Zentrum für Ökonomische und Soziologische Studien

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Leonard Raul Primavesi

The (dys)functionality of the aggregate production function

ZÖSS
ZENTRUM FÜR ÖKONOMISCHE UND SOZIOLOGISCHE STUDIEN

ExMa-Papers
ISSN 1868-5005/56
Exemplarische Master-Arbeiten
Hamburg 2024

The (dys)functionality of the aggregate production function

Leonard Raul Primavesi

ExMa-Papers
ISSN 1868-5005/56
Zentrum für Ökonomische und Soziologische Studien
Universität Hamburg
April 2024

Impressum:

Die hier aufgeführten ExMA-Papers (Exemplarische Master-Arbeiten) setzen sich (vornehmlich) aus ausgewählten Lernwerkstatt-, Master-, Bachelor- oder Seminararbeiten von Studierenden des Masterstudiengangs ‚Arbeit, Wirtschaft, Gesellschaft – Ökonomische und Soziologische Studien' und des Bachelorstudiengangs Sozialökonomie zusammen, die aufgrund ihrer exemplarischen Interdisziplinarität oder Qualität als Vorbild für andere Arbeiten gelten können und deshalb publikationswürdig sind.

Herausgeber/Redaktion:

Zentrum für Ökonomische und Soziologische Studien (ZÖSS)

rouven.reinke@uni-hamburg.de

Universität Hamburg

Fakultät für Wirtschafts- und Sozialwissenschaften

Fachbereich Sozialökonomie

Welckerstr. 8

20354 Hamburg

1. Introduction

The concept of production functions at the firm, plant, or industry level serves as a cornerstone in the neoclassical theory of production and distribution, constituting a fundamental element in the development of modern microeconomics. Similarly, in macroeconomics, aggregate production functions (APFs), which model the production of the entire economy, hold significant importance and are extensively utilized. Moreover, theories grounded in production functions, such as the marginalist theory of factor pricing or the IS-LM model, not only form an integral part of the foundational education for economics students but also serve as a crucial basis for economic research.

However, upon examining the historical evolution of production functions, Mongiovi (1999: p.7) observes that *"the principle of factor substitution [i.e. the central mechanic of the production function, L.P.] originated not in the observation of empirical regularities but in a process of deduction from axioms presumed by the early marginalists to be plausible. The notion of price-elastic factor and commodity demand functions has so deeply penetrated the economic intuition of our age that to doubt their existence seems to contradict the obvious. But of course these functions have never been, and can never be, directly observed"*.

While these concepts may appear intuitively plausible or even trivial within the framework of neoclassical or New Keynesian education, actual aggregate production functions cannot be directly observed. The dynamics of variables such as labor, capital, or output do not inherently imply specific substitution mechanisms or production processes. Consequently, the production function, as a theoretical construct, relies heavily on logical consistency. Should a logical flaw be identified in the construction of the theoretical model of production, the entire theoretical construct may not be empirically salvageable. However, this does not necessarily hold true for the production function as an empirical tool, as there exist methods to assess the validity of this model without directly observing the actual production function.

The justification for utilizing (neoclassical) aggregate production functions, whether as theoretical constructs or empirical tools, has sparked intense debate. This study aims to delve into the historical foundations of these debates and their current status to address the research question: **"Can (neoclassical) aggregate production functions serve as functional[1] tools for macroeconomic theory and practice?"**

In doing so, this study challenges the prevailing intuition, echoing Mongiovi, that questioning the existence of functional APFs and their implications runs counter to common sense. Initially, the focus will be on APFs as theoretical constructs and the underlying theory. Two primary inquiries will guide this exploration: "Can an APF be theoretically constructed in a consistent manner?" and "Does this production function exhibit neoclassical properties?" It will be demonstrated that APFs have been refuted theoretically.

Subsequently, attention will shift to debates surrounding the justification of employing the **neoclassical aggregate production function (NAPF)** as an empirical tool. The third section will center on a prominent method used to test the NAPF, which has historically been utilized to justify its application in numerous instances. However, this method will be shown to be flawed.

Following this, the fourth section will summarize additional debates beyond the aforementioned method and evaluate studies either supporting or attempting to refute the NAPF. The study will conclude with a brief overview of the implications drawn from the findings, a reflection on the limitations encountered, and prospects for future research endeavors.

2. APFs as Theoretical Constructs

2.1 The Aggregation of Micro Production Functions

Neoclassical macroeconomics previously leaned on the notion that aggregate production functions (APFs) could be directly derived from microeconomic single-firm production functions. Although this approach is no longer viable for constructing an APF today, it can serve as an intuitive starting point for theoretical discussions and sheds light on key aggregation challenges.

[1] "Functional" in the sense that they usefully model the production processes of the economy.

In neoclassical microeconomics, it is assumed that single-firm production functions possess certain **neoclassical properties**. These include positive and diminishing marginal products, a declining profit rate with increasing capital intensity, and convex isoquants.

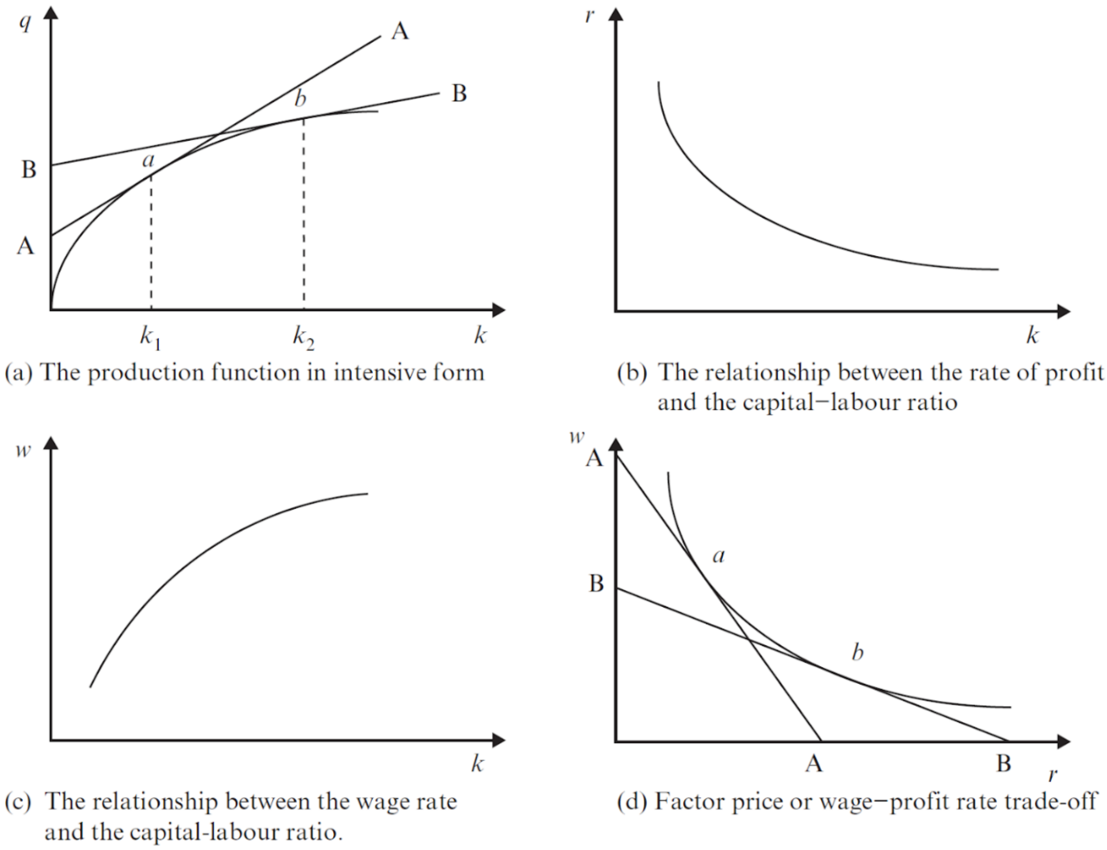

Figure 1: The neoclassical aggregate production function(NAPF) (Felipe and McCombie 2013, p. 36)

If all micro production functions adhere to these properties, aggregation should indeed result in an aggregate production function (APF) with neoclassical characteristics. This "well-behaved" APF is illustrated in Figure 1(a).

The **fundamental neoclassical postulate** asserting that lower interest rates correspond to higher capital intensity in production is considered central to all other neoclassical properties (Ferguson, 1969, p. 252). However, the straightforward aggregation of micro production functions encounters two main obstacles. Firstly, numerous assumptions are so restrictive that they fail to accurately represent any real-world economy. These limitations are critiqued in literature addressing what is termed "The Aggregation Problem" (for further discussion, see Felipe and McCombie, 2013; Brown, 1980; Felipe and Fisher, 2003).

One significant hindrance arises from the assumption that every production process can be expressed in a simple mathematical form, and thus, micro production functions exist for each firm, a condition not necessarily met (Felipe and McCombie, 2013, p. 4). Moreover, several mathematical conditions must be met for aggregation:

- Leontief (1947a, 1947b) posits that aggregation is feasible if and only if the marginal rate of substitution between two variables within the group is independent of any other input factor. Applied to production functions, this implies that the substitution possibilities of different capital goods must be unaffected by the amount of labor.
- Fisher and Monz (1992) identify a series of restrictive conditions necessary to derive an APF from micro-production functions. These conditions are so stringent that they are scarcely applicable to real economies. They require all micro-production functions to be identical, differing solely in the capital-efficiency coefficient. This entails that a labor aggregate L can only exist if a given set of relative wages induces all firms to employ an identical set of workers in the same proportion. Additionally, the absence of production specialization is mandated: all firms must produce the same set of products in the same proportions, differing only in scale. Given the disparity between these conditions and real-world economies, they can only be derived under the assumption of constant returns to scale. Beyond this scenario, deriving aggregation conditions becomes virtually impossible.

The second reason why the aggregation of micro production functions to form an Aggregate Production Function (APF) warrants critical scrutiny is the so-called **value problem**. This issue arises when attempting to aggregate a composite index of capital (*K*). In micro production functions, capital is typically measured in *physical terms*, representing the actual quantity of specific capital goods utilized in the production process. However, in order to aggregate these heterogeneous capital goods, it becomes necessary to utilize value-added terms, often expressed in *monetary units*.

In a simple production price model, Sraffa (1960) elucidates that the relative prices of capital goods are contingent upon the distribution of resources between capital and labor. In other words, the monetary values employed for aggregation are intricately linked to both the wage-rate and the profit-rate. This poses a significant challenge, as the original intent of the APF was to determine the optimal distribution of factors, namely labor and capital, to maximize production. However, by incorporating monetary terms, the value index of capital—an essential input factor of the production function—becomes subject to fluctuations in relative prices, as demonstrated by Sraffa.

Consequently, the functionality of the APF is compromised, as it no longer provides overarching insights into the optimal factor distribution based on the actual production relationships within the economy. Instead, it yields factor shares for labor and capital based on prevailing wage and profit rates at the time of analysis. This implies that the derived results may undergo significant changes with alterations in wage and profit rates.

Robert Solow (1976, p. 138) further emphasizes the impracticality of aggregating micro production functions to construct an APF:

> *"I have to insist again that anyone who reads my 1955 article (Solow 1955) will see that I invoke the formal conditions for rigorous aggregation not in the hope that they would be applicable but rather to suggest the hopelessness of any formal justification of an aggregate production function in capital and labor".*

While the idea of a simple aggregation of micro production functions to theoretically justify an APF with neoclassical properties has been rightfully abandoned mostly among economists, the APF itself, nearly always conceptualized with neoclassical properties, remained in use. An attempt for its justification, that finally led to its theoretical refutation will be described in the following section.

2.2 The General Aggregate Production Function

The microfoundation of Aggregate Production Functions (APFs) was first scrutinized in the **Cambridge Capital Controversies**, a series of debates on production theory involving scholars from Cambridge, US, and Cambridge, UK. The arguments summarized in section 2.1 highlighted the necessity for justification in continuing to utilize production functions in theoretical frameworks. Samuelson (1962) attempted to justify the APF by proposing a method to simplify a complex multiple-product, multiple-industry production system into a single mathematical function, without solely relying on aggregating micro-production functions. He initiated his analysis with the concept of a "book of blueprints," which represents a comprehensive repository of all production techniques known to society. Building upon this theoretical foundation, Zambelli (2018) follows Samuelson's argumentative framework to construct a general aggregate production function mathematically. This function, capable of mapping a unique path from input factors to output, does not necessarily exhibit neoclassical properties and warrants specific attention as it forms the theoretical basis for Samuelson's subsequent arguments, which will be reviewed in section 2.3.

The hypothetical *"book of blueprints"* is derived from input-output tables of a given economy and comprises all known production methods for each commodity. Consequently, the production of an economy is determined by the adopted methods of production and the level of employment. For a selected set of methods, the production prices necessary for the system to self-reproduce while maintaining a consistent profit rate across all sectors are determined[2] using a Sraffian production price model. This enables the definition of a **wage-profit curve** or w(r)-curve, which delineates the relationship between wages and profits associated with a given set of methods.

While there exists a wage-profit curve for every possible set of production methods, only a select few are important. This is because many of them are inherently less efficient than others across all wage and profit rates. The function that emerges from the most efficient wage-profit curves, thereby delineating the set of methods utilized at a specific profit rate, is termed the **wage-profit frontier**. When moving along this curve, the optimal set of methods shifts contingent upon the profit rate, with methods being replaced by more efficient alternatives at designated **switch points**[3]. By extracting the methods employed at any juncture of the wage-profit frontier, Zambelli elucidates the set of methods described by Samuelson as constituting the reality of heterogeneous production. The aim of Zambelli's production function is to closely approximate this reality. However, whether this production function exhibits neoclassical properties depends on the specific set of methods being considered.

[2] The latter is assumed because of market dynamics: if there were a higher profit in some industries, investors would move their capital and the difference would vanish.

[3] This can only occur when a changing rate of profit within one industry renders a method more effective than the one previously employed.

To derive an approximating production function, Zambelli utilizes the set of methods at the frontier to construct vectors for the physical frontier of net national product and physical capital. He then derives the values of capital and net national product and correlates them with the amount of labor to establish a general production function that does not necessarily imply neoclassical properties.

Even in this very general form, where the APF does not assume neoclassical properties, there are some points of criticism: First, while the concept of the **book of blueprints** avoids some problems of the “aggregating micro production functions”-approach, it still entails certain problematic assumptions. Foremost among these is the assumption that all techniques are accessible to every firm, implying that all firms within an industry are using the same most efficient method. However, this assumption is contestable, particularly in cases of monopolies or oligopolies, where firms may not have access to the same methods, or when there is incomplete information regarding production techniques. Empirical evidence, as highlighted by Leibenstein (1966, p. 401), demonstrates significant variations in producer efficiency among firms producing identical products. Additionally, as noted by Gandolfo (2008, p. 799), the assumption of perfect competition does not hold true for all industries. Consequently, the interest rate may not equal the profit rate in all cases. With the interest rate no longer determined solely by the profit rate, it becomes a component of production costs, and prices cannot be determined within the Sraffian model.

Moreover, the central issue of the value problem in aggregating capital remains unresolved. Zambelli (2018, p. 387) observes that even constructing a general aggregate production function necessitates assuming that the entries in input-output tables are measured in physical units rather than value units. However, this assumption does not hold true for capital and output, as physical units cannot be added without a scale of measurement. Samuelson (1962) not only makes this leap but also assumes neoclassical properties for the production function. He attempts to justify this with a model—the *surrogate production function*—which will be elucidated in the following section.

2.3 The Neoclassical Surrogate Production Function

2.3.1 Theory

The *surrogate* idea, already a necessary condition for the general production function, posits that the value-added aggregates for capital and output can be treated as if they were measured in physical units, even though in reality they are not. The claim is that the resulting production function can then serve as a **parable** or an **approximation** of the actual production function. However, this assumption needs justification, and Samuelson (1962, p. 196) attempts to do so by using a model that considers a "special subclass of realistic cases" within the realm of possible production functions. These cases are characterized by heterogeneous physical capital goods, each of which can be utilized with labor to produce output as well as a new stream of itself. Based on the assumption that every capital good produces itself and its associated output with the same capital-labor ratio, Samuelson derives that the wage-profit curves for each individual technique must be linear, as shown by α in *figure 2a*. Subsequently, it follows that the wage-profit frontier for the entire economy must be convex. The resulting profit frontier is illustrated in *figure 2b*. This implies a neoclassical production function with marginal products of labor and capital, a profit rate that is always negatively related to capital intensity, and a convex isoquant.

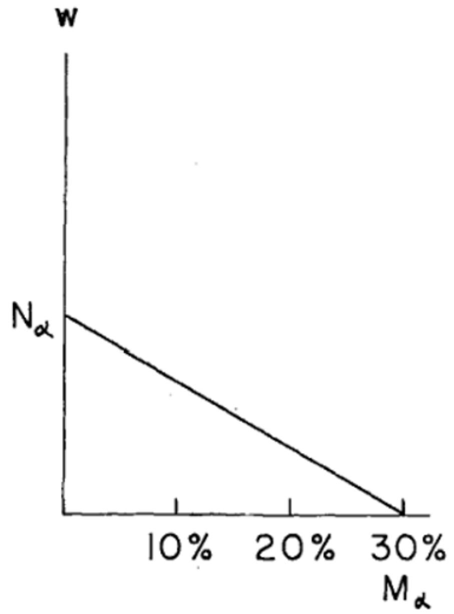

Figure 2a: The $w(r)$-curve for α (Samuelson 1962, p. 197)

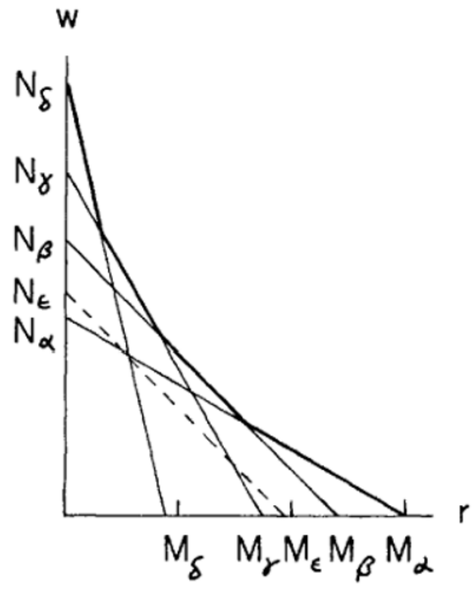

Figure 2b: the convex wage profit frontier resulting from straight $w(r)$- curves (Samuelson 1962, p. 197)

Therefore, Samuelson argues that a one-capital good world can be used as a parable for the underlying more complex economy with heterogeneous capital goods.

2.3.2 The Refutation

Samuelson's construction of a parable to theoretically justify the use of a NAPF (as shown in section 2.3.1) has been refuted in the course of the Cambridge Controversies. Felipe and McCombie (2013) break it down to the simple point that: *"Samuelson's result depends on the assumption that both sectors making up the single technique have identical capital-labor ratios. This means that, to all intents and purposes, we are still in a one-commodity world. As soon as any attempt is made to generalize this assumption, the neoclassical parable breaks down" (Felipe and McCombie 2013, p. 38)*

Heine and Herr (2013, p. 255) comment that in fact Samuelson has not, as he had originally planned, shown that a production function with neoclassical properties can depict a world with heterogeneous capital goods. On the contrary, he has unintentionally demonstrated under which unlikely circumstances a neoclassical production function can be assumed, namely in a one-commodity world.

Once the assumption of similar capital-labor ratios is dropped, wage-profit curves can be concave, convex, or linear. This has groundbreaking theoretical consequences, which will be illustrated with the example of a special case that has garnered much attention in the Cambridge debates. When wage-profit curves do not necessarily all share the same shape, two wage-profit curves can have more than one intersection. This means that one technique (see figure 3, upper quadrant, curve *CC*) can be more profitable than another (*DD*) at both high and low interest rates, while the other technique is more profitable at intermediate interest rates. As the same two wage-profit curves switch again at a second point, this phenomenon is known as **reswitching**.

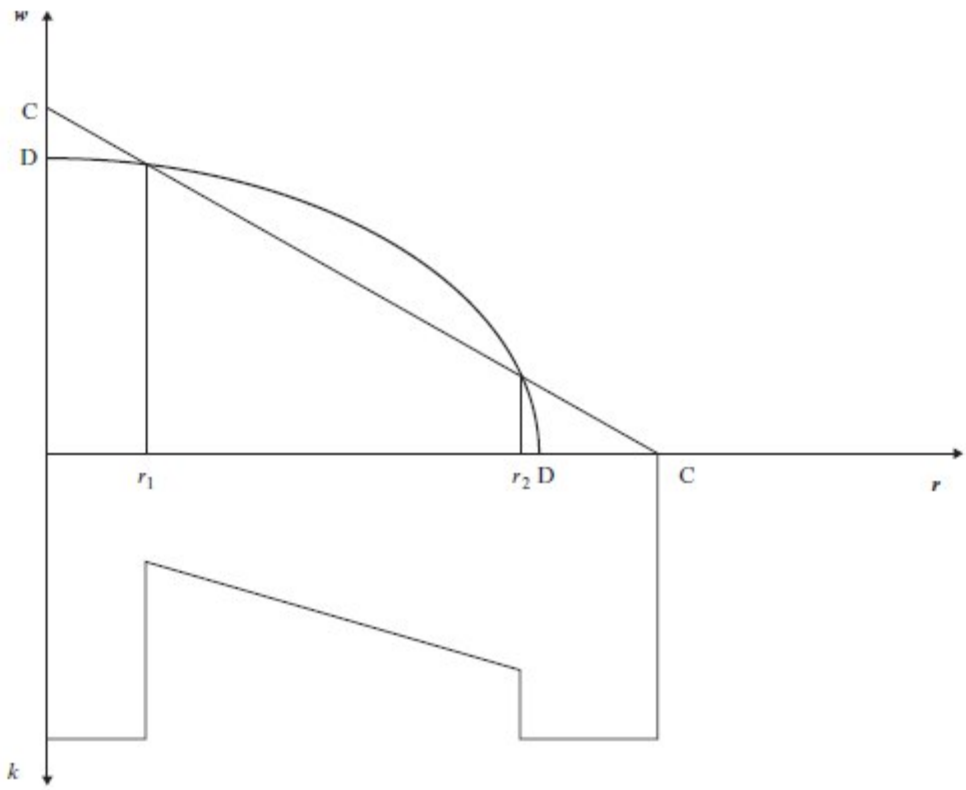

Figure 3, Reswitching and capital reversing (Felipe and McCombie 2013, p. 39)
(CC and DD being $w(r)$-curves, k being the amount of capital)

Reswitching causes but is not the only possible cause of **capital reversing**, which occurs when the physical (or value of) capital increases with a rising profit rate and/or quantity of labor (see figure 3, lower quadrant) [4]. This clearly contradicts the neoclassical postulates implied by the Neoclassical Aggregate Production Function (NAPF).

Reswitching became a pivotal point in the debate when Levhari (1965) attempted to demonstrate that it could not occur in a scenario where every output requires every other output as an input, directly or indirectly, in order to salvage the production function. However, as Pasinetti (1966) and others pointed out, this assumption holds true only in a few special

[4] The literature is somewhat unclear in its terminology, as we encounter various meanings for the terms *capital reversing*, *reverse capital deepening*, and *price Wicksell effects*. Zambelli (2018) defines the positive correlation between capital and the profit rate as *capital reversing* or *price Wicksell effects*, and a subset of cases where two wage-profit curves switch a second time as *reswitching* or *reverse capital deepening* (which implies the occurrence of *capital reversing*). Meanwhile, Petri (2011) employs *reverse capital deepening* for the positive correlation and *price Wicksell effects* for the subset of cases where the correlation is not caused by *reswitching*. This study will adopt the definitions provided by Zambelli (2018, p. 392).

cases. The debate concluded with Levhari and Samuelson (1966) conceding the logical validity of reswitching.[5]

Petri (2011, p. 404) notes that the mere possibility of reswitching suffices to invalidate the conception of capital as a single production factor[6] and, therefore, the theoretical validity of the APF. This immediately precludes the use of APFs to validate other theoretical relationships, constructs, or models on their own. It also eliminates the possibility of basing a general approach to value and distribution on this model, which is the primary method utilized in theoretical neoclassical macroeconomic theory to explain the level of production, the capital-labor ratio, and the distribution.

While the possibility of reswitching has not been heavily contested since 1966, the likelihood of reswitching has been the subject of some studies and disputes. These will be discussed later in this work (in section 4.1), as they are relevant when examining the usefulness of NAPFs as empirical tools. However, this debate is irrelevant here, as even a low to non-existent probability of reswitching could not salvage the theoretical construct of a production function.

2.4 Conclusion

The consistent construction of a theoretically sound Aggregate Production Function (APF) based on a world with heterogeneous capital goods has yet to be demonstrated and appears unlikely to be feasible. Zambelli (2018) comes close to this goal with the general aggregate production function constructed in section 2.2, but even for this model, he has to resort to assuming a surrogate capital, which ultimately overlooks the value problem, a simplification that is difficult to justify.

Even if a *surrogate capital* can be accepted, the APF cannot be assumed to possess neoclassical properties in every case. When transitioning to a model with multiple capital goods, reswitching and reverse capital deepening can occur. Samuelson's wage-profit frontier cannot be regarded as a justification for this, as it theoretically operates within a one-commodity world and collapses when heterogeneous capital goods are assumed.

[5] For a more in-depth summary of the debate see Birner (2002).

[6] For further expositions on why that is the case see Petri (2011).

While the APF as a theoretical construct is thus found to be unjustified, the Neoclassical Aggregate Production Function (NAPF) as an empirical tool might still retain functionality. The argumentative support for this conclusion must stem from empirical results demonstrating that production function tools can indeed prove useful in models. The primary defence presented in the course of the Cambridge Controversy follows precisely this instrumentalist path, as exemplified by Solow (1966, p. 1259 - 1260):

> *"I have never thought of the macroeconomic production function as a rigorously justifiable concept. In my mind, it is either an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it doesn't, or as soon as something better comes along."*

The defense for using the production function lies in its ability to provide reasonable approximations and plausible estimates of reality, thereby rendering a theoretical foundation unnecessary for it to serve as a useful empirical tool.

Subsequent sections will delve into a closer examination of the argument supporting this pragmatic justification. Firstly, methods for directly testing production functions will be reviewed. However, as these methods will be shown to either be dysfunctional or to provide insufficient evidence regarding the usefulness of production functions, the fourth section will explore various discussions on how to validate or invalidate the functionality of Aggregate Production Functions (APFs).

3 Directly testing NAPFs

Which production functions are mostly used in practice? Zambelli (2018, p. 384) finds that *"[d]espite a widespread acknowledgement that aggregation could be problematic, the (generalized) Cobb–Douglas production function is widely used in various models across theories and remains a fundamental tool for the empirical assessment of technological progress and productivity growth."*

The mentioned **Cobb-Douglas production function** generally takes the form of

(3.1) $$Q = AL^{\alpha}K^{\beta}$$

With A, L, K and Q being productivity growth, labor input, capital input and output respectively and α and β being the output elasticities of capital and labor (with $\alpha + \beta = 1$ for constant returns of scale to hold) (Douglas 1948, p. 6). The Cobb-Douglas production function assumes neoclassical properties and therefore suffers from the problems listed in the previous parts of this study.

As the historically most utilized and discussed production function, and also the foundation for many advanced production functions, the Cobb-Douglas production function will be thoroughly examined in this study. It will be used to illustrate the challenges associated with Neoclassical Aggregate Production Functions (NAPFs) in general.

While there exist other advanced NAPFs utilized in theories, such as the CES-production function, the translog-function, and the box cox transformation, delving deeply into whether and how the argument changes when these advanced production functions are employed exceeds the scope of this work. However, most of the arguments presented are applicable to all APFs that assume neoclassical properties. In cases where this assumption does not hold, a paragraph will be dedicated to addressing that issue. The subsequent chapters will solely focus on NAPFs because, to the best of my knowledge, there are no instances of non-neoclassical production functions being used as empirical tools[7].

3.1 How can NAPFs be tested? The most influential Method

First and foremost, the relevance of the value problem when testing production functions is a crucial consideration. Not only does the Aggregate Production Function (APF) face challenges, but the very notion of a single aggregate production factor—capital—is also undermined by the value problem. Capital cannot be consistently aggregated while maintaining its status as a physical coefficient. Therefore, the task of seeking argumentative support for Neoclassical Aggregate Production Functions (NAPFs) as empirical tools entails finding plausible reasons why a NAPF model, which maps a unique pathway from a *value-added* capital coefficient and

[7] While discussing the reasons for this phenomenon is beyond the scope of this work, a speculative explanation could be that production functions lacking neoclassical properties may not yield unambiguous solutions.

a labor coefficient to a *value-added* output, could yield accurate results. Given that the theory provides no indication of such plausibility, this becomes an empirical undertaking.

Various attempts have been made to test whether a NAPF such as the Cobb-Douglas function can accurately represent the production processes of an economy. Firstly, this work will explore a method commonly used by Douglas (1948, 1976) and others[8]. This method assesses how well data from input-output tables can be approximated with a Cobb-Douglas Production function. Studies utilizing this method have often been used to justify the application of a NAPF. However, questioning the validity of this justification strategy, section 3.2 will demonstrate why this method is not an effective way to test NAPFs.

In his 1948 paper titled "Are there laws of production?", Paul H. Douglas presents a collection of twenty-nine studies that attempt to estimate a Cobb-Douglas production function for various regions of the world across multiple time periods. In his subsequent work in 1976, Douglas replicates these studies for industries in Australia, spanning more recent years and encompassing a total of 4,681 cross-sectional observations. These studies assume a Cobb-Douglas production function in the following form to estimate the "underlying" production function:

(3.2) $$V_i = A_o {L_i}^{\alpha} {J_i}^{\beta}$$

Where V_i and J_i are output and capital for the i-th firm, both value-added at constant prices, L_i is the labor input and α and β are the output elasticities of labor and capital respectively. A_o, the technical progress of the economy, is assumed to be equal over all observations as we deal with cross-sectional data.

Then, data from input-output-tables is used in order to compute the values for A_o, α and β that result in the closest match to the data. This data is first used to draw conclusions from the production function estimates in terms of the estimated values of α and β as well as $\alpha + \beta$ which indicates whether there are increasing (when > 1), decreasing (when < 1), or constant (when $= 1$) returns to scale.

[8] Examples are Liu and Hildebrand (1965); Moroney (1972).

The aim is to evaluate the accuracy of the production function by comparing its estimate with the actual data. One simple measure to gauge how closely the estimate reflects the data is the R^2-value of the respective production function regression. This value indicates the proportion of variation in the output level that can be explained by changes in the input factors considered in the regression function, namely labor and capital. Douglas (1948, 1976) conducts a more detailed analysis of the goodness of fit by comparing the estimated output levels for each data point with the actual output levels. Additionally, he scrutinizes the regression residuals, which represent the points where the estimate deviates from the actual output.

Another test performed by Douglas (1948) involves comparing the output elasticities of labor and capital with the actual factor shares attained by labor and capital owners in the economy. This test is rooted in the neoclassical theory of factor pricing, which posits that, under ideal conditions, the marginal product of labor equals the wage rate (likewise for the marginal product of capital and the profit rate):

$$\frac{\partial V_i}{\partial L_i} = w_i \qquad (3.3)$$

$$\frac{\partial V_i}{\partial J_i} = r_i \qquad (3.4)$$

Consequently, it follows that the value-added factor shares are equal to the output elasticities[9]:

$$\alpha = wL/V \qquad (3.5)$$

$$\beta = rJ/V \qquad (3.6)$$

Under the conditions of perfect competition and constant returns to scale, comparing the estimated output elasticities with the actual observed factor shares can serve as a test for one of two scenarios. Firstly, if the production function estimate is assumed to accurately reflect

[9] These output elasticities are calculated under the assumption of $\alpha + \beta = 1$, which Douglas considers to represent constant returns to scale. This assumption is not arbitrary, as we will explore later. Constant returns to scale are implied in the accounting identity when constant factor shares are presumed. Additionally, Douglas observes that the $(\alpha + \beta)$ values are consistently close to 1 across all cases.

the actual output elasticities[10], similar factor shares and output elasticity estimates would provide empirical support for the marginalist theory of factor pricing, as the actual output elasticities would align with the factor shares. Secondly, assuming the validity of the marginalist theory of factor pricing, the method can be utilized to test the accuracy of the production function. In this case, the factor shares are presumed to represent the actual output elasticities, and when the estimated output elasticities closely match the actual ones, it validates the estimate.

Douglas finds values for α ranging from 0.55 to 0.85 across various regions, with some outliers, while β typically ranges between 0.15 and 0.45. Notably, $(\alpha + \beta)$ consistently approximates 1 across different studies, suggesting constant returns to scale.

Furthermore, the testing results for the NAPF appear promising. Douglas observes that actual output values fall within two standard deviations in 95% of cases and within three standard deviations in 99% of cases. Additionally, the R^2-values mentioned in the study indicate an exceptionally good fit, consistently above 0.9, aligning with results from similar studies[11].

When assessing the closeness between output elasticities and factor shares, Douglas finds strong fits. He interprets these results as confirmation of the marginalist theory of factor pricing, and suggests that the remarkable goodness of fit further validates the production function itself. However, Douglas acknowledges that the assumption of perfect competition does not align with reality, raising questions that require further research. While these results, replicated in similar studies, suggest that a NAPF can accurately depict the production process, it's important to note that this instrumentalist defence of the production function, based on the achieved results with this method, is fundamentally flawed. Its limitations will be elucidated in the following section.

[10] For instance, the values that describe the actual values that would be added on the output with an additional unit of input.
[11] See Felipe and McCombie (2013, p. 11)

3.2 Critique

A version of the critique that challenged the effectiveness of the method was initially proposed by Phelps Brown (1957) as a direct response to Douglas (1948). Later, Simon (1979) and Shaikh (1974, 1980) contributed significantly to its reinforcement.

This section will adhere to the arguments presented by Felipe (1998) and Felipe and McCombie (2013), outlining the case against the Cobb-Douglas production function, initially with cross-sectional data and then with time-series data.

In summary, the critique unfolds as follows: All value-added data must adhere to the underlying cost accounting identity *TC = V = wL + rJ*.[12] However, with the assumption of constant factor shares, the general Cobb-Douglas production function, employed to model the production function in Samuelson's model, essentially transforms into a reformulated version of the accounting identity. Consequently, regardless of the degree of competition, the presence of increasing or decreasing returns to scale, or the potential for data approximation with a NAPF, all value-added data must conform to the structure of the accounting identity. Therefore, when factor shares remain roughly constant, testing the estimated Cobb-Douglas production function invariably yields high R^2-values because the value-added Cobb-Douglas production function model simply mirrors the expected pattern dictated by the accounting identity.

Under these circumstances, this test fails to discern whether the Cobb-Douglas production function accurately reflects the actual physical reality of the economy, as consistently favorable results are obtained irrespective of its correspondence with reality. The subsequent sections will provide a mathematical exposition and delve deeper into the argument for both cross-sectional and time-series data.

[12] With TC, V, w, L, r and J being total costs, value-added output, wage rate, the amount of labor, profit rate and the value-added amount of capital respectively. It is important here to differentiate between the neoclassical accounting identity $pQ = wL + rK + \Omega$, which has to assume profits Ω to equal 0 to hold and the virtual accounting identity $TC = V = wL + rJ$ which always holds. In this case, the virtual accounting identity is used. The argument that the Simon/Shaikh critique only applies under the assumption of profits equal to 0 is therefore wrong. (Felipe and McCombie 2013, p. 63)

3.2.1 Cross-sectional data

The Cobb-Douglas constant returns to scale production function measured in value terms is defined as

(3.7) $$V_i = A_o {L_i}^{\alpha} {J_i}^{1-\alpha}$$

with A_o, the technical progress of the economy, being assumed equal over all observations[13]. Its output elasticity with respect to labor is defined as $\alpha = \frac{\partial V_i}{\partial L_i}\left(\frac{L_i}{V_i}\right)$

With the assumptions of the neoclassical theory of factor pricing, the marginal product of labor equals the wage rate:

(3.8) $$\frac{\partial V_i}{\partial L_i} = w_i$$

The cost accounting identity for the i-th firm is given by

(3.9) $$V_i = w_i L_i + r_i J_i$$

with V being the output, w and r being the wage rate and the *observed* rate of return. It simply states the fact that the value of all output that is generated by a firm has to be distributed between labor income and capital income. Partially differentiating the accounting identity with respect to L_i gives $\frac{\partial V_i}{\partial L_i} = w_i$, it follows that:

(3.10) $$ln\frac{Vi}{Vj} = \alpha ln\frac{Li}{Li\prime} + \beta ln\frac{Ji}{Ji\prime}$$

This result for the factor share for labor exactly equals the putative output elasticity for labor derived from the production function and the neoclassical theory of factor pricing. But this time, it is derived from an identity, that, again, is true independently from the degree of competition, no matter whether there are increasing, constant or decreasing returns to scale and no matter whether an APF, which can sufficiently approximate the complexities of an underlying economy, actually exists. Therefore, the data always will show the factor shares to

[13] This is because we deal with cross sectional data.

be approximately the same as the estimated output elasticities, which means this is not a rejectable hypothesis to test anything anymore. Douglas' confusion about the extremely good results of this test when the reality did not come close to match the assumption of perfect competition, can be resolved with this. He tested neither the marginalist theory of factor pricing nor his estimated NAPF, but he just pointed towards a tautology.

The next step is proving that the whole Cobb-Douglas production function is simply an alternative way of expressing the accounting identity and therefore has no implications for the underlying economy, following Felipe and McCombie (2013).
We start from a Cobb-Douglas-function:

(3.11) $$V_i = A_o {L_i}^{\alpha} {J_i}^{\beta}$$

To prepare this for a Taylor-series approximation, we divide it by $V_i' = A_o {L_i'}^{\alpha} {J_i'}^{\beta}$ and take logarithms:

(3.12) $$ln\frac{V_i}{V_i} = \alpha ln\frac{L_i}{L_i'} + \beta ln\frac{J_i}{J_i'}$$

X_i' denotes that the value of the variable in the neighbourhood of X_i where X is V, L and J. The ratio between X_i and X_i' is arbitrarily close to 1. Using the Taylor-series approximation that $ln\frac{X_i}{X_i'} \approx (\frac{X_i}{X_i'} - 1)$, at the tangency of a plane to equation (3.11) this equation can be expressed as:

(3.13) $$V_i = \left(\alpha ln\frac{V_i'}{L_i'}\right) L_i + \left(\beta ln\frac{V_i'}{J_i'}\right) J_i + (1 - \alpha - \beta) V_i'$$

At the same time, a Taylor-series expansion on the accounting identity (3.9) and the assumption of constant wages and a constant rate of profit gives:

(3.14) $$V_i = V_i' + w(L_i - L_i') + r(J_i - J_i') = wL_i + rJ_i + [1 - a - (1 - a)]V_i'$$

The comparison of equations (3.13) and (3.14) shows that $\alpha = wL/V$ and $\beta = rJ/V$. Also, it follows that $(1-\alpha-\beta)V = 0$ so $\alpha+\beta = 1$.

Therefore, the data will always suggest 'constant returns to scale', no matter the actual technological relationships of the economy. Furthermore, this result shows that the linear accounting identity will ensure that the method will always give a good fit to the Cobb-Douglas, even when a Cobb-Douglas relationship does not exist.
Felipe and McCombie visualize this:

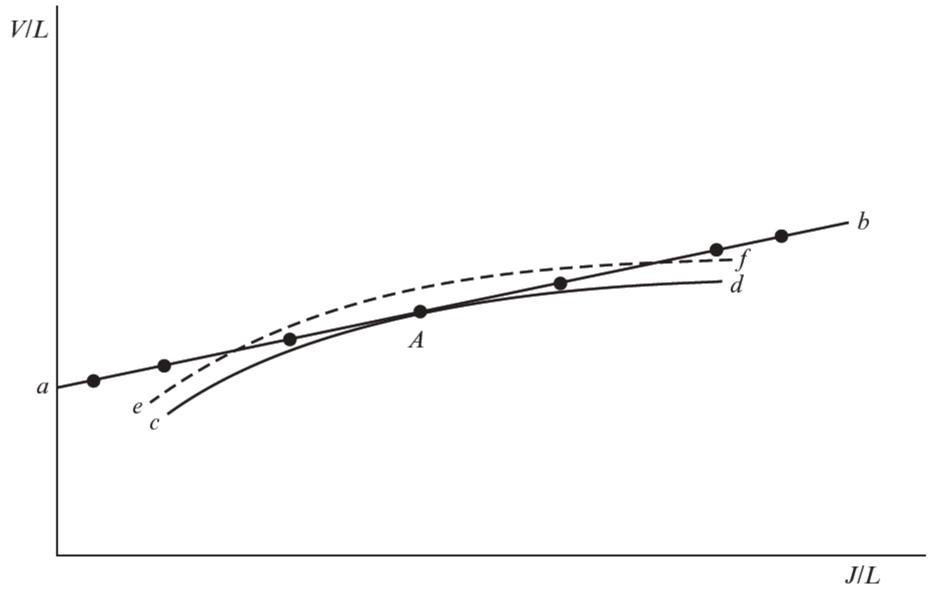

Figure 4: The Cobb-Douglas approximation to the linear accounting identity (Felipe and McCombie 2013, p. 55)

The line *ab* represents the accounting identity (simplified; in reality, there might be some roughly parallel lines above and below ab for different firm accounting identities), while the curve *cd* represents the Cobb-Douglas production function for an exemplified firm. The dotted line *ef* represents the Cobb-Douglas function likely to be estimated and considered a good fit to the data.

Consequently, the implication is that the ability to fit the data well with a Cobb-Douglas production function cannot be construed as evidence for anything substantive. Even in scenarios where the relationships in the physical data diverge from those assumed by the Cobb-Douglas model, the "test" would still yield high R^2-values and good fits when comparing actual outputs with estimated outputs, as the Cobb-Douglas model would merely approximate the accounting identity. Therefore, this method cannot effectively assess the validity of NAPFs.

The implication of this is that the fact that the data can be fit well by a Cobb-Douglas production function cannot be taken as evidence for anything anymore. This is because even when there would not be the relationships in the physical data that the Cobb-Douglas assumes, the 'test' would still give good R^2-values as well as good fits when comparing actual outputs with estimated outputs as the Cobb-Douglas would just approximate the accounting identity. This method therefore cannot be used to test NAPFs.

3.2.2 Time-series data

The argument follows through for time-series data as well. Following the reasoning of Felipe (1998), the cost accounting identity at time t is given by:

(3.15) $$TC_t = V_t = w_t L_t + r_t J_t$$

where TC, V, w, r, L and J denote total costs, value-added output, wage rate, profit rate, level of employment and value-added stock of capital, respectively.
In growth rates, this is:

(3.16) $$\dot{v}_t = v_t \dot{w}_t + (1 - a_t)\dot{r}_t + a_t \dot{l}_t + (1 - a_t)\dot{j}_t$$

where a denotes the labor share in output and the dots denote growth rates. If constant factor shares are assumed ($a_t = a$), after a Substitution and an Integration, we obtain:

(3.17) $$V_t = A w_t^{\alpha} r_t^{(1-\alpha)} L_t^{\alpha} J_t^{(1-\alpha)}$$

If it is further assumed that wages and profit rate grow at the constant rate ϕ_w and ϕ_r, respectively, then equation (3.17) can be written as

(3.18) $$V_t = B e^{[a\phi_w + (1-a)\phi_r]} L_t^{\alpha} J_t^{(1-\alpha)}$$

with A and B as constants. This is identical with the Cobb-Douglas function with constant returns to scale, but it is directly derived from the accounting identity. Just as with cross sectional data, if we estimate a Cobb-Douglas production function in logarithms, i.e.:

(3.19) $$lnV_t = c + \phi_t + \alpha lnL_t + \beta lnK_t + v_t$$

with v_t as the error term, α and β as the parameter elasticities ϕ_w and ϕ_t being interpreted as the growth of the total factor productivity, we should expect a perfect fit, with α and β being equal to the constant shares of labor and capital. This is given the two assumptions about constant factor shares and constant wages and rate of profit are true.

The assumption of constant factor shares would be expected to be fulfilled at a point in time across different industries if we would assume a neoclassical world and a working Cobb-Douglas aggregate production function. But there are many more reasons why constant factor shares should occur. Felipe and McCombie (2013, p. 57) suggest that if firms pursue a constant mark-up pricing policy, this is given regardless of the underlying production relations. Post-Keynesian macroeconomic Theory of Distribution will also give this result without the use of an APF.

3.3 Further Debates on this Method

A few further questions and debates will be covered in this section. First, some empirical confirmations of the refutation will be reviewed to strengthen the claim that Douglas's method is refuted. Then, some empirical results that might provide an indication of the functionality of NAPFs will be considered.

Anwar Shaikh conducted simulations of this method on multiple occasions to confirm the critique. In his works from 1974, 1980, and 1990, Shaikh demonstrates that a Cobb-Douglas production function can fit well even when the theoretical conditions for NAPFs are blatantly violated. For instance, in Shaikh (1990), he considers an anti-neoclassical "Robinsonian" economy with fixed proportion technology and Harrod-neutral technical change. Despite not aligning with the assumptions of the Cobb-Douglas function, the economy appears consistent with an APF due to the accounting identity. Shaikh concludes by highlighting that even physical data points spelling out the word "HUMBUG" can be well-fitted by a Cobb-Douglas production function, as long as their profit and wage rates remain constant, further demonstrating that this test lacks implications about the function's ability to represent the actual underlying economy.

A question arising from the argument that this method can only yield good results by design is why there are occasional outliers in the form of low R^2-values and poor fits of the Cobb-Douglas Function. Sylos Labini (1995, p. 490) provides examples of studies with poor fits, which may seem to challenge the argument. However, the point of the argument is not that it is impossible to obtain bad fits for a Cobb-Douglas production function, but rather that the attainment of good fits depends on factors other than whether the underlying economy conforms to the assumptions of the Cobb-Douglas function.

There are multiple reasons why there might be poor statistical fits. One possible reason is that the assumption of constant factor shares is not sufficiently met. While this could account for some instances, Felipe and McCombie (2013, p. 77) identify the main reason as the inadequacy of approximating the weighted logarithm of the wage rate and the rate of profit (or their growth rates) with a linear time trend (or a constant). This issue is further discussed in Chapter 10 of Felipe and McCombie (2013).

Felipe and Fisher (2003, p. 256) compile a variety of studies that test the discussed production function regressions, concluding that *"[a]t the empirical level, contrary to widespread belief, [...] production functions, when estimated econometrically, tend to yield, in general, poor results [...]"*.

They base this assertion on the findings of McCombie (1998) and Felipe and Adams (2005), who subjected the dataset originally used by Cobb and Douglas (1928) to numerous stability tests, revealing the regression to be highly fragile. Temple (1998) also demonstrated the regression by Mankiw et al. (1992) to lack robustness. Moreover, Felipe and Fisher (2003) observed that the regression produced very poor results when a linear time trend was added to account for technical progress. They conclude that *"[w]ith today's econometric tools, nobody would conclude that this data set indicates that the elasticity of labor was 0.75 and* *that of capital 0.25 in the USA during the period analyzed"* [by (Douglas 1976, 1948), L.P.] (Felipe and Fisher, 2003, p. 256).

Furthermore, the finding that the functions best fitting the data are not neoclassical has already been recognized as a common outcome by Shaikh (1990, p. 191), citing Fisher (1971) and Walters (1963).

Finally, it is worth considering how this critique applies to other production functions besides the Cobb-Douglas. Felipe and McCombie (2013, pp. 84–98) demonstrate mathematically that similar issues arise with CES and translog production functions as well as the box cox transformation. Surprisingly, these alternative functions often yield even better results than the Cobb-Douglas production function. Felipe and McCombie attribute this to their greater flexibility, allowing them to better approximate the accounting identity. The box cox transformation, for example, can provide good fits with varying factor shares. For further details, refer to Felipe and McCombie (2013), Appendix A2.

3.4 Conclusion

Let's reconsider the quote from Solow regarding the production function: *"... [E]ither an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it doesn't, or as soon as something better comes along."* (Solow, 1966, pp. 1259–1260)

The results discussed in section 3.2 strongly suggest that the method of comparing an estimate of a neoclassical production function with actual economic data is unable to provide a meaningful test for determining whether a production function yields good results. This method, while influential, produces the least ambiguous outcomes. Its refutation implies that theorists must rely on alternative methods or indicators to demonstrate that neoclassical aggregate production functions (NAPFs) yield the "good empirical results" referred to by Solow. However, this method cannot be used to disprove the validity of NAPFs. Conversely, stability tests yield unfavorable outcomes for the functionality of NAPFs. Further efforts to find empirical evidence supporting or refuting NAPFs will be examined in the following chapter.

4. The Neoclassical Postulate - a way to empirically support the NAPF?

So far, this study has demonstrated that directly attempting to compute a neoclassical aggregate production function (NAPF) through regression fails due to the constraints imposed by the accounting identity. This section will now explore alternative methods for testing the underlying neoclassical assumptions, particularly focusing on the postulate proposed by Ferguson (1969, p. 252) [14] and the derived conclusions. This approach represents a reversal of the traditional direction of justification seen in established theoretical discussions: whereas neoclassical theory typically uses the theoretical construct of the NAPF to justify the neoclassical postulate and construct the neoclassical system, here, the supposed empirical reality of the neoclassical postulate will be leveraged to justify the APF as an empirical tool.

Sato (1974), e.g., uses this justification when stating

> "[...] *that there is a not-too-small world in which the neoclassical postulate is perfectly valid. So long as we live in that world, we need not to give up the neoclassical postulate. In order to refute it, it is necessary to demonstrate that this world is imaginary. This demonstration has not been supplied in the literature. Nonetheless, it is important to realize that there is another world in which the neoclassical postulate may not fare well or is contradicted. An empirical question is which of the two models is more probable."*
> (Sato 1974, p. 383)

If the neoclassical postulate can be empirically validated within the framework established by Sato, then a NAPF might prove to be functional. However, this assertion hinges on whether the relationship between the interest rate and capital intensity is primarily driven by *substitution mechanisms within the production system*. Should alternative causes exist for this relationship, it would merely constitute a correlation, rendering the production function valuable only in illustrating that correlation. Moreover, erroneous assumptions regarding causality could potentially diminish the utility of a model.

To assess whether this argument can provide empirical support for or against NAPFs, we will first review the empirical data regarding the neoclassical postulate. This involves examining

[14] Which is that, "[…] the lower the rate of interest, the greater the capital intensity of production. All other neoclassical properties follow immediately from this simple relation." (Ferguson 1969, p. 252.)

the ongoing debate surrounding the likelihood of reswitching and capital reversing. Subsequently, we will analyze how this evidence may contribute to our understanding of the functionality of NAPFs.

4.1 Empirical Evidence on the Neoclassical Postulate

Studies on the probability of reswitching and reverse capital deepening serve as a means to test the neoclassical postulate, given that reverse capital deepening precisely contradicts this postulate. Reswitching, although historically contentious, is often examined as it is one potential cause of capital reversing, albeit not the only one. It's important to note that while studies revealing a high probability of reswitching are significant, as they inherently imply a high likelihood of capital reversing, the reverse is not necessarily true. A low probability of reswitching does not directly imply a low probability of capital reversing. Conversely, a low probability of **capital reversing** can be construed as supporting evidence for the neoclassical postulate.

Various approaches exist for measuring the likelihood of reswitching and capital reversing. One method involves a pseudo-empirical approach, where economies are simulated with randomly generated production sectors to observe the occurrence of reswitching and capital reversing. Zambelli (2004), for instance, undertook this method by simulating 21,000 economies. By constructing a book of blueprints from randomly chosen numbers and applying simple rules to ensure productivity and bounded physical surplus, Zambelli then tested these simulated economies against neoclassical postulates.

Zambelli found that reswitching occurred sporadically on the wage-profit frontier in only nine out of the 21,000 simulated economies. However, the probability of capital reversing was notably high, with the neoclassical postulate being violated in 50% of cases. In other words, in half of the simulated economies, the value of capital increased with a rising profit rate at certain points along the wage-profit frontier. Other simulation studies yield mixed results: Pertz (1980) observed a declining probability of reswitching with an increasing number of sectors in the modeled economy, while Ahmad (1991) found the opposite. Mainwaring and Steedman (2000), however, concluded that reswitching is highly improbable in a two-sector model.

Petri (2011, p. 401) casts doubt on the validity of results obtained through simulation methods due to the significant influence of arbitrary underlying assumptions on the study's outcome. He highlights the risk that altering these assumptions could yield almost any result, thus questioning the reliability of simulation-based findings.

Another method involves analyzing input-output tables to empirically test the likelihood of reswitching. Han and Schefold (2006) utilize OECD data for sectors of nine countries spanning from 1968 to 1990. Their analysis reveals reswitching to be extremely rare, with only one of the 32 input-output tables showing evidence of reswitching. Similarly, they find the probability of capital reversing to be quite low, at just over 1 percent.

Zambelli (2018), on the other hand, employs a broader dataset consisting of input-output tables covering 31 sectors, 30 countries, and 17 years, which includes the data used by Han and Schefold (2006). Zambelli computes and compares *w(r)*-curves, constructs a wage-profit frontier, and calculates aggregate values for capital, labor, and production using the most general production function introduced in section 2.2. However, Zambelli's results diverge significantly from those of Han and Schefold (2006), as he finds no reliable support for the neoclassical postulate to hold.

Results obtained from input-output tables are criticized by Gandolfo (2008, pp. 799–800) due to encountering the value problem once again. The input-output approach relies on assuming constant "technical coefficients of production," which are approximated by the value-added data from the input-output tables. However, with changes in the wage and profit rates, only the value-added data changes, while the technical coefficients remain constant. This raises concerns that findings supporting the neoclassical postulate may be merely artifacts of the cost accounting identity, which must apply to all value-added data under the assumption of constant factor shares.

Han and Schefold (2006, p. 750) defend their results by arguing that the monetary values in the input-output tables reflect the physical structure, as distribution and relative prices undergo minimal changes. However, Felipe and McCombie (2013, p. 42) find this defence unconvincing, as it relies on an assumption. The fundamental issue, wherein changes in the distribution of wages and profits impact the input-output tables, remains unresolved.

Furthermore, many studies observe *w(r)*-curves to exhibit near linearity (Ochoa 1989; Petrović 1991; Tsoulfidis and Maniatis 2002; Han and Schefold 2006). This suggests that the likelihood of capital reversing due to reswitching, as well as due to other reasons, is relatively low. This aligns with Samuelson's model of a concave wage-profit frontier, constructed from linear *w(r)*-curves[15].

4.2 How reinforcing are these Results for the NAPF?

While the results regarding the neoclassical postulate are varied, the extent to which positive findings could reinforce the application cases of the NAPF remains uncertain. Generally, this depends on the specific purposes for which the NAPF is employed, but some general observations can be made. The relationship posited by the neoclassical postulate might be influenced by factors other than the neoclassical substitution mechanisms presumed in the NAPF. The assumption that switches in techniques occur based on the interest rate is vigorously challenged by Keynesians such as Robinson (1979): *"Even if there was such a thing as a pseudo production function, there would be no movement along it to pass over switch points, and furthermore, in reality, there is no such thing as a pseudo-production function"* (Robinson 1979, p. 82)

If factors other than those presumed by the neoclassical postulate were responsible for the correlation, it would imply that a production function could only depict this correlation without adding any substantive value to the model. While merely illustrating such a correlation might not detrimentally affect the model, any attempt by the production function to imply causation between the purported causes and effects would render it flawed. There are several proposed explanations for the origin of this correlation. When utilizing value-added data, the cost accounting identity might underlie this relationship. Furthermore, Nell

[15] See 2.3.1; Indeed, while the evidence for linear wage-profit curves may appear to bolster the theoretical model of Samuelson's surrogate production function, its significance lies solely within the debate surrounding the production function as an empirical tool. It's crucial to note that the surrogate production function model has already been refuted with the undeniable possibility of reswitching. Therefore, the presence of linear wage-profit curves does not rehabilitate the surrogate production function model from this fundamental challenge.

(2022) contends that the correlation between the interest rate and capital intensity arises from the financial system rather than from technical shifts in the production sector.

Additionally, Petri (2011) observes that even if the neoclassical postulate holds true most of the time, primarily due to the substitution mechanisms presumed in the production function, the resulting effects may not align with those posited by neoclassical theory. This is because the investment function not only needs to exhibit a decreasing trend, indicating minimal signs of the neoclassical postulate being violated, but also requires *sufficient elasticity*. The rationale for this is illustrated in figure 5, where it becomes apparent that with lower elasticity, the potential equilibrium interest rates can vary significantly.

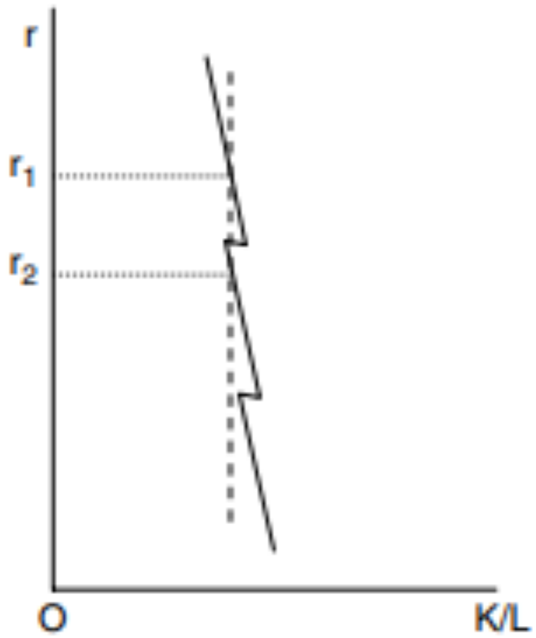

Figure 5: the investment curve with some cases of reswitching and a low elasticity (Petri 2011, p. 405)

While none of the authors who estimated the probability of reswitching have analyzed the elasticity of the value of capital per unit of labor with respect to the profit rate *r*, there is evidence on straight *w*(*r*)-curves that points towards a low elasticity. Additionally, the investment function merely determines the desired value-capital/labor ratio. This only transforms into a demand-for-value-capital function if full employment of labor is assumed. In the presence of unemployment, a given capital/labor ratio renders investment indeterminate.

In both scenarios—whether due to an insufficiently elastic investment function or unemployment—the NAPF cannot establish equilibrium in an investment-savings market. Whether this poses a problem for the NAPF depends on the context in which it is utilized.

5 Conclusion

For a final conclusion, let's revisit the research question: **"Can (neoclassical) aggregate production functions be a functional tool for macroeconomic theory and practice?"**

The theoretical construct of the production function has rightfully been abandoned by many macroeconomists in the course of the Cambridge Capital Controversy. The use of production functions in theory-building should be viewed critically, as, up to this point, it seems impossible to construct a logically consistent index of capital. The debate on the functional utility of NAPFs (with value-added capital) as empirical tools is more nuanced and lacks an unambiguous conclusion. While there hasn't been a clear refutation of NAPFs empirically, the methods reviewed also didn't provide unequivocal support for the production function.

The method of Paul Douglas, which involves regressing a general form of a neoclassical production function on data gathered from input-output-tables, is primarily used to justify the use of production functions on instrumentalist grounds. However, this method has been shown to be structurally flawed, rendering the results neither a refutation nor a support for the NAPF. Some studies find that the production function best fitting to the data does not possess neoclassical properties, which can be construed as evidence against the NAPF. The tests of the neoclassical postulate are also ambiguous, leaning slightly towards supporting the neoclassical postulate, but the methods used are also affected by the value problem. These results can only be regarded as empirical support for NAPFs if the correlation of the neoclassical postulate is indeed caused by the neoclassical technique-switching substitution mechanisms. However, this claim is contentious; other explanations include the financial system or the cost accounting identity.

Moreover, the functionality of some NAPF applications hinges on a sufficiently elastic investment curve as well as on the assumption of full employment. Overall, these limitations can be seen as empirical evidence against the functionality of NAPFs.

6 Limitations, Implications and further Research

As the scope of this study is limited, I cannot claim to provide a comprehensive overview of all relevant literature on the topic of APFs, especially considering the extensive and detailed discussions that have taken place over time. The conclusions drawn in the subsequent section are based solely on the review of the cited papers and should therefore be approached cautiously.

Given these limitations, the implications derived from the studied work must be significant, as NAPFs and aggregate capital are deeply entrenched in (neoclassical) macroeconomic theory. Firstly, NAPFs should not serve as the foundation for macroeconomic theories, as they have been demonstrated to be logically inconsistent. This poses a challenge for a vital part of neoclassical economics. While not all neoclassical macroeconomic theories rely on production functions as their basis, the untenability of consistently defining aggregated capital undermines the neoclassical approach as a whole. Without a reliable concept of aggregated capital, there is no basis for assuming the traditional view of investment as a decreasing function of the interest rate, which in turn disrupts the notion of investment adapting to savings and the idea of a mid- or long-term general equilibrium at full employment—a cornerstone of the neoclassical approach.

Consequently, the established narrative that market-based systems tend towards stabilizing in general equilibria and efficiently allocating resources must be abandoned, unless there is an alternative way to address the issues highlighted in this study or justify this narrative.

Real business cycle theory and the theory of marginal factor pricing, both reliant on the theoretical construct of APFs, are also called into question. Without a viable alternative justification, these theories have no theoretical basis anymore.

The implications for NAPFs as empirical tools are more nuanced, as there is neither a clear refutation nor clear empirical support for them. Beyond the arguments outlined in the conclusion, the discussion around alternatives for NAPFs is crucial in this context.

One common defense for APFs, as noted by Felipe and Fisher (2003, p. 246), is that there are **simply no alternative models available**. However, the applicability of this argument depends heavily on the specific use case of the NAPF. While alternatives exist for some applications, for others, where no alternatives are available, Felipe and Fisher suggest critically revisiting the legitimacy of the research itself rather than persisting in using flawed models:

> *"Of course, if one insists on a research program whose goal is, for example, to split overall growth into the alleged contribution of technical progress and factor accumulation (i.e. growth accounting) at the country level, surely one needs an aggregate production function in order to allegedly relate aggregate output to aggregate inputs (and thus to speak of a country's multi-factor or total factor productivity). But if one realizes that the whole meaning of aggregates such as investment, GDP, labor and capital is questionable, as Fisher (1987) pointed out, the legitimacy of the research program collapses."* (Felipe and Fisher 2003, p. 246)

This debate holds significant potential impact when considering the overarching paradigm within which macroeconomics operates, particularly in relation to the analytical conception of long-term equilibrium. This extends beyond the simple general equilibrium assumed by neoclassical theory, which has been notably undermined by the theoretical refutation of the NAPF. The analytical conception of equilibrium is also utilized in other schools of thought besides neoclassicism, and without it, both macroeconomic theory and the practice of macroeconomic modeling would necessitate substantial revisions. This conception relies on a method to describe the production process, and until viable alternatives to NAPFs emerge, two possible implications arise from the issues discussed in this study: either accepting the problems and persisting in using production functions to maintain the paradigm, or completely rejecting the heuristic notion of long-term equilibrium. While taking a definitive stance on this question exceeds the scope of this work, exploring potential alternatives and reevaluating macroeconomics without this conception could be an intriguing avenue for further research.

Other promising avenues for follow-up research could involve a more detailed examination of specific application cases of production functions and how the arguments presented might undermine these applications. Additionally, further investigation could delve into how the presented arguments extend to other production functions beyond the Cobb-Douglas model. While the overall conclusion regarding the functionality of NAPFs must remain somewhat ambiguous, the severity of the conceptual issues cannot be overlooked by mainstream macroeconomics. If the state of scientific knowledge, as far as it can be understood within this bachelor thesis, is taken seriously, debates regarding the viability of NAPFs and the associated challenges with neoclassical theory warrant broader discussion on a larger scale.

References

Ahmad, Syed (1991): Capital in Economic Theory. 5th ed. Cheltenham: Edward Elgar.

Birner, Jack (2002): The Cambridge controversies in capital theory. A study in the logic of theory development. London, New York: Routledge (Routledge studies in the history of economics, 47).

Brown, Murray (1980): The Measurement of Capital: A postreswitching problem. In D. Usher (Ed.): The Measurement of Capital. Chicago: University of Chicago Press, pp. 377–420.

Cobb, Charles W.; Douglas, Paul H. (1928): A Theory of Production. In American Economic Review 18 (1), pp. 139–165.

Douglas, Paul H. (1948): Are There Laws of Production? In The American Economic Review 38 (1), pp. 1–41.

Douglas, Paul H. (1976): The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some New Empirical Values. In Journal of Political Economy 84 (5), pp. 903–915. DOI: 10.1086/260489.

Felipe, Jesus (1998): Does the aggregate production function imply anything about the laws of production? A comment. In Applied Economics Letters 5 (9), pp. 543–547. DOI: 10.1080/758529496.

Felipe, Jesus; Adams, F. G. (2005): ‘A Theory of Production’. The Estimation of the Cobb-Douglas Production: A Retrospective View. In Eastern Economic Journal 31 (3), pp. 427–445.

Felipe, Jesus; Fisher, Franklin M. (2003): Aggregation in Production Functions: What Applied Economists should Know. In Metroeconomica 54 (2-3), pp. 208–262. DOI: 10.1111/1467-999X.00166.

Felipe, Jesus; McCombie, J.S.L. (2013): The Aggregate Production Function and the Measurement of Technological Change. 'Not Even Wrong'. Cheltenham: Edward Elgar.

Ferguson, C. (1969): The Neoclassical Theory of Production & Distribution. Cambridge: Cambridge University Press.

Fisher, Franklin M. (1971): Aggregate Production Functions and the Explanation of Wages: A Simulation Experiment. In The Review of Economics and Statistics 53 (4), pp. 305–325. DOI: 10.2307/1928732.

Fisher, Franklin M.; Monz, J. (1992): Aggregation: aggregate production functions and related topics. 1st ed. Cambridge: MIT Press.

Gandolfo, Giancarlo (2008): Comment on "C.E.S. production functions in the light of the Cambridge critique". In Journal of Macroeconomics 30 (2), pp. 798–800. DOI: 10.1016/j.jmacro.2007.12.006.

Han, Zonghie; Schefold, Bertram (2006): An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory. In Cambridge Journal of Economics 30 (5), pp. 737–765. DOI: 10.1093/cje/bei089.

Heine, Michael; Herr, Hansjörg (Eds.) (2013): Volkswirtschaftslehre. München: Oldenbourg Wissenschaftsverlag.

Leibenstein, Harvey (1966): Allocative Efficiency vs. 'X-Efficiency'. In The American Economic Review 56 (3), pp. 392–415.

Leontief, Wassily (1947a): A note on the interrelation of subsets of interdependent variables of a countinous function with continous first derivatives. In Bulletin of the American Mathematical Society 53, pp. 343–350.

Leontief, Wassily (1947b): Introduction to a Theory of the Internal Structure of Functional Relationships. In Econometrica 15 (4), pp. 361–373. DOI: 10.2307/1905335.

Levhari, David (1965): A Nonsubstitution Theorem and Switching of Techniques. In The Quarterly Journal of Economics 79 (1), pp. 98–105. DOI: 10.2307/1880514.

Levhari, David; Samuelson, Paul A. (1966): The Nonswitching Theorem is False. In The Quarterly Journal of Economics 80 (4), pp. 518–519. DOI: 10.2307/1882912.

Liu, T. C.; Hildebrand, George H. (1965): Manufacturing production functions in the United States, 1957. Ithaca: Cornell University Press.

Mainwaring, Lynn; Steedman, Ian (2000): On the probability of reswitching and reverse capital deepening in a two sector Sraffian model. In Critical Essays on Piero Sraffa's Legacy in Economics.

Mankiw, N. G.; Romer, D.; Weil, D. N. (1992): A Contribution to the Empirics of Economic Growth. In The Quarterly Journal of Economics 107 (2), pp. 407–437. DOI: 10.2307/2118477.

McCombie, J. S. L. (1998): 'Are There Laws of Production': an assessment of the early criticisms of the. In Review of Political Economy 10 (2), pp. 141–173. DOI: 10.1080/09538259800000023.

Mongiovi, Gary (1999): Introduction. In Gary Mongiovi, Fabio Petri (Eds.): Value, Distribution, and Capital: Essays in Honour of Pierangelo Garegnani. Oxfordshire: Routledge, pp. 1–16.

Moroney, John R. (1972): The structure of production in American Industry. Chapel Hill: University of North Carolina Press.

Nell, Edward (2022): Do Linear Wage-Profit Equations Support Neoclassical Theory? In Contributions to Political Economy 41 (1), pp. 153–167. DOI: 10.1093/cpe/bzac009.

Ochoa, Eduardo M. (1989): Values, prices, and wage-profit curves in the US economy. In Cambridge Journal of Economics 13, pp. 413–429. DOI: 10.1093/oxfordjournals.cje.a035101.

Pasinetti, Luigi L. (1966): Changes in the Rate of Profit and Switches of Techniques. In The Quarterly Journal of Economics 80 (4), pp. 503–517. DOI: 10.2307/1882911.

Pertz, Klaus (1980): Reswitching, Wicksell Effects, and the Neoclassical Production Function: Note. In The American Economic Review 70 (5), pp. 1015–1017.

Petri, Fabio (2011): On the likelihood and relevance of reswitching and reverse capital deepening. In C. Gehrke, Neri Salvadori (Eds.): Keynes, Sraffa and the neoclassical Theory: Essays in the Honour of Heinz Kurz, vol. 129. London, New York: Routledge, pp. 380–418.

Petri, Fabio (2019): Capital Theory 1873–2019 and the State of Macroeconomics. In History of Economics Review 74 (1), pp. 1–24. DOI: 10.1080/10370196.2020.1722411.

Petrović, P. (1991): SHAPE OF A WAGE-PROFIT CURVE, SOME METHODOLOGY AND EMPIRICAL EVIDENCE. In Metroeconomica 42 (2), pp. 93–112. DOI: 10.1111/j.1467-999X.1991.tb00361.x.

Phelps Brown, E. H. (1957): The Meaning of the Fitted Cobb-Douglas Function. In The Quarterly Journal of Economics 71 (4), pp. 546–560. DOI: 10.2307/1885710.

Robinson, Joan (1979): The Generalisation of the General Theory. In Joan Robinson (Ed.): The Generalisation of the General Theory and other Essays. London: Palgrave Macmillan UK, pp. 1–76.

Samuelson, Paul A. (1962): Parable and Realism in Capital Theory: The Surrogate Production Function. In The Review of Economic Studies 29 (3), pp. 193–206. DOI: 10.2307/2295954.

Sato, Kazuo (1974): The Neoclassical Postulate and the Technology Frontier in Capital Theory. In The Quarterly Journal of Economics 88 (3), pp. 353–384. DOI: 10.2307/1881941.

Shaikh, Anwar (1974): Laws of Production and Laws of Algebra: The Humbug Production Function. In The Review of Economics and Statistics 56 (61), pp. 115–120. DOI: 10.2307/1927538.

Shaikh, Anwar (1980): Laws of algebra and laws of production: Humbug II. In Edward Nell (Ed.): Growth, Profits and Property, Essays in the Revival of Political Economy. Cambridge: Cambridge University Press, pp. 80–95.

Shaikh, Anwar (1990): Humbug production function. In John Eatwell, Murray Milgate, Peter Newman (Eds.): Capital Theory. London: Palgrave Macmillan UK, pp. 191–194.

Simon, Herbert A. (1979): On Parsimonious Explanations of Production Relations. In The Scandinavian Journal of Economics 81 (4), pp. 459–474. DOI: 10.2307/3439661.

Solow, Robert M. (1955): The Production Function and the Theory of Capital. In The Review of Economic Studies 23 (2), pp. 101–108. DOI: 10.2307/2296293.

Solow, Robert M. (1966): Review of capital and growth. In The American Economic Review 56 (5), pp. 1257–1260.

Solow, Robert M. (1976): Discussion of Zarembra's Chapter-characterization of a technology in capital theory. In Murray Brown, Kazuo Sato, P. Zarembra (Eds.): Essays in Modern Capital Theory. Amsterdam: North-Holland, pp. 138–140.

Sraffa, Piero (1960): Production of commodities by means of commodities. A Prelude to a Critique of Economic Theory. Cambridge: Cambridge University Press.

Sylos Labini, Paolo (1995): Why the interpretation of the Cobb-Douglas production function must be radically changed. In Structural Change and Economic Dynamics 6 (4), pp. 485–504. DOI: 10.1016/0954-349X(95)00025-I.

Temple, Jonathan R. W. (1998): Robustness tests of the augmented Solow model. In J. Appl. Econ. 13 (4), pp. 361–375. DOI: 10.1002/(SICI)1099-1255(199807/08)13:4<361::AID-JAE483>3.0.CO;2-1.

Tsoulfidis, L.; Maniatis, T. (2002): Values, prices of production and market prices: some more evidence from the Greek economy. In Cambridge Journal of Economics 26 (3), pp. 359–369. DOI: 10.1093/cje/26.3.359.

Walters, A. A. (1963): Production and Cost Functions: An Econometric Survey. In Econometrica 31 (1-2), pp. 1–66. DOI: 10.2307/1910949.

Zambelli, S. (2004): The 40% neoclassical aggregate theory of production. In Cambridge Journal of Economics 28 (1), pp. 99–120. DOI: 10.1093/cje/28.1.99.

Zambelli, Stefano (2018): The aggregate production function is NOT neoclassical. In Cambridge Journal of Economics 42 (2), pp. 383–426. DOI: 10.1093/cje/bex011.