A C_F T_R \left[\frac{56}{9} - \zeta_2 - 4 \zeta_3 \right] \\ & + C_F T_R^2 (2N_F + 1) \frac{64}{9} \left[\frac{121}{9} - 4 \zeta_2 \right] + C_A T_R^2 (2N_F + 1) \frac{32}{9} \left[\frac{101}{9} - 8 \zeta_2 \right] . \end{aligned} \quad (51)$$

$$\begin{aligned} d_{2,F}^{(2)} = & -32 C_F^2 T_R \left[\frac{1}{3} \ln^2 \left(\frac{Q^2}{m^2} \right) - \frac{10}{9} \ln \left(\frac{Q^2}{m^2} \right) + \frac{28}{27} \right] \\ & + 32 C_A C_F T_R \left[-\frac{899}{27} + 7 \zeta_2 + \frac{20}{3} \zeta_3 \right] + \frac{64}{3} C_F^2 T_R \left[\frac{56}{9} - \zeta_2 - 4 \zeta_3 \right] \\ & + C_F T_R^2 (2N_F + 1) \frac{256}{9} \left[\frac{53}{9} - \zeta_2 \right] . \end{aligned} \quad (52)$$

Among the subleading terms not stemming from $\hat{c}_{L,g(q)}^{(3)}$ those $\propto T_R$ scale by the color factor. $H_{L,q}^{\text{NS},(3)}(x, Q^2/m^2; \mu^2/m^2)$ is regular for $N=1$. Similarly to the treatment in [34,35] one might consider its singular behaviour around $N=0$, however, the small- x resummation for these terms were not yet derived.

5 Numerical Results

In the following we will give some numerical illustrations of the effect of the heavy flavor contributions in the limit $Q^2 \gg m^2$. These results are, unfortunately, of limited phenomenological use, since one expects, similar to the case of the NLO corrections [24], that these corrections become effective at large values of $Q^2 \simeq 1000 \text{ GeV}^2$, where no data on F_L are available at present. At lower values of Q^2 power corrections do still contribute. In the case of $F_2(x, Q^2)$, a sufficient description by the asymptotic expression could be obtained for scales $Q^2 \gtrsim 30 \text{ GeV}^2$ already, cf. [24].

We illustrate the size of the contributions at NLO and NNLO for $F_L^{Q\bar{Q}}(x, Q^2)$ choosing the parton distributions as follows at $Q_0^2 = 30 \text{ GeV}^2$

$$xq_{\text{NS}}(x) = N_{\text{NS}} x^{a_{\text{NS}}} (1-x)^{b_{\text{NS}}} \quad (53)$$

$$xq_{\text{PS}}(x) = N_s x^{a_s} \left[(1-x)^{b_s} + c_s x^{d_s} \right] \quad (54)$$

$$xg(x) = N_g x^{a_g} \left[(1-x)^{b_g} + c_g x^{d_g} \right] \quad (55)$$

We apply this parameterization both for the NLO and NNLO effects for illustrative purposes. The parton distributions at higher scales are obtained by evolution. The values of

the parameters at Q_0^2 are listed in Table 1. The strong coupling constant $\alpha_s(Q^2)$ is calculated using the values of $\Lambda_{\text{QCD}}^{\overline{\text{MS}}, \text{NS}, (4)}$ determined in [36]. One obtains: $\alpha_s^{\text{NLO}}(30 \text{ GeV}^2) = 0.1977$, resp. $0.1708(Q^2 = 100 \text{ GeV}^2)$, $0.1132(Q^2 = 10^4 \text{ GeV}^2)$ and $\alpha_s^{\text{NNLO}}(30 \text{ GeV}^2) = 0.1928$, $0.1673(Q^2 = 100 \text{ GeV}^2)$, $0.1118(Q^2 = 10^4 \text{ GeV}^2)$. The charm quark mass was chosen to be $m_c = 1.5 \text{ GeV}$.

Parameter	N_i	a_i	b_i	c_i	d_i
NS	1.00000	0.50000	3.0000		
S	0.60000	-0.30000	3.5000	5.0000	0.80000
gluon	0.11518	-0.32230	6.0445	0.9618	0.00422

Table 1: The parameters of the quark non-singlet, singlet and gluon distribution at $Q^2 = 30 \text{ GeV}^2$.

The asymptotic heavy flavor non-singlet contributions together with the light flavor terms are shown in Figure 1 and turn out to be small due to the shape of the input distribution and since they emerge only at $O(a_s^2)$. Their contribution shrinks with growing Q^2 . The asymptotic heavy flavor pure singlet part added to the light-flavor contributions of $F_L(x, Q^2)$ are depicted in Figure 2. Also here the the first contribution is obtained at $O(a_s^2)$, but the effect is much larger if compared to the non-singlet part due to the small- x behaviour of the corresponding distribution function $\propto x^{-0.3}$. The contribution rises with Q^2 and amounts to $\sim O(1/5 \dots 1/6)$ of the gluon contributions, shown in Figure 3. The gluon contribution is the largest and emerges already at LO. It grows towards the small x region. In all cases the NNLO result is larger than that at NLO. To obtain a complete picture for the heavy flavor contributions to $F_L(x, Q^2)$ the corrections at scales of lower values of Q^2 have to be calculated.

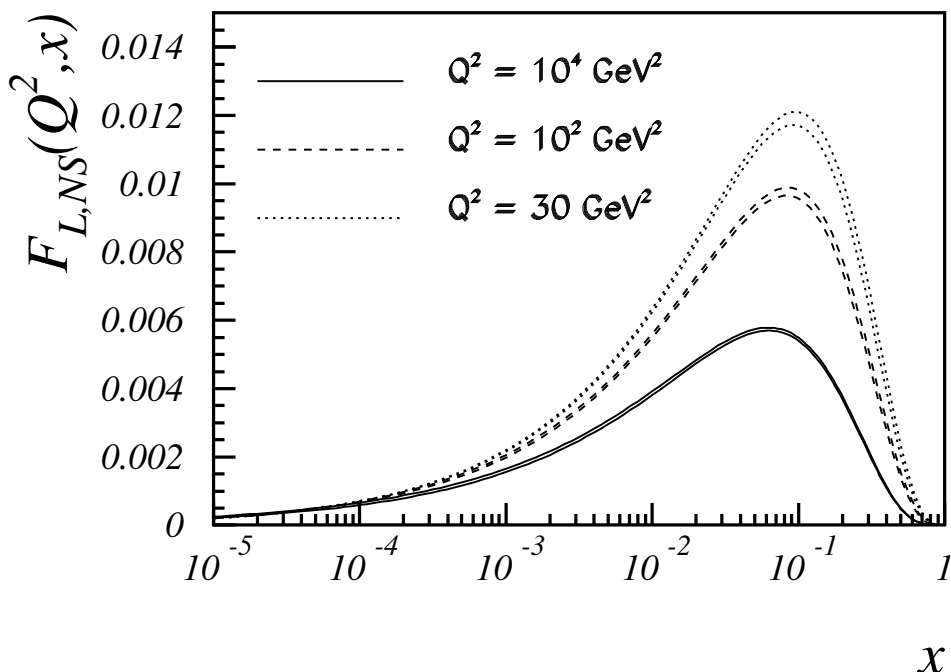


Figure 1: The light flavor and asymptotic heavy flavor non-singlet contributions due to charm to $F_L(x, Q^2)$ in NLO and NNLO. Upper lines: NNLO, lower lines NLO.

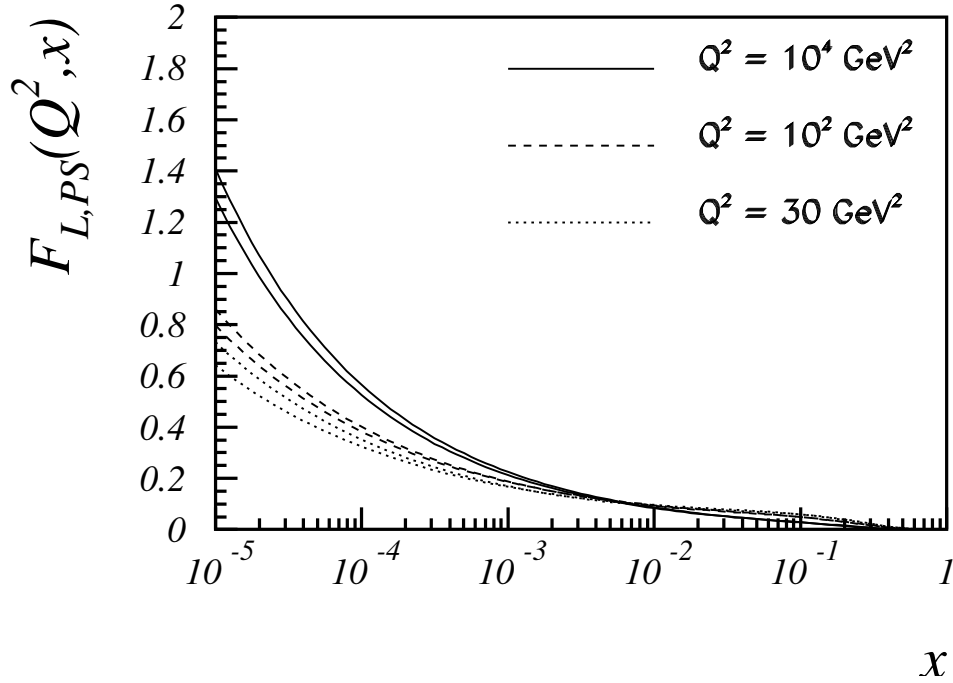


Figure 2: The light flavor and asymptotic heavy flavor pure singlet contributions due to charm to $F_L(x, Q^2)$ in NLO and NNLO. Upper lines: NNLO, lower lines NLO.

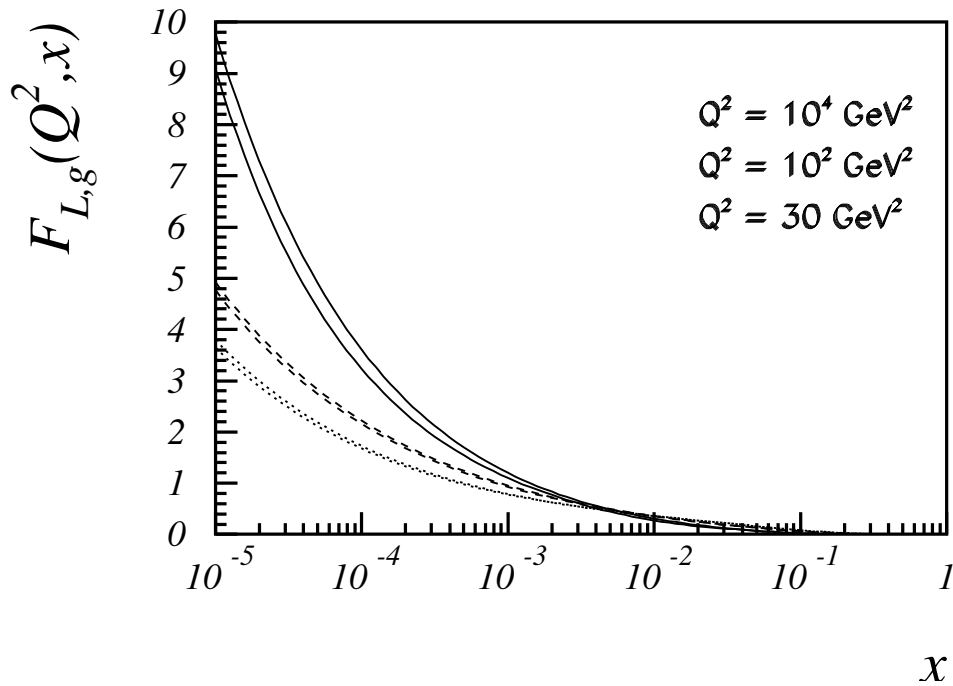


Figure 3: The light flavor and asymptotic heavy flavor gluon contributions due to charm to $F_L(x, Q^2)$ in NLO and NNLO. Upper lines: NNLO, lower lines NLO.

6 Conclusions

We have calculated the heavy flavor contributions to the structure function $F_L(x, Q^2)$ in the region $Q^2 \gg m_Q^2$ at $O(\alpha_s^3)$. In this kinematic regime the respective terms are obtained as the logarithmic orders $\propto \ln^k(Q^2/m_Q^2)$ and the constant term. Power corrections cannot be determined using the method of the present paper. In this approximation the heavy flavor Wilson coefficients are given by a convolution of the light-flavor Wilson coefficients and **universal** operator matrix elements, which contain the information on the heavy quarks. At NLO a numerical comparison of the complete calculation to the asymptotic case was possible and scales in the range $Q^2 \gtrsim 1000 \text{ GeV}^2$ were identified to apply the asymptotic relation for $F_L^{Q\bar{Q}}(x, Q^2)$. This is likely to be the case at NNLO too. We presented numerical results for the asymptotic $O(\alpha_s^3)$ corrections added to the light-flavor contributions. The largest contribution is due to the gluonic term, followed by the pure singlet term, which is a factor ~ 5 smaller in the small x region. The flavor non-singlet contribution is very small. The leading small x terms of the heavy flavor Wilson coefficients $H_{L,q(g)}^{\text{PS,(S)}}(x, Q^2/m^2)$ were determined. In $O(\alpha_s^3)$ the pure heavy flavor terms contribute to the next-to-leading small x terms. In part of the terms scaling by $C_F(C_A)$ is observed comparing the respective gluonic and quarkonic contributions.

7 Appendix

The Mellin transforms used in the present calculation may be found in [27]. Some expressions can be written in the more compact form given below.

$$\begin{aligned}\mathbf{M}[\ln(1+z)](N) &= \frac{1}{N} \left\{ \ln(2) - (-1)^N [S_{-1}(N) + \ln(2)] \right\} \\ &= \frac{1}{N} [\ln(2) - \beta(N+1)]\end{aligned}\quad (56)$$

$$\mathbf{M}[\ln(z) \ln(1+z)](N) = -\frac{1}{N^2} [\ln(2) - \beta(N+1)] - \frac{1}{N} \beta'(N+1) \quad (57)$$

$$\begin{aligned}\mathbf{M}[\ln^2(z) \ln(1+z)](N) &= \frac{2}{N^3} \left\{ \ln(2) - (-1)^N [S_{-1}(N) + \ln(2)] \right\} \\ &\quad - (-1)^N \frac{2}{N^2} \left[S_{-2}(N) + \frac{\zeta_2}{2} \right] - (-1)^N \frac{2}{N} \left[S_{-3}(N) + \frac{3}{4} \zeta_3 \right] \\ &= \frac{2}{N^3} [\ln(2) - \beta(N+1)] + \frac{2}{N^2} \beta'(N+1) - \frac{1}{N} \beta''(N+1)\end{aligned}\quad (58)$$

$$\begin{aligned}\mathbf{M}[\text{Li}_2(-z)](N) &= -\frac{\zeta_2}{2N} + \frac{1}{N^2} \left\{ \ln(2) - (-1)^N [S_{-1}(N) + \ln(2)] \right\} \\ &= -\frac{\zeta_2}{2N} + \frac{1}{N^2} [\ln(2) - \beta(N+1)]\end{aligned}\quad (59)$$

$$\mathbf{M}[\ln(z) \text{Li}_2(-z)](N) = \frac{\zeta_2}{2N^2} - \frac{2}{N^3} [\ln(2) - \beta(N+1)] - \frac{1}{N^2} \beta'(N+1) \quad (60)$$

$$\mathbf{M}[\text{Li}_2(-z) + \ln(z) \ln(1+z)](N) = -\frac{1}{2N} [\zeta_2 + 2\beta'(N+1)] \quad (61)$$

$$\mathbf{M}[\text{Li}_3(-z)](N) = -\frac{3}{4N} \zeta_3 + \frac{1}{2N^2} \zeta_2 - \frac{1}{N^3} [\ln(2) - \beta(N+1)] \quad (62)$$

$$\begin{aligned}\mathbf{M}[\Phi_1(z)](N) &= \frac{(-1)^{N+1}}{N} \left\{ 2S_{1,-2}(N) + \zeta_2 [S_1(N) - S_{-1}(N)] \right\} \\ &\quad + \frac{[1 + (-1)^{N+1}]}{N} \left[\frac{\zeta_3}{4} - \zeta_2 \ln(2) \right]\end{aligned}\quad (63)$$

$$\begin{aligned}&= \frac{1}{N} \left\{ 2\mathbf{M} \left[\frac{\text{Li}_2(x)}{1+x} \right] (N) - \frac{2}{N} \zeta_2 + \frac{2}{N^2} S_1(N) + 3\zeta_2 \beta(N+1) \right. \\ &\quad \left. + 2S_1(N) \beta'(N+1) - \beta''(N+1) + \frac{\zeta_3}{4} - \zeta_2 \ln(2) \right\},\end{aligned}\quad (64)$$

where

$$\Phi_1(z) = 2\text{Li}_2(-z) \ln(1+z) + \ln^2(1+z) \ln(z) + 2S_{1,2}(-z). \quad (65)$$

$\mathbf{M}[\text{Li}_2(x)/(1+x)](N)$ is a basic function, cf. [28,30].

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