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by

C.A. Dominguez

Deutsches Elektronen-Synchrotron DESY, Hamburg

J. Solà

Departament de Física Teòrica,
Universitat Autònoma de Barcelona

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Radiative pion decay: Determination of $F_A(0)$ from τ-lepton decay data

C.A.Dominguez[1]
Deutsches Elektronen-Synchrotron DESY, Hamburg, Fed. Rep. Germany

J. Solà
Departament de Física Teòrica, Universitat Autònoma de Barcelona, Catalonia,Spain

Abstract

The form factor $F_A(0)$ in $\pi \to e\nu\gamma$ is calculated,at the soft pion point,directly from data on the semileptonic τ-lepton decays $\tau \to \nu_\tau + n\pi$, which determine the relevant vector and axial-vector hadronic spectral functions. The reliability of this calculation is assessed by using these fitted spectral functions to check the saturation of the first two Weinberg sum rules. The results are: $F_A(0) = 0.017 \pm 0.001 \pm 0.004$, and $\gamma(0) \equiv F_A(0)/F_V(0) = 0.67 \pm 0.04 \pm 0.16$.

[1] Alexander von Humboldt Research Fellow. On leave of absence from: Facultad de Física, Pontificia Universidad Católica de Chile, Santiago, Chile.

Considerable experimental and theoretical attention has been devoted in the past to the radiative decay of the pion: $\pi^+(p) \to \nu_e(k) + e^+(k') + \gamma(\epsilon, q)$, as this process serves as a unique testing ground of the low energy structure of QCD (for reviews see [1]-[2]). Since the inner bremsstrahlung (IB) from e^+ or π^+ is calculable in QED , this decay essentially probes the structure dependent (SD) piece of the amplitude associated to the vector and axial-vector $\Delta S{=}0$ weak hadronic current.This SD amplitude involves two *a priori* unknown form factors F_V and F_A, to be defined more precisely below. Theoretically, F_V may be related through CVC to the form factor in $\pi^0 \to \gamma\gamma$ [3] which is accurately described by the chiral anomaly [4]. Concerning F_A, or equivalently $\gamma \equiv F_A/F_V$, there is a wide variety of predictions from e.g. the quark model [5], the linear σ model [6],etc. and, more pertinent to make contact with QCD, from QCD sum rules [7], chiral- symmetry [8] and current algebra [9]. From the last two methods one finds $\gamma \approx 0.5$, while other predictions span the range $-1 \leq \gamma \leq 1.5$. The latest two experimental results are (earlier experiments are reviewed in [2])

$$\gamma = 0.7 \pm 0.5 \tag{1}$$

from SIN [10], and

$$\gamma = 0.25 \pm 0.12 \tag{2}$$

from LAMPF [11], while the world average stands now at [11]

$$\gamma = 0.39 \pm 0.06 \tag{3}$$

In this paper we calculate F_A at the soft pion point,i.e. $F_A(0)$, directly from experimental data on the semileptonic decays $\tau \to \nu_\tau + n\pi$ [12]. These data determine the vector and axial-vector hadronic spectral functions entering the expression for $F_A(0)$. Previous current algebra estimates of $F_A(0)$ relied upon theory, e.g. tree-level chiral Lagrangians or model parametrizations of the spectral functions. Hence, our results should be useful to constrain some of the theoretical parameters appearing there, in addition to being a valuable test of the soft pion expression for $F_A(0)$ from experimental data *independent from radiative pion decay.* The reliability of our calculation may be assessed by analyzing how well do the fitted spectral functions saturate the first two Weinberg sum rules (WSR) [13]. This analysis was first done in [14], but we briefly reexamine it here on the light of some recent developments [15].

Apart from the IB contribution, radiative pion decay is described by the on-mass-shell limit of the hadronic amplitude

$$M_{\mu\nu}(p,q)|_{SD} = i\int d^4x\, e^{iqx} < 0|T(V_\mu^{EM}(x) J_\nu^{(-)}(0))|\pi^+(p) > \tag{4}$$

where V_μ^{EM} is the electromagnetic current and $J_\nu^{(-)}$ the $\Delta S{=}0$ weak (V-A) hadronic current. The dimensionless form factors $F_V(s)$ and $F_A(s)$, with s=p·q, are defined through

$$\sqrt{2}\, f_\pi M_{\mu\nu}(p,q)|_{SD} = \epsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma F_V(s) + i(s\, g_{\mu\nu} - q_\mu p_\nu) F_A(s) \tag{5}$$

where $f_\pi = 93.2MeV$, and the π-intermediate state piece in (5) has been removed to combine with the IB contribution. Using CVC and the chiral anomaly one obtains for $F_V(0)$ [3]

$$F_V(0) = \frac{1}{\sqrt{2}} F_{\pi^0\gamma\gamma}(0) = \frac{1}{4\pi^2} \tag{6}$$

Some time ago, claims were made [16] of possible large deviations of CVC in Eq.(6) but they have not survived closer scrutiny [17]. Corrections to Eq.(6) are expected to be of order $\mathcal{O}(m_d - m_u)$ and thus very small [17]. Turning to F_A, current algebra in the soft pion limit leads to [1]

$$F_A(0) = C(0) + \frac{2}{3} f_\pi^2 < r_\pi^2 > \tag{7}$$

where $< r_\pi^2 >$ is the EM r.m.s. radius of the pion, and

$$C(0) = -\frac{2}{\pi}\int_{4\mu_\pi^2}^{\infty} \frac{dt}{t}\,[Im\,\Pi_V(t) - Im\,\Pi_A(t)] \tag{8}$$

In Eq.(8) Π_V (Π_A) is the longitudinal plus transverse projection of the diagonal two-point function involving vector (axial-vector) currents,normalized such that the QCD asymptotic freedom limit is

$$\lim_{t\to\infty} \frac{1}{\pi} Im\,\Pi_{V,A}(t)|_{QCD} = \frac{1}{8\pi^2}\,[1 + \frac{\alpha_s(t)}{\pi} + \mathcal{O}(\alpha_s^2)] \tag{9}$$

where $\alpha_s(t)/\pi = -2/[\beta_1 \ln(t/\Lambda_{QCD}^2)]$ is the running coupling constant with $\beta_1 = -\frac{29}{6}$ for two flavours and $\Lambda_{QCD} \approx 100MeV$. With this normalization the first two Weinberg sum rules, *in the chiral limit*, read

$$\frac{1}{\pi}\int_0^\infty dt\,[Im\,\Pi_V(t) - Im\,\Pi_A(t)] = f_\pi^2 \tag{10}$$

$$\frac{1}{\pi}\int_0^\infty dt\,t\,[Im\,\Pi_V(t) - Im\,\Pi_A(t)] = 0 \tag{11}$$

where the longitudinal piece of $Im\,\Pi_A$, i.e. the π-pole, has been written explicitly in (10); notice that this term does not contribute to C(0) in Eq.(8).

Recent experimental data [12] on the semileptonic decays $\tau \to \nu_\tau + n\pi$, with n=even (odd), allow for a determination of the vector (axial-vector) hadronic spectral functions up to the kinematical phase space limit $t \approx 3\ GeV^2$. Fits to these data were done in [14] and used there to study the saturation of the WSR (10)-(11). Some improvements to these fits were performed in [15] by e.g. including background contributions to the axial-vector channel and using updated values for some of the branching ratios. The results of these latter fits are depicted in Fig.1, where for simplicity experimental errors are not shown.

Although the upper limit of integration in Eqs.(8),(10)-(11) extends to $t = \infty$, in practice it is only necessary to integrate up to some finite cutoff $t = t_c$. In fact, as it follows from duality, after some threshold $t = t_0^{V,A}$ (in general $t_0^V \neq t_0^A$) the hadronic spectral function is expected to merge into its asymptotic QCD value (9).Qualitatively, a visual inspection of Fig.1 shows that the data above resonance supports this expectation.Quantitatively,this

asymptotic freedom threshold is an eigenvalue solution to the following Finite Energy Sum Rule (FESR) [15],[18]

$$1 + F_2(t_0) \equiv I_0(t_0)|_{V,A} = \frac{8\pi^2}{t_0^{V,A}} \int_0^{t_0^{V,A}} dt\,\frac{1}{\pi} Im\,\Pi_{V,A}(t) \tag{12}$$

where

$$F_2(t_0) = \frac{\alpha_s(t_0)}{\pi} + [\frac{\alpha_s(t_0)}{\pi}]^2 (F_3 - \frac{\beta_1}{2} - \frac{\beta_2}{\beta_1} ln ln \frac{t_0}{\Lambda_{QCD}^2}) \tag{13}$$

with $F_3 = 1.7566...$, and $\beta_2 = -115/12$ for two flavours. In Fig.2 we show the behaviour of the r.h.s. and the l.h.s. of Eq.(12) as a function of t_0 for the spectral functions of Fig.1. Taking into account the experimental errors in the data (for a description of the error analysis see [18]) one finds stable eigenvalue solutions to Eq.(12) in the range $1.4GeV^2 \lesssim t_0^V \lesssim 1.7GeV^2$,and $1.75GeV^2 \lesssim t_0^A \lesssim 2.25GeV^2$.

Further constraints on the above duality regions may be obtained from an analysis of the WSR (10)-(11). In a first step we ignore the results from Eq.(12) just discussed,and simply integrate the spectral functions

$$\frac{1}{\pi} Im\,\Pi_{V,A}(t) = \frac{1}{\pi} Im\,\Pi_{V,A}(t)|_{FIT}\,\Theta(t_c - t) + \frac{1}{8\pi^2}\,[1 + \frac{\alpha_s(t)}{\pi} + ...]\,\Theta(t - t_c) \tag{14}$$

The results shown in Figs.(3.a)-(3.b) (solid curves) indicate a satisfactory saturation of the two WSR. Next, we make use of the information provided by the FESR (12) and integrate the spectral functions

$$\frac{1}{\pi} Im\,\Pi_V(t) = \frac{1}{\pi} Im\,\Pi_V(t)|_{FIT}\,\Theta(t_0^V - t) + \frac{1}{8\pi^2}\,[1 + \frac{\alpha_s(t)}{\pi} + ...]\,\Theta(t - t_0^V) \tag{15}$$

and $Im\,\Pi_A(t)$ still given by Eq.(14) since $t_0^A > t_0^V$. Clearly, in this case $t_c \equiv t_0^A$. The results from this procedure are shown in Figs.(3.a)-(3.b), as dashed curves, for the choice $t_0^V = 1.6GeV^2$. Allowing t_0^V to change within its duality range and inspecting the results as a function of $t_c \equiv t_0^A$ one finds that the WSR are saturated to a very good accuracy for $t_0^V \approx (1.50 - 1.65)GeV^2$ and $t_c \equiv t_0^A \approx (1.75 - 2.25)GeV^2$.

Turning to C(0) and performing the integrals in Eq.(8) for the above ranges of $t_0^{V,A}$ one finds quite stable results as shown in Fig.4. Numerically we obtain

$$C(0) = -(4.78 \pm 0.13) \times 10^{-2} \tag{16}$$

which upon using $< r_\pi^2 >|_{EXP} = 0.44 \pm 0.03 fm^2$ [19] in Eq.(7) leads to

$$F_A(0) = 0.017 \pm 0.001 \pm 0.004 \tag{17}$$

$$\gamma(0) = 0.67 \pm 0.04 \pm 0.16 \tag{18}$$

where the first error comes from Eq.(16) and the second one is due to the experimental uncertainty in $< r_\pi^2 >$.The different power weights of the spectral functions in Eqs.(10)-(11)

lead us to expect the first WSR to be more accurately saturated than the second.Notice that uncertainties in the τ-decay data increase with energy,as the number of events approaches zero at the phase space end point.This expectation is fully confirmed by the analysis.To this extent we expect the result from Eq.(8),i.e.Eq.(16), to have a similar (or better) accuracy as the first WSR. The fair agreement between $\gamma(0)$,Eq.(18), and the current experimental world average (3) appears to support the view that corrections to CVC in Eq.(6), and to the soft pion theorem (7), should be small. A more precise comparison would have to await improvements in the measurements of γ and $< r_\pi^2 >$.

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Figure Captions

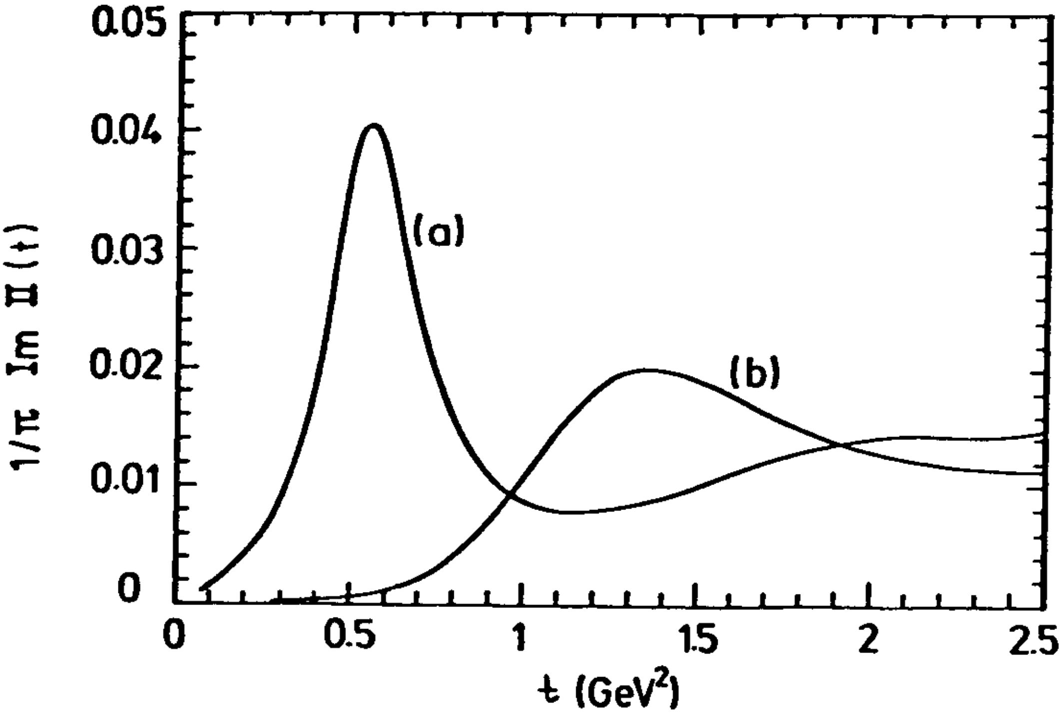
Fig.1 : Hadronic spectral functions in the vector (curve (a)) and axial-vector (curve (b)) channels (Fit errors are not shown).

Fig.2 : The r.h.s. of the FESR (12) in the vector (curve (a)) and axial-vector (curve (b)) channels, together with the l.h.s. (curve (c)).

Fig.3.a : The first WSR (10) multiplied on both sides by 4π. Curves (a) and (b) are obtained using Eqs.(14) and (15),respectively.Straight solid line (c) is the value $4\pi\ f_\pi^2$.

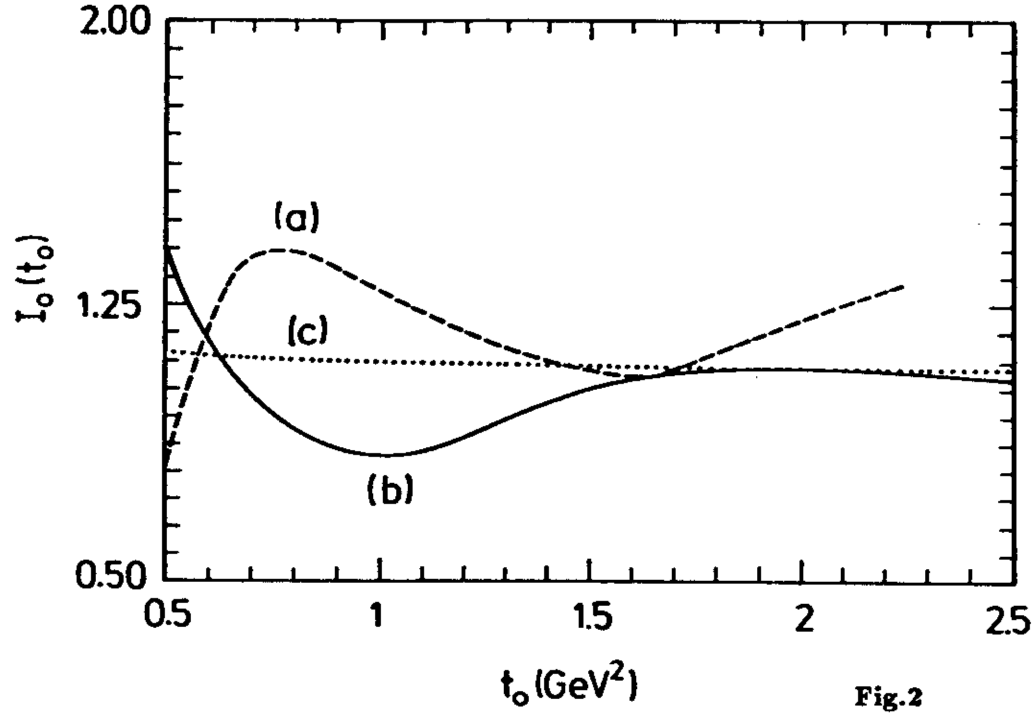
Fig.3.b : The second WSR (11) multiplied by 4π.Curves (a) and (b) are obtained using Eqs.(14) and (15),respectively. Straight solid line (c) is the value 0.

Fig.4 : The function C(0),Eq.(8),using Eq.(15) with $t_0^V\!=\!1.65GeV^2$ (curve (a)), and $t_0^V\!=\!1.50GeV^2$ (curve (b)),both for $t_c\!\equiv\!t_0^A$.



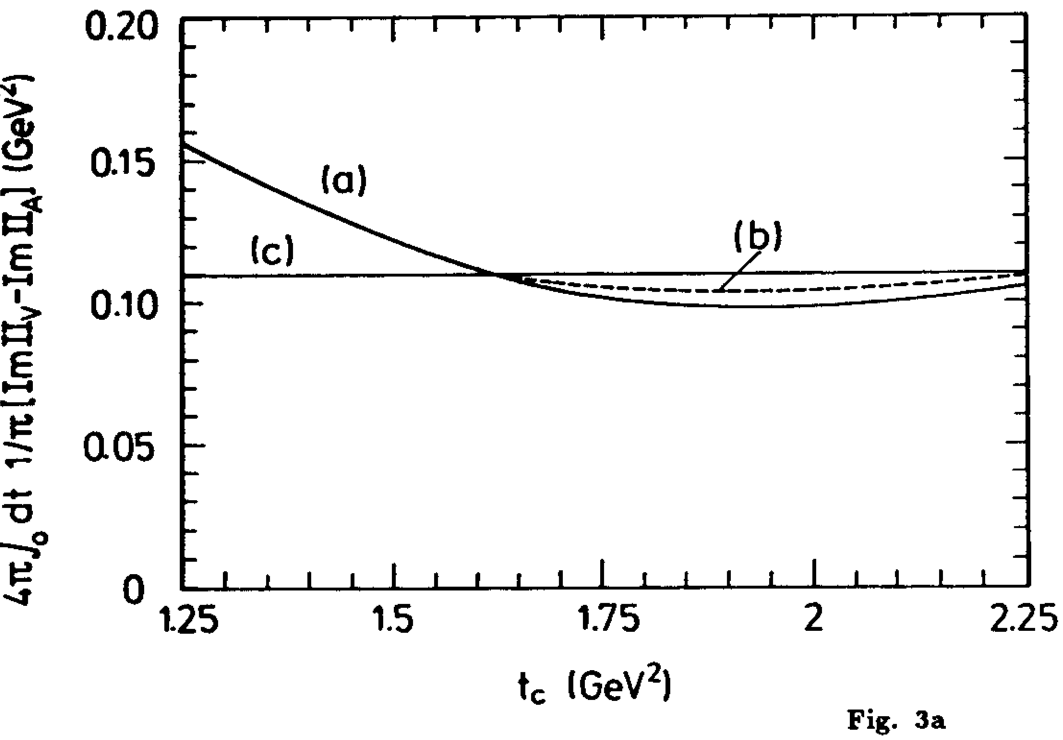
0.05
0.04
0.03
0.02
0.01
0
1/π Im Π(t)
(a)
(b)
0
0.5
1
1.5
2
2.5
t (GeV²)

Fig.1



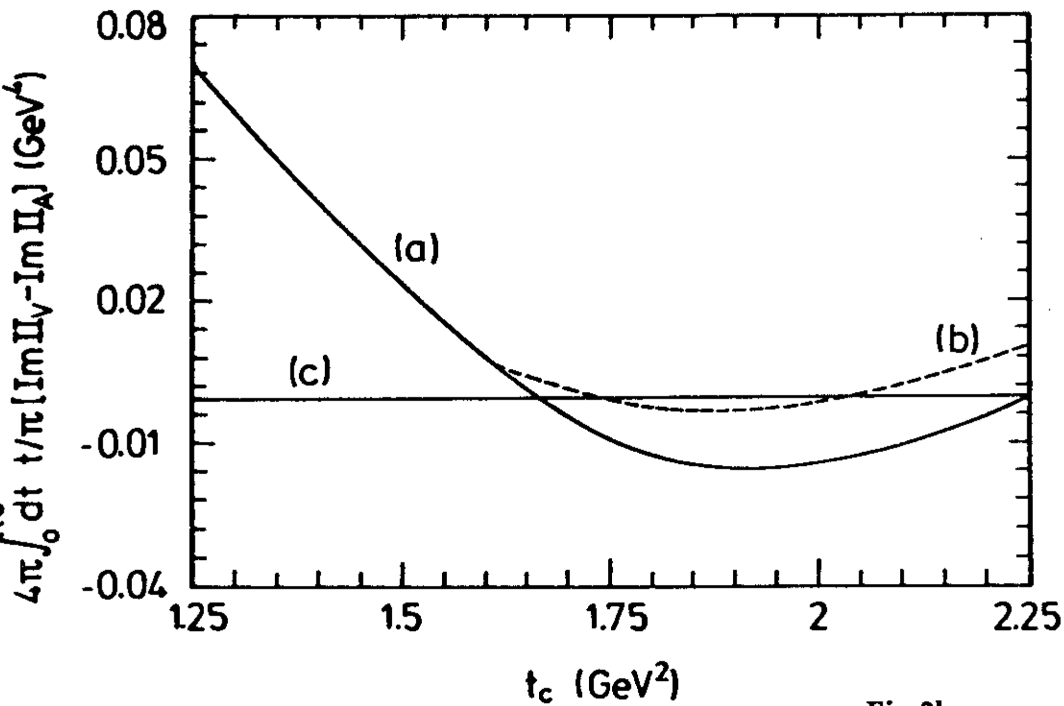
2.00
1.25
0.50
I₀(t₀)
(a)
(c)
(b)
0.5
1
1.5
2
2.5
t₀ (GeV²)

Fig.2



0.20
0.15
0.10
0.05
0
1.25
1.5
1.75
2
2.25
(a)
(b)
(c)
t_c (GeV^2)
$4\pi\int_0^{t_c} dt\ 1/\pi\,[\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A]$ (GeV^2)

Fig. 3a



0.08
0.05
0.02
-0.01
-0.04
1.25
1.5
1.75
2
2.25
(a)
(b)
(c)
t_c (GeV^2)
$4\pi\int_0^{t_c} dt\ t/\pi\,[\mathrm{Im}\Pi_V - \mathrm{Im}\Pi_A]$ (GeV^4)

Fig.3b

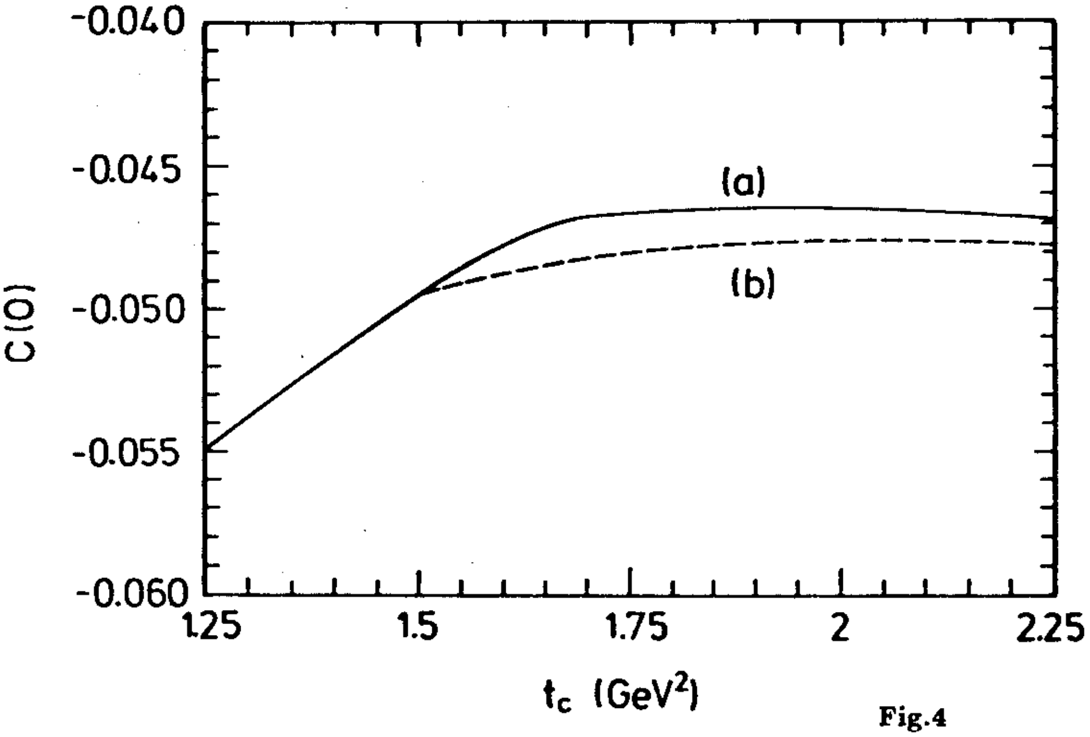


Fig.4