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Abstract

Fermionic propagators and the Wilson loop of the gauge invariant chiral Schwinger model are compared with their counterparts in the Schwinger model. It is made evident that in the chiral Schwinger model the charges are also shielded as in the ordinary Schwinger model. Furthermore we show that the Schwinger model can be reformulated in such a way that it becomes the chiral Schwinger model endowed with a special regularization scheme.

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I. Introduction

In the last years the chiral Schwinger model (CSM) has been proven to be an example for a theory which is consistent in spite of being anomalous /1/. The CSM has been treated in two different approaches: the original "anomalous" formulation /1 - 10/ and the "gauge invariant" formulation /7 - 15/. They differ in the existence of a Wess-Zumino scalar θ in the latter which renders the theory gauge invariant /11, 16, 17/. Later it has been shown that the two formulations are not very different, the "anomalous" formulation turned out to be a special gauge of the gauge invariant formulation, namely the $\theta = 0$ gauge /13/.

Though there have been many investigations in the CSM /1 - 15/, as well as in its nonabelian extension /18 - 21/, the physics of the model is still unclear. The reason is that most of the treatments use the bosonized version of the model, which is not suitable to uncover the behaviour of the fermions. In the ordinary Schwinger model (SM) the most interesting feature is the complete shielding of charges /22 - 25/. Therefore it is desirable to know how the fermions behave in the CSM, too.

There are some efforts to clarify this point. In ref. /3/ the exact fermion propagator has been calculated in the "anomalous" formulation. The result suggests the existence of asymptotically free charges (i.e. no screening), in contradistinction with the SM. In this ref. the calculated propagator is considered physical, because it is thought to belong to a theory without gauge invariance. However, in our opinion the "anomalous" formulation is just a specific gauge of a gauge invariant theory. Hence it is not allowed to draw physical conclusions from the fermion propagator which is a gauge dependent object *. In fact, we are going to show that screening occurs also in the CSM. In ref. /6/ evidence is given to a confining character of the CSM in a specific regularization (a = 0). Since the

* The same feature has been observed in the SM: the fermion propagators suggest different behaviour in different gauges /4/.

operator solution of the model with a = 0 is inconsistent /2/, we do not want to consider this case. Instead, we confine ourselves to regularizations which are undoutedly reasonable, namely a > 1. After completion of the present work we noticed a paper by Miyake and Shizuya /26/, who investigated the fermions in the CSM. They used the operator solution of the model in its bosonized form to reconstruct the fermions from the bose fields. Their conclusion (namely that there is shielding) coincides with ours, however, we consider their argument not to be totally convincing (see below).

It is the purpose of the present paper to study the fermion behaviour from the path-integral point of view. To this aim we investigate gauge dependent and gauge invariant fermion propagators and the Wilson loop. We use the striking similarity of the results in the SM and the CSM to conclude that the charged fermions behave in the same way in both models, i.e. that charges are shielded. This statement is also supported by the close relationship between the SM and the CSM, which we are going to establish.

This paper is organized as follows: in section II we explain the models and set up our notation. In section III the fermion two point functions are calculated. We present the calculation for the SM and explain how this has to be modified to reach the corresponding results for the CSM. In section IV we show that the Wilson loops of the two models coincide and in section V we demonstrate that the SM may be reformulated in such a way it looks like the CSM with a special regularization. Section VI contains the conclusions.

II. The Schwinger Model and the Chiral Schwinger Model

Classically, the models are defined by their classical actions. Upon quantization, it is also necessary to fix the fermionic measure for the specification of the model. In the SM, having no anomaly, it is fixed by the requirement of gauge invariance /27/. In the CSM, such a requirement cannot be fulfilled. Hence quantization of the fermions spoils gauge invariance. A detailed analysis of the Faddeev-Popov procedure has shown that gauge invariance automatically is restored by the quantization of the gauge field /11, 16, 17/. The mechanism is such that the gauge degree of freedom contained in the gauge field becomes alive as a Wess-Zumino scalar field, which cancels the fermionic anomaly. In this way the classical action of an apparently anomalous theory is modified by a Wess-Zumino term at the quantum level, leading to an anomaly free quantum theory. This quantum action is called the standard action /11/.

Our notation is:

$$g_{00} = -g_{11} = \varepsilon^{01} = -\varepsilon_{01} = 1 \quad , \quad \gamma_5 = i\gamma^0\gamma^1 \; ,$$
$$P_{L,R} = \frac{1}{2}(1 \pm i\gamma_5) \quad , \quad X^{\pm}_{\mu} = \frac{1}{2}(g_{\mu\nu} \pm \varepsilon_{\mu\nu})X^{\nu} \; , \qquad (1)$$

where X is an arbitrary vector. In this notation the models are specified by the actions:

$$S^{SM} = \int [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma_{\mu}(i\partial^{\mu} + eA^{\mu})\psi \,]\, d^2x \; , \qquad (2)$$

$$S^{CSM} = \int \{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma_{\mu}(i\partial^{\mu} + eA^{\mu}P_L)\psi - \frac{a-1}{8\pi}\theta\Box\theta + \frac{e}{4\pi}\theta\partial_{\mu}[(a-1)g^{\mu\nu} + \varepsilon^{\mu\nu}]A_{\nu}\}\, d^2x \; , \qquad (3)$$

where the CSM is endowed with a regularization prescription which renders the theory gauge invariant. Note that this is only possible in the "gauge invariant" formulation, where the Wess-Zumino field is present. The parameter "a" in the standard action of the CSM reflects the ambiguity in the definition of the fermionic measure /28 - 32/. It also occurs in the Wess-Zumino action since this is just the Jacobian for a transformation of the Fermions with θ as (finite) parameter /11/. The generating functionals for the models read

$$Z_{SM} = \int dA\, d\psi\, d\bar{\psi}\; e^{i(S^{SM} + S_{GF})} \quad , \qquad (4)$$

$$Z_{CSM} = \int dA\, d\theta\, d\psi\, d\bar{\psi}\; e^{i(S^{CSM} + S_{GF})} \quad . \qquad (5)$$

Here S_{GF} denotes the gauge fixing part of the action. For reasons of comparison we assume the same gauge fixing condition for both models.

In two dimensions it is possible to write A_μ in terms of two scalar fields according to

$$A_\mu = \frac{1}{e} (\partial_\mu \sigma + \varepsilon_{\mu\nu} \partial^\nu \rho) , \qquad (6)$$

The fermions can be integrated out by rotating the interaction away via the transformation:

$$SM: \; \chi = e^{-i(\sigma - i\gamma_5 \rho)} \psi \quad , \quad \bar{\chi} = \bar{\psi}\, e^{i(\sigma + i\gamma_5 \rho)} \quad , \qquad (7)$$

$$CSM: \; \chi = e^{-i(\sigma - \rho) P_L} \psi \quad , \quad \bar{\chi} = \bar{\psi}\, e^{i(\sigma - \rho) P_R} \quad . \qquad (8)$$

Inserting this into the fermionic actions shows that χ is a free (noninteracting) fermion field. The resulting effective actions are:

$$S^{SM}_{eff}[A] = \int [-\tfrac{1}{4} F_{\mu\nu} (1 + \tfrac{m^2}{\Box}) F^{\mu\nu} + \mathcal{L}_{GF}] \, d^2x \qquad (9)$$

$$S^{CSM}_{eff}[A,\theta] = \int \{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{e^2 a}{8\pi} A_\mu A^\mu - \tfrac{e^2}{2\pi} A^-_\mu \tfrac{\partial^\mu \partial^\nu}{\Box} A^-_\nu + \mathcal{L}_{GF}$$
$$- \tfrac{a-1}{8\pi} \theta \Box \theta + \tfrac{e}{4\pi} \theta\, \partial_\mu [(a-1) g^{\mu\nu} + \varepsilon^{\mu\nu}] A_\nu \} \, d^2x \, . \qquad (10)$$

As we pointed out above, the parameter a coming from the fermion integration has to coincide with the regularization parameter in the Wess-Zumino action. For our purpose it is allowed to integrate over the Wess-Zumino field, too. Then the effective action for the CSM becomes very similar to eq. (9) /11/:

$$S^{CSM}_{eff}[A] = \int [-\tfrac{1}{4} F_{\mu\nu} (1 + \tfrac{\tilde{m}^2}{\Box}) F^{\mu\nu} + \mathcal{L}_{GF}] \, d^2x \, . \qquad (11)$$

The masses in the SM and in the CSM are given by /33, 1/:

$$m^2 = \frac{e^2}{\pi} \quad , \qquad (12)$$

$$\tilde{m}^2 = \frac{e^2 a^2}{4\pi(a-1)} \quad , \qquad (13)$$

respectively. This means that the two actions (9) and (11) differ only in the masses m and $\tilde{m}$ of the bose field. The effective actions may be used to read off the exact photon propagators, in the Lorentz gauge $\left(\mathcal{L}_{GF} = \frac{-1}{2\alpha}(\partial_\mu A^\mu)^2\right)$ they are found to be

$$D_{\mu\nu} = \frac{-i}{k^2 - M^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - i\alpha\frac{k_\mu k_\nu}{k^4} \tag{14}$$

with $M^2 = m^2(\tilde{m}^2)$ in the SM (CSM).

III. Fermion Two-Point Functions

In this section we compare the fermion propagators of the CSM with those of the SM. We investigate the usual (gauge dependent) propagator as well as the gauge invariant propagators, where the fermions are connected by a string. In both cases we shall present the calculation for the SM as detailed as necessary in order to point out which modifications have to be made in the CSM.

a) gauge dependent propagator

As usual, the propagators are defined by:

$$G_{SM}(x-y)_{\alpha\beta} = \int dA\, d\psi\, d\bar\psi \;\; \psi_\alpha(x)\,\bar\psi_\beta(y)\; e^{i(S^{SM} + S_{GF})}, \tag{15}$$

$$G_{CSM}(x-y)_{\alpha\beta} = \int dA\, d\theta\, d\psi\, d\bar\psi \;\; \psi_\alpha(x)\,\bar\psi_\beta(y)\; e^{i(S^{CSM} + S_{GF})}. \tag{16}$$

Using the transformations of eqs. (7) and (8), respectively, these propagators can be calculated in terms of the free ones. Since these transformations are precisely the ones which have been used to calculate the effective actions, the propagators can be written as

$$G_{SM}(x-y)_{\alpha\beta} = G^0(x-y)_{\gamma\delta} \cdot \int dA \left(e^{i(\tau - i\gamma_5\rho)(x)}\right)_{\alpha\gamma} \cdot \left(e^{-i(\tau + i\gamma_5\rho)(y)}\right)_{\delta\beta} e^{iS_{eff}^{SM}[A]} \tag{17}$$

$$G_{CSM}(x-y)_{\alpha\beta} = G^0(x-y)_{\gamma\delta} \cdot \int dA \left(e^{i(\tau-\rho)(x)P_L}\right)_{\alpha\gamma} \cdot \left(e^{-i(\tau-\rho)(y)P_R}\right)_{\delta\beta} e^{iS_{eff}^{CSM}[A]}. \tag{18}$$

The matrix character can be eliminated by projection onto left and right handed states:

$$G^{L,R}(x-y) = P_{L,R}\; G(x-y)\; P_{R,L} \quad , \tag{19}$$

the other components of G vanish. The right handed part of G_{CSM} is just the free propagator, the left handed part (G^L_{CSM}) is the same as G^L_{SM} (with the appropriate rescaling of the mass). Hence it is sufficient to calculate the propagators for the SM:

$$G^{L,R}_{SM}(x-y) = G^{0;L,R}(x-y)\int dA\; e^{i(\tau\mp\rho)(x)}\, e^{-i(\tau\mp\rho)(y)}\, e^{iS_{eff}^{SM}[A]}, \tag{20}$$

This can be evaluated following the lines of ref. /6/, where the left handed propagator for the CSM is calculated in the $\theta = 0$ gauge. The result is a universal correction for the left and right handed propagator, in the Lorentz gauge it reads:

$$G_{SM}(x) = G^0(x)\,\exp\left\{ie^2\int\frac{d^2k}{4\pi^2}\left(\frac{-1}{k^2(k^2-m^2)} + \frac{\alpha}{k^4}\right)\left(1 - e^{-ik\cdot x}\right)\right\}. \tag{21}$$

For $\alpha = 0$ this can already be found in Schwinger's paper /33/. As indicated above, eq. (21) implies for the CSM:

$$G^{R}_{CSM}(x) = G^{0;R}(x) \; , \qquad (22)$$

$$G^{L}_{CSM}(x) = G^{0;L}(x)\, \exp\left\{ ie^2 \int \frac{d^2k}{4\pi^2}\left(\frac{-1}{k^2(k^2-\tilde{m}^2)} + \frac{\alpha}{k^4}\right)\left(1-e^{-ik\cdot x}\right)\right\} .$$

Eqs. (21) and (22) contain the exact fermion propagators of the SM and CSM, respectively. We want to note two observations concerning the CSM: Firstly, the right handed fermions remain free in spite of the fact that the regularization of the fermion determinant uses a coupling of the gauge field to the right handed fermion /29 - 32/. Secondly, for the left handed fermions the only difference between the SM and the CSM is the modification of the gauge boson mass. Eqs. (17) and (18) show that the behaviour of the left handed fermions is only influenced by the effective features of the gauge field. In the CSM the interacting right handed fermions of the SM are mimicked by the Wess-Zumino field. This leads to the similarity of the effective actions of both models, see eqs. (9) and (11). Hence it has to be expected that the left handed fermions in the CSM behave precisely in the same way as they do in the SM. This means that fermionic charges are shielded in the CSM, too, since they are shielded in the SM /23 - 25/.

For $\alpha = 0$, eq. (22) has also been calculated in ref. /26/. There the behaviour for $(x-y)^2 \to -\infty$ has been used as an argument for confinement. This, however, contradicts the interpretation of ref. /3/, where the fermion propagators (in $\theta = 0$ gauge) has been shown to become the free one as $(x-y)^2 \to -\infty$, suggesting the existence of free left handed fermions. Therefore it seems to be insufficient to study the large x behaviour of the (gauge dependent) fermion propagator in order to draw physical conclusions.

b) gauge invariant propagator

The preceeding discussion leads us to look at gauge invariant objects. An example is built out of two fermions connected by a gauge string. In the SM, this has been employed in the construction of the gauge invariant algebra /23/ and in the definition of the fermionic current via point splitting /22, 34/. The corresponding time ordered expectation value is the gauge invariant propagator, in the SM it reads:

$$\tilde{G}_{SM}(x-y)_{\alpha\beta} = \int dA\, d\psi\, d\bar{\psi}\;\; \psi_\alpha(x)\, e^{ie\int_x^y A_\mu dz^\mu}\, \bar{\psi}_\beta(y) \cdot e^{i(S^{SM}+S_{GF})} \; . \qquad (23)$$

In the CSM, only the left handed propagator needs a string because the right handed fermions are gauge invariant anyway:

$$\tilde{G}^{L}_{CSM}(x-y) = P_L \int dA\, d\psi\, d\bar{\psi}\, d\theta\;\; \psi(x)\, e^{ie\int_x^y A_\mu dz^\mu}\, \bar{\psi}(y)\; e^{i(S^{CSM}+S_{GF})}\; P_R \; , \qquad (24)$$

$$\tilde{G}^{R}_{CSM}(x) = G^{R}_{CSM}(x) = G^{0;R}(x) \; . \qquad (25)$$

The phase factor is not unique /35/, for definiteness we understand the line integral to be taken along the straight path between x and y. The gauge invariant propagators can be evaluated in the same way as in the preceeding section: the fermion fields are rotated into free ones (eqs. (7) and (8)), then all fields except the gauge field are integrated out. This results in:

$$\tilde{G}^{L,R}_{SM}(x-y) = G^{0;L,R}(x-y)\cdot\int dA\; e^{i(\sigma\mp\rho)(x)}\; e^{ie\int_x^y A_\mu dz^\mu}\; e^{-i(\sigma\mp\rho)(y)}\; e^{iS^{SM}_{eff}[A]}\;, \tag{26}$$

$$\tilde{G}^{L}_{CSM}(x-y) = G^{0;L}(x-y)\cdot\int dA\; e^{i(\sigma-\rho)(x)}\; e^{ie\int_x^y A_\mu dz^\mu}\; e^{-i(\sigma-\rho)(y)}\; e^{iS^{CSM}_{eff}[A]}\;. \tag{27}$$

Expressing σ and ρ in terms of A and parametrizing z(t) = t · (y-x) + x, $\tilde{G}^{L,R}_{SM}$ can be evaluated. In contrast to the calculation of the gauge dependent propagator, here only the (gauge invariant) $g^{\mu\nu}$-part of the exact photon propagator contributes. The result is

$$\tilde{G}_{SM}(x) = G^{0}(x)\,\exp\left\{ie^2\int\frac{d^2k}{4\pi^2}\left(\frac{2}{k^2}-\frac{x^2}{(k\cdot x)^2}\right)\frac{1-e^{-ik\cdot x}}{(k^2-m^2)}\right\}\;, \tag{28}$$

i.e. the correction is the same for the left and right handed parts. For the CSM, a new calculation is not necessary, the result can be read off from eq. (28):

$$\tilde{G}^{L}_{CSM}(x) = G^{0;L}(x)\,\exp\left\{ie^2\int\frac{d^2k}{4\pi^2}\left(\frac{2}{k^2}-\frac{x^2}{(k\cdot x)^2}\right)\frac{1-e^{-ik\cdot x}}{(k^2-\tilde{m}^2)}\right\}\;. \tag{29}$$

$\tilde{G}^{L}_{CSM}$ has already been calculated in ref. /6/, where the anomalous formulation has been used which agrees with the $\theta = 0$ gauge in our approach. Due to gauge invariance of $\tilde{G}^{L}_{CSM}$ the results coincide. Again, there is no difference between the behaviour of the left handed fermions in the two models (apart from the different photon mass).

IV. Wilson Loop

Usually, the standard test for confinement is the calculation of the Wilson loop /36/, which is defined by the expectation value

$$W = \left\langle e^{ie\oint A_\mu dz^\mu}\right\rangle\;. \tag{30}$$

Taking the limit that the temporal extension of the integration contour tends to infinity, W can be used to define the potential between two static fermionic charges. This shows whether it is possible to separate the fermions or not. Unfortunately, in chiral gauge theories there seems to be a problem: static charges have to be infinitely massive. A mass term for the fermions, however, is in conflict with chiral gauge invariance. Hence one has to invent new massive fermions with non-chiral coupling to the gauge field which can serve as test charges *. Nevertheless it can be hoped that these additional fermions do not change the interpretation, since screening is caused by the polarizability of the vacuum. In the relevant infinite mass limit the heavy fermions cannot be pair-produced, thus they do not contribute to the polarizability.

For the evaluation of the Wilson loop all fields but the gauge field are integrated out, this yields

$$W_{SM} = \int dA\; e^{ie\oint A_\mu dz^\mu}\; e^{iS^{SM}_{eff}[A]}\;, \tag{31}$$

$$W_{CSM} = \int dA\; e^{ie\oint A_\mu dz^\mu}\; e^{iS^{CSM}_{eff}[A]}\;. \tag{32}$$

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* In fact, this has to be done in the ordinary SM as well because the massless fermions in the model cannot be used as test charges, either.

Since the only difference between the effective actions is the photon mass, the result for the Wilson loop is the same in both models. For the SM, it has been calculated in refs. /37, 38/. There the potential between two static charges in the distance d has been found to be:

$$V_{SM} = \frac{2e^2}{m^2}\left(1 - e^{-md}\right) , \tag{33}$$

which can easily be converted to the corresponding potential in the CSM:

$$V_{CSM} = \frac{2e^2}{\tilde{m}^2}\left(1 - e^{-\tilde{m}d}\right) . \tag{34}$$

Both potentials show confinement for small d (linearly rising potential) and complete screening for large distances (constant potential). This shows that (apart from the difference between m and $\tilde{m}$) in the CSM the polarizability of the vacuum is sufficiently strong to screen the test charges completely, precisely as it is the case in the SM.

V. Relation between the Schwinger Model and the Chiral Schwinger Model

Now we want to show that the SM can be reformulated in such a way that it looks like a CSM with a special regularization prescription. This is done by decoupling the right handed fermions only. Then the resulting fermionic action is the same as in the CSM. The fermionic Jacobian for the transformation $\psi \rightarrow e^{i\Lambda P_R}\psi$ reads

$$\ln J_R[A,\Lambda] = \frac{-ie}{8\pi}\int d^2x \left[\Lambda\, \varepsilon_{\mu\nu}\tilde{F}^{\mu\nu} + (a-1)\frac{1}{e}\Lambda \Box \Lambda\right] , \tag{35}$$

with /32/

$$\tilde{F}^{\mu\nu} = \partial^\mu \tilde{A}^\nu - \partial^\nu \tilde{A}^\mu \quad , \tilde{A}_\mu = a A^+_\mu + (2-a) A^-_\mu . \tag{36}$$

Here we have fixed the regularization for the right handed Jacobian such that i) the left handed Jacobian is regularized as in the CSM /32/ and ii) vector-like transformations are treated as in the SM /27/. The right handed fermions are decoupled if $\Lambda = -(\sigma + \rho)$. Then eq. (35) becomes

$$\ln J_R[A,-(\sigma+\rho)] = \frac{-ie^2}{8\pi}\int d^2x \left[(2-a)A_\mu A^\mu - 4A^+_\mu \frac{\partial^\lambda \partial^\nu}{\Box} A^+_\nu\right] . \tag{37}$$

It may easily be verified that the combination of $J_R[A,-(\sigma+\rho)]$ with the result of integrating out the left handed fermions (eq. (10) with $\theta = 0$) just gives the effective action of the SM (eq. (11)). As usual, the nonlocal term is removed by introducing an auxiliary scalar field φ . This gives for the generating functional of the SM:

$$Z_{SM} = \int dA\, d\psi\, d\bar{\psi}\, d\varphi\, e^{iS} \tag{38}$$

with

$$S = \int \Big\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma_\mu (i\partial^\mu + eA^\mu P_L)\psi + (2-a)\frac{e^2}{8\pi}A_\mu A^\mu - \frac{1}{8\pi}\varphi \Box \varphi + \frac{e}{2\pi}\varphi \partial^\mu A^+_\mu + \mathcal{L}_{GF} \Big\} d^2x . \tag{39}$$

Identification of φ with θ (eq. (3)) shows that the ordinary SM can be viewed upon as the special case a = 2 of the CSM (note that the SM is independent of a and that the CSM does depend on a)[*]. So it is not astonishing at all that the left handed fermions in both models show the same behaviour.

* N.K. Falck thanks B. Schroer for useful discussions on this point.

VI. Conclusion

We have studied fermion propagators and the Wilson loop in the CSM. As a result, we have found that the left handed fermions behave in the same way as they do in the SM. This is evident by the striking similarity of the considered quantities in the two models. This coincidence is true for all Green's functions containing only left handed fermions and gauge bosons, as can be proven in the same way as we showed it for the left handed propagators. Hence also in the CSM the polarizability of the vacuum is strong enough to shield charges completely, as it is the case in the SM /22 - 24/. This proves that the left handed fermions do not show up in the physical spectrum.

The CSM defines a whole class of quantum theories which only differ by the value of the regularization parameter "a". We have shown explicitly that this class contains the ordinary SM by bosonizing the right handed fermions. The CSM, in its bosonized version, containes a massless state in addition to the massive state of the SM /1/. This does not harm because the noninteracting state $\varphi - 2e\frac{\partial^{\mu}}{\Box}A^{+}_{\mu}$ in eq. (39) can be added at will without altering physics. Hence it is due to the bosonization procedure, which is also evident from the fact that in the fermionic CSM the exact boson propagators do not show such a massless state /14/.

Finally, we want to remark that our investigations concern the topologically trivial sector only. For a treatment of the nontrivial sectors in the SM, see /39, 40/; in the CSM up to now not much progress has been made into this direction.

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References:

/1/ R. Jackiw and R. Rajaraman, Phys. Rev. Lett. 54 (1985) 1219

/2/ H.O. Girotti, H.J. Rothe and K.D. Rothe, Phys. Rev. D33 (1986) 514

/3/ H.O. Girotti, H.J. Rothe and K.D. Rothe, Phys. Rev. D34 (1986) 592

/4/ C. Aragao de Carvalho, K.D. Rothe, C.A. Linhares and H.J. Rothe, Phys. Lett. B (to appear)

/5/ R. Link, A. Roberge and C.W. Semenoff, Z. Phys. C (to appear)

/6/ C.A. Linhares, H.J. Rothe and K.D. Rothe, Heidelberg preprint HD-THEP-87-17 (1987)

/7/ I.G. Halliday, E. Rabinovici, A. Schwimmer and M. Chanowitz, Nucl. Phys. B268 (1986) 413

/8/ M. Chanowitz, Phys. Lett. B171 (1986) 280

/9/ C.A. Linhares, H.J. Rothe and K.D. Rothe, Phys. Rev. D35 (1987) 2501

/10/ D. Boyanowski, Stanford preprint SLAC-Pub 4198

/11/ K. Harada and I. Tsutsui, Phys. Lett. B183 (1987) 311

/12/ K. Harada and I. Tsutsui, Prog. Theor. Phys. 78 (1987) 878, Tokyo preprint TIT/HEP-102 (1986)

/13/ N.K. Falck and G. Kramer, Ann. Phys. (N.Y.) 176 (1987) 330

/14/ N.K. Falck and G. Kramer, Z. Phys. C (to appear)

/15/ F.A. Schaposnik and J.N. Webb, Z. Phys. C34 (1987) 367

/16/ O. Babelon, F.A. Schaposnik and C. Viallet, Phys. Lett. B177 (1986) 385

/17/ A. Kulikov, Serpukhov preprint IHEP 86-83 (1986)

/18/ R. Rajaraman, Phys. Lett. B162 (1985) 148

/19/ N.K. Falck and G. Kramer, Phys. Lett. B193 (1987) 257

/20/ E. Abdalla and K.D. Rothe, Phys. Rev. D36 (1987) 3190

/21/ V.M. Manias, F.A. Schaposnik and M. Trobo, Phys. Lett. B195 (1987) 209

/22/ J. Schwinger, Theoretical Physics, Trieste Lectures, 1962, I.A.E.A., Vienna 1963

/23/ J. Lowenstein and J. Swieca, Ann. Phys. (N.Y.) 68 (1971) 172

/24/ A. Casher, J. Kogut and L. Susskind, Phys. Rev. D10 (1974) 732

/25/ K.D. Rothe and B. Schroer, Field Theoretical Methods in Particle Physics, Univ. Kaiserslautern, 1979 (ed. W. Rühl), Plenum Press, New York 1980

/26/ S. Miyake and K. Shizuya, Phys. Rev. D36 (1987) 3781

/27/ R. Jackiw, Relativity, Groups and Topology II, Les Houches, 1983 (eds. B. DeWitt and R. Stora), North Holland, Amsterdam, 1984

/28/ K. Fujikawa, Phys. Rev. D21 (1980) 2848, D22 (E) (1980) 1499

/29/ K. Harada, H. Kubota and I. Tsutsui, Phys. Lett. B173 (1986) 77

/30/ J.N. Webb, Z. Phys. C31 (1986) 301

/31/ J. Barcelos-Neto and C. Farina de Souza, Rio de Janeiro preprint IF/UFRJ/87/06 (1987)

/32/ N.K. Falck, Hamburg preprint DESY 88/x (in preparation)

/33/ J. Schwinger, Phys. Rev. 128 (1962) 2425

/34/ N.K. Nielsen and B. Schroer, Phys. Lett. B66 (1977) 475

/35/ H. Banerjee and R. Banerjee, Z. Phys. C (to appear)

/36/ K. Wilson, Phys. Rev. D10 (1974) 2445

/37/ H.J. Rothe, K.D. Rothe and J.A. Swieca, Phys. Rev. D19 (1979) 3020

/38/ H.A. Falomir, R.E. Gamboa-Saravi and F.A. Schaposnik, Phys. Rev. D25 (1982) 547

/39/ N.K. Nielsen and B. Schroer, Nucl. Phys. B120 (1977) 62

/40/ C. Jayewardena, Ph.D. thesis, Bern (1987)