David Kollosche (Ed.)

Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference

Volume 1

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Further information about the Mathematics Education and Society research group can be found online at https://www.mescommunity.info.

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Editorial

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This editorial of the Proceedings of the Eleventh International Mathematics Education and Society Conference explains the circumstances and organisation of the conference, presents the contents of this book and the review process that lies behind its production, and acknowledges the contributors who invested their resources to make this conference and this book a reality.

The conference: Exploring new ways to connect

These are the Proceedings of the Eleventh International Mathematics Education and Society (MES) Conference, to be held in September 2021. Although this is already the eleventh MES conference, this is the first one that is held entirely online, as so many conferences are at times of the COVID-19 pandemic. It is a huge challenge to organise a conference, which is traditionally held in a physical form, purely virtually – even more so in the case of an MES conference, which does not only feature presentations, which can easily be streamed, but includes intense discussions in various formats that cannot be easily copied to the digital realm. In this sense, the conference organisers and participants face the task of 'exploring new ways to connect'.

The conference organisers sticked as closely as possible to the traditional Principles and Guidelines of the MES community (https://www.mescommunity.info). This includes

– ensuring regional, ethical, and gender diversity concerning the choice of the four plenary speakers, of their two respondents each, and of the four plenary discussants,
– providing space for intense small-group and plenary feedback discussions on each plenary presentation,
– allowing for thematic specialisation through symposia, and
– facilitating lively discussions of individual research papers, project presentations, and posters.

A special challenge was the organisation of the times slots in which conference activities would be scheduled. Our experience from other online conferences, where the usual conference day was organised in eight consecutive hours, was that this solution suits people in a few time zones very well but makes participation for people in other time zones nearly impossible. We decided to distribute three three-hour time slots around each day, so that only one time slot would be at night at any time zone on Earth. Although this approach largly fragments the conference days and makes it very complicated to enjoy all programme points of the conference, we hope that it will stimulate participation from a large varierty of places.

The difficulty of scheduling meetings across time zones motivated us to introduce the rule that live meetings should be used for exchange, while the reception of information should be possible on an individual schedule. Thus, we do not only follow the traditional

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MES policy to publish the proceedings before the conference, so that the papers will be only for reading in advance. We also invited contributors to create short videos of their paper presentations and to host them for the conference participants, so that participants can come to the live sessions well prepared and use them mostely for feedback and discussion. We asked our participants to include the tag 'MES11' in the videos they host on video hosting platforms such as YouTube, and as long as authors followed this request and left their videos online, the reader might be able to still find some presentations searching for this tag.

Although switching to an online conference brought the obvious challenges, it also broad new possibilities, which we are eager to explore. Apart from technological innovations such as discussions on Padlet and informal meetings on SpatialChat, the online character of the conference allowed for a more inclusive participation from around the world. While physical conferences cause expenses for travelling, accommodation, eating out, and hosting, which not every scholar from any cultural and economic background can easily shoulder, our online conference only asked for a conference fee in three different income-dependent tariffs. Consequently, we could witness a wide variety of places from where contributions were submitted, and from where participants registered. At the time of the publication of these proceedings (7 September 2021), we had 46 registrations from the United States, 31 from Brazil, 21 from Sweden, 17 from India, 14 from Canada, 13 from the United Kingdom, 11 from Norway, 9 from Austria, 7 from Germany, 7 from New Zealand, 5 from Greece, 4 from Israel, 3 from Colombia, 3 from Japan, 2 from Nepal, 2 from Spain, 1 from Armenia, 1 from Australia, 1 from Chile, 1 from Egypt, 1 from Ghana, 1 from Indonesia, 1 from Rwanda, 1 from Saudi Arabia, 1 from South Africa, and 1 from Turkey.

The contents of this book

In this book, you find the manuscripts of nearly all contributors to the conference. Thereby, MES aims at facilitating a 'wider discussion of the social, ethical, and political dimensions of mathematics education for disseminating theoretical frameworks, discussing methodological issues, sharing and discussing research, planning for action and the development of a strong research network on mathematics education and society' ('Mathematics education and society', n.d.).

In the *first part*, titled 'Plenaries and Responses', you find the papers which accompany three of the four plenary presentations. We were not able to include the paper of plenary speaker Maisie L. Gholson, nor the papers of the respondents Ana Carolina Faustino and Luz Valoyes-Chávez, but we hope to be able to provide them later on the MES website. In all cases, both plenary speakers and respondents were chosen and invited by the MES11 organising team, and we are very happy and grateful that they agreed to invest their qualities in MES.

In the *second part* of this book, titled 'Symposia', you find the texts of the symposium proposals. These papers merely outline the contents of the symposia and were used to judge the quality of each proposal for its acceptance for the conference. The *third part* of this book, titled 'Project Presentations', contains the short papers in which envisaged or currently running research projects are being presented. The *fourth part* of this book, titled 'Poster Descriptions', comprises the descriptions of the posters that were submitted to the conference. The last and largest *fifth part*, titled 'Research Papers', includes the full-length manuscripts of the research papers submitted for presentation and discussion at the conference.

Symposia proposals, project presentations, poster descriptions, and research papers were each reviewed by two peers, who were already acquainted with the goals and policies of MES, and finally evaluated by a member of the International Committee of MES. Although the reviewing process of MES aims at productive feedback that allows manuscripts to reach the quality necessary for publication, it still checks the academic quality of each submission and its fit to the interests and aim of the MES community. In a few cases, where contributors challenged the boundaries of what would usually be accepted as academic work, we strived to tolerate and accept these contributions, also in the interest of further developing the forms of academic inquiry and exchange through experimental formats.

Nevertheless, in the reviewing process, some submissions had to be withdrawn or rejected because of a lack of fitting to the interests and aims of the MES community or because of a lack of academic quality. Other submissions were withdrawn by their authors for personal reasons or rejected because revisions were not sent even after a widely extended deadline. Eventually, this book contains 133 of 139 submitted research papers, 29 of 35 submitted project presentations, 7 of 8 submitted symposium proposals, and 4 of 5 submitted poser descriptions.

Acknowledgements

Organising a high-quality conference for more than 200 participants is a huge task that cannot be shouldered by a few. Consequently, there are many people to give thanks to.

First of all, I would like to thank the MES11 Organising Team, that is Yasmine Abtahi, Lisa Darragh, David Kollosche, Renato Marcone, Amanda Queiroz Moura, João Pedro Antunes de Paulo, Luz Valoyes-Chávez, and David Wagner, for our weekly online meetings, where countless issues were discussed and decisions were reached as some drank their morning coffee while others had their evening drink. It was a pleasure to learn and work with you, and your help is unvalueable.

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Further, I owe a debt of gratitude to my local supporters, that is Amanda Queiroz Moura, Margit Pirker-Zedlacher, Gordana Gajić, and especially Martin Köfer, for shouldering much of the work that often goes unnoticed but is urgently necessary for the administration of such a big event. In this sense, I also thank our institution, the University of Klagenfurt, whose administration was always eager to help and meet our needs.

Eventually, I would like to thank all the participants for bringing this intellectual richness to the conference. Special thanks go to those 46 participants who donated money to support socio-economically unpriviledged scholars to participate at future physical MES conferences.

References

‘Mathematics education and society’. (n.d.). [Website]. Retrieved September 1, 2021, from https://www.mescommunity.info

Plenaries and Responses

Innovative learning environments and the digital era: Finding space for mathematics identity

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The dichotomy of traditional versus reform mathematics classrooms has been of much research interest, including how students and teachers perform agentic identities in these contexts. Nowadays, however, mathematics learning in Aotearoa New Zealand plays out on a stage that may be a vast departure from either of these classrooms. So called "innovative learning environments" are characterised by fluid seating arrangements, multiple teachers, many digital devices; and they may be predominantly 'online'. In fact, we might argue that mathematics instruction today is 'device-centred' more than being teacher- or student-centred, a trend exacerbated by distance-learning during the pandemic. How then do teachers and students develop mathematics identities in this new era? A performative definition for identity allows us to see identity more easily as existing outside the individual; not only produced in and by social contexts, but also in wider societal narratives. In this paper I will discuss how 'innovative' learning environments are situated in neoliberal ideology and "twenty-first century" discourses, and I consider the production of teacher and learner identity scripts in these spaces.

Setting the scene

Let me start by painting a picture of a primary classroom in Aotearoa New Zealand. First you must delete your image of four walls, a desk for each child, and a forward-facing orientation. The floor plan of this classroom is hexagonal – an irregular polygon that would fit three or more traditional classroom squares. There are sectioned off spaces or smaller rooms with walls of glass. The furniture is varied and optional; children may sit on the floor, on cushions, on beanbags, at low tables, at higher tables, or they may stand or even lie down. Their belongings are in 'tote trays' so that they are mobile. The devices are mobile as well – children may bring their own laptops, Chromebooks, or tablets/I-pads; and the classroom also has its own collection of these for the children to use. There are multiple teachers but they may not be easy to spot, not being situated front and centre. I suspect for some it will be difficult to reconcile this scene with a more traditional image of the mathematics classroom that would otherwise automatically spring to mind. I invite you to click on the link below and watch the short video titled "Understanding pedagogy" (from: Te Kete Ipurangi (TKI): Ministry of Education, 2021).

https://www.inclusive.tki.org.nz/guides/planning-innovative-learning-environments-iles/

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The scene above does not describe *every* classroom in Aotearoa New Zealand, there is certainly a great deal of variety in classroom types, however it (and the embedded video) represent the direction taken in designing educational contexts over the past decade. Let us call these examples the "new" classroom, in order to differentiate and acknowledge that there are certainly other, more traditional classrooms to be found elsewhere throughout the country.

Of course, I have described a pre-pandemic classroom. Even after the return from lockdowns and emergency remote teaching it was certainly inappropriate to have such a high level of interaction between children and free movement through the space. On the other hand, the use of digital technologies that were already prevalent has increased. In Aotearoa New Zealand we are fortunate to have returned (at the time of writing) to 'normal' classroom interaction, but there remains the question of what is normal, or what might be the new normal? As Borba and colleagues suggest, the pandemic may entirely change the agenda for mathematics education (Borba, 2021; Engelbrecht et al., 2020). However, we have already seen how crises may be harnessed as a rationale for wide-scale educational change (Mutch, 2017; Williamson, Eynon & Potter, 2020), and I suspect it is safe to assume that these technology-rich, innovative learning spaces, such as seen in the link above, will become increasingly common, at least in Aotearoa New Zealand.

One might ask what *mathematics* teaching and learning look like in the 'new' classroom space. Whilst it seems clear that the 'new' classroom is a vast departure from the traditional classrooms of last century, my question is not whether the innovation constitutes an improvement to mathematics teaching and learning, rather, I am interested in how these environments (including online environments) produce identities as teachers and learners of mathematics. This is an important question given the value of identity research in understanding teacher change (e.g., Chronaki & Matos, 2013; Lutovac & Kaasila, 2017) and students' relationships with and participation in mathematics (e.g., Mendick, Moreau, & Epstein, 2009; Radovic, Black, Salas, & Williams, 2017).

In this paper I first outline a definition of identity as performative, following Judith Butler's work on gender identity. I extend her theatrical metaphor to propose a method by which we may more easily understand identity as beyond the individual, and produced by the wider socio-cultural and political context. Next, I will briefly give an account of the historical, educational context of Aotearoa New Zealand – the 'theatre'. Then I will discuss the 'stage' for learning mathematics (and producing identity), in this case innovative learning environments (ILEs) that include online programs for mathematics instruction, as I term the 'new' classroom. The discussion is based on my current research into the phenomenon of learning mathematics via online instructional programs and my personal observations of ILE classroom spaces. Additionally, in lieu of data, I invite you to consider the video linked above, and also the website for *Mathletics* (3P Learning, 2021, https://www.mathletics.com/nz), both of which I will refer to throughout.

Performative identity and extending the theatre metaphor

A few years ago, Darinka Radovic and I were invited to write a definition for mathematics learner identity for the *Encyclopedia of Mathematics Education* (Lerman, 2020). We were tasked to give a definition that *reflected* (rather than advanced) the work on identity in mathematics education currently. We finally settled on the following:

> A socially produced way of being, as enacted and recognized in relation to learning mathematics. It involves stories, discourses and actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities. This incorporates how they are treated and seen by others, how the local practice is defined and what social discourses are drawn upon regarding mathematics and the self. (Darragh & Radovic, 2018, para. 1)

Although the above definition focuses on mathematics learner identity, a mathematics teacher identity may be thought of in a similar way. We aimed to write a definition that encompassed the various ways that mathematics learner identity is defined and operationalised in the discipline. Each aspect: (e.g., socially produced, enacted, recognized, multiple) is evident in the wider literature, but each of these aspects may be understood slightly differently depending on the perspective taken. There are certainly many ways to understand identity, and sometimes definitions are vague or absent altogether in literature published in the field. Elsewhere we have both argued that authors must be explicit in the way they define and operationalise identity for the purpose of their research (Darragh, 2016; Radovic, Black, Williams, & Salas, 2018), and so I will attempt to be explicit here. I take a *performative* view of mathematics identity, drawn from Butler (1988), that explains the social production of identity acts and highlights the role of recognition as part of identity. Others in mathematics education have seen value in Butler's work for identity (e.g., Chronaki, 2011; de Freitas, 2008; Gholson & Martin, 2019; Mendick, 2017); I find it useful myself because it allows an operationalisation of identity that incorporates more than interview narratives, and focuses the gaze beyond the individual to look at identity enactment and the production of identity within various layers of context. This enables a more thorough understanding of the relationships people form with mathematics learning and teaching and of the decisions they make regarding future participation in higher education or engagement with professional learning.

Judith Butler defines *gender* identity as performative, that is, a "stylised repetition of acts" (Butler, 1988, p. 519). Butler argues "a body becomes its gender through a series of acts which are renewed, revised, and consolidated through time"; thus identity is not predetermined but rather "the legacy of sedimented acts" (p. 523). I find it very useful to consider mathematics learner or teacher identity in the same way, that is, a series of acts which are renewed, revised, and consolidated over time. Butler wrote this particular definition in the '*Theatre Journal*', which may explain the theatrical emphasis on the idea of an 'act'; however, it is worth noting the theatre metaphor has generated some confusion as it tends to imply a separation between the actor and the act, as opposed to a poststructuralist understanding of subjectivity where the act produces the actor (Jagger, 2008), which is key to performativity. Yet, I see much potential in exploiting the theatre metaphor in order to make explicit the wider social and political context in which identities are performed and performatively produced. We may consider the socio-political context to be the *theatre* and the immediate social context (e.g. the classroom) as the *stage*. We might ask ourselves what the typical *scripts* or normative ways of being are, and whether *improvisations* are possible. Finally, the *audience* takes up the important role of recognising (and validating) identity performances. I contend that understanding how identity is performatively produced requires a consideration of the wider temporal, ideological, and physical contexts.

I would like to unpack and operationalise this theatrical metaphor as I consider how the 'new' classroom context produces mathematics identity. I invite you to contemplate this metaphor alongside me as you view the linked video, the website from one popular online mathematics instructional program, *Mathletics* (3P Learning, 2021); or you may like to reflect on your own context. First of all, I situate us within the wider socio-political context of Aotearoa New Zealand's neoliberal educational system.

The 'theatre': Socio-political-historical context of education in Aotearoa New Zealand

In Aotearoa New Zealand, like many other countries around the world, neoliberal policies have dominated the political scene over the past three decades thus impacting the education system considerably (Ladd & Fiske, 2003; McMaster, 2013). Neoliberalism is an economic and political ideology that proposes individual, entrepreneurial freedom and is characterised by free markets (Harvey, 2005). Some of the ways we see neoliberalism at work in education is the devolution of control from central government to individual schools; 'free choice' for parents in where they send their children to school; voucher systems where the money follows the student; and outsourcing of educational provision to private providers (Thrupp, O'Neill, Powell, & Butler, 2020). Although an early adopter of neoliberal education, Aotearoa New Zealand has resisted some of the policies that have become entrenched in other nation's educational systems (McMaster, 2013). However, some neoliberal features that remain are: devolution of governance and curriculum to local schools (Lange, 1988; Ministry of Education, 2007; O'Neill, 2011), and private providers being allowed to make profit in public schools via educational provision of particular subject areas, including mathematics (Thrupp et al., 2020).

The beginning of neoliberalism within education for Aotearoa New Zealand was the policy document "Tomorrow's Schools" (Lange, 1988). *Tomorrow's Schools* devolved much responsibility for education to individual schools, managed by 'Boards of Trustees', which were made up of parents in the community. One intention of the policy was to give greater voice to parents and the community (McMaster, 2013). This allowed a great deal of autonomy to individual schools and, together with the 1991 abolition of school enrolment zones, encouraged school choice and competition for students (Ladd & Fiske, 2003; McMaster, 2013). Curriculum reform swiftly followed, with new curricula in 1992, and again in 2007. The 2007 curriculum is remarkable for the small size of the document. All eight content areas and all 13 years of schooling are contained in a document of only 65 pages, of which mathematics has no more than 10 pages (Ministry of Education, 2007). In other words, the national curriculum contains only a list of achievement objectives without guidelines of how to teach them. The impetus was given to schools to create their own localised curriculum based on this document. On one hand this meant schools were able to cater educational experiences to the local community's wants and needs (see also McMurchy-Pilkington, Trinick, & Meaney, 2013). On the other hand, some schools struggled to develop a curriculum that was sufficient for the needs of their students. In either case, the variety of different practices in schools has increased under this policy.

Another prominent feature of the 2007 curriculum is the focus on producing "twenty-first century learners" (Benade, 2015; Ministry of Education, 2007). This is a global movement, driven in part by the Organisation for Economic Cooperation and Development (OECD), which has questioned "'outmoded' transmission models of teaching" and called for reform of educational systems (Benade, 2019, p. 58; see also OECD, 2013). In Aotearoa New Zealand this "transformation is increasingly evident in new technology-rich flexible learning environments, characterised by large open spaces, permeable boundaries and diverse furnishings emphasising student comfort, health and flexibility" (Benade, 2019, p. 58). Benade also notes that the design reflects the "neoliberal concern with ensuring that education is relevant to the realities of the twenty-first century workplace" (Benade, 2017, pp. 804–805), as shall be elaborated further later.

A more recent event that had a significant impact on schooling in Aotearoa New Zealand was the 2010 earthquake in Christchurch. The disaster was used as a justification for widespread permanent closure of some schools, to the devastation and surprise of their local communities (Mutch, 2017). With the post-earthquake re-building of schools, many were designed using "Innovative Learning Environment" (ILE) guidelines – with classroom spaces as described in the opening paragraphs. Subsequently, all school builds, expansions, and renovations in the entire country have been required to follow the ILE design in order to receive funding (Bradbeer et al., 2017). The ILE space certainly looks innovative and modern, but it has been criticised for not being based on research evidence (Bradbeer et al., 2017), motivated either by lower cost or neoliberal ideology, and evidence of the way a crisis can be harnessed by politics for educational change (Mutch, 2017). Over the past decade this has meant many schools have realigned their classrooms to the ILE model, although others remain traditionally 'single cell'.

Within this neoliberal and 'technology rich' context, mathematics education has increasingly drawn on digital technologies as a regular part of instruction. A recent OECD survey found that children in Aotearoa had the fastest growing rate of computer use in mathematics classes in the OECD with 89% of students using this technology as part of their mathematics learning (Vincent-Lancrin, Urgel, Kar, & Jacotin, 2019). With such high availability of the technology, it is hardly surprising that the majority of schools have turned to commercial programs to provide instructional material using these computers and mobile devices. As many as 80% of primary schools in Aotearoa New Zealand subscribe to at least one online mathematics instructional program (OMIP), such as *Mathletics, Study Ladder, Mathsbuddy*, and *Sumdog* (Darragh & Franke, 2021). These online learning platforms are run by private corporations; they are typically of international origin but adapted to the national curriculum. Schools or parents may purchase an annual subscription and there is sufficient content on the platforms to provide mathematics learning for the full year. Although the COVID-19 pandemic exposed inequities of access to internet and digital devices at home (Riwai-Couch et al., 2020), children in Aotearoa New Zealand now have considerable access to devices and the internet at school (Vincent-Lancrin et al., 2019).

Aotearoa New Zealand was fortunate to avoid the health crisis experienced by many other nations during COVID-19, however the *educational* impact has been widely felt as we had repeated (albeit brief) lockdowns to halt community transmission and children were in and out of school depending on the health alert levels. Given that some parents were desperate to provide extra learning activities for their children during lockdown (RNZ, 2020, 28 July) those companies that provide online instructional programs may have had the opportunity to cement themselves further into the educational landscape; indeed, the marketing of such programs worldwide has ramped up considerably since the pandemic began (Williamson et al., 2020).

To summarise, the context of ILEs and online learning platforms for mathematics instruction can only be understood within the neoliberal policy agenda of education in Aotearoa New Zealand. This agenda includes: the necessity for schools to create their own local curriculum and decide whether to use private providers for part of the mathematics instruction, and the governmental requirement to create ILE spaces with new builds. In the next section I zoom in so that we may examine more closely this particular stage for the learning and teaching of mathematics. As we do this, I invite you to consider your own particular ‘person of interest’. This may be the newly qualified mathematics teacher, the teacher who works in poverty-stricken areas, the ‘out-of-field’ teacher, or the white, middle-class teacher in diverse contexts. Or the person of interest may be the neuro-diverse child, the high achiever, the hearing impaired, the Black girl, the recently arrived immigrant, or the child who engages with schooling to revitalise their indigenous language.[1] As I present the ‘new’ classroom stage for mathematics teaching and learning in Aotearoa New Zealand, I invite you to ponder this question: Can you picture your person in this space?

[1] My apologies here – this list is clearly non-exhaustive and I am aware that I may have missed your particular ‘person of interest’ thus marginalising them further (literally) in this footnote.

The 'stage': Innovative learning environments and online mathematics learning

Whilst the 'theatre' attends to the broader socio-political context, the 'stage' may describe the context at the local level, i.e., the mathematics classroom. There are a number of studies on identity at this level of the 'zoom lens' (Lerman, 2001), but most are based in single-cell classrooms, whether they incorporate 'traditional' or 'reform' instruction (see for example Esmonde, 2009; Heyd-Metzuyanim & Shabtay, 2019; Ma & Singer-Gabella, 2011; Valoyes-Chávez, 2019). In this section I will first describe the 'new' stage for learning mathematics – the ILE space and online instructional programs. I follow with an explanation of how students may negotiate the space when learning mathematics and then discuss a couple of features of this 'new' stage that set it apart from classrooms of the past.

In a typical ILE environment, children learn in classroom spaces that are designed to fit 40-160 students together with 2-6 teachers (Everatt, Fletcher, & Fickel, 2019). Whilst the teachers are supposed to work collaboratively to teach the entire group, children are required to navigate the space independently. Benade gives an evocative description based on research within Aotearoa New Zealand schools:

> The placement of tables and chairs, often boardroom style, is a place where a 'workshop' can take place, facilitated by a teacher, or, more appropriately, a 'learning advisor' or 'coach'. A private space off to the side for a small group to work together is a 'breakout space'. Along with this neoliberal language of the business conference is the imagery of the future hunter-gatherers of the twenty-first century knowledge economy gathering at the 'campfire' (a circular formation of Ottomans). Redolent of captivating tales or fellowship, this is a space of gathering together before expedition, or debriefing after. Thirsting for knowledge, some young cubs work intently at a 'watering hole', a circular arrangement of seats and tables, where they plan their next project. For those who are required to work on complex tasks (such as numeracy) there are the high tables and chairs that provide a 'lookout', allowing these students to gaze intently into the long distance, as they solve challenging problems. (Benade, 2017, p. 802)

Where might we see the teacher in this space? There is little room for the traditional 'chalk and talk'. The teacher becomes a facilitator - assigning learning activities and managing the learning environment. And at times we may see the teacher engaging with a small group of children (at the "campfire") and teaching them some mathematics content.

The hunter-gatherer imagery employs a different metaphor to the idea of the stage within a theatre, yet it is scenery that is taken up by a number who write about ILE classrooms. The metaphor comes from Thornburg (e.g., 2004), who also describes the 'cave' as being a space for internal reflection, in addition to the more populated learning spaces of waterhole and campfire. In the above scene, the place given to mathematics appears isolated – children at high tables (rather than the low, collaborative spaces) deep in thought as they solve challenging problems. However, mathematics learning may be seen in the other spaces too. Children might be able to engage in small group, collaborative problem solving (at the watering hole) – but we also see many children seated either individually or in pairs and with some kind of digital device (laptop, chromebook, or tablet, for example). What mathematics learning do these devices provide? In Aotearoa New Zealand it is likely the children will be engaged with commercial platforms that provide individualised learning online (Darragh & Franke, 2021).

Thornburg suggests that each of the neolithic scenes (the campfire, the watering hole and the cave) may be replicated in the online environment. Indeed, one thing missing from Benade's description above (though he refers to it elsewhere, e.g., Benade, 2015) is the presence of modern technologies in these spaces. As previously discussed, a key aspect of twenty first-century learning and ILE spaces are their "technology-rich" nature. Children and teachers in Aotearoa New Zealand have ready access to computers and hand-held devices and the internet at school (Vincent-Lancrin et al., 2019), and this makes the use of OMIPs during mathematics somewhat unsurprising. The most popular platform is *Mathletics* (3P Learning, 2021), originally designed in Australia, and subscribed by approximately 40% of the schools in Aotearoa New Zealand that use OMIPs (Darragh & Franke, 2021); there are dozens of other OMIPs to choose from.

The OMIP platforms draw on behaviourist techniques to motivate children to do maths (Jablonka, 2017); for example, completing a number of exercises is rewarded with a game, or the mathematics work is 'disguised' in a gaming format. Jablonka (2017) discussed this gamification in the context of OMIP "Sumdog" at a previous Mathematics Education in Society conference. Many of the platforms offer 'payment' for doing the mathematics in the form of 'tokens', 'coins', or 'points' that may be spent on designing avatars (or purchasing related paraphernalia) during the reward phase. This is the case for *Mathletics* (3P Learning, 2021): Figure 1 below shows the points, gold bars, and certificates achieved by the depicted (cartoon) 'learner'. Figure 2 depicts information directed to the teacher – learner analytics, with an example of the type of statistical information given about the learner's performance and progress.

In general, research within mathematics education is remarkably silent on the use of OMIPs. The plethora of studies into technology use in mathematics tend to focus on issues of teaching and learning (Young, 2017), and they tend to take an uncritical view of the technology itself; greater attention is given to the benefits to learning or the challenges of having teachers adopt the technology. By contrast, outside of mathematics education and particularly in the field of Media Studies we find much critique of the EdTech (educational technology) industry. The aspects of EdTech of concern centre on the personalisation of learning (Roberts-Mahoney, Means, & Garrison, 2016), and 'big data' collected via 'learner analytics' (Knox, Williamson, & Bayne, 2020). Personalising learning may be argued a valid goal for education but, as pointed out by McRae (2013), the 'hyper-individualisation' of the programmes is reductionist (mathematics becomes basic facts, for example) rather than providing personalised learning based on student interest or cultural background. In other words, learning is individualistic and individualised (Biesta, 2012), that is, centered on the *individual learner,* rather than being centred on students as a group. Such an approach does not consider the backgrounds and interests of the children as a community of learners in the classroom and thus the teachers' situated knowledge of them is de-valued in this model.

Figure 1: Rewards earned by the *Mathletics* student (3P Learning, 2021) see: https://www.mathletics.com/nz/features/

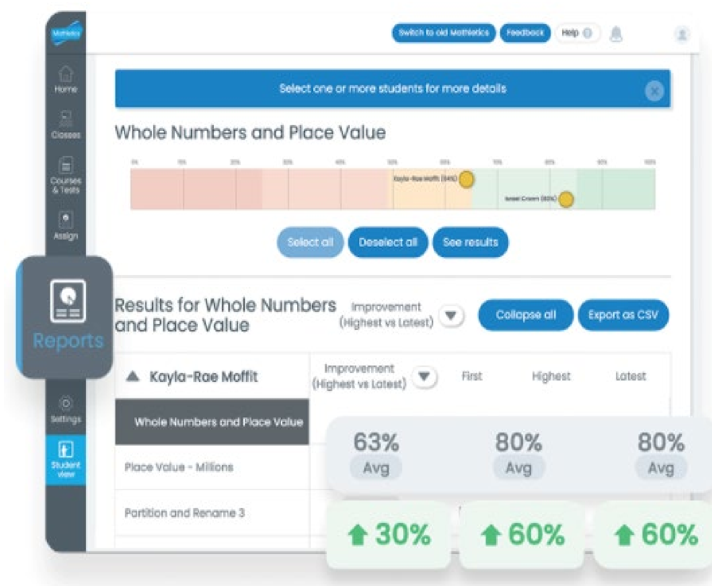

Figure 2: Learner analytics for one student
(3P Learning, 2021) see: https://www.mathletics.com/nz/features/

Teaching and learning mathematics on the 'new' classroom stage

It may be challenging to see clearly the teaching and learning of mathematics in ILE or OMIP spaces, and so it is perhaps useful to explore how a mathematics 'lesson' may proceed. In most Aotearoa New Zealand classrooms, a group rotation is used in mathematics whereby one group works with the teacher and the other groups are assigned other activities. In many schools the groups are ability based (Anthony & Hunter, 2017), despite research critiquing this practice. In some schools, however, groupings are instead fluid, and membership changes according to the needs of the child. In either case, because the class is not taught all together as a whole, communication as to what each student should be doing becomes complex. The illustration below (taken from the embedded video) depicts an "Action Stations" board (see Figure 3). The various activities on offer are represented by pictures and underneath each activity are the names of a group or individual children. The students find their allocated activity, locate the resources they need, and find a space in which to do their task, and often there is the option of 'free choice activities' included.

Figure 3: "*Action-stations" board: Children find their name and allocated* activity (Ministry of Education, 2021) see: https://www.tki.org.nz/

Typically, some students are directed to a digital device, and likely an activity on an OMIP such as *Mathletics*. When engaging with mathematics on the OMIP, students might have some choice as to which activity to complete, or they may have been assigned an activity - either by the teacher or by the OMIP's learning analytics.

(Co-)performing agentic mathematics teacher or learner identities in the 'new' mathematics classroom

Returning to the question of your particular person of interest, how do you imagine they co-perform mathematics teacher/learner alongside their other identities on this 'new' classroom stage? Might they be agentic in these performances, or would the context constrain how they may *be* a mathematics teacher or learner? To answer these questions, it may help to first look more closely at the way in which the 'new' classroom stage may constrain (or enable) particular identity performances, and secondly, consider what the *identity scripts* are that normalise performances on such a stage.

There are a couple of features of the 'new' classroom stage that set it apart from classrooms of the past. These may be seen in both the ILE space and the OMIP platforms; *free choice*, and the use of *surveillance*. These two features work to produce the mathematics teacher and learner in particular ways, as discussed below.

Free choice

One notable aspect of the 'new' classroom stage is the bodily freedom allowed to students as they perform the learner role. The children in the video certainly seemed in control of their own movement through the classroom space, bodies were not constrained in chairs facing in one direction. The space for being a mathematics learner is considerably expanded; in fact, it even extends beyond the classroom walls, as children may take a device elsewhere or log on to their learning portal at home. Agency for the learner is emphasised in both the ILE environment and the OMIP platforms with this notion of free choice. The school principal speaking in the video linked above mentioned students' freedom of choice and student agency. The online instructional programs also claim to allow student agency in selecting their own pathways through the available lessons (see *Mathletics* example in Figure 4 below). Within OMIPs, the students' freedom of choice is further evident in their choice of which game to play or what they might 'buy' for their avatar (using the credit points awarded for their mathematics work – see Figure 5).

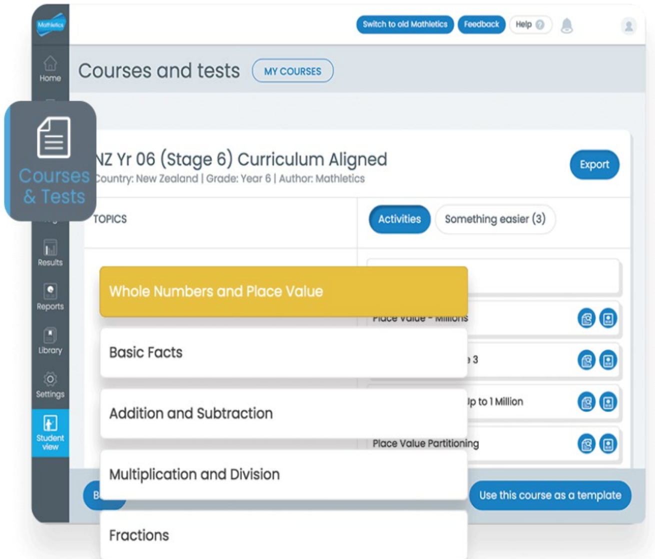

Figure 4: Choice of learning activities (3P Learning, 2021), see: https://www.mathletics.com/nz/

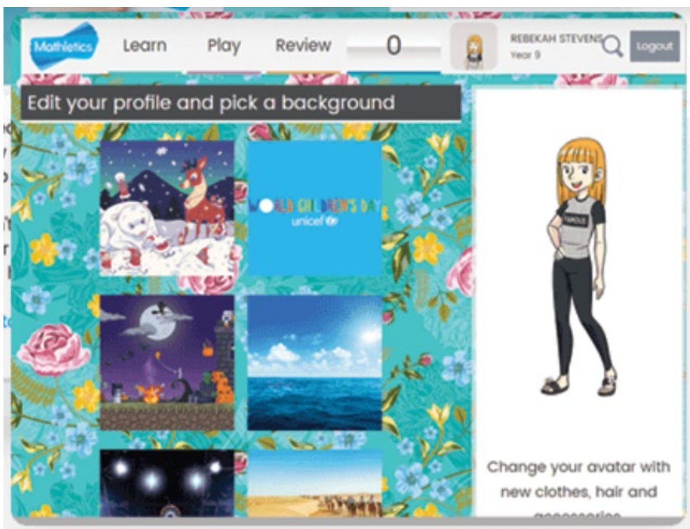

Figure 5: Choosing an avatar (3P Learning, 2021), see: https://www.mathletics.com/nz/

Such choice should be understood within neoliberal ideology, where free choice is a key feature of a market model of education. In Figure 5 we see how OMIPs further promote ‘marketisation’ by producing the mathematics learner as being a consumer in capitalist society, what Jablonka (2017) calls a 'token economy', as students are invited to purchase items for their avatar based on their points 'earned' by doing the mathematics work.

Of course, we might question the level to which choices are in fact 'free'. Whilst children may be able to choose for themselves what to do and how/where to do it, their choices may affect the way in which they are recognised as a learner. Recall Walkerdine's (1990, 1998) research into mathematics of girls and boys in working-class schools: girls taking on the overt messages about good behaviour and following the rules were seen as hard-working (but not very bright), whereas the boys following the covert messages of exploration were attributed with having 'real understanding' and 'potential' despite lack of achievement (Walkerdine, 1990). I wonder what might be the overt and covert messages in these 'free choice' ILE spaces, and which children might be recognised as normal or pathological due to the messages they follow. Relatedly, Benade (2019) considered whether the ILE space is an inclusive design and identified the challenges faced by those with auditory, sensory and socio-cognitive issues; including the noise, the self-regulation required, anxiety, and getting 'lost' in the space. How might such children be recognised as mathematics learners in these cases? Where does your 'person of interest' fit here?

The notion of agency when using the OMIPs may certainly be problematised also. The complex learning analytics generate an assessment of students' next steps for their mathematics learning, and provide activity options based on this assessment (Knox et al., 2020). What appears to be freedom is in fact a tightly constrained choice based on the program's analysis of required next steps. As cautioned by Lupton & Williamson (2017):

> […] learning analytics platforms appear to displace the embodied expert judgement of the teacher to the disembodied pattern detection of data analytics algorithms […] A significant risk that children's opportunities might be narrowed by the assumptions encoded in algorithmic processes is raised by such techniques (p. 787).

In other words, the assessment capability of the OMIPs means that decision-making may be taken out of the teachers' hands and this power placed with the OMIP instead. This fact constitutes a key area of criticism from the field of Media Studies, as mentioned earlier. Therefore, not only is the child constrained in their choice, but the teacher is de-professionalised (Roberts-Mahoney et al., 2016) through their reduced ability to select their own learning pathways for their students. The mythical free choice in the 'new' classroom that appeared to be on the agenda for students is even less evident for teachers. Here we see a further shift in who (or what) is centred on the stage of the mathematics classroom.

In short, the fiction of free choice, promoted by neoliberal ideology, appears to produce agentic performances of mathematics learner or teacher, but this agency is limited and it is controlled - as shall be seen further in the section to follow.

Surveilled performance

Another key feature of the 'new' stage of mathematics classrooms is *surveillance*. Researchers in the field have often used Foucault's (1977/1991) *Discipline and Punish*, applying the "panoptican" to the classroom setting (e.g., Hardy, 2004; Jablonka, 2017; Walshaw, 2010).

Typically, it is the student who is surveilled, but in the modern ILE, full of glass in place of walls, the teacher faces an increased surveillance, their teaching displayed to an extent not possible in the single cell classroom. In the linked video we could spy the teacher in a breakout room behind glass - her mathematics teaching easily visible. The principal in the video used the term “transparency” to describe the teaching practices, conveying a sense of openness both in terms of the physical and the pedagogical. Benade’s (2017) study of teachers’ work in ILE spaces juxtaposes the collaboration and transparency in teacher practice with the “stress of making collaboration work, the feelings of vulnerability, and a sense of always being on show” (p. 803).

The students face a very different kind of surveillance. On the one hand their physical selves may avoid surveillance – they might disappear into a dark corner or beneath a tent – but their learner selves are continually tracked through data collected. Lupton and Williamson (2017) call this “dataveillance”, that is, digital surveillance. In the ILE space, students’ learning and behaviour are firstly tracked via platforms such as Seesaw, ClassDojo, or Google classroom; both teachers and parents can view children’s ‘learning progress’ on these platforms. The OMIPs go further with the sophisticated ‘learning analytics’ as seen earlier in Figure 2, (see also Jablonka, 2017). It is worth noting that this kind of data surveillance forms a violation of an unwritten rule of assessment: that students should know when an activity is being assessed. Pepin and colleagues call this “stealth assessment” (Pepin, Choppin, Ruthven, & Sinclair, 2017), when *everything* the child does is assessed, but they do not necessarily know it. This may be problematic as it denies them the opportunity to knowingly deliver their best effort for the assessed tasks.

What is also important to note here is that a huge amount of data may be collected from children engaging with the OMIP. For example, every keystroke, the length of every pause, the ratio of productive vs unproductive screen time, all may be recorded and used to build up a picture not only of the learner, but also to form a massive database of many thousands of learners.

> Performing such analysis depends not just on surveillance of the individual but also on massive dataveillance of millions of data subjects to generate the kinds of big databases from which accurate predictions are made by comparing individuals against norms derived algorithmically from the masses. (Lupton & Williamson, 2017, p. 787)

Ultimately this ‘big data’ (McRae, 2013; Roberts-Mahoney et al., 2016) is used to make the program more addictive, marketable, and profitable. This has been called “colonial design” technology, whereby the providers of the learning platform learn about the learners rather than the learners learning about themselves (Macgilchrist, 2018).

The emphasis on free choice from both the ILE and OMIP spaces also contributes to a form of surveillance. The students are required to be self-managing as they exercise their choices in selecting the activity likely to optimise their learning. For example, students often must manage their own learning to ensure they have completed a list of weekly tasks, and have made progress towards their own individual goals. This kind of self-management is a form of surveillance, whereby the child surveils themselves (see also Jablonka 2017).

"Learning personalization ensures, in this sense, the learner's individualized and responsibilized investment in their anticipated future self" (Macgilchrist, 2018, p. 242). Being a mathematics learner is about being future-focused via self-management.

Being a mathematics teacher and a mathematics learner therefore includes the performance of self-governance (Foucault, 1991) that results from surveillance. Being a mathematics learner additionally means being datafied, and continually subject to measurement against a norm. To summarise, the stage of the 'new' mathematics classroom in Aotearoa New Zealand produces particular identity acts that involve performances of agency, being surveilled, and self-governance. Such acts may be further understood by considering what the identity scripts are that describe normative ways of being the mathematics teacher or learner.

Dominant scripts for identity on the 'new' stage

The various identity performances as enabled and normalised on the stage and theatre constitute the *performance scripts*. These are expected or idealised ways of enacting identity that may be taken up by the individual, or ignored/altered via 'improvisation'. However, I wish to point out that neither the 'taking up' of a script nor rejecting it, is an easy (or even available) decision. Scripts, as normative models for identity performance, are powerful and impactful because they privilege a particular way of being. Scripts draw from societal narratives – in this case neoliberal ideology, twenty-first century narratives and the EdTech discourse of educational corporations – which all speak to how one should *be* a learner or a teacher of mathematics. Scripts, whilst produced in wider narratives, are made available in the local context – the classroom stage. On any stage there may be a variety of scripts available for individuals to take up (or there may be very few). Popkewitz's problem solving child is an example of a script; he shows how for some the desirable script is not available and these learners are "those left behind" (Popkewitz, 2004). In other words, the concept of 'scripts' answers the question of how one *should be* - in this case a mathematics teacher or learner in the 'new' mathematics classroom. In this section I present two dominant scripts produced in neoliberal and EdTech narratives, which are made available on the 'new' classroom stage. We might name these scripts: '*twenty-first century mathematics teacher*' and '*twenty-first century mathematics learner'*.

The twenty-first century mathematics teacher

Performing the twenty-first century mathematics teacher means being a coach/guide, being a collaborator, and having an audience (being surveilled). Whilst these performance aspects may be applied to teachers of any subject, they each have implications specifically for the mathematics teacher identity.

The production of the teacher as a coach or guide is evident in OECD texts about the ILE space, where the term "learning professional" (OECD, 2013) may equally be used. Here the teacher is made invisible, in a manner also evident in reform mathematics discourse (Valoyes-Chávez, 2019). Within EdTech discourse they even lose the name 'teacher', reduced instead to ‘coach’, ‘facilitator’, or ‘data-analyst’ (Ideland, 2021). A student-centred emphasis means that the teacher is sidelined; their job is to guide students to their learning, but they are no longer required to teach. Similarly, when using the OMIPs, teachers need only direct students to the computer and then let the program do the teaching – including actual mathematical instruction (some OMIPs have instructional videos for this purpose), assessment, and assigning of tasks. In the case of primary school teachers – who are typically generalists (rather than having a speciality in mathematics teaching) and often described as having a problematic relationship with mathematics (e.g., Boylan & Povey, 2009; Hardy, 2009; Hodgen & Askew, 2011) – the OMIP space encourages them to sidle away from the responsibility for mathematics instruction.

The distancing of teachers from mathematics may be further exacerbated by the notion of teacher as collaborator (Benade, 2017). Collaborations enable teachers to divide out responsibility for various subject areas. In this situation just one of the teachers in the ILE space may take on the mathematics teaching, whilst the others may be responsible for other subject areas instead. Here the reduction of the teacher role goes further; for some, their mathematics teaching services are no longer required at all.

Yet a third aspect of the twenty-first century teacher is somewhat in contrast to the previous two. Whilst the mathematics teaching may be side-lined, the teachers themselves are not at all invisible. As discussed earlier, teachers are visible in a way not typically experienced in a single-cell classroom: their mathematics teaching performance may be observed and judged at any time. Given the literature about primary teacher anxiety regarding mathematics (Hodgen & Askew, 2011; Intawati & Abdurrahman, 2019; Jenßen et al., 2020) this visibility may indeed be intimidating as teachers are forced to put their mathematics teaching on display when they may prefer to avoid it altogether. In this *physical* aspect the contrast between the ILE and the OMIP spaces becomes apparent. In one the mathematics teacher is on show, in the other they are completely hidden. However, *pedagogically* the two spaces generate a similar effect.

The twenty-first century mathematics learner

Performance scripts for the twenty-first century mathematics learner are abundant. This type of learner is produced globally in OECD texts (e.g., OECD, 2013) and produced locally in Ministry of Education documents, for example:

> New Zealand needs an education system that provides its people with the skills and knowledge they require to be successful in life and in an increasingly global economy. An effective education system provides qualifications that open doors to future opportunities and the skills needed in today’s society and the modern workplace. Equipping learners for a digitally enabled future is a key goal of our Four Year Plan. Demand for future-focused learning is increasing – the Ministry’s ICT strategy and our twenty-first century practice in teaching and learning priority ensure we have the right focus to meet this need. (Ministry of Education, 2016, p. 10)

The discursive emphasis is very clear in this excerpt that prioritises “success” in a “global economy” and ties together “future-focused learning” with digital skills. The twenty-first century learner is also produced in EdTech discourse, and marketed via the OMIP websites (Darragh, 2020). As Macgilchrist explains, to be successful in the twenty-first century, according to the mainstream argument, requires the "skills of creativity, collaboration, critique, and communication [...] This type of success is thoroughly entangled with the neoliberal, self-optimizing, 'entrepreneurial self'" (Macgilchrist, 2018, p. 242). Accordingly, I argue that performing the twenty-first century mathematics learner means to be entrepreneurial, self-managing, and individual.

Both the ILE space and OMIP platforms position the mathematics learner as a *self-managing entrepreneur*. If the 20th century classroom was designed to produce factory workers then the ILE space clearly prepares for a very different workplace, one that likely matches up with our mental images of the Google offices - themselves having become something of a trope. The entrepreneur is also produced in the OMIP platforms as children are encouraged to engage in the capitalist behaviour of buying products for their avatars. The twenty-first century mathematics learner is self-managing: they must meet their own learning goals and manage not only their own behaviour but also take responsibility for their own learning. It was notable that the Action Station task board in the video contained pictures to direct children to learning tasks, meaning that even children who cannot yet read are responsible for their own learning. The OMIPs encourage the children in this self-management, offering rewards for those who engage in large quantities of activities.

Finally, the twenty-first century mathematics learner is an *individual*, as made explicit in the hyper-personalised learning emphasised on both ILE and OMIP spaces. The ethos of twenty first-century education is very much student-centered (OECD, 2013), and we could certainly see this in the video linked earlier. The field of mathematics education has long been a proponent of 'reform' mathematics, with a de-emphasis on teacher-centred, traditional practices. However, there are some differences between the student-centred, problem-solving reform mathematics class (see also Lundin, 2012; Popkewitz & Lindblad, 2004) and the student-centred, individualised learning promoted by neoliberal ideology. The OMIPs and ILEs are firmly situated within the neoliberal version of 'personalised learning' (Darragh, 2020). Learning within this ideology entails receiving a separate and individualised learning plan – a task much more achievable by the artificial intelligence of learning analytics (and making the teacher ever more irrelevant). Further, there is a competitive aspect to this script; competition in games becomes competition for jobs in the future workplace.

To summarise, the twenty-first century teacher and learner are scripts produced by the neoliberal theatre, on the 'new' student- or device-centred classroom stage, and in the narratives of EdTech and educational policy documents. However, I wish to reiterate that performing mathematics teacher or learner is not about the exact following of a script. The script forms a notion of the ideal mathematics teacher or learner, against which an identity performance might be measured, recognised, or found lacking. Whilst any individual's performance of mathematics teacher or learner identity is formed at least in part by the available scripts, following the script is not always equally available for all people, and divergences from the script may be differently recognised depending on the person also. In this co-performance of identity emerges issues of intersectionality (Bullock, 2017; Leyva, 2016, 2017) and where we may see the impact of the power of recognition (or 'recognition power'). Therefore, scripts are problematic in a number of ways: firstly, because they are not equally available to all, secondly, because improvisation from the script may be seen as pathological, and finally, scripts narrow the possibilities for ways of being.

Improvising the mathematics teacher or learner?

The scripts for mathematics teacher and learner seem inevitable given the neoliberal theatre of education, and the 'new' classroom stage that emulates the twenty-first century workplace. However, considering identity as performative, we might ask what divergence from these scripts are even possible. Butler's notion of the repetition of acts allows a certain agency here. Each time we perform mathematics teacher or mathematics learner we may renew the performance differently – we may improvise. It is precisely in the repetition of identity that an individual may revise the act, and any act may follow scripts closely or deviate. While performances are always constrained by the context, there is a whisper of agency here; the possibility of revision raises the question of what the alternative ways of being a mathematics teacher or learner are. Already we see alternatives to the mathematics teacher script; it may be sidestepped via collaboration with other teachers, or the use of online programs enable teachers to avoid the role completely. What of the mathematics learner? Is there a way to be a mathematics learner that denies the self-managing, independent entrepreneur? What sort of mathematics learner would this improvisation produce? And, finally, how might the improvisation be recognised?

To conclude, performative identity as a stylised repetition of acts means that we need to take seriously the physical, temporal, and ideological space in which these acts are made. A mathematics teacher or learner identity is made, renewed, and revised in every individual performance of 'teacher' or 'learner'. In this paper, I have shown how ILE classroom design and online mathematics platforms dictate certain behaviours for mathematics teaching and learning and make available scripts for the normal (twenty-first century) mathematics teacher/learner. A few questions remain, and I ask the reader to once again consider their person of interest. For whom are these scripts more available? How might improvisation away from the script be recognised? Finally, what alternative ways of being a mathematics teacher or learner are imaginable beyond these scripts?

References

3P Learning (2021). *Mathletics*. https://www.mathletics.com/nz/

Anthony, G., & Hunter, R. (2017). Grouping practices in New Zealand mathematics classrooms: Where are we at and where should we be? *New Zealand Journal of Educational Studies*, *52*, 73–92.

Benade, L. (2015). Bits, bytes and dinosaurs: Using Levinas and Freire to address the concept of "twenty-first century learning." *Educational Philosophy and Theory*, *47*(9), 935–948. https://doi.org/10.1080/00131857.2015.1035159

Benade, L. (2017). Is the classroom obsolete in the twenty-firt century? *Educational Philosophy and Theory*, *49*(8), 796–807. https://doi.org/10.1080/00131857.2016.1269631

Benade, L. (2019). Flexible learning spaces: Inclusive by design? *New Zealand Journal of Educational Studies*, *54*, 53–68. https://doi.org/10.1007/s40841-019-00127-2

Biesta, G. (2012). Giving teaching back to education: Responding to the disappearance of the teacher. *Phenomenology & Practice*, *6*(2), 35–49. https://www.education.govt.nz/assets/Uploads/4-Year-Plan-2016-WEB.pdf

Borba, M. C. (2021). The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics.* Advance online publication. https://doi.org/10.1007/s10649-021-10043-2

Boylan, M., & Povey, H. (2009). Telling stories about mathematics. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships in education: Identities and participation* (pp. 47–57). Routledge.

Bradbeer, C., Mahat, M., Byers, T., Cleveland, B., Kvan, T., & Imms, W. (2017). The "state of play" concerning New Zealand's transition to innovative learning environments: Preliminary results from phase one of the ILETC project. *Journal of Educational Leadership Policy and Practice, 32*(1), 22–38.

Bullock, E. (2017). Beyond "ism" groups and figure hiding: Intersectional analysis and critical mathematics education. In A. Chronaki (Ed.), *Mathematics education and life at times of crisis* (pp. 29–44). MES9. https://www.mescommunity.info/#conferences

Butler, J. (1988). Performative acts and gender constitution: An essay in phenomenology and feminist theory. *Theatre Journal*, *40*(4), 519–531.

Chronaki, A. (2011). Disrupting "development" as the quality/equity discourse: Cyborgs and subalterns in school technoscience. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 3–19). Springer.

Chronaki, A., & Matos, A. (2013). Technology use and mathematics teaching: Teacher change as discursive identity work. *Learning, Media and Technology, 39*(1), 107–125. http://dx.doi.org/10.1080/17439884.2013.776076

Darragh, L. (2020). "Seductive promises" and the use of online mathematics instructional programmes in New Zealand primary schools. *New Zealand Annual Review of Education*, *26*, 162–167. https://doi.org/10.26686/nzaroe.v26.6931

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*(1), 19–33. https://doi.org/10.1007/s10649-016-9696-5

Darragh, L., & Franke, N. (2021). Online mathematics programs and the figured world of primary school mathematics in the digital era. *Mathematics Education Research Journal.* Advance online publication. https://doi.org/10.1007/s13394-021-00384-9

Darragh, L., & Radovic, D. (2018). Mathematics learner identity. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Springer. https://doi.org/10.1007/978-3-319-77487-9_100017-1

de Freitas, E. (2008). Mathematics and its other: (Dis)locating the feminine. *Gender and Education*, *20*(3), 281–290.

Engelbrecht, J., Borba, M., Llinares, S., & Kaiser, G. (2020). Will 2020 be remembered as the year in which education was changed? *ZDM Mathematics Education*, *52*, 821–824.

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. *Review of Educational Research*, *79*(2), 1008–1043. https://doi.org/10.3102/0034654309332562

Everatt, J., Fletcher, J., & Fickel, L. (2019). School leaders' perceptions on reading, writing and mathematics in innovative learning environments. *Education 3-13*, *47*(8), 906–919. https://doi.org/10.1080/03004279.2018.1538256

Foucault, M. (1991). *Discipline and punish: The birth of the prison*. Penguin. (Original work published 1977)

Gholson, M. L., & Martin, D. B. (2019). Blackgirl face: racialized and gendered performativity in mathematical contexts. *ZDM Mathematics Education*, *51*, 391–404.

Hardy, T. (2004). "There's no hiding place": Foucault's notion of normalization at work in a mathematics lesson. In M. Walshaw (Ed.), *Mathematics education within the postmodern*. Information Age.

Hardy, T. (2009). What does a discourse-oriented examination have to offer teacher development? The problem with primary mathematics teachers. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships in education: Identities and participation* (pp. 185–197). Routledge.

Heyd-Metzuyanim, E., & Shabtay, G. (2019). Narratives of "good" instruction: Teachers' identities as drawing on exploration vs. acquisition pedagogical discourses. *ZDM Mathematics Education, 51*, 541–554. https://doi.org/10.1007/s11858-018-01019-3

Hodgen, J., & Askew, M. (2011). Emotion, identity and teacher learning: Becoming a primary mathematics teacher. In C. Day & C.-K. Lee (Eds.), *New understandings of teacher's work* (pp. 165–183). Springer. https://doi.org/DOI 10.1007/978-94-007-0545-6_11

Ideland, M. (2021). Google and the end of the teacher? How a figuration of the teacher is produced through an ed-tech discourse. *Learning, Media and Technology*, *46*(1), 33–46.

Intawati, M., & Abdurrahman, G. (2019). Correlation between teaching readiness with math anxiety of primary pre-service teacher. In *Proceedings of the First International Conference on Progressive Civil Society*, 187–192. https://dx.doi.org/10.2991/iconprocs-19.2019.39

Jablonka, E. (2017). Gamification, standards and surveillance in mathematics education: An illustrative example. In A. Chronaki (Ed.), *Mathematics education and life at times of crisis: Proceedings of the ninth international mathematics education and society conference* (pp. 544–553). University of Thessaly Press.

Jagger, G. (2008). *Judith Butler: Sexual politics, social change and the power of the performative*. Routledge.

Jenßen, L., Hosoya, G., Jegodtka, A., Eilerts, K., Eid, M., & Blömeke, S. (2020). Effects of early childhood teachers' mathematics anxiety on the development of children's mathematical competencies. In O. Zlatkin-Troitschanskaia, H. A. Pant, M. Toepper, & C. Lautenbach (Eds.), *Student learning in German higher education* (pp. 141–162). Springer. https://doi.org/10.1007/978-3-658-27886-1_8

Knox, J., Williamson, B., & Bayne, S. (2020). Machine behaviourism: Future visions of "learnification" and "datafication" across humans and digital technologies. *Learning, Media and Technology*, *45*(1), 31–45.

Ladd, H., & Fiske, E. B. (2003). Does competition improve teaching and learning? Evidence from New Zealand. *Educational Evaluation and Policy Analysis*, *25*(1), 97–112.

Lange, D. (1988). *Tomorrow's schools*. New Zealand Government Printer.

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics*, *46*(1), 87–113. https://doi.org/10.2307/3483241

Lerman, S. (2020). *Encyclopedia of mathematics education*. Springer.

Leyva, L. (2016). An intersectional analysis of Latin@ college women's counter-stories in mathematics. *Journal of Urban Mathematics Education*, *9*(2), 81–121.

Leyva, L. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. *Journal for Research in Mathematics Education*, *48*(4), 397–433.

Lundin, S. (2012). Hating school, loving mathematics: On the ideological function of critique and reform in mathematics education. *Educational Studies in Mathematics*, *80*, 73–85. https://doi.org/10.1007/s10649-011-9366-6

Lupton, D., & Williamson, B. (2017). The datafied child: The dataveillance of children and implications for their rights. *New Media & Society*, *19*(5), 780–794.

Lutovac, S., & Kaasila, R. (2017). Future directions in research on mathematics-related teacher identity. *International Journal of Science & Mathematics Education*. https://doi.org/10.1007/s10763-017-9796-4

Ma, J. Y., & Singer-Gabella, M. (2011). Learning to teach in the figured world of reform mathematics: Negotiating new models of identity. *Journal of Teacher Education*, *62*(1), 8–22. https://doi.org/10.1177/0022487110378851

Macgilchrist, F. (2018). The "digital subjects" of twenty-first-century education: On datafication, educational technology and subject formation. In P. P. Trifonas & S. Jagger (Eds.), *Handbook of cultural studies and Education* (pp. 239–256). Routledge.

McMurchy-Pilkington, Trinick, T., & Meaney, T. (2013). Mathematics curriculum development and indigenous language revitalisation: Contested spaces. *Mathematics Education Research Journal*, *25*, 341–360. https://doi.org/10.1007/s13394-013-0074-7

McMaster, C. (2013). Working the "shady spaces": Resisting neoliberal hegemony in New Zealand education. *Policy Futures in Education*, *11*(5), 523–531.

McRae, P. (2013). *Rebirth of the Teaching Machine through the seduction of data analytics: This time it's personal* [Online resource]. https://nepc.colorado.edu/blog/rebirth-teaching-machine-through-seduction-data-analytics-time-its-personal

Mendick, H. (2017). Mathematical futures: Discourses of mathematics in fictions of the post-2008 financial crisis. In A. Chronaki (Ed.), *Mathematics education and life at times of crisis: Proceedings of the ninth international mathematics education and society conference* (pp. 74–89). University o Thessaly Press.

Mendick, H., Moreau, M.-P., & Epstein, D. (2009). Special cases: Neoliberalism, choice, and mathematics. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships in education: Identities and participation*. Routledge.

Ministry of Education (2021) Te Kete Ipurangi (TKI). https://www.tki.org.nz/

Ministry of Education. (2007). *The New Zealand Curriculum*. Learning Media. https://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum

Mutch, C. (2017). Winners and losers: School closures in post-earthquake Canterbury and the dissolution of community. *Waikato Journal of Education*, *22*(1), 73–95.

OECD. (2013). *Innovative Learning Environments*. OECD. https://doi.org/10.1787/9789264203488-en

O'Neill, J. (2011). The privatisation of public schooling in New Zealand. *Journal of Education Policy*, *26*(1), 17–31. https://doi.org/10.1080/02680939.2010.493227

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: Foundations for change. *ZDM Mathematics Education*, *49*, 645–661.

Popkewitz, T. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Research Journal*, *41*(1), 3–34.

Popkewitz, T., & Lindblad, S. (2004). Historicising the future: Educational reform, systems of reason, and the making of children who are future citizens. *Journal of Educational Change*, *5*(3), 229–247.

Radovic, D., Black, L., Salas, C. E., & Williams, J. (2017). Being a girl mathematician: Diversity of positive mathematical identities in a secondary classroom. *Journal for Research in Mathematics Education*, *48*(4), 434–464.

Riwai-Couch, M., Bull, A., Ellis, B., Hall, K., Nicholls, J., Taleni, T., & Watkinson, R. (2020). *School-led learning at home: Voices of parents of Māori and Pasifika students*. Evaluation Associates Ltd.

RNZ. (2020, 28 July). Teaching maths in lockdown: Some parents "felt somewhat abandoned." *The New Zealand Herald*. https://www.nzherald.co.nz/nz/teaching-maths-in-lockdown-some-parents-felt-somewhat-abandoned/WNXLYYCXQZXTMKUN5CUKX4MFMI

Roberts-Mahoney, H., Means, A. J., & Garrison, M. J. (2016). Netflixing human capital development: Personalized learning technology and the corporatization of K-12 education. *Journal of Educational Policy*, *31*(4), 405–420. https://doi.org/10.1080/02680939.2015.1132774

Thornburg, D. D. (2004). Campfires in cyberspace: Primordial metaphors for learning in the 21st Century. *Instructional Technology and Distance Learning*, *1*(10), 3–10.

Thrupp, M., O'Neill, J., Powell, D., & Butler, P. (2020). Private actors in New Zealand schooling: The path to saturation. *Journal of Educational Administration and History*, *52*(4), 385–402.

Valoyes-Chávez, L. (2019). On the making of a new mathematics teacher: Professional development, subjectivation, and resistance to change. *Educational Studies in Mathematcs*, *100*, 177–191.

Vincent-Lancrin, S., Urgel, J., Kar, S., & Jacotin, G. (2019). *Measuring innovation in education 2019: What has changed in the classroom? Educational Research and Innovation*. OECD Publishing. https://doi.org/10.1787/9789264311671-en

Walkerdine, V. (1990). *Schoolgirl fictions*. Verso.

Walkerdine, V. (1998). *Counting girls out: Girls and mathematics*. Falmer.

Walshaw, M. (2010). Learning to teach: Powerful practices at work during the practicum. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics classrooms* (pp. 109–128). Information Age.

Williamson, B., Eynon, R., & Potter, J. (2020). Pandemic politics, pedagogies and practices: Digital technologies and distance education during the coronavirus emergency. *Learning, Media and Technology*, *45*(2), 107–114. https://doi.org/10.1080/17439884.2020.1761641

Young, J. (2017). Technology-enhanced mathematics instruction: A second-order meta-analysis of 30 years of research. *Educational Research Review*, *22*, 19–33.

Identity as a significant concept in mobilising collective action in mathematics education and beyond: A response to Lisa Darragh

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It is my pleasure to be invited to offer a response to Lisa Darragh's plenary on 'Innovative learning environments and the digital era: Finding space for mathematics identity'. In doing so, I will argue that the concept of identity is not only useful in understanding students' and teachers' relationships with and participation in mathematics. Identities, like critical knowledge, are also potentially powerful tools in mobilising collective action that can be transformative of practice.

The myth of free choice and individualism

At first glance, Lisa provides an account of mathematics teaching and learning in Aotearoa that appears quite idyllic. This is especially so given my experience of an education system that is heavily regulated by central government (e.g. through standardised testing, a national curriculum etc.) and increasingly run in the interests of private businesses through the joining of schools into academy trusts (run by private trustees from business/charities), a predisposition towards consultocracy (Gunter, Hall, & Mills, 2015) and the outsourcing of curricula, resources and professional development to commercial enterprises. Whilst Lisa's account resonates with some of this, she also describes a localised curriculum that is devolved to the school level offering schools, teachers, parents (and students?) agency over what is learnt with potential to serve the needs of the community. With colleagues at Manchester, I have been involved in projects inspired by the Funds of Knowledge (FOK) approach (Moll, Amanti, Neff, & Gonzalez, 1992) and it's more recent adaptation Funds of Identity (FOI) (Esteban-Guitart & Moll, 2014). Both of these approaches involve teachers working with oppressed/minoritized groups to locate knowledge and identities in the home and community as a resource for developing curriculum projects in schools. The aim is to connect the school curriculum to the 'everyday' knowledge, practices and needs students experience in their communities. So in one sense Lisa's description of educational policy in New Zealand seems to align with the fundamental principles of a FOK approach and its overarching aim of developing curricula that challenge the privileging of elite forms of knowledge (and identification) in the academic curriculum.

Yet Lisa critiques educational policies in New Zealand as maintaining the fiction of 'free choice' in pursuit of a neo-liberal subject who is then held accountable for such choices. Indeed parental choice has been long identified as an 'essential circuit' of neoliberal policy

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in education (Ball, 1994), which assumes notions of self definition, self actualisation and thereby individual responsibility for the success and/or failure of the child. According to this kind of critique, the fiction lies in the myth that choice proffers agency when in reality it is restricted to the privileged few – e.g., in the mathematics classroom this may be those in the right position to accept or reject what Lisa refers to as 'identity scripts' associated with success.

The concept of identity here is pertinent since, as Holland et al. (1998) note, any moment of (or engagement in) social activity not only involves what one does 'in practice' but also a self-authoring[1] of the subject in ways that are culturally or socially recognised for and by others. Thus learners in the mathematics classroom are not only subjectively experiencing mathematical practices they are also self-authoring as mathematics learners - or not - as the case may be. Indeed, Lisa notes how this self-authoring is framed by power relations when she states that "following the script is not always equally available for all people, and divergences from the script may be differently recognised depending on the person also" (Darragh, 2021, p. 23, in this volume). So how or why do policies that aim to give greater voice to students/parents/teachers become, 'in practice', a learning environment that is potentially stratifying and exclusive?

In the aforementioned work on FOK, we have offered a critique (using Bourdieu) of so called 'domesticated' versions of the approach for their propensity to surface capital in students' homes in the interests of serving the needs of the school or the educational field (Black et al., 2019; Williams, 2016). Clearly those who have access to such capital are more able to offer the kinds of resources that the school might want or the kinds of identity scripts that are ascribed value in the mathematics classroom. However, the fundamental concern here must be with the structure of the educational field and the way its social, political and economic function is refracted through pedagogic practices. Such a critique suggests a more radical agenda is necessary whereby critical pedagogies are employed to challenge even transform (rather than serve) education as a process of reproduction.

The commodification of learning

Lisa's focus on the identity scripts produced through Innovative Learning Environments (ILE) and commercial Online Mathematical Instruction Programmes (OMIPs) links acutely to debates around privatisation in education and the economic commodification of learning and teaching which I refer to above. Initiatives such as the introduction of Mastery Mathematics in England, exemplify the complex fuzziness of the private/public distinction in relation to forms of curricula/pedagogy innovation. Notions of pedagogy, curricula and learners are constructed in the name of 'public value' whereby responsibility for reform is shifted away from the institution (education system) but towards complex relationships involving 'local publics'[2] (Newman, 2013) and private actors. Such initiatives involve the

[1] Note there are fundamental differences between self-authoring and neo-liberal concepts of self-actualisation as outlined by Holland et al (1998).

[2] Newman (2013) refers to new localism – where central functions of the state are devolved, fragmenting a unitary public in the name of flexibility, responsiveness and goals.

production/consumption of particular methods and resources which are in part public (i.e., openly stated as for the public good with public funds attached) but are also mobilized through both public (e.g., national organisations like NCETM) and private interests (e.g., paid consultants, academy trusts etc.). This complexity then produces particular relations of power between various stakeholders, and between organisations which permeate at every level. Quite literally this involves the commodification of students' educational labour for profit by private providers, which adds another layer to previous economic analyses of education whereby student labour is commodified in the form of qualifications that have value to be exchanged in the world of labour relations (Williams 2012).

However, Newman (2013) proposes a reconceptualization of 'public value' to incorporate a concept of 'public action', which involve new forms of alliance between activists, academics and policy actors. This arguably means such initiatives can potentially provide space for diverse collective forms of social agency and empowerment - that can lend themselves to more radical or progressive appropriations than we might otherwise see ventriloquated through policy discourse[3]. For instance: (i) alliance between teachers, academics and professional organisations which can produce collective agency for change (e.g., primary assessment reform); (ii) alliance between teachers within/across schools enabling some increase in professional autonomy locally; (iii) pedagogic practices which offer 'voice' to the mathematical learner and diverse forms of learner identity. Arguably, these forms of alliance engender ideas of voice and activism which are to be distinguished from the neo-liberal notions of 'free choice' and 'individualism' which Lisa alludes to. Public action engenders ideas of collective agency rather than the pursuit of private or individual gain.

Harnessing contradictions

Nevertheless, it is not my intention to dichotomise concepts here such as private/public, individual/collective and capital gain/human need. In Black et al (2021) we discussed how ideas associated with private capital gain (individual) and public good, collective agency and human need can be understood as a dialectic relation of exchange value – use value, drawing on Marx's concept of the commodity relation. In line with others in CHAT (Blunden, 2009), we argue for a unit of analysis which preserves the living dynamic unity of exchange value - use value and which, recognises this relation as fundamentally one of contradictory moments. Recognising and harnessing such contradictory moments can be developmental which can bring about transformations in practice. In the description Lisa offers, I suggest we might see such contradictory moments when the needs of a school community (run by parents) come into conflict or tension with the overarching 'need' of the school system to grow 'cultural capital' (exchange value) for those privileged enough to access it. For example, Lisa points out how some OMIPs promote a hyper individualisation of the learner (presumably to foster performances/actions/knowings that are transferable to standardised

[3] Newman (2013) points out that marginalised or politically less powerful publics are more likely to be mobilised through autonomous groups rather than via official consultation and participation in established government.

tests) that conflicts with the diverse needs and interests of a community of learners as a group rather than as a sum of individuals. This raises a question: at what point might this community collectively decide to challenge the system as it stands? In CHAT terms, this is where the harnessing of contradictions can be productive through the creation of a shared joint object that motivates a group towards collective action through the kinds of alliances Newman refers to above.

And so to identity…

With the above in mind, I argue that the concept of identity is not only useful for exploring students' relationships and participation in mathematics education, identity or identities are also potentially powerful in mobilising collective action. Think, for instance, of the identity work taken on by oppressed groups that is central to so many struggles for change (e.g., we recently analysed the case of the Mexican American Studies programme in Tucson, Arizona; Black et al., 2021). But this requires a concept of identity that not only emphasises individual repetitive acts that come to carry cultural significance (as identity scripts) - it also requires some concept of motive (collective-individual) 'to act' and a sense of reflection on how our subjective experiences make sense in terms of who we are and what we are becoming (Holland et al., 1998). Change occurs not only through stylised individual performances (improvisation) but also through collective action and I argue that is through latter that imagined alternative identity scripts can be realised.

References

Ball, S. (1994). *Education reform.* McGraw-Hill Education.

Black, L., Choudry, S., Howker, E., Phillips, R., Swanson, D., & Williams, J. (2021). Realigning Funds of Identity with struggle against capital: The contradictory unity of use and exchange value in cultural fields. *Mind, Culture, and Activity, 28*(2), 97–110.

Blunden, A. (2009). An interdisciplinary concept of activity. *Outlines: Critical Practice Studies, 11*(1), 1–26.

Darragh, L. (2021). Innovative learning environments and the digital era: Finding space for mathematics identity. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 32–28). Tredition. https://doi.org/10.5281/zenodo.5469648

Esteban-Guitart, M. & Moll, L.C. (2014). Funds of identity: A new concept based on the funds of knowledge approach. *Culture & Psychology, 20*(1), 31–48.

Gunter, H. M., Hall, D., & Mills, C. (2015). Consultants, consultancy and consultocracy in education policymaking in England. *Journal of Education Policy, 30*(4), 518–539.

Holland, D. C., Lachicotte, W., Jr., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds.* Harvard University Press.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory Into Practice, 31*(2), 132–141.

Williams, J. (2012). Use and exchange value in mathematics education: Contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics, 80*(1-2), 57–72.

Williams, J. (2016). Alienation in mathematics education: Critique and development of neo-Vygotskian perspectives. *Educational Studies in Mathematics, 92*(1), 59–73.

Innovation in school mathematics? Historical iterations and other enduring dangers: A response to Lisa Darragh

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In this response, I interrogate the limits of innovation in school mathematics within a historical context. I explore the continuities as well as shifts in the normalizing practices of school mathematics. I argue that the notions of "free choice" and "surveillance" are not only specific to neoliberal regimes but also are embedded in histories of modern schooling. The historical context enables us to explore the dangers of innovative learning environments such as ordering the differences on a hierarchy in addition to the production of particular identities.

Introduction

Seeking a change in teaching and learning practices has long been a concern for mathematics education. Innovative pedagogical methods and curricular ideas are always presented to ensure 'better' learning environments for all students. While these 'innovative' approaches are considered to improve teaching and learning mathematics, they do more as argued in Darragh's paper: Identities for students and teachers are produced, regulated, and normalized by the multiplicity of societal narratives such as neoliberalism, colonialism, racism, sexism and so on.

Darragh's paper revisits how the identities of mathematics learners and mathematics teachers are being produced and regulated in "technology-rich, innovative learning spaces". These learning environments are located in Aotearoa New Zealand; but she also situates the processes of identity formation of learners and teachers within neoliberal ideology, twenty-first-century narratives, and the EdTech discourse of educational corporations. Rather than positioning teachers or learners as fully agentic humans, her conceptualization of identity enables an analysis of the multiplicity of discourses that regulate the identities and normalize particular actions and participation in "ILE (innovative learning environment) spaces".

My response draws on the historical background of normalization practices, including "free choice" and "the use of surveillance" in learning spaces. Although Darragh notes that these two normalization practices are the features of "new" classrooms or online learning platforms in our digital era, I discuss how these practices historically have been part of the modern world, particularly they are embedded within the practices of schooling and school mathematics. In my response, first, I explicate the historical emergence of sciences of

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decision-making, which is beyond neoliberal ideology, that makes discourses of "free choice" possible and reasonable. Here, I also bring historical shifts in the practices that organize and regulate uncertain learning spaces that are presumed to be planned, stabilized, and secured. Following these, I consider the dangers of the common way of thinking about change and innovation, including the differentiating mechanisms in school mathematics.

Historical continuities of normalization in educational spaces and shifts in the practices of educational decision making

Educational spaces are complex, dynamic, and uncertain. Social actors of (mathematics) education experience several predicaments when they are asked to make choices among a range of options. While one decides different choices, the notion of uncertainty embedded in decision-making processes is not always subject to endless possibilities. Rather, decisions are produced in systems that include scenario planning, risk profiling, algorithmic modelling, and data analysis (Amoore, 2011). Are these emerging practices of data collection, analysis, and representation new to social and educational spaces? How might we historically think about these processes and their exacerbation with the increase of online education?

How to act and participate in the real world under uncertain conditions is not a new problem. Hacking (1990), for example, explored how statistics and probability became technologies to formulate complicated realities into stabilized entities to tame the chance in the modern world. These technologies of data collection and analysis have been concerned with "making up people" as administrable citizens of the state. With the avalanche of printed numbers, future society became designable through counting people and their habits. The enumerations resulted in populational categories that constitute human kinds (Hacking, 2007). The categories for humans such as effective housekeeper, intelligent adult, or democratic citizen have been placed into enclosed and disciplinary spaces to order, differentiate, classify and normalize proper and improper modes of actions and participations in the world (see Foucault, 1995).

One of the most familiar examples from schooling has been the wide circulation of intelligence tests in the late 19th century modern nation-states, a particular context that can be remembered as a major breakthrough in education with the industrialization, public education, and waves of migration. Schools were seen as an effective technology that prepared children for industrial work and average adult life (Danziger, 1997). While later these tests were to compare the 'national' IQ level of countries and classify the regions along a continuum of values (Valero, 2017), the widespread adoption of standardized tests were linked to eugenic projects that aimed to purify population as well as maintenance of a White supremacist society (Davis & Martin, 2018). Back then, ability groups were considered as an innovative strategy to plan effective learning environments. The societal hope of dividing students was not only about economic development and progress but also was concerned with race betterment and ensuring the well-being of population(s) (Yolcu & Popkewitz, 2019). Commitment to the knowledge produced through multiple data points, including scores of standardized math tests and time on solving mathematics questions, instead of arbitrary decisions, was a tactic to rationalize the tracking of students.

The contemporary calculations of the future and uncertainty have shifted. They are less about spatial classifications but more related to the configurations of spaces of security and control (Foucault, 2007). These spaces are not enclosed in the disciplinary sense: Rather than spatial distribution of individuals in advance, there is a widespread installation of control technologies across spaces and possibilities (Deleuze, 1992). The tools like robots, smartphones, or networked machines enable perpetual training, frequent and faster surveillance, and continual monitoring of communities to maintain the safety and stability of the world. Within the data produced through these devices, practices of algorithms, data analytics, or risk profiling become “the authoritative knowledge of choice” to anticipate the future uncertainty (Amoore, 2011). Here, the notion of free choice would not simply be constrained by data, but data analytics is part of what we call ‘free choice’ or ‘informed decisions’ that we make under uncertain conditions. Despite the changes in the tools and technologies, the uncertainty of educational spaces was resolved through apparently precise, specific, and quantitative data networks in which reasonable and rational choices could be made. The explosive interest in data based decision-making can be framed as a historical reiteration of the hope for a safe and stable world (Heyck, 2015).

In contemporary educational research, while tracking and assessing students’ IQ levels become unwanted, old-fashioned practices, we do still have standardized exams. However, today, standardized assessment items emphasize 21st-century skills such as problem solving, modelling, or systems thinking. That is, despite the changes, there is persistent trust in the data produced through the standardized tests. Nevertheless, contemporary educational choices could no longer rely only on the tests. There should be more to attend to the contextuality and uncertainty of learning environments.

In addition to contemporary modified testing practices, students and teachers are asked to produce their data in their contexts. For example, continual in-class tracking of children’s mathematical learning trajectories is considered as active agents to close the “education gap” between ambitious goals of reform and actual student mathematical thinking (Daro, et al., 2011, p. 11). With the tools of the digital age, ongoing classroom assessment of mathematical trajectory becomes possible (e.g., Confrey & Maloney, 2012). Installation of these tools into the classrooms does not only provide rapid and frequent feedback for teachers who make instructional decisions but also contributes to the ongoing surveillance of learning environments.

The historical desire for stable and secure world orders the calculation of uncertain educational spaces. As I have briefly discussed, and as Darragh argues in her paper, these social processes have normalizing effects in educational settings. Nevertheless, the normalization has long occupied the landscape of school mathematics despite the changes in technologies and tools such as IQ tests, skill-based assessment items, or classroom trajectories. So, it is possible to refer to the process of normalization as a historical spiral, moving from layer to layer, never stabilizing itself and the practices are always open to modification and adjustment with the changing conditions.

Dangers of normalization practices: Differentiating axes of school mathematics

Administration of the landscape of school mathematics with normalizing practices has been a way to make the children as a particular kind people. Darragh discusses this process as production of identities that are "scripted" by the contemporary neoliberal regime. Particularly, she talks about the 21st-century mathematics learner (and the teacher) who embodies capitalist behaviour in online platforms, takes responsibility for their own learning, and performs identities as an entrepreneurial. With the discourses of "free choice", she takes our attention toward the generation of agentic performances that are controlled through ongoing surveillance and data collection.

The network of school mathematics practices produces a normative and regulatory space for 21st-century mathematics learners and it simultaneously generates axes of differentiation. Children are no longer categorized as mathematically defective, disable, slow or remedial, but they are profiled as "at-risk" not only through the generalizations of national or international exam score but also through ongoing classroom assessment results. The children who are outside of the normative accounts of educational spaces are categorized as at risk, disadvantaged or underrepresented and become the objects of interventions, such as teaching, research, or reform to conserve the historically planned order and stability of the world.

While the normative accounts regulate and produce particular human kinds, they simultaneously generate the "others". The differentiated spaces for children are configured as the laboratories of experimenting the innovative or new ideas of school mathematics. In order to be prepared to the shifts in the educational spaces, novel psychological categories are generated in addition to the desired identities. This includes, for example, the interest and willingness of students to persist on mathematical tasks (Organization for Economic Co-operation and Development [OECD], 2018). While willingness to do mathematics is formulated as one of the desired distinctions of 21st-century mathematics learners in these accounts, the differentiated spaces are simultaneously generated for others who are seen and perceived as 'unwilling' to do mathematics.

At the end of the plenary paper, there is an important question that Darragh raises: "For whom are these [identity] scripts more available?" Taking into account the differentiated axes embedded in school mathematics, I want to take this question a step further. I wonder, what specific technological devices are available for whom? Are there any additional and modified pedagogical strategies for those who act outside of the boundaries of produced identities? What differentiated categories are designed for those who push against the boundaries of 'innovative learning environments'?

Conclusion

In mathematics education, everybody wants to make a change and innovation: Teachers, researchers, students, parents, policymakers, and curriculum reformers to name a few. These innovations are not only concerned with teaching and learning mathematics but also with producing identities, normalize particular subjectivities and also generate spaces for others. Exploring the history behind the reform and change offers ways to problematize what is given as natural, sensible, and necessary part of mathematics education including those rules and conventions that configure what we perceive as “change” within the boundaries of how we conventionally reason about school mathematics.

If we think of the normalization processes in the innovative learning spaces as historical, the identity “scripts” for 21st-century mathematical learners are also embedded in the numerical practices of testing, visualizing, or modelling the big data. As more teachers and learners get enumerated, the complicated realities of learning spaces are formulated into stabilized entities. Application and production of data are to render classrooms certain, secure, and stable with rational decisions. The stabilizations do not only make up people but also enable axes of differentiation. It is a simultaneous process of production of identities and their differential constitution.

Despite the shifts in the tools and practices of normalization and differentiation, the historical reiterations to secure the uncertainty in learning environments reveal that there is something sticky in the ‘reason’ of school mathematics. How we think about change in mathematics education is embedded in a style of reasoning that normalizes particular subjectivities while differentiates the others. Despite the shifts in the tools and technologies, mathematical learning environments have been occupied with the production of objects of teaching, research, and policy. Then, the snapshots of learning environments, which were narrated at the beginning of the plenary paper, are not a change in the premises that constitute objects in educational spaces. Rather, it is a historical iteration of ‘reason’ of school mathematics that makes, normalizes, and differentiates particular human kinds.

One might ask: Isn’t there a possibility to perform any agentic identities in this digital era? Is there no space to be free in our choices? Is nothing changing at all? Are we going to give up inventing digital technologies or searching for possibilities of change in mathematics education? I would say no. “What is given up”, as Popkewitz (2008) writes, “is the notion of planning people” that “stabilizes and fixes the boundaries of freedom” (p. 184). So, the change is never deadlocked. On the contrary, the spaces for performing freedom and other potentialities could be found in the very act of exploring historical shifts and iterations, where the resistance can become the continual interrogation of what is think-able and say-able within the boundaries of current practices.

References

Amoore, L. (2011). Data derivatives on the emergence of a security risk calculus for our times. *Theory, Culture & Society, 28*(6), 24–43.

Confrey, J., & Maloney, A. P. (2012). Next generation digital classroom assessment based on learning trajectories in mathematics. In C. Dede & J. Richards (Eds.), *Steps toward a digital teaching platform* (pp. 134–152). Teachers College Press.

Danziger, K. (1997). *Naming the mind: How psychology found its language*. Sage.

Daro, P., Mosher, F., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Consortium Policy Research Education.

Davis, J. & Martin, D. B. (2018). Racism, assessment, and instructional practices: Implications for mathematics teachers of African American students. *Journal of Urban Mathematics Education, 11*(1-2), 45–68

Deleuze, G. (1992). Postscript on the societies of control. *October, 59*, 3–7.

Foucault, M. (1995). *Discipline and punish: The birth of the prison*. Vintage Books. (Original work published 1975).

Foucault, M. (2007). *Society must be defended: Lectures at the College de France, 1975–76*. Picador.

Hacking, I. (1990). *Taming of chance*. Cambridge University Press.

Hacking, I. (2007). Kinds of people: Moving targets. *Proceedings of the British Academy, 151*, 285–318.

Heyck, H. (2015). *Age of system: Understanding the development of modern social science*. Johns Hopkins University Press.

Organization for Economic Co-operation and Development. (2018). *PISA 2021 Mathematics Framework (Draft)*. OECD Publishing.

Popkewitz, T. S. (2008). *Cosmopolitanism and the age of school reform: Science, education, and making society by making the child*. Routledge.

Valero, P. (2017). Mathematics for all, economic growth, and the making of the citizen-worker. In T. S. Popkewitz, J. Diaz, & C. Kirchgasler (Eds.), *A political sociology of educational knowledge: Studies of exclusions and difference* (pp. 117–132). Routledge.

Yolcu, A., & Popkewitz, T. S. (2019). Making the able body: School mathematics as a cultural practice. *ZDM Mathematics Education, 51*(2), 251–261.

Rethinking exemplification in mathematics teacher education multilingual classrooms

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Examples that teachers choose and use are fundamental to what mathematics is taught and learned, and what opportunities for learning are created in mathematics classrooms. In this paper, I bring together three frameworks which have been used separately in mathematics education research – variation theory, meaning making as a dialogic process framework, and the notion of interacting/multifarious facets/dimensions within teacher education. The emergent framework consists of a triadic approach to understanding exemplifying practices within teacher education, and in particular, within multilingual teacher education classrooms. Lesson transcript data from an introductory class in probability in one teacher education multilingual classroom is used to illustrate how working with the amalgamated framework conduces to a powerful way of examining the choice and use of examples in mathematics teacher education multilingual classrooms, and how the three frameworks work together to attend to three critical layers involved in the complexity of teaching and learning in mathematics teacher education multilingual classrooms.

Introduction

As a teacher educator involved with both pre-service and in-service teacher education (TE), I have often tried to model how to teach using different practices in my class. These practices in themselves were never the object of attention in our discussions beyond their mere definitions. I became more sensitised to the importance of engaging my students on what makes for a good practice in multilingual classrooms in the course of my study (See Essien, 2014) using Wenger's (1998) communities of practice theory to engage with teacher preparedness for teaching mathematics in multilingual pre-service teacher education classrooms. Subsequently, how teacher educators in multilingual classrooms choose and use examples – that is, exemplifying as a practice – became an important focus of my research and practice. The importance of mathematical examples – that is, of tasks which are used to illustrate concepts in mathematics (Essien, 2021) – cannot be overemphasised. My focus on exemplifying as a mathematics practice was motivated by the centrality of examples in the teaching and learning of mathematics. Research has shown that the examples which teachers choose and how these examples are used play an important role in what mathematics is taught and how students learn and understand the mathematics that is taught in class

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(Arzarello, Ascari, & Sabena, 2011; Bills, Dreyfus, Mason, Tsamir, Watson, & Zaslavsky, 2006; Goldenberg & Mason, 2008; Zaslavsky, 2010; Zodik & Zaslavsky, 2008).

In my attempt to better understand the choice and use of examples, the following questions became important: what examples can teachers use in multilingual mathematics classrooms to help mediate knowledge in mathematics? And fundamentally, how can pre-service teachers be enculturated into how to choose and use examples that would help their future (multilingual) students better understand mathematics? What affordances do the selection and use of examples offer to pre-service teachers regarding the multiple dimensions of TE in mathematics? And what should a "good" (use of) example be in multilingual pre-service teacher education mathematics classrooms – given that these pre-service teachers themselves are most likely to teach in multilingual classrooms at the end of their qualifications? While it is not the intention of this paper to answer all the above questions, these questions were, for me, the driving force behind my quest for an all-encompassing framework which has the ability to not only provide a gaze into the types of examples that are chosen and used within TE, but also a gaze into how the choice and use of examples in teacher education can attend to the complexity involved in enculturating pre-service multilingual teachers into the intricacies of teaching in multilingual mathematics school classrooms. In doing this, the words of Lester (2010, p. 83) who argues that "rather than adhering to one particular theoretical perspective, [that] we act as bricoleurs by adapting ideas from a range of theoretical sources to suit our goals", came to mind. In an earlier paper for ZDM (Essien, 2021), I attempted to do this using a dyadic framework that accounted for both the examples chosen and used and the interactional pattern during the enactment of the examples. My contention is that using the dyadic framework did not account sufficiently for how pre-service teachers (PSTs) were enculturated into the multifaceted dimensions within teacher education.

In analysing the nature of examples used in mathematics classrooms, variation theory as a theory of learning has become ubiquitous in research about how the structure of an example space is not only an important mathematical process but also a critical didactical goal (Arzarello et al., 2011). This paper is premised on the notion that variation theory, which is commonplace in research involving the choice of examples in mathematics classrooms, may be insufficient to provide adequate perspectives on the quality of instructional examples (Zaslavsky, 2010) in teacher education classrooms of pre-service teachers who are themselves multilingual and who will teach in multilingual classrooms at the end of their qualification. Using a multifocal framework designed to unpack such complexities, I argue that in order to take into account the full extent of the multifaceted nature of teacher preparation for teaching in multilingual context, exemplifying as a mathematical practice in teacher education needs necessarily to also account for 1) how language is used to enable what some authors (e.g., Stein, Engle, Smith, & Hughes, 2008) have referred to as productive mathematical discussions in the class, and others (e.g., Engle & Conant, 2002) as productive disciplinary engagement, and 2) how the different facets involved in pre-service teachers education are (co-)constructed in multilingual context.

I start the next section by a brief exposition of variation theory subsequently bring in two ‘bricoleurs’, – meaning making as a dialogic process framework and the notion of interacting/ multifarious facets or dimensions within TE. Then I present the amalgamated framework. Although this paper is a conceptually rather than an empirically driven piece, using transcribed data of an introductory probability lesson, I show the applicability of the framework (so formed) and discuss the relevance of the framework both conceptually and empirically and how the interactional process in the enactment of examples bring into focus (or not) the multiple dimensions characteristic of teacher education in multilingual contexts.

Variation theory in mathematics teaching and learning

There have been several theoretical expositions on variation theory as a pedagogic theory in mathematics classrooms (see Kullberg, Runesson, Kempe, & Marton, 2017; Marton & Booth, 1997; Pang & Marton, 2005; Watson & Mason, 2006). For this paper, suffice it to indicate that variation theory holds that learning is a function of discernment, that is, of seeing or experiencing critical aspects of what is to be learned (Marton & Booth, 1997). Three core tenets of variation theory are important to my overall framework: Object of learning, Critical features, and Patterns of variation. The object of learning is what is to be learnt (Pang & Marton, 2005) – it is the focus of attention. In mathematics, this would be the mathematical object of learning. Pang and Marton (2003, 2005) and Runesson (2005) make a distinction between the intended, the enacted, and the lived objects of learning. The intended object of learning is the capabilities the teacher wants the learners to develop and the enacted object of learning is how these capabilities are realised in the classroom. As such, the enacted object of learning “is co-constituted in the interaction between learners and the teacher or between the learners themselves” (Runesson, 2005, p. 70). The lived object of learning is what is actually learned, that is, how the object of learning is experienced by the students. This brings to focus the important role the teacher plays in how his/her chosen examples are used in relation to the context in which the teaching is imbedded so that the intended object of learning aligns as much as possible to the lived object of learning.

Variation theory defines critical features are those aspects of a phenomenon that are necessary for the learner to discern in order for the learner to develop a particular understanding of the object of learning in focus. Variation theory holds that learners own experience of certain patterns of variation and invariance of novel situations and discernment or awareness thereof of the critical features of the object of learning is a *sine qua non* condition of learning. This means that discernment is not possible without the experience of difference between two of more different things/situations and without the experience of difference, it would not be possible to discern similarities. The kinds of awareness brought about by patterns of variation include contrast, separation, generalisation and fusion. Regarding contrast, variation theory holds that learners are more readily able to discern the critical features of an object if they are able to contrast it with other objects or another object. Similarity is what is kept invariant (Watson & Mason, 2006) in the mathematics structure within the sequence of mathematics examples. Drawing on Marton and Tsui (2004), Olteanu and Olteanu (2013) assert that separation "refers to the other dimensions of variation that need to be kept invariant or varying at a different rate in order to discern a dimension of variation that can take on different values" (p. 515). Variation theorists argue that in order to fully understand a concept, it is necessary to also experience varying features of the concept so as to separate the features that are not critical (that is, that are not defining features of the concept). Fusion is when there is simultaneous variation of several critical aspects of the object of learning in an example space (Lo & Chik, 2016; Olteanu, 2018). The interrelationship between the different constructs of variation theory is represented in Figure 1 below:

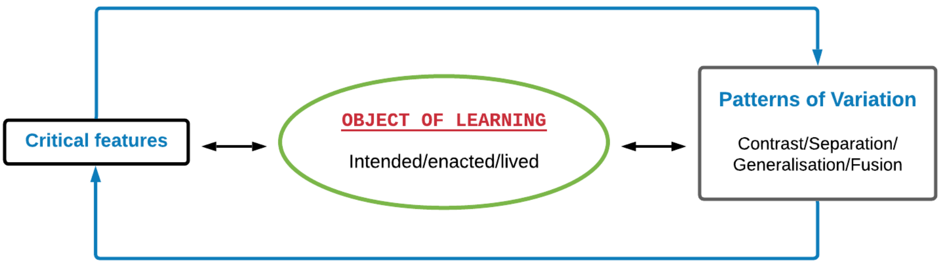

Figure 1: How the various concepts in variation theory interconnect

Perhaps, a good way to summarise variation theory is in the words of Lo and Chik (2016, p. 296) who assert that "necessary conditions for learning include focusing on the object of learning [hence the central position occupied by the object of learning in Figure 1], identifying which of its aspects or features are critical, and exposing learners to appropriate patterns of variation that help them discern these critical aspects or features".

'Bricolaging' variation theory

What does it mean to use variation theory in multilingual classrooms? More specifically, how can variation theory be used in multilingual teacher education classrooms so that it accounts for the complexity involved in preparing teachers for teaching in multilingual classrooms?

To better understand the choice and use of examples in teacher education, I draw on Mortimer and Scott's (2003) notion of meaning making as a dialogic process, and the notion of interacting identities within teacher education (Essien, 2014) and bring these to bear on variation theory. Lo (2012) argues that variation theory has two aspects: "the specific aspect, which refers to the subject matter, knowledge or skill that we wish students to learn (short-term goal), and the general aspect, which refers to the capabilities that can be developed through the learning of the specific aspect (long-term goal)" (p. 25). The long-term educational goal of pre-service teachers needs necessarily to go beyond knowledge acquisition and knowledge of subject matter. In any mathematics classroom, not only are examples used in the teaching and learning process important, but also important is the discourse that is used to engage with the chosen examples and how language is used to negotiate meaning in the interactional process leading to the construction of the mathematical knowledge and the development of mathematical thought (Jung & Schütte, 2018; Scott, Mortimer, & Aguiar, 2006). Through such interaction, the multiple dimensions within teacher education are attended to (or not) as the class engages with the examples at hand. My use of the term 'discourse' resonates with Monaghan's (2009, p. 15; drawing on Morgan, 2007) understanding of discourse to mean the "patterned uses of language and other forms of communication whose deployment identifies the user as belonging to a particular community at a particular time in a particular setting". In what follows, I engage with meaning making as a dialogic process.

Meaning making as a dialogic process

Mortimer and Scott (2003) conceive of meaning making as fundamentally a dialogic process, where different ideas are expressed and acknowledged by the teacher, and worked upon. The framework focuses specifically on ways in which the teacher acts in order to guide meaning making interactions within the classroom. The framework comprises of five different, but interconnected aspects of interactions in the classroom 1) teaching purposes; 2) content; 3) communicative approach; 4) patterns of discourse; and 5) teacher interventions. Mortimer and Scott's framework positions dialogue as connecting participants to meaningful, purposeful and valuable processes of knowledge construction.

In the current framework, I have focused more on the aspects of the framework that deal with classroom interaction, namely, the communicative approach and patterns of discourse as it is my contention that variation theory, in its focus on the object of learning, already engages with the mathematics content within the classroom. It is also my contention that in the mathematics teacher education classroom, the framework on the multifarious dimensions of teacher education (which I engage with in the subsequent section) is more adequate for delineating the 'teaching purposes' aspect of Mortimer and Scott's framework.

Communicative approach focuses on "how the teacher works with students to develop ideas in the classroom" (Mortimer & Scott, 2003, p. 33). For Mortimer and Scott, talk can be dialogic or authoritative, but it can also be interactive or non-interactive. A dialogic talk allows for different points of view even if the talk is orchestrated by one person, while an authoritative talk focuses on one point of view – usually that of the teacher. Talk can also be interactive which means it is structured to allow for the participation of other people, or non-interactive when it excludes the participation of other people. Mortimer and Scott conceive of the meaning making process as two continuums in which in the first continuum, at one extreme there is the dialogic communication approach and at the other extreme, there is the authoritative communication approach (see Scott et al., 2006). In the second continuum, there is interactive talk at one end and non-interactive talk at the other.

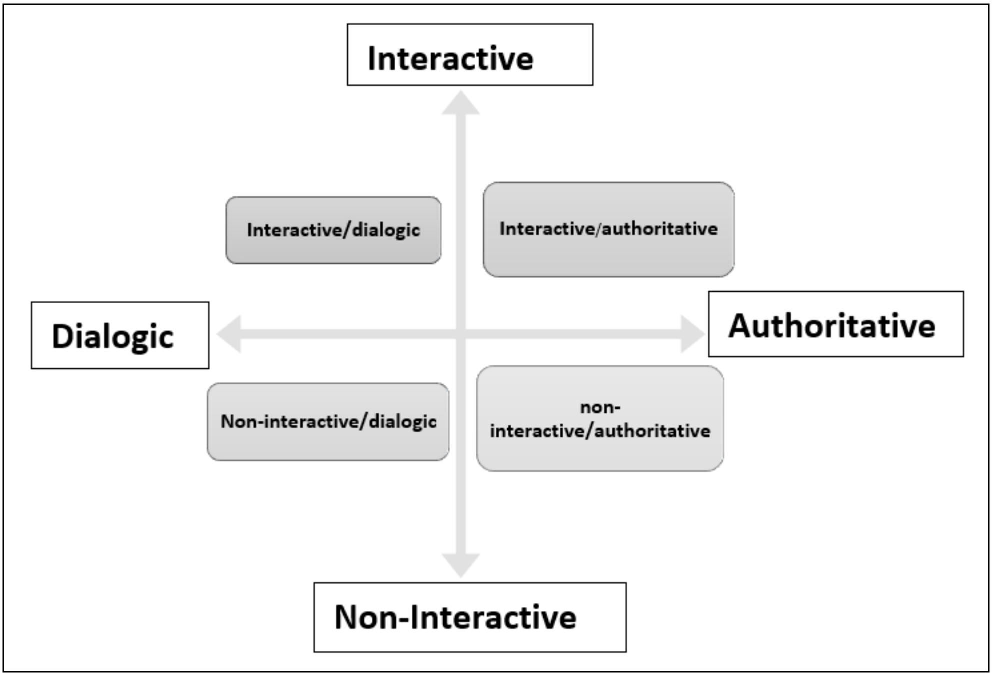

Figure 2: The dialogic-authoritative dimensions of discourse on an interactive–non-interactive continuum (adapted from Mortimer & Scott, 2003, p. 35).

The two dimensions of communicative approach (dialogic–authoritative and interactive–non-interactive) can be categorised into four classes of communicative approach with which discourse can be analysed (see Figure 2). The four classes are: 1) *Interactive/dialogic* where the teacher seeks to elicit and explore different ideas about a particular issue or concept and involves the students in the interactional process through, for example, questions which probe students' points of view; 2) *Non-interactive/dialogic* where the teacher is involved in presenting a specific (mathematics) point of view in a presentational mode (non-interactive), but at the same time, explicitly considering and drawing attention to different points of views (dialogic); 3) *Interactive/authoritative* where the focus is on one specific point of view that leads students through a question and answer routine with the aim of establishing and consolidating that point of view; 4) *Non-interactive/authoritative* which involves the teacher presenting a specific mathematics point of view or concept in a formal lecture mode (Scott et al., 2006).

Each communicative approach is put into action through specific *patterns of discourse* used by the teacher. Mortimer and Scott (2003) introduced the Initiation, Response, and Prompt (I-R-P-) pattern of discourse (Aguiar, Mortimer, & Scott, 2010; Mortimer & Scott, 2003; Scott et al., 2006). They argue that this pattern of discourse can also occur either in form of a closed chain or open chain in the interactional process. For closed chain, the pattern takes the I-R-P-R-P-R-…E form, where the prompt (P) by the teacher is followed by a further response from the student (R), and so on until the chain is closed by an evaluation (E) by the teacher. In the open chain, there is no final evaluation by the teachers, and so the interactional process takes the I-R-P-R-P-R-P-R- form (Aguiar et al., 2010; Scott et al., 2006). This pattern may be different if students (rather than teachers) are the initiators of the question in the above chain. They pattern would then be in the form I-R_{s1}-R_{s2}-R_{s3}- (where S_1 would be student 1, S_2 student 2, etc). Mortimer and Scott argue that this pattern of discourse can be used to support dialogic interaction while most authoritative interactions are played out through the I-R-E pattern.

Multifaceted dimensions of teacher education

For teachers of mathematics intending to use variation theory in their classrooms, variation theory provides a way of structuring their lessons to maximise the chances of students' lived object of learning aligning as closely as possible to the teacher's intended object of learning. In teacher education multilingual classrooms, however, both the intended object of learning and the lived object of learning are more complex because they both necessarily need to go beyond the acquisition and/or the construction of disciplinary (content) knowledge. Hence, it is not simply about choosing examples that will achieve the target objectives as this could very easily place an emphasis solely on content. In pre-service teacher education mathematics classrooms, in addition to being knowledgeable about the mathematics content the Pre-service teachers (PSTs) will teach at the end of their qualification, PSTs need to develop an awareness of the context in which they will teach and have knowledge about instructional practices that are pertinent for this context. In this vein, research in multilingual classrooms has argued for the need to attend to linguistic aspects of mathematics teaching and learning and for attention to be paid to the language needs of multilingual learners (Barwell, 2020; Erath, Prediger, Quasthoff, & Heller, 2018; Schleppegrell, 2007; Smit & van Eerde, 2011). Moschkovich (2013) and Moschkovich and Zahner (2018) also argue that in mathematics classrooms, attention needs to be paid to enculturating students into participating in valued mathematical practices. In the specific context of teacher education for teaching mathematics in multilingual contexts, it can be argued that the multifacetedness of teacher education necessitates that PSTs need to at once be enculturated into becoming teachers of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics content, becoming learners of mathematical practices and becoming proficient LoLT Users for the purpose of teaching/learning mathematics (Essien, 2014). My elaboration of these multiple dimensions involved in TE draws from both the field of mathematics education (as indicated above) and from Wenger's (1998) notion of identity as a 'constant becoming', as trajectories which are not necessarily linear, and which has no fixed destination.

Becoming teachers of mathematics is about teaching, and the teacher educator sees herself/himself as developing this dimension within teacher education in the PSTs, while the PSTs see themselves as imbibing this identity. By the same token, in *becoming learners of* *mathematics content*, the teacher educator sees herself/himself as responsible for the development of disciplinary knowledge in the pre-service teachers, and the pre-service teachers see themselves as learners of mathematics content. *Becoming learners of mathematical practices* relates to becoming knowledgeable about mathematical processes such as the processes of coming to define, exemplify, code switch, revoice, etc. If, for example, a teacher educator teaches a particular content using her/his chosen examples, and the focus is on the mathematics content, the teacher educator is enculturating the PSTs into becoming learners of mathematics content. But if the focus is also on the logic behind the examples that have been selected for use and what makes for a good set of examples in the topic at hand, then the teacher educator is enculturating the PSTs into becoming learners of mathematical practices/processes. In *becoming teachers of mathematics in multilingual classrooms*, there is something specific about teaching in multilingual contexts, and as such, attention is not only paid to the fact that the pre-service teachers would become teachers, but that they would become teachers in multilingual contexts. *Becoming proficient language users for the purpose of teaching/learning mathematics* describes a situation in which attention is paid to how the mathematics language and the language of learning and teaching (LoLT) in which it (mathematics language) is imbedded, are used in class.

A triadic framework for understanding examples in multilingual classrooms

How do these different frameworks come together to provide a gaze into examples and example spaces used in teacher education multilingual classrooms?

I contend that while variation theory provides perspective into the choice of examples by the teacher educator and the mathematics made possible to learn, Mortimer and Scott's (2003) framework illuminates how language is used to engage with these examples in practice, and finally the framework on the multifarious dimensions within teacher education provides perspective on how through the teacher educator's use of language, these dimensions are either attended to (or not) in teacher education multilingual classrooms. In Figure 3 below, I present the resultant triadic framework.

At the centre of the merger framework is the mathematical object of learning. This is because the object of learning is the focus of attention for the lesson. The teacher educator needs to first determine what the object of learning is for a class, and what critical features within an example set are best suited to achieve her/his object of learning. It is at this point that the patterns of variation become key (hence the question: "what examples would best bring out the critical features" is crucial). But also, it is essential for the teacher educator to engage with the kinds of discourse (patterns of discourse), communicative approach (collectively called interactional process), and teacher moves that are more adequate not only in the teaching process, but also in the discernment of pre-service teachers' understanding of the object of learning. The question as to which combination of interactional process and teacher moves will best enable pre-service teachers to develop a relevance structure on the topic at hand so that learning is made more meaningful to them becomes important.

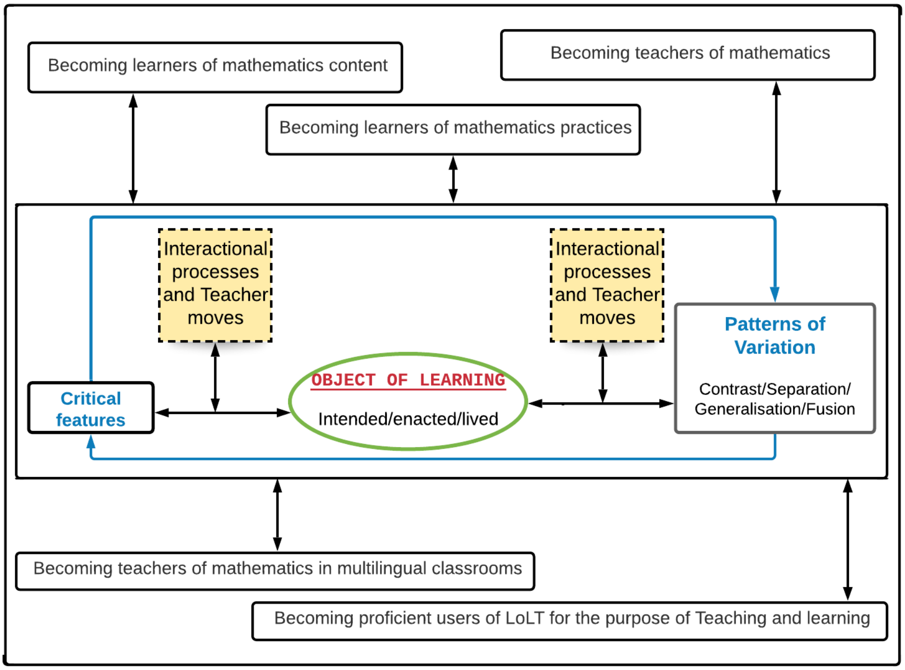

Figure 3: A triadic framework for understanding examples in mathematics teacher education multilingual classrooms

The bidirectional arrows between the 5 dimensions within TE and the core concepts of variation theory, and those from Mortimer and Scott framework are an indication that through the interactional process in engaging with the examples, multiple facets in TE are attended to (or not). But also, the awareness of the context of teaching and learning and the context the PSTs will teach at the end of their qualifications need to also inform the interactional process in class.

These three frameworks have been used separately by researchers to analyse data. I bring all three frameworks together and show an example (using empirical data) of how it can be used holistically to gain insight into not only the exemplifying practices within teacher education, but into how through the interactional process associated with the examples used in the classroom, opportunities for the enculturation of PSTs into the different facets within TE can be teased out. Zaslavsky (2010) argues that some examples have more explanatory power than others depending on the context and the classroom activities surrounding these examples. In this sense, the amalgamated framework provides tools for analysis of the instructional examples in classrooms which are both pre-service and multilingual in nature. It must be noted that in this framework, the examples that are chosen and used, the pattern of discourse that is enacted need not necessarily be fixed before the lesson.

Using the triadic framework to understand examples in mathematics teacher education multilingual classrooms

In the transcripts that follow, I bring the triadic framework (henceforth ‘the framework’) to bear on the example space used by a teacher educator in an introductory class on probability in order to show the value of the framework in analysing classroom data in multilingual teacher education classrooms. I engage with the classroom interactional process that occurred during the enactment of these examples and also engage with what facets of teacher education were attended to in the course of the enactment. For brief background, the teacher educator is a monolingual first language English speaker and does not share a common first language with most of her PSTs, most of whom are multilingual. It is important to note that as a product of the old South African high school curriculum, the PSTs in this class were encountering the topic probability for the first time, having not done it previously at high school. In the introductory class, focused on teaching the meaning of probability and its scale, the teacher educator provided the class with these four examples:

> **Example 1**: Chances of the teacher educator coming to class in the subsequent lesson
>
> **Example 2**: Tossing a coin. The game of football is used in which the referee tosses a coin after two captains have chosen their side of the coin
>
> **Example 3**: Throwing a dice: Throwing a dice and finding:
> 3.1 P(4)
> 3.2 P(Even number)
> 3.3 P(number less than 5)
>
> **Example 4**: Pack of cards and finding:
> 4.1 P(Jack)
> 4.2 P(10 Diamond)
> 4.3 P(Odd number)
> 4.4 P(Heart)
> 4.5 P(Black/Suit)

After this, in the next lesson, the teacher educator performed an experiment involving the law of large numbers and subsequently explained theoretical probability. Due to space limitations, in this paper, using classroom observation transcripts, I focus on the four examples above used in the introductory lesson. Using the merger framework, I start by analysing the example set based on variation theory as it concerns the object of learning, patterns of variation and critical features before engaging with the teacher moves and the interactional process in the enactment phase of the examples.

While this lesson was not theory-driven based on variation theory, a number of observations can be made on the teacher educator’s choice of examples using variation theory as a lens, and in terms of what the examples make possible to learn. First, in considering the set of examples used in this introductory lesson on probability, it can be deduced that even though the initial object of learning was the definition and the meaning of probability at the start of the class, this object of learning shifted to the relevance of probability in everyday context. In terms of variance and invariance, all four examples are similar in the sense that they relate to (and were taught in such a way that they related to) the PSTs' real-life context. Examples 1 and 2 are also invariant in as much as there are two possible outcomes. What is possible to discern in Examples 1 and 2 is that even though both have two possible outcomes (in terms of probability), the desired outcome can be an "either/or" situation. In Example 1, the teacher educator will either be in class or not be in class, and in Example 2, one captain either wins or loses. But beyond this, the two examples also allow the PSTs to discern the difference between the two scenarios in both examples in that while the probability of coming to class for the teacher educator is either 0 or 1 (for theoretical probability) or dependent on the relative frequency in terms of the number of times the teacher educator has been present/absent from class (relative probability), the probability of either winning the toss or not in Example 2 is $\frac{1}{2}$. The two examples make it possible for the PSTs to be able to discern and generalise that situations of "either/or" are not necessarily 0 or 1 in probability. The question that can be asked here is to what extent the teacher moves and interactional process enabled the PSTs to discern the above from the examples.

What contrasts Example 3 from the first two examples is the fact that there are six possible outcomes in Example 3. In Example 4, the structure of the question is invariant with Example 3 but can also be differentiated by the fact that Example 4 has 52 possible outcomes. Overall, four aspects or dimensions of variation are present across Examples 1 to 4.

Aspects	Examples of Critical Features
Events	Tossing a die, tossing a coin, or Obtaining a particular suit from a pack of cards
Sample Space	Coin: 2; Dice: 6; Cards: 52
Sample points	P(H), P(4) or P(10 diamonds)
Conditions	P(less than four) or P(Black or Suit)

Table 1: Critical features evident in the teacher educator's example set

These are events and their corresponding sample space and/or sample points and the stated conditions around the desired outcomes. The critical features or the values related to these aspects (see Table 1) are different and vary according to the nature of the presented event. So the example space can be described as simultaneously varying and thus Fusion.

So far, my analysis has focused on using variation theory to analyse the examples chosen for the introductory class on probability. In what follows, using transcripts of the interactional process in the class, I show a snippet of how the teacher educator enacted Examples 2 (see Essien, 2021, for analysis of Example 3).

Transcript 1: Tossing the coin

1	TE:	So for example *[writes:* e.g.*]* …has anyone got a coin here please? I didn't bring one in. *[gets a coin].* Right. We are now about to kick off with the Confederation Cup. I'm the captain of South Africa Bafana Bafana and you *[points to a student]* are the captain of Iraq *[everyone laughs].* And the referee comes along. Now before that you know that the coin is tossed. What do you call it in your language? How do you call that?

2	PSTs:	*(Students shout out answers)*
3	TE:	Anyway, so he [Referee] comes along *[hands the coin to a student]* You are the referee. Now before he does the activity, the referee is going to toss the coin. This is the event. Now whoever gets like a head or a tail, whoever gets it, what happens then?
4	PST1:	Reward
5	TE:	Ja, but what is the reward for getting the… you know, if I call 'heads' and she tosses heads
6	PST1:	Then you get to kick off
7	TE:	Then I get the kick-off, don't I? OK.
8	PST2:	Choose sides.
9	TE:	Now, oh hang on, before you toss the coin, listen to me, what are the chances that heads are going to come up?
10	PSTs:	*[in chorus]* Half-half
11	T:	Half-half
12	PST3:	2 to 1
13	TE:	2 to 1. What else?
14	PST2:	*[softly]* Unlikely
15	PST4:	Even odds
16	TE:	How do we speak?
17	PST5:	Equally likely
18	TE:	Equally likely chances that heads will come up. OK, do you see how I'm using the language with the number, with this? What's the number? You said it just now? Choose one of the numbers. […]
19	TE:	So basically you can read this in mathematics just like this and immediately, instead of saying, 'Well it's half-half' or 'it's equally likely' what you can do is give me a number that goes with this event. What are the chances of getting heads? So please put equals *[next to* P (heads) *writes =].* And after that you said … *[after = writes ½] …*It's 1 out of 2 chances. Now I'd like you please to put the 2 in another colour. In fact let's just put the 1 in another colour *[goes over and writes the* 1 (numerator) *in pink and the* 2 (denominator) *in yellow)*
20	TE:	I'm doing this for a reason.
21	TE:	If you create a fraction like this, people, the numerator… here *[writes N]*, the numerator tells me something and the denominator tells me something. *[writes* D = *with a line pointing to the* 2*].* Now think of the coin, think of the ½, the 1 and the 2 and tell me what they tell me. *[TE cleans the scale off the board]*
22	TE:	You see a half in fraction work we teach the little ones it's 1 out of 2 equal pieces, isn't it? So if I cut an apple into 2, half-half, that's it. In Probability, this fraction is telling me more in English because it's linked to an activity, OK. So what's it telling you? Right, what's the numerator telling me?

23	PST:	I think the numerator is telling you that out of the 2 chances available of(?) which(?) the denominator, there's a probability of you getting only one. So of which it can be either the head or the tail.
24	TE:	OK, so the numerator, do you see that the numerator is linked to what I want? *[points to* N *and then to* heads*].* That's very very important you understand that. [...]

In Transcript 1, the teacher educator taught the concept of tossing a coin (the object of learning) as something that is connected to the PSTs' everyday life by first using a physical coin but also by using soccer as an example. Using the triadic framework, it is possible to see how the pattern of discourse and the communicative approach enabled the teacher educator to bring into focus (or not) the different facets of teacher education. Interactive/authoritative approach is clearly the predominant communicative approach in the transcript. As indicated previously, using the interactive/authoritative communicative approach entails that even though the teacher educator welcomes PSTs' viewpoints, the interactional process is directed to one viewpoint – in this case, the viewpoint of the teacher educator. In Turn 9 where the teacher educator asks the PSTs what the chances are of obtaining a Head even before commencing with the experiment of tossing a coin, the teacher educator uses and I-R-P-R-P-R-P-E pattern of interaction to allow for different views to be offered (Turns 12, 14 and 15) on the probability of tossing a coin. The different viewpoints however converge to the teacher educator's viewpoint (hence interactive/authoritative as opposed to interactive/dialogic). It is important to note that through this interactional process, the teacher educator attempts to incorporate cognitive academic language opportunities after noting the PSTs' one- to two-word responses. She therefore asked in Turn 16, *how do we speak*?, and provides the answer in Turns 18 and 19 drawing the PSTs' attention to the correct way of expressing the probability of obtaining a Head when a coin is tossed. By so doing, attention is paid to the development of the PSTs into becoming proficient users of the LoLT.

A parallel is seen later in the transcript where the teacher educator instructor draws on language to explain the meaning of the answer (1/2) obtained in the probability question. Here, we see a focus on meaning rather than on procedures (for arriving at the correct answer) where the interactional process around the solution to the example attempts to give meaning to the mathematics symbol. Attention is paid to how the mathematical representation and the register around the mathematics symbol (1/2 in this case) are related. Such approach of interweaving content and language is well documented in recent literature (example, Wessel, 2019; Erath et al., 2018). But what the teacher educator does in addition to weaving the content and the mathematical language associated with the content is to draw on both content and the interactional context of the content to, in Turn 22, allude to how fraction is taught in earlier grades thus paying attention to the fact that the PSTs are becoming teachers of mathematics.

Discussions in relation to the triadic framework

Going back to the example space provided by the teacher educator, it is my contention that what the teacher educator could do further in enacting the examples is to focus on explanations that allow for the PSTs to see structure in the example space that makes generality possible. While this generality is evident using variation theory as a lens, in reality, it is mainly through the interactional processes and the teacher moves that this generality can become evident to PSTs. If the teacher educator had drawn attention to the pattern of variation and invariance in the questions she posed, she would have provided opportunity for the development of PSTs as learners of the mathematical practice of exemplifying. This is because in so doing, she would have drawn attention to exemplifying as a mathematical process.

While the framework allows for teasing out which dimensions of teacher preparation for teaching mathematics in multilingual context come into focus, at the same time, it also provides perspectives on which dimensions are backgrounded. In Transcript 1, through the interactional process around the examples at hand, it is easy to see attention paid to (and the PSTs enculturated into) 1) becoming learners of mathematics content, 2) becoming teachers of mathematics, and 3) becoming proficient users of the language of learning and teaching. Using the framework as a lens, it also becomes evident that in the enactment of the mathematical object of learning, becoming teachers of mathematics in multilingual context, and becoming learners of mathematical practices are not attended to in the presented transcript. A missed opportunity for the development of becoming teachers of mathematics in multilingual context is seen in Turn 1 of the transcript where the teacher educator asks the PSTs what they call "tossing a coin" in their home language. It is a missed opportunity because this question could have been used by the teacher educator to enculturate the PSTs into becoming teachers of mathematics in multilingual classrooms. The different ways of naming 'tossing a coin' in the different languages present in the class and their meanings in English could have been interrogated in class. This would have, no doubt, enriched their discussion around the meaning of tossing a coin in mathematics.

Variation theory provides conceptual learning opportunities but such opportunities in multilingual teacher education classrooms should go hand-in-hand with language learning opportunities deliberately tied into the interactional process that occur in class. The distinction between the two roles of language as a learning medium and language as a learning goal (Lampert & Cobb, 2003; Erath et al., 2018) comes to mind. Language learning opportunities are evident when using the amalgamated framework as opposed to using only variation theory either to teach or to analyse teaching.

Finally, the question could be asked as to how the class dynamics would have been affected if, for example, a different type of communication approach or a different type of pattern of discourse was used in enacting the examples. An I-Rs_1-Rs_2-Rs_3- pattern of discourse where PSTs are given the opportunity to engage or critique one another's solution to the problem would have enculturated the PSTs into practices that deal with judgments about what are mathematically legitimate claims, and practices such as providing justification, proving, critiquing conjectures, critiquing solutions, etc.

Concluding remarks

The triadic framework provides a multifocal conceptual lens for the exploration of three critical layers involved in the complexity of teaching and learning in mathematics teacher education multilingual classrooms. First, the mathematical object of learning needs to be central to the teaching and learning process. Regarding the object of learning, the choice of examples (using variance and invariance) needs to be such that it attends to the intended object of learning. Second, attention needs to be paid to the interactional process that takes place in the course of the enactment of the object of learning. Finally, in the context of teacher education, the framework provides for attention to be given to how the different dimensions involved in teacher preparation are (co-) constructed (or not).

The analysis in this paper has basically shown an inside-out approach (see Figure 3) in teacher preparation for teaching mathematics in multilingual classrooms where the starting point is the object of learning, culminating in the multiple facets of teacher education. My contention is that the choice of examples, and the accompanying interactional process need not necessarily gear towards developing the multifaceted dimensions involved in teacher education. The framework also provides for an outside-in approach where the teacher educator decides in advance what facets of teacher education to attend to at a particular point and then decides on which interactional process to use to give focus to these facets. This will in turn inform the type of examples that the teacher educator would choose to bring the object of learning into focus. In such a case, the framework suggested in this paper could be extended to professional development programmes involving multilingual mathematics teachers.

References

Aguiar, O. G., Mortimer, E. F., & Scott, P. H. (2010). Learning from and responding to students' questions: The authoritative and dialogic tension. *Journal of Research in Science Teaching, 47*(2), 174-193.

Arzarello, F., Ascari, M. & Sabena, C. (2011). A model for developing students' example space: The key role of the teacher. *ZDM Mathematics Education 43,* 295–306. https://doi.org/10.1007/s11858-011-0312-y

Barwell, R. (2020). Learning mathematics in a second language: Language positive and language neutral classrooms. *Journal for Research in Mathematics Education*, *51*(2), 150–178

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. In J. Novotna (Ed.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education.* Prague, Czech Republic: PME.

Engle, R. A., & Conant, F. R. (2002). Guiding principles for ostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and Instruction, 20*(4), 399–483.

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: The case of explaining to learn and learning to explain. *Educational Studies in Mathematics, 99,* 161–179. https://doi.org/10.1007/s10649-018-9830-7

Essien, A. A. (2014). Examining opportunities for the development of interacting identities within pre-service teacher education classrooms. *Perspectives in Education, 32*(3), 62–77.

Essien, A. A. (2017). Dialogic and argumentation structures in one quadratic inequalities lesson. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 82–99). London: Routledge.

Essien, A. A. (2021). Understanding the choice and use of examples in mathematics teacher education multilingual classrooms. *ZDM Mathematics Education, 53*, 475–488. https://doi.org/10.1007/s11858-021-01241-6

Goldenberg, P., & Mason, J. (2008). Shedding light on and with example spaces. *Educational Studies in Mathematics, 69*(2), 183–194. https://doi.org/10.1007/s10649-008-9143-3

Jung, J., & Schütte, M. (2018). An interactionist perspective on mathematics learning: Conditions of learning opportunities in mixed-ability groups within linguistic negotiation processes. *ZDM Mathematics Education 50*, 1089–1099. https://doi.org/10.1007/s11858-018-0999-0

Kullberg, A., Runesson Kempe, U., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM Mathematics Education, 49*(4), 559–569. https://doi.org/10.1007/s11858-017-0858-4

Lampert, M., & Cobb, P. (2003). Communication and learning in the mathematics classroom. In J. Kilpatrick & D. Shifter (Eds.), *Research companion to the NCTM standards* (pp. 237–249). Reston: NCTM.

Lester, F. K. (2010). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 67–85). Springer.

Lo, M. L. (2012). *Variation theory and the improvement of teaching and learning*. Göteborg, Sweden: Acta Universitatis Gothoburgensis.

Lo, M. L., & Chik, P. P. M. (2016). Two horizons of fusion. *Scandinavian Journal of Educational Research, 60*(3), 296–308. https://doi.org/10.1080/00313831.2015.1119730

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum.

Marton, F., & Tsui, A. (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum.

Monaghan, F. (2009). Mapping the mathematical langscape. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 14–31). Bristol: Multilingual Matters.

Mortimer, E. F., & Scott, P. H. (2003). *Meaning making in secondary science classrooms*. Philadelphia: Open University Press.

Moschkovich, J. (2013). Issues regarding the concept of mathematical practices. In Y. Li & J. N. Moschkovich (Eds.), *Proficiency and beliefs in learning and teaching mathematics: Learning from Alan Schoenfeld and Günter Toerner* (pp. 257–275). Rotterdam: Sense.

Moschkovich, J., Zahner, W. (2018). Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions. *ZDM Mathematics Education 50,* 999–1011. https://doi.org/10.1007/s11858-018-0982-9

Olteanu, C. (2018). Designing examples to create variation patterns in teaching. *International Journal of Mathematical Education in Science and Technology, 49*(8), 1219–1234. https://doi.org/10.1080/0020739X.2018.1447151

Olteanu, C., & Olteanu, L. (2013). Enhancing mathematics communication using critical aspects and dimensions of variation. *International Journal of Mathematical Education in Science and Technology, 44*(4), 513–522. https://doi.org/10.1080/0020739X.2012.742153

Pang, M. F., & Marton, F. (2003). Beyond "lesson study": Comparing two ways of facilitating the grasp of some economic concepts. *Instructional Science, 31*(3), 175–194. https://doi.org/10.1023/A:1023280619632

Pang, M. F., & Marton, F. (2005). Learning theory as teaching resource: Enhancing students' understanding of economic concepts. *Instructional Science, 33*(2), 159–191. https://doi.org/10.1007/s11251-005-2811-0

Runesson, U. (2005). Beyond discourse and interaction. Variation: A critical aspect for teaching and learning mathematics. *Cambridge Journal of Education, 35*(1), 69–87. https://doi.org/10.1080/0305764042000332506

Schleppegrell, M. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139–159.

Scott, P., Mortimer, E., & Aguiar, O. (2006). The tension between authoritative and dialogic discourse: A fundamental characteristic of meaning making interactions in high school science lessons. *Science Education, 90*(4), 605–631.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teacher move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340.

Smit, J., & van Eerde, H. (2011). A teacher's learning process in dual design research: Learning to scaffold language in a multilingual mathematics classroom. *ZDM Mathematics Education, 43*(6), 889–900.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8*(2), 91–111.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge: Cambridge University Press.

Wessel, L. (2019). Vocabulary in learning processes towards conceptual understanding of equivalent fractions: Specifying students' language demands on the basis of lexical trace analyses. *Mathematics Education Research Journal, 32*, 653–681. https://doi.org/10.1007/s13394-019-00284-z

Zaslavsky, O. (2010). The explanatory power of examples in mathematics: Challenges for teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 107–128). Springer.

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics, 69*(2), 165–182. https://doi.org/10.1007/s10649-008-9140-6

Variation and dialogic communication in maths teacher education in a multilingual context: A response to Anthony Essien

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Anthony Essien, in his plenary session, presented a triadic Framework to analyse the choice of "exemplification" in teachers education mathematics classrooms in a multilingual context, with a focus on the role of the teacher educator, who is seldom the object of research in education. In my response, I will be reviewing his paper on two counts. A. I will be reflecting on the presentation of the triadic framework developed and B. reviewing how the selection of examples and the classroom interaction can be further extended to bring a greater multilingual and multicultural input into maths teacher education, with a greater focus on dialogue. I will end with a few suggestions as to how the framework may be extended and enriched to incorporate multilinguality in a stronger way.

The triadic framework for analysing choice of examples

As the elements of choice of examples in a multilingual mathematics classroom are complex, choice is determined by complex interrelated factors. A framework putting together these elements that can act as criteria of choice, is, indeed, important. It is, heartening that Anthony has taken upon himself this challenging task.

Another important aspect is that the site for the framework is teacher education classrooms and not school classrooms. This is a crucial element that is often neglected. Even though we're talking of Student Teachers here, the assumption that the student teacher has understood school mathematics concepts does not hold for developing countries like South Africa and India.

It is in the context of recognising its importance and its strengths that I now critique it. Maths is a subject that takes one through to an abstract realm. However, the human mind learns through concrete examples, from experience and reflections. Hence, exemplification is a necessary part of learning. It is interesting that Anthony tries to see "exemplification" through trying to make a triad of three different yet inter - related frameworks. This is an important exercise. He has chosen to do it theoretically rather than empirically. In the attempt to develop this framework theoretically, the elements of the framework have not been explicated enough and have become somewhat opaque. A little more elaboration with examples would have been more helpful.

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The triadic framework developed by merging variation theory, the dialogic interactive communicative process and the multifaceted aspects of multilingual contexts has captured a lot of important aspects and placed them in relation to each other. However, it seems to put multilingual aspects and contexts as an add on at the end, whereas this needs to be an integral and overarching part of the framework – like a canvas in a painting. Because language and multilinguality is what is the lifeblood of the children learning the mathematics.

The paper does not deal with the language aspects and its relation to mathematization at all. This is an important weakness. Different languages not only have different words, but different idioms and cultural contexts that need to be taken into account. A more centre-stage position to multilingualism would have brought out the immense potential of intertwining it with variation theory. This would map the examples from different languages and cultures onto the critical features of a mathematical concept and procedure.

Variation and dialogue: Two important elements for maths education in a multilingual context

The element of variation, in mathematics classrooms has two aspects to it – a) content variation and b) procedural variation. The latter is equally, if not more important in mathematization of concepts, but has not been dealt with at all. Procedural variation gives a lot of scope for multilingual and multi-cultural ways of solving problems, particularly oral ways – classical examples are using grouped addition from hundreds to units for multiplication – rather than the written algorithm of multiplying from the unit side and carrying over – this is procedural variation and is present in almost all algorithms. If connected with multilingual cultures, it would give a lot of mathematical power to deprived communities, whose procedural variations are excluded from mainstream mathematics.

Equally important are the communication and dialogue aspects, which have not been elaborated – the collective, reciprocal, supportive, cumulative and purposeful aspects of dialogue. It is not only language that is different in a multilingual context, but the cultural differences reflected by the language are much deeper and wider.

Those tribes and community groups that have developed with livelihoods which interact with numbers in certain ways, often give rise to quinary, quinary-decimal and vigesimal number systems in traditional societies. Even though other aspects of tribal living may decline – sometimes, when the livelihood remains the same, such number systems, ways of calculations, words, phrases, riddles continue. They make for a stronger procedure in the heads of such communities than the formal metric system introduced through school. Incorporating these in the examples and dialogues in the classroom has a number of advantages.

1. Most such groups are deprived communities and their language and ways of thinking have been excluded from the mainstream classroom. This invisibilises them, undermines their identity and demotivates them from continuing their studies. Including conceptual and procedural variation on mathemetisation from their lives, will enhance their visibility and motivate them to learn mathematics better and own that learning.

2. A diversity of variations will also help other students in class become familiar with a greater diversity of possibilities, other cultures and other words and meanings. This will develop cognitive robustness as well as empathy in all students.

Using the triadic framework to understand examples in multilingual mathematics classrooms

The concept chosen is an introduction to probability. While it is mentioned that the language background of the teacher educator and the student teachers is different, it is not clear which languages, tribes and cultures the student teachers hail from.

The choice of the four examples is mentioned, but not the bases of their choice. It is not clear whether they have been contributed by the student teachers, or the teacher educator decided on them by himself. Enough background is not available to say how they were chosen and any links to the cultures and languages of the student teachers are not mentioned. Football, dice games and cards all seem western contexts.

The aspect of probability being dependent on ambient conditions on a large scale is not mentioned nor incorporated in the activity or dialogue, or how this probability can be played around with – weighting the dice or coin. Just discussing one example does not bring out the conceptual and contextual features.

The teacher educator in the transcript excerpt also doesn't give ample space to discussing the student teachers' own conceptual, cultural ideas on probability and reveal their own language of probability. I don't know enough about South Africa, but in India the idea of probability is integrally linked with the idea of fate. Another area is gambling. We, in our alternative science curriculum have taken up the issue of odds in gambling, at the middle school level, and shown how it is impossible to have a winning situation in the end.

Secondly, this example did not have much dialogue or real interaction, somewhat vitiating the point of the dialogue part of the framework. It was presented by the TE in a relatively closed ended way – the coin was not tossed even once in the excerpt, whereas to get a hang of probability – you need to toss it a fairly large number of times, then go to the numerator denominator issue. Had it been followed up by a number of throws, the name for the throws, the use of coin throwing discussed, it may have brought forth a better dialogue necessary for better understanding. As presented both the activity and dialogue seem cosmetic rather than authentic.

A lot of mathematics – number, operations, time, weight, length, algebra can be dealt with in a multilingual way, using variation of examples and procedures from different cultures. This would benefit both the different language speakers as well as the dominant language speakers as through dialogue they could collectively bring consonance. This would require a framework for multilinguality which maps on to the variation theory framework.

In conclusion

In order to integrate a framework for multilinguality along with variation theory and communication, elements of language and culture, that could form criteria for selection of examples need to be laid out.

Some of these could be

- Words and phrases closest to the concept, that are available in the learners' context (e.g., number names in a quinary system, how number names for larger numbers are constructed in the language, or names for units of time, weight distance, etc., and for past, future, long short).
- Real life contexts where the concept is used (e.g., when teaching percentages look for examples of percentage in the community and how it is expressed – e.g., in poor communities and rural contexts when in the context of lending the phrase x rs. *prati saikda* (per hundred) is used not *pratishat* (percentage)).
- The local procedures for solving problems of that concept need to be looked for or extracted from students.

Secondly, a way of dialogue that brings out students' culture and language needs to be outlined. Elements of dialogue mentioned above also need to explicitly form part of the framework. These could then form a checklist and rubric for selection of examples to be incorporated when selecting examples according to variation theory. A revised framework could incorporate these aspects in a more authentic manner.

The ethical significance of exemplifying: A response to Anthony Essien

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In his plenary paper, Tony Essien wrote that "The importance of mathematical examples […] cannot be overemphasised". Tony's paper opened my eyes to the significance of examples and of the complex practice of exemplifying in multilingual mathematics education in general, and in teacher education in particular. The triadic framework that Tony proposed captures exemplifying as illustrating concepts from the perspectives of (a) grasping the illustrated concept by discernment; (b) communicating about the illustration as a fundamental idea of teaching and learning; and (c) attending to the multilingual context (of pre-service teachers) in which the illustration is submerged.

Multilingualism in mathematics education

Before moving further, I would like to recognise that multilingual pre-service teachers and multilingual students in general are far from a homogeneous group either in, or across contexts (Barwell, 2016; Civil, 2012). However, multilingual students are often from marginalised groups whose opportunities to learn mathematics are diminished compared to their peers from majority/dominant groups whose first language is the language of power and hegemony and of learning and teaching (e.g., Chronaki, 2009; Källberg, 2018). I consider a multilingual classroom to be any classroom where more than one language is present. Hence, some students in a multilingual classroom may be multilingual, while others are not.

In multilingual classrooms some languages might be silenced due to monolingual norms and/or policies, but they are silently present in every instance of the interaction (García & Wei, 2014). Different languages in which different experiences, knowledges and worldviews are embedded (Knijnik, 2012; Radford, 2012) are brought into the learning spaces. This means that students may experience non-hegemonic and hegemonic cultural and language resources against each other at school, often finding some of their cultural, epistemological, and language resources and identities ignored or perhaps even rejected. Ultimately, this may influence multilingual students' future prospects. Therefore, multilingualism in mathematics classrooms is a matter of social, political, cultural, and ethical issues.

Ethical dimensions of a 'good' example

Initially, Tony shared some questions that inspired him to conduct research about the practice of exemplifying in multilingual mathematics teacher education. One such question

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was: "What should a "good" (use of) example be in multilingual pre-service teacher education mathematics classrooms?" This question invited me to consider value-laden or ethical dimensions of the practice of exemplifying in multilingual learning spaces. Indeed, the word example can be used to express conducts of behaviour; an example is one that/who serves as a desired pattern to be imitated – a role model. An example may also be a punishment inflicted on someone as a warning to others (Merriam-Webster Thesaurus Online, n.d.).

The philosophical branch of ethics is concerned with questions of right or wrong, of benevolence and harm, and about what is proper conduct. Ethics is not a monolithic concept. Rather, it covers a broad range of ideas that span from normative, utilitarian conceptions of virtue and duty, to postmodern ethics that embrace dimensions that may capture the ecological, the social and cultural, others, and self (Dubbs, 2020). Skovsmose (2020) suggested that adding an ethical dimension to mathematics opens up a space for reflection on the ways in which mathematical activity impacts upon society. He suggested that critical mathematics education offers the capacity to embrace such ethical reflection.

Opening up a space for ethical reflection

The triadic framework that Tony proposed helped me to identify one instance in the excerpt that he provided that has the potential to open up a space for ethical reflection on political effects of monolingual mathematics education in multilingual societies. In line 14 of the excerpt, a pre-service teacher softly replied "Unlikely" when the teacher asked what the chances of "winning" are when tossing a coin. It appears as if the coin-tossing example allowed the pre-service teacher who replied "Unlikely" to discern some critical aspects that relates to injustice. There ought to be equal opportunities that the heads or the tails appear when the coin is tossed, and the chances of winning should be 50 percent. The "unlikely" reply may be a reply that challenges the idea of equal opportunities. It could suggest a disadvantaged position where chances in life and opportunities, such as learning mathematics, are unequally distributed. In the interaction, the "Unlikely" reply seems to pass unnoticed by the teacher and by the other pre-service teachers because no one comments on it. The "Unlikely" reply could provide an opportunity to open up a space for reflection on the ways in which mathematical (school) activity impacts upon societies that are multilingual, which most societies are. In this space, reflections may capture and move forward a critique in multilingual mathematics education that revolve around discourses such as those discussed by Chronaki and Planas (2018) regarding representation and politics, language as resource, Eurocentrism, racism and epistemic violence, deficits, benevolence, dichotomies (formal-informal mathematics, first–second language, global-local, foregrounds-backgrounds) and othering, *et cetera*. These are discourses that pre-service teachers need to navigate as they are at once "enculturated into becoming teachers of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics content, becoming learners of mathematical practices and becoming proficient LoLT users for the purpose of teaching/learning mathematics" (Essien, 2021, in this volume, p. 45).

A relational perspective on ethics

What are good examples of how to navigate such spaces? How do we deal with the idea that, for instance, a 50–50 chance of "winning" might be true for some students in some situations, while winning might be very "unlikely" for other students, without reproducing stigmatising categorizations or deficit discourses (Norén & Boistrup, 2013)? Or without putting an extra burden on marginalised groups, demanding that they explain or share their experiences (Hand et al., 2021)?

The questions invite me to consider Swanson's (2017) writings:

> Perhaps it is time for us to remember what the intentions of mathematics education should be, to live well with mathematics education in order to live well with others; to live and research well with mathematics education in order to make possible futures of radical hope. (Swanson, 2017, p. 13)

The above quote shifts the attention from an ethical reflection on how mathematical activity may impact upon society to how we (for example educators and pre-service teachers) inhabit mathematics learning spaces in ways that allow us to live well with mathematics and with each other. In other words, the attention shifts from socio-political projects to a critique that centres considerations in multilingual mathematics activities from a relational perspective of ethics.

Swansson (2017) reminded me that for us to live well with mathematics, mathematics education and research, we may need to expand theories of ethics from non-Euro-centred perspectives. She called for a move towards, for example, indigenous, decolonial, posthuman ontologies, epistemologies, and theories to live better with each other as we engage mathematics. Drawing on indigenous knowledges (Ermine, 2007), Russell and Chernoff (2013) proposed ethics as a capacity and/or will to know what may harm or enhance ecological, social, and cultural well-being, and the well-being of others and self. While it may be challenging to grow a capacity to know what may harm or enhance the well-being of multilingual pre-service teachers or multilingual students in mathematics learning spaces, it is crucial.

Closing remarks

The triadic framework that Tony presented concurrently captures the significance of ensuring access to dominant mathematics for multilingual pre-service teachers *and* the significance of highlighting the presence of language and thereby also cultural and epistemological diversity, in exemplifying practices as part of language diverse learning spaces. In other words, the framework allows us to move away from dichotomising stands that revolves around either-or issues such as knowing and learning dominant vs non-dominant languages and/or mathematics. The capacity to recognise the socio-political significance of multilingual pre-service teachers and students getting access to dominant mathematics and dominant languages may be a matter of enhancing well-being. It could provide multilingual pre-service teachers with the language and mathematical resources they need in order to be heard and listened to by others such as parents, students, and colleagues. However, without pausing for ethical reflection and critique (Hand et al., 2021), dominant languages and mathematics will simply be (re)produced, which could potentially cause harm (Le Roux & Rughubar-Reddy, 2021).

For example, ethical reflection could comprise illuminating the significance of the presence of non-dominant languages and mathematics in multilingual learning spaces. A both–and stand – that is, a stand that embraces both dominant languages and mathematics and non-dominant languages and mathematics, which I believe Tony's triadic framework does – is a good example that could help us inhabit mathematics learning spaces in ways that allow us to live well, or at least a little bit better with mathematics and with each other.

References

Barwell, R. (2016). Mathematics education, language and superdiversity. In A. Halai & P. Clarkson, (Eds.), *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education* (pp. 25–39). Sense.

Chronaki, A. (2009). An entry to dialogicality in the maths classroom: Encouraging hybrid learning identities. In M. César & K. Kumpulainen (Eds.), *Social interactions in multicultural settings* (pp. 117–143). Brill.

Chronaki, A., & Planas, N. (2018). Language diversity in mathematics education research: A move from language as representation to politics of representation. *ZDM Mathematics Education*, *50*(6), 1101–1111.

Civil, M. (2012). Mathematics teaching and learning of immigrant students: An overview of the research field across multiple settings. In O. Skovsmose & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 125–142). Sense.

Dubbs, C. (2020). Whose ethics? Toward clarifying ethics in mathematics education research. *Journal of Philosophy of Education*, *54*(3), 521–540.

Ermine, W. (2007). The ethical space of engagement. *Indigenous Law Journal*, *6*(1), 193–203.

Essien, A. A. (2021). Rethinking exemplification in mathematics teacher education multilingual classrooms. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 63–55). Tredition. https://doi.org/10.5281/zenodo.5457116

García, O., & Wei, L. (2014). *Translanguaging: Language, bilingualism, and education.* Palgrave Macmillan.

Hand, V., Herbel-Eisenmann, B., Byun, S., Koestler, C., & Bartell, T. (2021). Tensions and failures in an analysis of whiteness among a racially and socially diverse group of mathematics teacher educators. In A. Andresson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 123–143). Brill.

Knijnik, G. (2012). Differentially positioned language games: Ethnomathematics from a philosophical perspective. *Educational Studies in Mathematics, 80*(1–2), 87–100.

Källberg, P. S. (2018). Immigrant students' opportunities to learn mathematics [Doctoral dissertation, Stockholm University].

le Roux, K., & Rughubar-Reddy, S. (2021). The potential of an Africa-centred approach to theory-use in critical mathematics education. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 100–122). Brill.

Norén, E., Boistrup, L. B. (2013). What is a group? Theoretical considerations when researching affordances for multilingual students' mathematical learning. In M. Berger, K. Brodie, V. Frith & K. de Roux (Eds.). *Proceedings of the Seventh International Mathematics Education and Society Conference* (pp. 431–440). Cape Town: MES7.

Radford, L. (2012). Commentary on the chapter by Richard Barwell, "Heteroglossia in multilingual mathematics classrooms". In H. Forgasz & F. Rivera (Eds.), *Towards equity in mathematics education* (pp. 339–342). Springer.

Russell, G. L., & Chernoff, E. J. (2013). The marginalisation of Indigenous students within school mathematics and the math wars: Seeking resolutions within ethical spaces. *Mathematics Education Research Journal*, *25*(1), 109–127.

Skovsmose, O. (2020). Banality of mathematical expertise. *ZDM Mathematics Education*, *52*, 1187–1197.

Swanson, D. M. (2017). Mathematics education and the problem of political forgetting: In search of research methodologies for global crisis. *Journal of Urban Mathematics Education, 10*(1), 7–15.

Mathematics education, researchers and local communities: A critical encounter in times of pandemic, pareidolia and post-factualism

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Pandemic have increased the importance of social phenomenon involving mathematics and making crisis even more critical. This text studies the current global situation in order to enquire the role of mathematics education researchers in times of crises. A review of research partnerships with communities facing long-term critical situations is done, in order to reflect on the possibilities that mathematics education researchers have nowadays. Theoretical notions from theater anthropology and decolonial studies are borrowed to trigger insights on mathematics education as a field of practice and research.

If we keep quiet they kill us, and if we talk they do so anyways. So, we talk.
Cristina Bautista Taquinás, Indigenous Nasa leader, killed in 2019

In this plenary I want to invite the MES-community to revisit a recurring old theme: The relationship between communities and researchers in mathematics education research driven by social, cultural, and/or political interests. Researchers refer to diverse communities: local communities, cultural groups, teachers, or even students. In almost all previous conferences we can find a plenary talk in which this relationship is explicitly addressed or at least implied. The study of this relationship has almost become a genre within our community, in which issues such as the role of the researcher, coherence among methods, principles and ends, impact, validation, agency and reflexivity, just to mention a few, are deployed.

Knowing that the debates about these issues are far from being closed, I want to address some of them with two particularities that can bring new insights and concerns: 1) the contemporary situation assembled in 2020, and 2) features of certain experiences of long-term work with communities. These two particularities will allow me to unfold ways in which mathematics education and political endeavors can nurture each other. My strategy consists of sharing a series of images from research experiences that I am familiar with, in order to formulate questions that mobilize discussion.

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The haze dazes the gaze

Opening MES 8 in Portland, Ubiratan D'Ambrosio foresaw a new era for MES issues. He was almost prophetic: "*there may be no future. Our existence, as a species, is threatened.* Our objectives must be even more than social justice and dignity for the human species, must be the survival of our species that is threatened by a societal breakdown" (D'Ambrosio, 2015, p. 20, original emphasis).

Just six years after we are watching cruel examples of what Ubiratan called social and environmental degradation. MES 9 and MES 10 served to highlight the new threats of post-factualism and the naturalization of crisis. We were experiencing similar or analogous cases of proliferation of (mis)information at different times of the year and (economical) crisis emerged at national scale, as if a ghost visited countries one at a time. First Greece, then Portugal, and so on. Nowadays, with the COVID-19 pandemic those threats get amplified. Crisis gained a global scale and citizens worldwide face the same problems almost simultaneously. Now it is undeniable that we all are on the same boat (although not in the same seat) and more people are conscious of the interconnectedness of our decisions.

Inside this evolving critical situation, mathematics has also amplified its power and agency within the assemblage of things. Mathematics-heavy entities appear everywhere: Models for virus propagation, estimation of vaccines impact, geo-refencing of hot spots of contagious. The incidence of past, current and future public policies at national and global scale, discussions of what metrics to use to assess and compare mortality, lethality, testing, treatments, vaccines and vaccinations. Rates, probabilities, charts, epidemic curves, cost-effectiveness analysis, forecast, simulation models and many more mathematical objects have invaded the public sphere. Debates on objectivity, bias and political consequences of mathematical models exploded everywhere, connecting politicians, scientists, philosophers, groups of interests and layman citizens. All of those debates require having some proficiency in mathematics and, simultaneously, put in doubt the affordances and assumptions that any mathematical model of a pandemic conveys.

I could not imagine a more powerful example of Ole Skovsmose's ideas on the critical nature of mathematics in society than the COVID-19 situation. Risk, uncertainty, responsibility, mathemacy and several types of mathematical practices in a globalized world are vividly exposed. Knowledge and truth have gone beyond modernist conceptions (Skovsmose, 2009, 2011, 2021). All in one single explosive situation. Decision making processes need to be accomplished in real time at every level (national, regional, municipal, and even personal) with the public scrutiny and turning accountability as a major issue for decision makers. Citizens can be (in/ex)cluded, engaged or discouraged to intervene according to their capacity to understand the mathematical discourse unfolded.

Past and present mathematics education practices exhibit their power within the COVID-19 showcase. Charts with several indicators are used and misused to sustain and criticize public policies. Laypeople assess and question the validity of nation-State measurements of the situation. Personal experiences with the virus, the treatment and the vaccine are used to mistrust the statistical populational tendencies pointed by experts. Possibility gets equated with certainty (probablity equal to 1). Sample results get immediately extrapolated for the whole population. Stochastic thinking is needed not only for citizenship, but for survival, and its absence is evident. Public health offices try to create awareness campaigns and fail miserably. Some people cannot understand the information provided in real-time. Others do not want to understand. Improvements in the propagation models due to new information and subsequent changes in the official policies are labelled as symptoms of improvisation. Hard times to explain rules of scientific method and Popperian falsifiability. Official and dubious information about lethality, infectivity and treatments exploded social networks several times. Conspiracy theories flourish and spread faster than the virus itself, and therefore there emerges the clamor to increase the mathematical-scientific literacy needed to act as a barrier against this tsunami of (mis)information.

Uncertainty in the facts lead to uncertainty in the interpretation of the facts. This is connatural to any state of crisis, and it creates a certain entropy to make the crisis even more critical. However, it is important to inquire the mechanisms that allow the constitution of this situation. Why are fake news so easily propagated? How post-factualism can lead the scene? This is not only because the technological possibilities of sharing information worldwide in real time. It is also because a very old impulse got unleashed: *Pareidolia*, that is the tendency to perceive a specific, often meaningful image, in a random or ambiguous visual pattern. Pareidolia is the basis for psychological exams such as the Rorschach inkblot test, which tries to infer a person's mental state by studying the thoughts or feelings that the person projects into some images (Wikipedia, n.d.).

Pareidolia is one type of *apophenia*, defined as a "tendency to perceive a connection or meaningful pattern between unrelated or random things (such as objects or ideas)." (Merriam-Webster.com, n.d.) There are other kinds of apophenia such as the Gambler's fallacy and the confirmation bias. They are related to situations like echo chambers, or cognitive bias such as the Dunning-Kruger effect. Although these tendencies have been deployed historically to segregate people by political or religious reasons, the internet and the fostering of digital social networks in the last decades have been functional to their unlimited growth and use in generating other type of divisions. One of the results of this dynamic is the political radicalization and polarization that escalate differences, justify violence and damage the social fabric that sustains a community. These worrying *apophanies*[1] seem to be omnipresent since the pandemic exploded and echo chambers seem to be shaping communication more than ever: virus deniers combined with anti-vaccine movements are radicalized by politicians attempting to promote biological passports.

I want to mention just two factors (among many) sustaining the proliferation of echo chambers and mistrust: the historic role of governments and the ways in which information widespreads. The first one is about the eroded confidence of many citizens in private and public institutions. At least in poor countries like mine, the government is historically

[1] An apophany refers to an instance of apophenia that produces the exact opposite of an epiphany.

characterized by the systemic and permanent denial of the most basic human rights of the population; and by the rampant corruption in the state institutions that deviates money from taxes to politicians and the private sector. How can a state like Colombia, that has kidnapped and killed more than 6400 civilians to present them as guerrilla soldiers, ask for credibility? How can the Peruvian government request confidence in medical procedures, after the forced sterilizations of more than 2000 indigenous women? When the official version of facts and events is in the hands of mythomaniacs, citizens are eager to look at other sources.

A second factor is the way in which information is being accessed and filtered. For instance, how search-engines operate in the Internet. It is well known that content creators obtain their profits by the amount of clicks, likes and visits. Algorithms create lists based on the features of the content, registered in meta-data and tags. In order to appear in the top ranks of a list, creators need to make-up their content to look more urgent, paradoxical and scandalous; so they are urged to produce and label content that appeals to the morbid curiosity of the audience, that is, to create clickbait. As a reaction, some educational youtubers have buckled under the pressure and decided to quit their channels because of the induced trivialization of their content. This means that the quality and diversity of contents are constrained and biased towards particular types of information.

Just to summarize this overview, I characterize the contemporary conjuncture as the mingle of new challenges emerging (the current pandemic), not so recent situations (post-factualism), and very old situations (pareidolia); all of these repowered. This explosive cocktail is clearly critical, as it is uncertain, risky and demands responsibility (Skovsmose, 2011). The COVID-19 cocktail operates as anabolic steroids for ongoing social, economic and ecological processes of oppression: Economic exploitation and massive unemployment, surveillance projects of fascist nature, social inequality, ecological deprivation, defunding of public schooling, and many others. All these threats to basic dignity got boosted in the last two years.

Turning on the fog lights

Nonetheless, the pandemic, postfactual and pareidolic (3P) situation also creates possibilities. For instance, Colombian public universities used the pandemics to negotiate with the national government until reaching agreements on full tuition waving for students due to the pandemic, and to enhance part of the digital infrastructure. Of course they did not obtain all resources that public universities deserve, and the conditions are still far from reaching a minimal decency. But certainly students forced the government to do things that were systematically denied for three decades of neoliberal policies of dismantling public education.

Indigenous Nasa people started a self-imposed lockdown in their territory, and gathered healers from different shelters to carry research on the COVID-19 virus. The team of healers experimented with several combination of plants and, after a couple of months, released to the whole community two different recipes: one for prevention and other for treating the symptoms. More recently, when the national government created official vaccination campaigns, Indigenous communities were fearing that the government would provide them with low quality vaccines or even introduce other diseases among indigenous communities. As a result, indigenous organizations decided not to participate, arguing against the lack of clarity on the procedures, expressing concerns on vaccine effectivity, and defending their indigenous protocols of biosecurity.

During the extended lockdown, the Colombian government also tried to pass a regressive tax law, that would increase the taxation burden on the poorest. That provoked a national strike. In April 28th 2021, there started a social revolt unseen in 50 years. Anger and indignation expressed in unexpected ways. Statues of colonizers were pulled down, public places were renamed, young people made blockages and barricades in strategic points of several main cities. They get organized through social networks and broadcasted live the confrontation with police squads, revealing police brutality.[2] That was useful to contest the fake news that proliferated through stablished mainstream media. Television news showed only the riots and looting, prompting the narrative that cities needed to declare martial law to control the protesters.

Internet activism served to made visible other dimensions of the protest. People organized in the neighborhoods and created several lines of resistance. While some conformed the front line of the confrontation, a side line of mothers went to the streets to take care of their kids in the frontline; lines of musicians went to play and serve as incidental soundtrack of the riot! Collectives of lawyers worked, free of charge!, to represent people detained and guarantee their civil rights. Even an anonymous guy who dressed with the Colombian flag was named “Captain Colombia”, in a sarcastic real emulation of the fictional U.S.A. hero. People organized a “communitarian pot” to provide food to front-line protesters. This “pot” consists in closing a street and turning it into a kitchen for whoever needs to be fed. Some of these initiatives became regular and generalized, while some other disappeared or became replaced by others side lines.

Outside the streets, “pedagogical” lines of resistance were also created. In Cali, collectives of lecturers at universities joined high school teachers, engineers and other professionals to create “Pre-Icfes pa’l barrio” [Pre high-stake-test training for the “hood”]. This initiative created a set of free online resources and personal tutoring in mathematics, chemistry, physics and biology, aiming to prepare last year high school students to obtain better grades in the national high-stake test. Variations of this type of initiative has emerged in other cities. In all, the initiative gathers 1.700 volunteers, dispersed around the country and even in other countries in the world, and reaches more than 14.000 youngsters, interested in gaining access to university. Probably the initiative will not succeed in improving the test scores. But

[2] During the period from April 28 until June 26, 2021 Colombian NGO’s have reported violence in the hands of public force: 1,617 victims of physical violence, 44 homicides presumably perpetrated by the Public Force, 2005 arbitrary arrests against protesters, 748 violent interventions of peaceful protests, 82 victims of eye aggressions, 228 cases of firing of weapons, 28 victims of sexual violence and 9 victims of gender-based violence (Temblores ONG et al., 2021).

certainly it mobilizes an important amount of people that usually did not play a part in political actions.

Beyond assessing the efficiency, continuity or coherence of these samples of applied pataphysics, it is important to stress their success in activating spaces of encounter among diverse people who were not were engaged before or were disarticulated. Their success is to be found in the care for the social fabric that pareidolia targets to damage.

Love at third sight

When trying to relate mathematics education (ME) with the recent communitarian contestations mentioned before, some experiences came to my mind. Mônica Mesquita developed, together with groups of fishermen of the Costa de Caparica in Portugal, several projects of communitarian education, using critical ethnography and participatory action research. They first devoted their efforts to obtain unpolluted water to a semi-illegal settlement. More recently they have worked to improve sustainable and safe fishing practices. These experiences have created spaces of formal and informal education that empower communities around issues of collaborative governance, participatory ICT, and fishing legislation. A 20-year process of critical encounters with fisherman, academics, and government representatives has impacted positively on the social conflicts of the Caparica region, working for spatial justice through intellectual justice (Franco & Mesquita, 2019; Mesquita, 2014, 2016).

Since 1998, Adailton Alves da Silva, a Brazilian mathematics educator, has been working with A'uwê/Xavante communities. He began as an advisor in educational projects for this Indigenous group. He still collaborates with the production of didactic materials based on rituals and ceremonies, professional development workshops, and the design and advice on the secondary education curriculum for the Xavante people (da Silva, 2006; da Silva et al., 2021).

Since 2004, Gentil Wejxia, an indigenous educator from the Nasa people in Colombia, has led several educational processes in the sacred region of Tierradentro. In 2008 Gentil became the team leader of an indigenous research group on Nasa mathematics, which studied past and current vernacular practices, relating them with the six universal proto-mathematical activities enunciated by Alan Bishop (Caicedo et al., 2009). Nowadays he is part of the educational experience of *Kiwe Uma,* one of the many enactments of "educación propia" or own indigenous education (Parra & Valero, 2021). This initiative is carried out by a small but growing group of indigenous families that want to develop a culture-rooted education, nurturing their seed (the children) in order to safeguard the indigenous territory, land and worldview. They have structured several development stages for the kids, according to Nasa cultural values and practices. The equivalent to the content in a curriculum is strictly related with the set of skills and knowledge that an indigenous person needs to manage as adult in indigenous and non-indigenous contexts. Gentil has developed further his insights on the cultural dimensions of mathematics. The mathematical component of *Kiwe Uma* includes both the spirituality behind weaving of some traditional handbags, hats and clothes, as well as the study of statistics and algebra, in order to understand the process of economic and ecologic deprivation that Nasa people have been suffering. This confirms that the *Kiwe Uma* collective decided to teach mathematics for life, but an indigenous life of cultural and political resistance. Kids were deliberately not registered to participate in the national educational system and thus, they would not sit the mandatory high-stake test.

I also acknowledge that MES conferences have been a rich space to meet inspiring experiences. I remember Munir Fasheh telling his story of healing from modern superstitions and the explorations on Mujaawarah in Palestine (Fasheh, 2015) ; the group on Crisis in MES shared Brazilian, Colombian, Indian and Palestinian strategies to deal with educational crises (Marcone et al., 2019; Parra et al., 2017). Gelsa Knijnik has also reported on the educational projects of the land-less movement (MST) in Brazil (Knijnik, 2004, 2006).

It is important to propose a thread that can bond all these experiences, and that sheds light on some of the ways that research and practice on social, cultural and political dimensions of mathematics education has been conducted. To do so, I want to introduce an insight coming from theatre theory, that can be used in many fields of human sciences, and that I find useful for mathematics education in particular. Eugenio Barba is the director of Odin Theatre, a group located in Holstebro, Denmark. He and his group are worldwide recognized for formulating the idea of theater anthropology. When reflecting about the several ways in which performative arts are developed around the world, Barba identifies a special kind of theatre, a “third theatre”, that is:

> Almost unknown, it is rarely subject to reflection, it is not presented at festivals and critics do not write about it.
>
> It seems to constitute the anonymous extreme of the theatres recognised by the world of culture: on the one hand, the institutionalised theatre, protected and subsidised because of the cultural values that it seems to transmit, appearing as a living image of a creative confrontation with the texts of the past and the present, or even as a “noble” version of the entertainment business; on the other hand, the avant-garde theatre, experimenting, researching, arduous or iconoclastic, a theatre of changes, in search of a new originality, defended in the name of the necessity to transcend tradition, and open to novelty in the artistic field and within society. (Barba, 1986, p. 193)

From now on, I invite you to play a game of analogies. A “first ME” would be devoted to the ideology of improvement, trying to find and disseminate the “best practices” for mathematics classrooms, in line with official curricula. Large scale projects get funded. A “second ME” would be concerned with studying the complexities of the field, trying to comprehend the dynamics of classroom practices and public policies. Small scale projects receive some funds. Getting back to Barba:

> The Third Theatre lives on the fringe, often outside or on the outskirts of the centres and capitals of culture. It is a theatre created by people who define themselves as actors, directors, theatre workers, although they have seldom undergone a traditional theatrical education and therefore are not recognised as professionals.

> But they are not amateurs. Their entire day is filled with theatrical experience, sometimes by what they call training, or by the preparation of performances for which they must fight to find spectators. According to traditional theatre standards, the phenomenon might seem insignificant. But from a different point of view, the Third Theatre provides food for thought. (Barba, 1986, p. 193)

Could we think of a "third ME" as one that lives on the margins, one that has to face armed conflict, poverty, famines, cultural extermination, illegal migration? How does a ME that does not locate its strength in institutionalized funding or prestige look like? Although the ordinal third could refer to the "third world", I think that this kind of mathematics education is being conducted everywhere by teachers, scholars, activists or even enthusiasts, who do not to wait for approval or acknowledgement coming from funding agencies or fine academic circles to research and intervene reality. A "third ME" would conceive itself not as the education *for* mathematics, but an education *through* mathematics, just as Ubiratan D'Ambrosio proposed many times.

The third space of ME is what bonds Mônica, Gentil, Adailton, Munir and Gelsa. The third space is inhabited by them and many other people and doings that often are not considered as legitimate ME. They have in common an inner force to experience ME "as a bridge, constantly threatened, between the affirmation of their personal needs, and the necessity of extending them into the surrounding reality" (Barba, 1986, p. 194). A third ME encompasses the diversity of possibilities of adopting a conception of mathematics education research living at the margins. Many of their experiences are developed outside the school, but within politically organized groups; and others happen within the school system.

Third thoughts

In this section I engage in some further considerations (as remarks or extended aphorisms) that will elucidate the affordances of noticing this third region of ME. Some of them paraphrase what Eugenio Barba has proposed about the third theater in (Barba, 1992, 2002). I begin observing that the nomination of "third ME" could be new, but the concerns and purposes subsumed in the nomination are certainly not new. So, it is not aiming to propose a new pedagogic style, neither a new turn. Rather, it performs a sociology of the absences that "amplifies the present by adding to the existing reality what was subtracted from it" (de Sousa Santos, 2012, p. 56): A decolonial move to make visible knowledge, people and stories that were assumed as non-existent.

A third space of ME is not an educational style, nor an alliance of research groups, still less a movement or international association; nor is it a school, a paradigm, or set of techniques or methods. Instead, it points to a way of giving meaning to mathematics education. So, whereas many people wonder if school (and ME) make any sense nowadays (as if it were a thing capable to have or lack anything), a third space assumes that such problem belongs to the people who inhabits school (and ME). It prefers to deal with the question: "Are we able to give meaning to what we do?" This is far from a solipsistic, isolated approach to the problem. It is an awareness that answers need to be singular and embodied in actions.

A search for meaning signifies above all a singular discovery of a craft. It invites to a "patient building up of our own physical, mental, intellectual, and emotional relationship with [teachers, students and knowledge], without conforming to those balanced and proved relationships current at the centre of [mathematics education]" (Barba, 1992, p. 9, paraphrases in brackets). A craft does not fall entirely in techniques or routines, and what is crucial in a craft is the *ethos* that sustain it. An ethos is the ensemble of social, linguistic, political, existential, communitarian and ethical behavior, that expresses the ways in which we want to relate our local history with History. Our field is used to be thought in two dimensions, as if what mattered was only policies, ideological tendencies, institutions, learning outcomes, or different methodologies (Barba, 2002). Such framing falls short when inquiring for ethos and its concomitant craft. When mathematics education is addressed as solitude, craft, and revolt. Just like Paulus Gerdes did his work until his last day of life. I wonder if this paragraph meets with what Sonia Clareto and Roger Miarka have delineated as "minor mathematics education" (Clareto & Miarka, 2015).

The quest(ion) for a craft opens up an entire new terrain. Who does research? As far as everyone is in the race to create meaning for their own practice, the conventional colonial frontier among the one who thinks and the ones that are thought, gets blurred. It happens a displacement in the *locus of enunciation*[3]. Such displacement puts in doubt the very concept of research, and also its impact, pertinence and validation. For instance, in the indigenous regions of Cauca, Colombia, educational projects must be formulated and justified in regards of local community agendas of political and cultural resistance. They must involve members as active researchers (and not merely informants). This is to say, the academic outcomes are not the leading force, even as the research goals grow from the inside of community and reach the outside of legislative funding agencies. Mônica Mesquita has had similar experiences in Costa de Caparica. She has shared with me the need to have in mind the "hidden agenda", setting collectively, first, what would be the gain for the fishing community and, then, making adjustments to present projects to academic instances. The term "hidden" here point to the hybridity in the task. While the agenda is explicit, open, and co-constructed with the community, it is hidden because the bureaucratic eye of institutions is incapable to see it and grasp the vibrant desires for dignity of those involved. This creative role stresses the capacities of communities to generate knowledge for survival, enlarging the horizon of possibilities beyond what heterodox and disciplinary practices can do. These possibilities are part of a sociology of emergences (de Sousa Santos, 2012).

Once the geopolitics of knowledge is noticed, attempts to "include" by "giving voice", or by "acting on behalf of" should be reconsidered in research. The core idea of research as representation of the *other* is alien to a third ME, because una cosa es el indio, y otra cosa es la antropología. Instead, research becomes an experience of encounter among different entities (individual, groups and even non-human agents), that can be performative and transformative only as far as the experience nurtures the craft of parties involved.

[3] Mignolo (2000) introduced this concept first, but we take it here as "the geo-political and body-political location of the subject that speaks" (Grosfoguel, 2011, p. 4).

Just like the emulated third theater, a third ME circulates in a diversity of scenarios, audiences, procedures and outcomes. For instance, grandparents and healers have a central role in the *Kiwe Uma* activities. The simplest notion of school as fixed place operating at certain regular hours is completely detonated by the Nasa proposals of "educación propia". Participants travel across several *resguardos* (a type of indigenous reservation) and gather according to the moon phases and the healer' spiritual assessment that determines if the group of kids is spiritually clean and strong to embrace the work session. Mathematics educators visiting the third space embrace their accountability to instances far outside of the OECD-PISA domain.

When the first and second ME describe themselves as exploring a field, they really use the term "field" in a metaphorical way, because they are seldom located and immersed in a particular concrete space. Instead, incarnations of third ME explore a real, concrete field, a land in dispute, such as Palestine, the invaded *fazendas* near Porto Alegre, in Brazil, the illegal settlements of Costa de Caparica in Portugal, or the indigenous *resguardos* in Cauca, Colombia. This is not a pure coincidence. Fighting for the land means fighting for the material conditions of existence, a fight for survival with dignity. The complex articulation between materiality, knowledge and culture is expressed by the Colombian leader Francia Márquez:

> Territory for the black people is the real possibility of giving birth to freedom, autonomy, self-determination, it is our space for being. That is why we often harangue: territory is life and life is not for sale, it is loved and defended, likewise community wisdom shows us that *territory is life and life is not possible without territory.* (Marquez, 2020, italics added)

Solidarity with social processes and land, lead us to this enigmatic sentence: with mathematics we can inquire on/in time and space, with a mathematics education we can inquire on the rights to have a time and space.

At this point of the text, I aspire that this paragraph would be a truism: a third ME addresses the political nature of mathematics education in a particular way. Whereas conscious of the structural constraints that shapes school, education, mathematics and research, it decides to focus in the here-and-now of its particular political agency. Paraphrasing the anthropologist Joan Rappaport: "while some scholars engage in [political] description with an eye to analyzing it, [third ME researchers] study [politics] to act upon it" (Rappaport, 2008, pp. 20–21, paraphrases in brackets)

A last remark here could be the first to be said. The three types of regions in mathematics education research are not exhaustive nor exclusionary. There could be other unknown types of regions and many shared spaces among regions. It is important to stress that a third space is not conceived as superior or better than the others; it mainly indicates a different arrangement of priorities. Namely, whereas a first type is devoted to understand "how you do mathematics education", and the second will study "what mathematics education you do", a third will ask "why you do mathematics education". It is natural that each one of the three questions conveys part of the others, but depending on which one is prioritized, the other two questions are answered differently.

Close encounters of the third kind

This section explores the question of how we can address research and practices of mathematics education (ME) in the mingle of the 3Ps (pandemic, post-factualism and pareidolia). So far, I have focused on a particular stance about the field, called the third ME, in which collaborative practices of research with communities are central, and a permanent observance on ME research agency leads the practice.

I contend that the current context of threats to human survival and transcendence demands a reframing of the terms and purposes in which mathematics education researchers conceive their practice, especially when they encounter communities. The concept of *craft* can guide such reframing.

An appeal to consider the theoretical and methodological possibilities of including practitioners' agency into academic research projects has been raised by Renuka Vithal in MES 2 and by Orlando Fals-Borda in MES 3 as part of a paradigm shift in the social sciences. They discussed the dichotomy subject/object of study and explored the political relevance of participatory methods, ending with an invitation to seek possibilities to make more horizontal the research relation[4] (Fals-Borda, 2002; Vithal, 2000).

Almost 20 years later, the framework of decolonial studies (Castro-Gómez & Grosfoguel, 2007; de Sousa Santos, 2010; Smith, 2013; Zavala, 2013) lead us to go further in the paradigm shift. What would happen if not only local practitioners are welcomed on board to design and execute academic research projects on mathematics education, but also scholars try to join communitarian social processes that involve mathematics? What if we turn the inclusive action to ourselves? What if we do not assume to have the power to include, but rather have the aim of being included? What would happen if academic scholars decided not to have the main role in the play? My invitation, then, is to explore again (and still differently) the possibilities to make the research relation more horizontal.

A first thing that could happen is that scholars will need to consider the existence of mathematics education research outside the school, conducted by people interested in learning mathematics in action. Scopus-free researchers, working with PISA-free students! Many of them live on the margins, in the third ME. Some others live in between.

Consciousness on the responsibilities of contesting the matrix of hierarchizations that sustain modern rationality will help ME researchers to establish respectful relationships with communities. Collaborative and participatory approaches could be compatible with his/her decisions, unfolding exercises of co-theorization, mutual interrogation or endogenous research methods that build trust among different stakeholders involved. Instead of reducing accountability to heterodox criteria (such as consistency, scope or validity) or to the problems of reflexivity, academic researchers could also be aware of the problems of *symmetry*. The latter can be thought as "the study of how the participation of both researchers and practitioners has been conceived, unfolded, enacted, registered, and assessed in research"

[4] Concerns on practices of intellectual extractivism were raised by Setati in MES2.

(Parra, 2018, p. 150). Symmetry is also crossed by the complex power relationships around representativeness, legitimacy and intellectual property that surround any research. Integral part of mathematics education researcher's craft is the management of this complexity.

As Vithal and Fals-Borda noted, collaborative processes are inevitably critical. They demand intense interactions among different expectations, skills and backgrounds. Join communitarian processes attempt to congregate heterogenous agents. This is precisely the opposite of the dissociative power of pareidolia and echo chambers, which segregate homogenous agents. 3P does not like these kinds of barters!

In a very paradoxical way, this type of detours and escapes from the normal flux of academic practices could bring us to a renovated mathematics: One like the envisioned by Orlando Fals-Borda and Ubiratan D'Ambrosio; one that studies and expands the ways in which local communities have survived and transcended for many years (some of them for more than 5 centuries!). The third ME came to learn ethnomathematics in action. It is possible to conceive a kind of mathematics that turns its efforts to safeguard life, and that bring answers to the urgent needs of people; a mathematics that can support the struggle of people organized around the defense of a territory, like the experiences in Porto Alegre, Costa de Caparica, Cauca, or any other illegal[5] settlement.

Decoloniality has stated it clear and loud: An *ecology of knowledge(s)* rejects dichotomic thinking; it understands the relevance of disciplinary knowledge to contribute to the well-being. And for that reason, it does not endorse anti-intellectual gestures, like the monster currently in charge of Brazil, who has taken advantage of the COVID-19 cocktail as a biological weapon for exterminating indigenous people in the Brazilian Amazonia. Indigenous people kick back and share the developments of their struggle to their Brazilian brothers. The scientists of Instituto Butantan in São Paulo are doing the same. Every help counts now to stop the current genocide. A third ME researcher necessarily contests post-factualism.

In the time of the 3Ps, humanity needs to learn how to establish sustainable relations with non-human existences[6]. Capitalists and colonialists never understood how to make it (they only want to see measures, profits and borders in the land). The ones who understood that the Earth is not a resource to exploit but the very condition of our existence have been living and resisting for centuries. These are the communities of indigenous peoples, farmers, fishermen, among others. Decoloniality articulates diverse types of knowledge, trough cultural encounters that multiply experiences and share wisdom. A third ME researcher knows the importance of entering and committing to communitarian territories to listen, share, care and learn how to resist.

[5] Sometimes its illegality means that it is managed collectively and has not been measured (devoured) by capitalism.

[6] Paola Valero (2019) asked already what would be the mathematics education compatible with the imminent "new climatic regime", because that regime challenge many assumptions that sustain the current mathematics education.

Third time's the charm (or maybe not)

An important contribution that an ethical stance can bring to the field of ME is the chance to elaborate on certain insights that are gaining currency within the MES community. Colleagues (for instance, Andrade-Molina, Baldino, Pais, Valero) have raised warnings on the chances that ME research has to truly impact the school system in contemporary neoliberal societies. They have pointed out several constraints, like the capitalist system that needs to produce surplus-value at any time, or the subjectification processes that occur for students, teachers and researchers. Broadly speaking, colleagues refer to the *mechanisms of capture* (a.k.a Foucaultian dispositif), and point out that there are several mechanisms of capture jeopardizing the possibilities of "changing" the system.

> So we are always facing the danger of being deceived, as we think that we are struggling against oppression, when in fact we are being allowed by the dominant class to do so, just to cool down the rebellion. As I said before, the core things, like assessment and school, are here to stay. (Pais, 2008)

In that line of thought, when addressing the role of mathematics (and school) within the new normal state of global crisis due to the 3Ps (pandemia, postfactualism and pareidolia), ME researchers will need to be very careful not to get trapped by the mechanisms of capture, and not to end up subsuming their efforts to make a critical mathematics education instrumental to the increase of human exploitation. I could not agree more; that is definitely a possibility. Furthermore, I am sure that such a thing is happening already. However, if we have the analytical acumen to perceive how transnational forces of political and economic domination enter the school or enact themselves in mathematics education research, then we should also have the insight to notice how local forces of resistance also seek school and university spaces. Because education is just one more set-up in this antagonistic relationship, and since we inhabit that set-up, it is up to us to address the situation.

Previous sections showed us people around the world using the pandemic to create spaces and lines of cultural and political resistance. They appeal to an ethos of resistance, to activate *mechanisms of escape* (a.k.a Foucaultian counter-conduct or Deleuzian lines of escape or flight). My point is that ME researchers can indeed find resources to block the mechanisms of capture by engaging with sectors of society that are activating the mechanisms of escape. Playing with the words of Barba, one could ask:

> Why do they choose [mathematics education] in particular as a means of change, when we are well aware that other factors determine the reality in which we live? Is it a question of blindness, of self-delusion?
>
> Perhaps for them, [mathematics education] is a means to find their own way of being present and seeking more human relationships with the purpose of creating a social cell in which intentions, aspirations and personal needs begin to be transformed into actions. (Barba, 1986, p. 194, paraphrases in brackets)

A disenchanted but powerful idea of utopia is suggested here. There is no possible salvation, neither a hedonistic approach is attempted. I do not expect to "solve" the societal problems of post-factualism or pareidolia when I proposed to work more closely with communities. I intend to do something different, closer to the concept of *hope* from Václav Havel: “It is not the conviction that something will turn out well, but the certainty that something makes sense, regardless of how it turns out” (Havel, 1990, p. 181).

Then, we should not wait for ME to make sense, but we should give it meaning. This in itself is a political act that is neither romantic nor useless. This is precisely the main task: to develop a craft, a care of the self:

> The hell of the living is not something that will be. If there is one, it is what is already here, the hell we live in every day, that we make by being together. There are two ways to escape suffering it. The first is easy for many: accept the hell, and become such a part of it that you can no longer see it. The second is risky and demands constant vigilance and apprehension: seek and learn to recognize who and what, in the midst of hell, are not hell, then make them endure, give them space (Calvino, 1978).

It is to develop a revolt, a rebellion, conceived as the stubbornness of doing something that is doomed to fail, of doing what is needed to do, or of doing what we are required to do in order to find/build meaning to our path.

Pataphysics is not romanticism, insofar as it boycotts the status of the thinkable. It is a corrosive force that denaturalizes normality.

With broken temples and arms beaten,
We sound the fanfare of those never surrendered
and of the always defeated!
(Leon de Greiff, Sarabanda, 1929)

O [assessment], where is your victory?
O [mandatory curriculum], where is your sting?
(1 Corinthians, 15: 55-57)

I look for life in death,
for health in sickness,
for freedom in prison,
a way out from the impasse,
and loyalty in the Judas.
But my destiny, from which I would
never expect anything good,
has decreed with the Gods
that, since I ask for the impossible,
they won’t even give me the possible.
(Miguel de Cervantes Saavedra)

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References

Barba, E. (1986). *Beyond the floating islands.* PAJ.

Barba, E. (1992). The third theatre: A legacy from us to ourselves. *New Theatre Quarterly*, *8*(29), 3–9.

Barba, E. (2002). The house with two doors. In I. Watson (Ed.), *Negotiating cultures: Eugenio Barba and the intercultural debate* (pp. 183–196). Manchester University Press.

Caicedo, N., Guegia, G., Parra, A., Guegia, A., Guegia, C., Calambas, L., Castro, H., Pacho, C., Diaz, E., Caicedo, N., & Parra, A. (2009). *Matemáticas en el mundo Nasa.* CIIIT.

Calvino, I. (1978). *Invisible cities*. Houghton Mifflin Harcourt.

Castro-Gómez, S., Grosfoguel, R. (2007). *El giro decolonial: Reflexiones para una diversidad epistémica más allá del capitalismo global.* Siglo del Hombre Editores.

Clareto, S. M., & Miarka, R. (2015). EDucAçÃo MAteMátiCA AefeTIvA: Nomes e movimentos em avessos. *Boletim de Educação Matemática*, *29*(53), 794–808. https://doi.org/10.1590/1980-4415v29n53a01

da Silva, A. A. (2006). A Organização Espacial AUWẽ: Xavante Um olhar qualitativo sobre o espaço. *Boletim de Educação Matemática*, *19*(26), 163–164.

da Silva, A. A., Severino-Filho, J., & Ferreira, L. L. (2021). *Etnomatemática: Os múltiplos olhares para os saberes locais.* Editora Tangará, UNEMAT.

D'Ambrosio, U. (2015). From mathematics education and society to mathematics education and a sustainable civilization: A threat, an appeal, and a proposal. In S. Mukhopadhyay & B. Greer (Eds.), *Proceedings of the Eighth International Mathematics Education and Society Conference* (pp. 19–30). MES8

de Sousa Santos, B. (2010). *Descolonizar el saber, reinventar el poder*. Ediciones Trilce.

de Sousa Santos, B. (2012). Public sphere and epistemologies of the South. *Africa Development*, *37*(1), 43–67.

Fals-Borda, O. (2002). Research tensions and paradigm shifts in action sciences. In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (pp. 46–55).

Fasheh, M. (2015). Over 68 years with mathematics: My story of healing from modern superstitions and reclaiming my sense of being and well-being. In S. Mukhopadhyay & B. Greer (Eds.), *Proceedings of the Eighth International Mathematics Education and Society Conference* (Vol. 1, pp. 33–60). MES8.

Franco, S., & Mesquita, M. (2019). Observatório de Literacia Oceânica: uma abordagem etnomatemática na ecologia local. *Revista de Educação Técnica e Tecnológica Em Ciências Agrícolas*, *10*(19), 129–138.

Grosfoguel, R. (2011). Decolonizing post-colonial studies and paradigms of political-economy: Transmodernity, decolonial thinking, and global coloniality. *Transmodernity: Journal of Peripheral Cultural Production of the Luso-Hispanic World*, *1*(1).

Havel, V. (1990). *Disturbing the peace: A conversation with Karel Hvížďala.* New York: Vintage.

Knijnik, G. (2004). Lessons from research with a social movement. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 125–141). Kluwer Academic Publishers.

Knijnik, G. (2006). *Educação matemática, culturas e cohecimento na luta pela terra.* Edunisc.

Marcone, R., Parra, A., Bose, A., Alshwaikh, J., Gonzalez, M., & d'Souza, R. (2019). "Crisis" - The new normal: Fake (Post-Factual) mathematics education. *Proceedings of the Tenth International Mathematics Education and Society Conference.*

Marquez, F. (2020). El territorio es la vida. In Alonso Sánchez Baute (Ed.), *Territorio (Futuro en tránsito)* (pp. 9–20). Rey Naranjo Editores.

Merriam-Webster.com. (n.d.). Apophenia. https://www.merriam-webster.com/dictionary/apophenia

Mesquita, M. (2016). Urban boundaries and love: The rebirth of political thought within the disturbing education. *International Journal of Žižek Studies*, *9*(2).

Mesquita, M. (2014). Fronteiras urbanas. Ensaios sobre a humanização do espaço. *Anonymage: Viseu.*

Mignolo, W. (2000). *Local histories/global designs: Coloniality, subaltern knowledges, and border thinking.* Princeton University Press.

Pais, A. (2008). Reinventing school? Reaction to Eric Gutstein's "Reinventing Freire: Mathematics Education for Social Transformation". In J. Matos, P. Valero, & K. Yasukawa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 25–30). Universidade de Lisboa.

Parra, A. (2018). *Curupira's walk: prowling ethnomathematics theory through decoloniality.* Aalborg Universitetsforlag.

Parra, A., Bose, A., Alshwaikh, J., González, M., Marcone, R., & D'Souza, R. (2017). "Crisis" and interface with mathematics education research and practice: An everyday issue. In A. Chronnaki (Ed.), *Mathematics education and life at times of crisis* (pp. 164–178). University of Thessaly Press.

Parra, A., & Valero, P. (2021). Propio as a decolonising tool for mathematics education. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 71–99). Brill.

Rappaport, J. (2008). Beyond participant observation: Collaborative ethnography as theoretical innovation. *Collaborative Anthropologies*, *1*(1), 1–31. https://doi.org/10.1353/cla.0.0014

Skovsmose, O. (2009). *In doubt: About language, mathematics, knowledge and life-worlds.* Sense.

Skovsmose, O. (2011). *An invitation to critical mathematics education.* Sense.

Skovsmose, O. (2021). A philosophy of critical mathematics education. *Philosophy of Mathematics Education Journal*, *37.*

Smith, L. T. (2013). *Decolonizing methodologies: Research and indigenous peoples.* Zed Books.

Temblores ONG, Indepaz, & PAIIS. (2021). *Executive summary: Temblores ONG, Indepaz and PAIIS report to IACHR on the systematic violation of the American Convention and the jurisprudential scope of the Interamerican Court of Human Rights (IACHR) with respect to the use of public force against the civil population in Colombia, during the protests that took place between April 28 and June 26 of 2021.* https://4ed5c6d6-a3c0-4a68-8191-92ab5d1ca365.filesusr.com/ugd/7bbd97_3ff4e9c4b0f14b3ea288049e2985d0e2.pdf

Valero, P. (2019). Mathematics education in the "New Climatic Regime." In L. Harbison & A. Twohill (Eds.), *Seventh Conference on Research in Mathematics Education in Ireland (MEI7)* (pp. 7–13). Dublin City University. https://doi.org/10.5281/zenodo.3581031

Vithal, R. (2000). Re-searching mathematics education from a critical perspective. In J. F. Matos & M. Santos (Eds.), *2nd International Conference Mathematics Education and Society.* (pp. 87–116). Centro de Investigação em Educação da Faculdade de Ciências, Universidade de Lisboa. https://www.mescommunity.info/#conferences

Wikipedia. (n.d.). *Pareidolia.* Retrieved August 17, 2021, from https://en.wikipedia.org/wiki/Pareidolia

Zavala, M. (2013). What do we mean by decolonizing research strategies? Lessons from decolonizing, Indigenous research projects in New Zealand and Latin America. *Decolonization: Indigeneity, Education & Society*, *2*(1), 55–71.

Urgency and the shameful escape of privilege: We move differently when we refuse to set aside the weight: A response to Aldo Parra

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In his plenary paper, Parra adopts a tone that is simultaneously kindly and urgent, at once friendly and keenly critical, as he surfaces deeply visceral aspects of the world as it exists today and the untapped power Mathematics Education has to respond to seemingly omnipresent violences. His observations and urgency culminate in a call to action: That we should not let the world define Mathematics Education, but instead embrace our communal power to define it ourselves in ways that defy the white supremacist, cis-hetero patriarchal, abled, parochial, neoliberal, late Capitalist systems that currently confine us. Here, I aim to expand on and amplify what I perceive as the essence of Parra's message, with particular focus on unsettling anticipated neoliberal critiques/evasions of Parra's call to action.

> *The white conservatives aren't friends of the Negro either, but they at least don't try to hide it. They are like wolves; they show their teeth in a snarl that keeps the Negro always aware of where he stands with them. But the white liberals are foxes, who also show their teeth to the Negro but pretend that they are smiling. The white liberals are more dangerous than the conservatives; they lure the Negro, and as the Negro runs from the growling wolf, he flees into the open jaws of the "smiling" fox. One is the wolf, the other is a fox. No matter what, they'll both eat you. (Malcolm X)*

In his plenary address and manuscript, Parra (2021, in this volume) makes a series of thoughtful and keenly incisive observations regarding the state of the world as it exists today as well as regarding the often untapped power of communal/community organizing, culminating in a simple but extremely powerful call to action: "Then, we should not wait for [mathematics education] to make sense, but we should give it meaning [...] to develop a revolt, a rebellion, conceived as the stubbornness of doing something that is doomed to fail, of doing what is needed to do, or of doing what we are required to do in order to find/build meaning to our path." In this brief response paper, I aim to make use of my positionality as a member of the "global north," as well as my experience as an anarchist political activist in said global context (Bowers, 2021; Bowers & Lawler, 2021a, 2021b), in order to anticipate and subvert certain critiques and rhetorical evasions that I anticipate Parra's argument will prompt

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among many members of that broadly privileged community (and particularly among members with multitudinous intersectional privileged identities). None of this should be taken as a critique of persons, but instead as a surfacing of the tacit ethical weight that privilege allows us to choose either to consciously bear or set aside in our choices of response. This is not simple iconoclasm, but instead an invitation to join us in a "hyper- and pessimistic activism," (Foucault, 1983) an invitation to embrace "not the conviction that something will turn out well, but the certainty that something makes sense, regardless of how it turns out" (Havel, 1990, p. 181, as cited in Parra, 2021, in this volume).

To begin, I will briefly recapitulate the broad shape and sense of Parra's plenary address (or, rather, the key thread that I am addressing in this paper), in order to help ensure we (writer/speaker and readers/listeners) begin from a place of some shared understanding, even if my sense of Parra's words does not match your own. I will then outline some of the critiques and rhetorical evasions that I anticipate will comprise and confine the reactions of some, and especially many of those who have lived and been enculturated wholly or primarily into western neoliberal lines of thought (these lines of thought are also inextricably rooted in white supremacy, cis-hetero patriarchy, parochialism, abled supremacy, and so forth–these intransigent systems are part and parcel to western neoliberalism as surfaced in work such as Kendi, 2016). I will then respond to these anticipated reactions, not with an eye towards debunking or refuting per se, but instead with an eye towards consciously surfacing the ethical weight that they bypass or ignore. I conclude by restating and amplifying Parra's call to action: Let us leverage our communal power to give mathematics education, a rebellion of doing what is needed and "good" regardless or in spite of whether such efforts are doomed to failure.

Recapitulation of Parra: Urgency and ethics

Parra's text begins at the end, or rather an end: Apocalypse. Pandemic, postfactualism, and pareidolia, acting in concert, represent an existential threat, not just in the sense that the existence of many people/populations is literally threatened through violence (police brutality; Capitalists holding hostage water, housing, medical treatment, and other means of survival; etc.), but also in the insidious sense that the already distant possibility of a life (relatively) free of oppression is being crushed under the deleterious explosion of mounting exploitation, unemployment, inequality, deprivation, and so forth. We could add much more to this already horrifying list of existential threats, such as the accelerating climate catastrophe (IPCC, 2021) or the threat and risks associated with artificial intelligence, but even without expanding the list of threats it seems clear that there is an *ethical weight* tied to how we choose to respond (or not respond) to these threats, as well as an *urgency* to when we respond. It is also worth noting at this juncture that for many people, *apocalypse* is not a hypothetical future, but a reality of their present and past. "Dystopia" and "post-apocalypse" are common language used in privileged spaces to describe privileged people coming to experience what so many others across, for example, Indigenous and Black diasporic communities have already experienced (note that we could add to this list innumerable queer, disabled/neurodivergent, and non-Christian communities). In other words, the ethical weight and urgency Parra describes already existed prior to the present moment, but the present moment has invited a growing group of people to consciously notice this weight and urgency.

From this apocalyptic beginning, Parra moves towards a more hopeful aspect of this line of inquiry: Extant resistance of omnipresent violence. Through a variety of examples, Parra discusses the potentially revolutionary power of redirecting our values and actions away from larger systems of oppression and towards something more local and communal. He describes examples of a vision of mathematics education that already exists in the margins, one rooted in resistance and emancipation. When the values of communities drive our work, when communities are able to determine the metrics of success, there is the potential to fundamentally change the nature of what we can accomplish. When communities are understood to include both human and non-human existences, a perspective which necessarily moves beyond viewing all symbolic and material assemblages as mere resources to be exploited, this revolutionary potential grows. Parra is careful to temper these claims with reference to work describing the innumerable ways oppressive systems capture and claim mechanisms intended as emancipatory (e.g., Cabral & Baldino, 2019; Pais, 2012). – Consider, for example, the way "strengths-based pedagogy" was/is intended to surface value in the margins of society, but is commonly enacted as "what can the center take from the margins to strengthen itself." This *capture* happens when the values ultimately being served are still the values of the oppressive systems (well symbolized by the absurd trend of rainbow-colored hostile architecture; the queer are disproportionately rendered homeless, so painting an anti-homeless bench in Pride colors represents a grimly fascinating material representation of this *capture*), and the value in the margin is simply a resource. Parra ends with a challenge: One piece of power that we inarguably have is the power of self- and communal-definition. What mathematics education *is*, broadly speaking, does not make ethical sense. If we want it to make sense, we have to leverage our power to make it make sense. We may still be doomed to fail against the overwhelming Authoritarian force we face, but even without the conviction that our efforts will turn out well, we can at least have "the certainty that something makes sense, regardless of how it turns out" (Havel, 1990, p. 181, as cited in Parra, 2021, in this volume). We can know that we are *doing good* even if we are doomed to fail.

Rhetorical evasion: Anticipating and responding

Based in my experiences as an Anarchist activist in the cultural context of the United States, there are a variety of common forms of rhetorical evasion or pushback I anticipate Parra's messaging prompting from the well-meaning privileged. I obviously can't name or respond to these exhaustively, but given the *ethical weight* and *urgency* of Parra's messaging, there are several that may warrant explicit naming and critical reflection. In particular, I will acknowledge and offer a reflective counter to these three responses, each of which have been extremely common roadblocks in efforts to organize with the intersectionally privileged: (1) I wish I could do more, but I can't; (2) Reform is a better path than revolution; and (3) I can *both* satisfy what the system, as it exists, demands of me, *and* oppose the self-same system. In each case, my goal is not to offer an exhaustive response, but instead to invite critical self-reflection/discussion and advance more emancipatory futurities in the context of the *ethical weight* and *urgency* surfaced by Parra.

"I wish I could do more, but I can't…"

When I hear people voice this concern, what I actually hear them expressing is the tension they feel between their desire to *do good* and their anxiety or fear of the vulnerable position that *doing good* would ultimately demand of them. If you are white (as I am), then opposing white Supremacy ultimately demands that you let go of the safety net that privilege affords; if you are a cis-heterosexual man (as I am not), then opposing cis-hetero patriarchy demands that oppose the very ideologies and mechanisms that keep you safe; if you are neurotypical (as I am not), then opposing neurotypical supremacy demands that you let go of cultural norms that privilege your ways of being and thinking; etc. I do not blame you for feeling this tension. However, it is worth noting that this vulnerability that you can opt to ignore along your privileged identities is something that the marginalized can not opt out of. Thus, I ask: If you were to sit with this vulnerability, day in and day out, for days and months and years at a time, how would that affect your choices and the way you move through life?

"Reform is a better path than revolution…"

Is it possible to reform the system/institution to the point of emancipation without fundamentally changing it? If fundamental change is necessary, then what we are doing is revolution, and what you are distinguishing is a slow revolution from a faster one. When is it appropriate to allow a slow revolution? Certainly not in a moment of *urgency*. In contexts where your positionality is privileged, would you feel differently about how quickly we need change if you instead imagined yourself as one of the marginalized?

Consider police "reform." If you are relatively safe around police (my whiteness offers some safety, but my queerness and neurodivergence still put me in a great deal of danger when engaging with police), maybe a slow revolution sounds fine. However, if you or your children are at risk of extrajudicial execution, perhaps that is less likely to seem like a reasonable option.

"I can both satisfy the system and oppose it…"

I conclude at the same place conversations such as these so often seem to culminate: "Both/And." To be clear, it is not unreasonable to believe (or to advance the belief) that we can act in some ways at some times that empower the system, and in other ways at other times that oppose it – I certainly agree. However, "both/and" responses encounter an ethical problem when the two categories under consideration are not constructed as equal, as when one focus is more/less oppressive than the other. In such cases, this response advances equity only symbolically and not materially. In parallel to Dubbs' (2020) observations regarding Sfard's (1998) metaphors for learning, who diverges from the claim that "it is essential that we try to live with both" (p. 8), this is a situation where it is uniquely trivial to observe that one focus can be said to be better than the other, with better being that which is more equitable. Uncritically suggesting that we can do "both/and" without a critical lens reifies oppression and marginalization, and it will continue to do so until such time as "We… normalize (and expect) the full taking up the philosophical and theoretical underpinnings of all of our work (even work that is not considered 'philosophical')" (Bakker, Cai, & Zenger, 2021, p. 12). In other words, if we do not consciously wrestle with the often tacit ways that our beliefs and consequent methods advance the interests of a violent system, we stand little chance of meaningfully opposing that violence. As it stands, "Both/And" is a product of late abled cis-hetero patriarchal white supremacist capitalism and neoliberalism, a signifier for the confrontation of postmodernity contra modernity, and as such it enables a disarmament of ideas that would otherwise be directed at critiquing late capitalism. In contrast to Hegelian synthesis, "Both/And" bypasses the negative moment of determination. Thus, "Both/And" isn't really "Both/And…" it's just an exercise in neoliberal thought (Bowers, 2021, p. 80).

Conclusion

I opened this paper with the words of Malcolm X, an evocative and deeply visceral attack on Centrism. That description of those words may sound strange to some who perceive Liberalism (e.g. the Democratic political party in the United States) as the "Left," but Liberalism is not and has never been the Left – instead, it is only the leftmost arm of the establishment, the leftmost position tolerated by the entangled authoritarian systems we occupy. In a word, it is the *captured* left. To be clear, if you currently identify as a Liberal or a Democrat, I do not think you are cruel or unreasonable; instead, I think only that you are *human*. In this culture of internalized white supremacy, cis-hetero patriarchy, abled supremacy, and neoliberal Capitalism, I would be deeply suprised by any relatively privileged member of society who did not identify in this way at some point. However, just because it is reasonable and human to find yourself here at some point in your journey of rhizomatic growth does not mean that it is an adequate place to stop. The fear, frustration, and rage we hear in Malcolm's words are real, and are not unique to him, nor unique to matters of race. The more marginalized the community I find myself learning with and from, the more likely I am to hear exactly Malcolm's sentiments. Now, in these final moments, I leave you with one final synthesis statement, one final call to action: Be not satisfied with centrism or the status quo, and force yourself to sit with the tensions that you have the privileged capacity to escape. We move differently when we force ourselves to carry the weight of the violences that privilege allows us to ignore (Bowers, Forthcoming).

> If you have come here to help me, you are wasting your time. But if you have come because your liberation is bound up with mine, then let us work together. (Lilla Watson in community with other Aboriginal Rights activists)

References

Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. *Educational Studies in Mathematics, 107*, 1–24.

Bowers, D. M. (2021). Anarchist mathematics education: Ethic, motivation, and praxis [Doctoral dissertation, Michigan State University]. Proquest.

Bowers, D. M. (forthcoming). Examples, exceptions, and monster barring: Unsettling centrism in mathematics education. *Journal for Theoretical & Marginal Mathematics Education.*

Bowers, D. M., & Lawler, B. R. (2021a). Anarchism as a methodological foundation in mathematics education: A portrait of resistance. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 86–330). Tredition. https://doi.org/10.5281/zenodo.5393560

Bowers, D. M. & Lawler, B. R. (2021b). Reconciling tensions in equity discourse through an anti-hierarchical (anarchist) theory of action. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), *Mathematics education across cultures: Proceedings of the Forty-Second Meeting of the North American Chapter of the International Group for Psychology in Mathematics Education* (pp. 518–524). PME-NA.

Cabral, T. C. B., & Baldino, R. R. (2019). The social turn and its big enemy: A leap forward. In J. Subramanian (Ed.), *Proceeding of the Tenth International Mathematics Education and Society Conference* (pp. 32–46). MES10.

Dubbs, C. H. (2020). Learner, student, mathematician: What we call the subjects of mathematics education. *Philosophy of Mathematics Education, 36*.

Foucault, M. (1983). On the genealogy of ethics: An overview of work in progress. In H. Dreyfus & P. Rabinow (Eds.), *Michel Foucault: Beyond structuralism and hermeneutics* (pp. 229–264). University of Chicago Press.

Havel, V. (1990). *Disturbing the peace: A conversation with Karel Hvížďala*. Vintage.

Intergovernmental Panel on Climate Change. (2021). *Climate change 2021: The physical science basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press.

Kendi, I. X. (2016). *Stamped from the beginning: The definitive history of racist ideas in America.* Nation Books.

Parra, A. (2021). Mathematics education, researchers and local communities: A critical encounter in times of pandemic, pareidolia and post-factualism. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 86–80). Tredition. https://doi.org/10.5281/zenodo.5457124

Pais, A. (2012). A critical approach to equity. In O. Skovmose & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 49–92). Sense.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4–13.

Activism in mathematics education research: Stopping epistemicide by confronting and resisting modern forms of epistemic violence: A response to Aldo Parra

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In this paper, I will share my reaction and evoked thoughts inspired by Aldo's paper. I will argue about modern forms of epistemicide and how particular groups of people have historically lost the condition of humans to the eye of privileged groups. From a Postcolonial perspective, epistemicide problematizes how knowledge is, and has been used, to exert power according to national and global ideas for progress. Such ideas invalidate knowledge produced outside the boundaries of well-founded western forms of scientific practice. Violence against the Other in the path of organizing behaviour implies not only epistemological injustice but a systematic destruction—involving colonization, oppression and genocide. Here, the role of math education researchers should take a turn to give voice to those that have been silenced.

Have you watched the anime *The Promised Neverland*? This anime unravels the story of an orphanage in the year 2045. The orphanage was settled as part of "The promise" agreement made in 1045. Humans and demons set this agreement to keep both worlds—the human world and the demon world—separated. Demons eat humans, so the agreement was to create human breeding farms disguised as orphanages to provide food for the demons. In these orphanages, children believe they are orphans and have to stay in these houses until they get adopted. They are taught that they will be qualified for adoption if they acquire a certain level of intelligence and age. They learn all kinds of things, and, of course, they learn mathematics. When they get "adopted", they pack their belongings, have a farewell party, and leave the household thinking they will finally be free. Although you can probably see where this is going, they are murdered and sold as meat. They learn how to act and behave, how to eat and what to know, only to be profitable for the meat industry. And, well, this analogy could be as literal as your frame of reference makes you believe. But there is something I've been struggling with for quite some time now—and Aldo Parra's (2021) paper invites me to problematize—: how do we end up believing that mathematics is and should be universal at the cost of annihilating other forms of knowledge? How do we end up believing that epistemicide was the safest bet for economic progress? How do we end up using mathematically justified models to commit genocide—i.e., war math models to calculate how many people can be killed if a bomb is detonated in a certain location? How do we end up using mathematic

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illiteracy as an advantage to commit abuse—i.e., Aldo's example of tax increase in Colombia? And so, this is my reaction and evoked thoughts inspired by Aldo's paper.

One of the firsts things that come to my mind when trying to understand how we move in life through desire is the capitalist ideology that leads us to think we have to do what we do to be successful in the global world. One has to have a good job to have a good life. To have a good job, one has to have a promising career. To have a promising career, one has to have good grades and high levels of proficiency in particular school subjects, such as mathematics and science. Did we ever think as kids that the path we take was to become a productive citizen? Probably you didn't, me neither. But here we are, privileged by knowledge, socially validated as productive. Žižek talks about this with this example, people are more aware of the environmental impact of their choices. Hence, they decide to buy products that won't generate much waste (no plastic involved, ethically made, cruelty-free, and so on). The industry then starts marketing products that should respond to people's needs. If someone goes to the supermarket and buys a fair-trade chocolate bar with a paper wrap over a bar of plastic-wrapped chocolate, did this person do something to combat the environmental impact of food consumption? The industry with fancy labels—and so on—doesn't tell that, sometimes, paper wraps have a plastic coat to preserve some foods and to avoid moisture (who will buy a chocolate bar with a wet paper wrap?). This is why desire is intriguing. I watched the documentary *Seaspiracy* and saw how "dolphin-safe" labels are put into tuna cans when there is no certainty if dolphins were killed or not in the process. But people buy these products because of sustainable wildlife preservation. In this regard, Žižek says desire comes from fantasy. The fantasy keeps us in a constant state of desiring, probably feeling incomplete or an impulse of belonging.

We are constantly validated according to what we can give to society intellectually, financially, you name it! For instance, I don't see myself doing anything else than teaching, researching and being the best mom I can possibly be for my daughter. Do I talk to anyone about how epistemicide leads to perceive diversity as a threat to school mathematics? I write papers about it (and receive my monetary incentive for publications), go to conferences, give lectures without much result of being labelled as a pessimist by some mathematics educators or be well-received by my peers and mentors (probably reading this reaction). Aldo's paper presents experiences from other fellow researchers worldwide resisting and confronting epistemological violence and the use of mathematically justified forms to commit abuse to segregated and historically marginalized groups. This is done not behind a desk and a screen as I do, but in the front line. In academia, what we do is silenced, arguing what we do is not valid scientific research. They do not publish our work, even if we have enough data to make our claims clear and justified. What we do is not considered proper math education research. We live marginalized in the shadows of mathematics education. One of the main conclusions I got from Aldo's discussion is that it is not enough by recognizing forms of violence and the interplay they have with mathematics (either to help mathematically or to show how school mathematics alienates groups of people). It is not enough by doing research or problematizing the *status quo*. One should become a mathematics educator activist, as David Bowers (2021, in this volume, also commenting on Aldo's paper).

The second thing that came to mind is how power has been historically exerted through knowledge. I mean, it is not about how many things someone knows (for example, Nasa's knowledge in Aldo's discussion), but if people learn the right things to be validated in contemporaneity (if people qualify for "adoption" or not). Historically, a systematic epistemicide of particular groups of people have led them to lose the condition of humans to the eye of privileged groups, for example, indigenous people to the eye of colonizers. From a Postcolonial perspective, epistemicide problematizes how knowledge is, and has been used, to exert power according to national and global ideas for progress. Such ideas invalidate knowledge that is produced outside the boundaries of well-founded western forms of scientific practice: the taken as valid forms of knowledge. Here, the role of mathematics education researchers should take a turn to give voice to the abused, segregated, marginalized and invisibilized.

Globalization does not set equitable knowledge levels to all but to produce and naturalize forms of epistemic violence to communities that don't share validated capitalistic practices, conducts, and desires. Then, they must strike for survival only because they share other cosmologies than the "normal" and conceive mathematics in other ways. Here, a fine line between the right of remaining silent and the silencing of voices (either rhetorical or literal) decides people's future and attitude to engage with formal forms of schooling and towards learning school mathematics. I wonder how much mathematics a Mapuche kid could learn if their families and communities are portrayed as terrorists by the media and if their communities are constantly attacked and killed. Then, it is a bit obvious, at least for me, that they are not going to perform well in national standardized tests. (Please, I invite you to search the internet for "police brutality against Mapuche children" or "Mapuche children injured by police".) Their fight is not about equitable access to education but the right to live without life-threatening attacks. In such contexts, school mathematics is far away from saving Mapuche children lives. Is it enough by publishing about their cultural richness and mathematical knowledge they have? Should we partake in helping to solve the conflict between authorities taking over Mapuche lands and Mapuche families defending their territories? What is our role as mathematic researcher activists? Is it enough by researching multiculturalism, multilingualism, inclusion, equity, agency, social justice, and so on? Is it enough by giving voice to the voiceless (as Aldo once said when we were PhD students)? Should I share Mapuche experiences to make visible the lack of conditions and foregrounds, in Ole Skovsmose' terms, they have?

> En un allanamiento la agresividad es impactante, pocas veces hemos podido registrar imágenes donde Carabineros golpea a un niño, golpea a una mujer con una niña en brazos. Que tu casa esté llena de lacrimógenas al interior y tengas que salir producto del ahogo o ver a tu padre lleno de perdigones y sangrando por todos lados, son cosas que impactan mucho a un niño. Impactan a cualquier persona", dice Mijael Carbone. (Unrepresented Nations & Peoples Organization [UNPO], 2015)

> In a raid, the aggressiveness is shocking; we have rarely been able to record images where the Police hit a child, hits a woman with a girl in her arms. That your house is full of tear gas inside, and you must come out because you cannot breathe or seeing your father full of pellets and bleeding from all sides. These are things that greatly impact a child. They impact anyone," says Mijael Carbone. (UNPO, 2015, my translation)

In 2011, three ONGs demanded Chile for violence and child abuse, so the Supreme Court ordered not to apply the Antiterrorist Law to children. Although, everything remained the same.

> En el último informe anual de la institución, publicado a comienzos de 2015, se lee: "las violaciones de derechos –en particular de niños/as y adolescentes mapuche– producto del uso excesivo de la fuerza por parte de personal de Carabineros, se ha mantenido a lo largo del año sin mayores cambios en un patrón sobre el cual el INDH ha reclamado, y al cual se ha referido la justicia, en el sentido de cuestionar y sancionar el actuar de las fuerzas de seguridad, particularmente respecto de niños y niñas mapuche, exigiendo su apego irrestricto a las normas y reglamentos vigentes". El INDH ya expresó su preocupación al Comité de los Derechos del Niño de la ONU, que hizo público el recién pasado 2 de octubre [2015] un documento con observaciones y sugerencias respecto a la infancia al Estado de Chile. Allí declara que "El Comité sigue profundamente preocupado por la situación permanente de la desigualdad, la discriminación y la violencia contra los niños indígenas, en particular los niños mapuches". (UNPO, 2015)

> In the last institution's annual report, published at the beginning of 2015, it is written: "the violations of rights –particularly of Mapuche children and adolescents– as a result of the excessive use of force by Carabineros, have been maintained throughout the year without major changes in such a way that the NHRI has complained, and to which justice has referred, in the sense of questioning and sanctioning the actions of the police, particularly with regard to Mapuche children, demanding their unrestricted adherence to the current rules and regulations". The INDH has already expressed its concern to the UN Committee on the Rights of the Child, which published on October 2 [2015] a document with observations and suggestions regarding children to the State of Chile. There it states that "The Committee remains deeply concerned about the permanent situation of inequality, discrimination and violence against indigenous children, in particular Mapuche children". (UNPO, 2015, my translation)

Violence against the *Other* in the path of organizing behaviour and the quest for social order implies not only epistemological injustice but a systematic destruction—involving colonization, oppression and even genocide. So, what role can we play as mathematic education activists against epistemological violence? I still wonder.

Not only have indigenous groups experienced these practices, but women have also been alienated from mathematic practices, believing that women should not be educated. In Chile, the firsts school for women were not meant for them to actively participate in society but in learning how to be a good mother and wife. The mathematics they knew were not meant for them to pursue an academic life but to be able to help their kids with their homework. What contributions could women make if they were not authorized to engage in mathematician discussions? Women's mathematical knowledge was not allowed to exist. Although, when it comes to ethnic differences, other marginalized groups suffered brutal forms of violence (not only epistemic violence). Black slaves who came to Chile, not by choice (sold as products), were not even considered humans. They were reduced to serve the white men and lose the opportunity to access the spaces reserved only for the elite, such as schools. There were even discussions about "reproduction of Black populations" that proposed forced sterilization of most Black people. In other parts of the globe, these white men supremacy included also forced sterilization of indigenous groups.

Losing humans' condition and being taken as a body (without agency or else) from which other people can decide (i.e., pro-choice versus pro-life debates about abortion) is much more than just school mathematics. But it should concern mathematics education activists. Mathematics is constantly used as a form of disseminating statistical results about everything, and people read it. More often, those numbers or graphs are manipulated for people to take directed decisions—this is when math illiteracy is used to commit abuse. Meanwhile, math is advertised as the key to escape from these unlivable spaces in educational policies and transnational reports. People should be empowered by mathematics; then, they will be able to read news and not be fooled by the media and take an informed decision. However, people wouldn't have to be empowered by math if the media didn't use maths to misinform and scandalized the public. What is our role as mathematics education activists here? Should we point to every time the press commits these abuses and explain why the numbers and graphs are adulterated? Does anyone care about charts at this point? As Aldo says, clickbait is created for the morbid audience to obtain profit from some random website: why informing people when you can make money from urgent, paradoxical and scandalous content, right? The wonders of school mathematics are another form of click bait. We have to ask who benefits from these classroom practices, is it children? I believe they don't.

Finally, I'm not comfortable with the idea of doing something if it makes sense for us, even if it doesn't turn out well, because that is the same train of thought that evangelization in Latin America was framed. They were saving us from our doom, from going to hell, whatever. We need to find a middle ground between not imposing our beliefs on *Others*, not alienating the *Other*. Probably Emmanuel Levinas could help in setting school mathematics from *Otherness* (*alteridad* in Spanish). We have so many possible paths to take from here, but I will start by challenging the assumption that all people need mathematics (and the same type of mathematics) for success. I mean, why mathematics? What mathematics? When mathematics? And for whom? We should challenge those political and economic decisions as mathematics education research activists to confront and resist modern forms of epistemic violence.

References

Bowers, D. M. (2021). Urgency and the shameful escape of privilege: We move differently when we refuse to set aside the weight: A response to Aldo Parra. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 91–86). Tredition. https://doi.org/10.5281/zenodo.5469692

Parra, A. (2021). Mathematics education, researchers and local communities: A critical encounter in times of pandemic, pareidolia and post-factualism. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 91–80). Tredition. https://doi.org/10.5281/zenodo.5457124

Unrepresented Nations & Peoples Organization (UNPO). (2015, December 3). Mapuche: Police brutality against Mapuche children. https://unpo.org/article/18766

Symposia

Mathematics teacher agency

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Mark Boylan, Sheffield Hallam University
Anna Chronaki, University of Malmö and University of Thessaly
Herine Otieno, African Institute for Mathematical Sciences
Pete Wright, UCL Institute of Education

Neoliberal policies dominate in many parts of the world, setting a frame within which education practices are frequently constrained. In mathematics, perhaps more than other subjects, these constraints seem to be more keenly felt, not least because of the economic value placed on mathematics expertise and the related effects of performativity and accountability. In this symposium we explore potential sources of support that may enable mathematics teachers to challenge orthodox practices, facilitate creative responses to and/or rejections of policy constraints as they negotiate agency over their practice and their learning.

Introduction and aims

Neoliberal discourses of mathematics practice and of teachers' professional learning are often framed in terms of quality, with teachers viewed as deficient, their skills, knowledge and practice in need of improvement. Responses to these perceived deficits include large-scale, cascade models of professional development, albeit with increasing attempts to incorporate knowledge of what makes for effective teacher learning experiences. Such responses can constrain opportunities for teacher agency and contribute to teacher dissatisfaction, as teachers experience a lack of autonomy over their learning and their work.

To counter these discourses, in this symposium we share examples of and perspectives on the achievement of mathematics teacher agency, exploring implications for teacher learning, foregrounding questions of equity. Our interest lies in the creative ways that individuals and groups of mathematics teachers resist predominantly neoliberal discourses, determining their own learning goals and how they are supported to do this. We reflect on our own efforts to support teachers to transform practice, the challenges this raises and how teachers respond.

Exploring mathematics teacher agency

Agency is understood as a 'situated achievement' (Priestly et al. 2015, p. 29) – temporally embedded within a socio-cultural context. One approach to understanding agency focuses

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on three temporal dimensions, an 'iterational element', providing a stabilising influence from the past, a practical-evaluative element that focuses on an actor's capacity for making reasoned decisions, and a projective element where creative possibilities for future are imagined (Emirbayer & Mische, 1998). Threading through these three dimensions, we identify a variety of *teacher roles and practices* (e.g., practitioner researcher, 'champion' teacher, ...), *modes of collectivity* (e.g., research group, collaboration, peer learning), engagement with *external stimuli and support* (e.g., texts, networks, researchers, teacher educators…) and *tools* (e.g., resource design, pedagogic experimentation, video stimulated reflection, social media).

Symposium plan

Through sharing alternative perspectives and accounts of teachers' agency over their practice the session aims to stimulate discussion on our roles (as teacher educators/ researchers) in supporting teachers' efforts towards transformative practices in mathematics. Each presentation will explore one or more of the themes of teachers' roles and practice, collectivity, external resources and tools in relation to agency. Presentations will be followed by small group discussions, inviting participants to reflect on their experiences in relation to these questions:

- How are possibilities for mathematics teacher agency supported or constrained in different contexts? Which groups/individuals are included/excluded?
- What responsibilities do we have (as teacher educators/researchers) in relation to supporting teachers' achievement of agency?
- What do we learn from teachers and students concerning 'agency'?

Introduction to the symposium and presenters (5 minutes)

Paper 1 Possibilities for mathematics teacher agency in England: Historical policy traces. Gill Adams & Mark Boylan (12 mins)

In this paper, we explore the state's changing role in shaping professional learning activities within the neo-liberal context, focussing on England. By examining ways that policy (and its absence) in relation to mathematics teacher learning influences broader socio-cultural conditions thereby offering shifting possibilities for teacher agency we outline the way that power relations, cultures and materialities operate across four time periods characterised by clear differences in policy and structures. These four are broadly: first, prior to the introduction of the National Curriculum (1970-1990); the second period from 1990 marked by national initiatives aimed at driving up standards in mathematics; the third centred on the introduction and early years of the National Centre for Excellence in Mathematics together with increased support for teacher led professional development and a fourth, marked by a reassertion of a central national agenda. We consider the roles and practices available to teachers in these times, drawing out possibilities for collaboration and examine how teachers (individually and collectively) interact with external influences.

Paper 2 AIMS Teacher Training Program: Working WITH Teachers and not ON teachers. Herine Otieno (12 mins)

In this paper, I reflect on the efforts I have made as a team lead for a Teacher Training Program for mathematics & science teachers in Rwanda, to transform a teacher training model originally shaped as a top-down cascade model to one which is largely hinged on teachers' individual and collective contributions. Citing specific examples, I reflect on the process of shifting the training program from using University lecturers as master trainers and a pre-defined, externally developed teacher training curriculum to promoting peer learning amongst teachers and drawing on individual teachers and teacher collectives referred to as champion teachers to organically identify key training content and interventions for improving quality of teaching & learning of mathematics in Rwanda secondary schools. Finally, drawing on observations and excerpts from two different threads of WhatsApp conversations with champion teachers and some of the participating teachers and employing the transformative professional learning framework (Jones & Charteris, 2017) I will explore the emerging 'impact' on the teachers' relationship with each other, teaching, and key stakeholders in their teaching 'environment'.

Paper 3 Participatory action research (PAR): A critical model for transforming classroom practice through developing collective agency. Pete Wright (12 mins)

PAR offers an alternative paradigm for research/ professional development in which teacher researchers (TRs) and academic researchers (ARs) collaborate in bringing about changes in classroom practice. Skovsmose and Borba (2004) outline a critical model of PAR which recognises the essential/ complementary roles played by both TRs (with their in-depth knowledge of the classroom situation) and ARs (with their expertise in research methods) in the research process. This model was adopted for the Teaching Maths for Social Justice (Wright, 2020) and Visible Maths Pedagogy (Wright, Carvalho, & Fejzo, 2020) research projects. Both projects involved the author as AR and sought to develop engaging and empowering practices in the mathematics classroom, a site that has historically proved highly resistant to change. The projects demonstrated how the mutual support and collective agency generated by a research group, or network of teachers, enables TRs to take risks and overcome constraints they face in developing their practice in line with a commitment to equity and social justice. The research groups provided opportunities for TRs to: engage with CME research literature; collaboratively plan, trial and evaluate classroom activities; design and implement their own data collection tools; and critique existing/new practices through video-stimulated reflection.

Paper 4 Teachers' relational agency: Affective bodying with children, materials, concepts and difference. Anna Chronaki (12 mins)

The purpose of this paper is to discuss teacher agency as a relational matter that grows through affective bodying with children, teachers, concepts and difference in the community revealing a process of minoritarian becoming(s) (Chronaki, 2019). It is based on the analysis of recent experiences through collaborative work amongst children, teachers, student-teachers and researchers. In the project context, the author was involved in a process of creative design addressing mathematics in the context of 'the commons' of a specific community through radical pedagogic experimentations (i.e., playing and making mathematical games and crafts: spaces for coming together, global crises and local solidarity: debt vs money as common good and money, see http://www.citizenship-and-mathematics.eu). In a series of seminars and school-based work with participant teachers and children, these materials moved from the researcher's desk out to the public space of school classrooms, communal areas, the streets or the kafeneion. Here, we aim to denote aspects concerning this transformative move and to discuss teachers' agency as relational in multiple layers of research-creation with teachers, children and the community.

Group discussion & plenary (25 minutes)

References

Biesta, G., Priestley, M., & Robinson, S. (2016). *Teacher agency an ecological approach*. London: Bloomsbury Academic.

Chronaki, A. (2019). Affective bodying of mathematics, children and difference: Choreographing 'sad affects' as affirmative politics in early mathematics teacher education. *ZDM Mathematics Education 51*, 319–330.

Emirbayer, M., & Mische, A. (1998). What is agency? *The American Journal of Sociology*, *103*(4), 962–1023.

Marguerite Jones & Jennifer Charteris (2017) Transformative professional learning: An ecological approach to agency through critical reflection. *Reflective Practice*, *18*(4), 496-513.

Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematics education. In P. Valero, & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 207–226). Dordrecht: Kluwer.

Wright, P. (2020). Transforming mathematics classroom practice through participatory action research. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-019-09452-1

Wright, P., Carvalho, T., & Fejzo, A. (2020). Visible mathematics pedagogy: A model for transforming classroom practice. *Educational Action Research*. https://doi.org/10.1080/09650792.2020.1850497

Parenting and educating in mathematics: Parental engagement during and beyond the COVID-19 pandemic

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In this symposium we engage participants in actively reflecting on our work with/as parents and caregivers in mathematics education and on their own experiences within their local contexts during the COVID-19 pandemic. Our goal is to reimagine parental engagement and plan for future collaborations to support new possibilities for and address exacerbated barriers to parental engagement in mathematics education.

Focus of the symposium

Parents' and caregivers' role in shaping children's mathematics education often goes overlooked, and the intersecting identities of mathematics teachers and teacher educators as parents is rarely considered. The COVID-19 pandemic, however, has placed parents in the spotlight. As nations aimed to slow the spread of the virus, 1.5 billion students worldwide abruptly transitioned to emergency remote mathematics instruction (UNESCO, 2020). A year later, the COVID-19 outbreak continues to cause significant disruptions to education. Parents have been thrust into unprecedented levels of engagement with school mathematics, and their responsibility for ensuring their child's mathematics learning has increased significantly. At the same time, as the work of educators has shifted into their homes, mathematics teachers and teacher educators are educating their students and parenting their children simultaneously.

Our intention for this symposium is to consider how COVID-19 might *enable* parental engagement by explicitly increasing expectations for involvement of parents in school mathematics and by drawing attention to the intersections of mathematics teaching and parenting. Symposium organizers will offer perspectives on parental engagement in mathematics during COVID-19 from two countries – the United States and New Zealand. By asking participants to reflect on our work with/as parents and on their own experiences in local contexts, we hope to build a global network of scholars committed to parental engagement and to plan for future collaborations.

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Parental engagement in mathematics education

Parents are uniquely positioned to support children's mathematics education across formal schooling. Unsurprisingly, parental involvement is strongly linked to children's mathematics achievement (Knapp et al., 2017). Yet, efforts to broaden mathematics learning opportunities largely ignore parents, which pits schools, teachers, and parents against each other rather than fostering partnerships for learning. Differences between parents' own mathematics education, which likely emphasized rules and procedures (Jackson & Remillard, 2005), and the curriculum and instruction experienced by their children present barriers to parental involvement, even in early grade levels (Muir, 2012). This challenge was intensified by the pandemic as parents struggled with or resisted the continuation of mathematical reasoning through child-centered strategies when learning shifted into the home. Despite these challenges, we see potential in blurring the line between parenting and educating in mathematics and seek to imagine possibilities for parental engagement. To that end, we ask questions such as:

3. What were the experiences of parents in diverse local contexts during the initial transition to emergency remote instruction? Presently?
4. How can we (re)envision parental engagement in mathematics education? How can we spark and sustain such efforts to engage parents in our local contexts?
5. How have parent experiences with mathematics education during the global pandemic created new barriers to engagement? How might we address those?

Symposium structure

After an overview (5 minutes), each organizer will present key ideas from their work with/as parents (20 minutes). Then, participants will engage in an interactive session to reflect on the experiences of parents with mathematics education in their local contexts in relation to the questions raised (40 minutes). We will conclude by building visions for parental engagement and planning for future collaborations (25 minutes).

Piata Allen and **Tony Trinick** both lecture at the University of Auckland where they focus on Māori-medium mathematics education. COVID-19 impacted differently on Māori-medium schools and whanau (extended families) than English-medium in Aotearoa New Zealand. This was in part due to the historic legacy of under resourcing of Māori-medium education by the State, in digital, human and print resources since the emergence of Māori-medium education 40 years ago. We discuss, our experiences as teacher educators, professional development facilitators and, in the case of Piata, a Māori-medium parent to support teachers and schools to teach mathematics in the medium of te reo Māori (the Māori language) during COVID-19 school closures. Our findings indicate that Māori-medium schools due to their legacy of being self-reliant, used their own agency to ensure a modicum of continuity in mathematics learning programmes. This was supported by wider community led initiatives that utilised social media and community networks to create, distribute and share resources quickly and efficiently in the face of inadequate state support.

Lisa Darragh lectures at the University of Auckland and her main research interest relates to learner and teacher identity in mathematics education. She will present results from a recent study she conducted with Dr. Nike Franke during the first couple of months of the nationwide lockdown in April 2020. The survey invitation was sent via Facebook community groups around Aotearoa New Zealand and received 634 responses over a three-week period. Parents were generally very engaged in the home learning of mathematics. They reported a range of opinions about quality of mathematics work and teacher support, and there was a correlation between general stress levels and negative opinions. To further support their child's mathematics learning, many parents turned to online mathematics programs, about which they were very positive, but the crisis brought to the fore a number of pre-existing issues. We argue that these findings have implications for all forms of mathematics home learning in the future, and suggest that schools need to listen to parental feedback regarding the quality, level and quantity of mathematics work. Additionally, schools could consider ways to deliver effective teacher support and to foster parental agency in helping their children with mathematics learning.

Frances K. Harper is an assistant professor at the University of Tennessee, Knoxville, where she focuses on how parent-teacher-community partnerships shift traditional power dynamics in mathematics education. She will present findings from a recent study conducted with Dr. Joshua Rosenberg, Sara Comperry, Kay Howell, and Sierra Womble during the initial transition to emergency remote instruction in 2020. Examining 100 survey responses and over 200 posts from Twitter, we saw a commitment among parents to engaging children with a range of mathematics topics from the elementary/primary school curriculum and a strong desire for collaborations among parents and teachers. We argue that these findings have implications for how we renew efforts to engage parents, such as placing greater value on the authentic ways families already engage in mathematics in the home and inviting parents into conversations about school mathematics on social media or in other accessible spaces. Harper will share how these implications inform the ongoing development of two networks of parent-teacher-community partners in her current projects, namely, networks aimed at fostering computational thinking among Black and Latinx pre-schoolers and at advancing racial justice in elementary/primary mathematics.

Naomi Jessup is an assistant professor at Georgia State University and one of her research interests examines the impact of COVID-19 on mathematics teaching and learning for Black communities as well as rehumanizing and culturally responsive approaches by parents. She will present preliminary findings of an ongoing two-year study that examined how Black parents in the United States responded to supporting their children's elementary mathematics learning with resources provided by schools throughout the middle of the 2019-2020 school year and subsequent school year. Initial findings indicate a range of approaches and strategies used by Black parents given that many of the strategies desired for learning mathematics were not common to them. In addition, parents voiced frustrations regarding the quality, rigor, and lack of cultural responsiveness of the mathematics tasks provided. Some parents modified and adapted the mathematics resources provided by the school to better accommodate their child's learning and allow them to make cultural and real-work connections. Parents discussed increased agency in supporting their child's mathematics learning and due to their shift in roles and responsibilities from pre-pandemic to now. Implications from this study provide a counternarrative regarding Black parents' engagement in their children's mathematics learning and highlights their social and cultural capitol that is often not invited, noticed, or silenced in schools.

Mary Candace Raygoza is an assistant professor and STEMinist teacher educator at Saint Mary's College of California. This presentation examines her mathematical motherscholar praxis during the COVID-19 pandemic and present global reckoning for racial justice. In March 2020, her child's pre-school closed, as schools far and wide shuttered for in-person instruction. Alongside her paid work, she was also an informal pre-school teacher of one pupil, joining many parents, especially mothers, around the world whose roles in relation to schooling shifted suddenly and with little societal support. Informed by scholarship that highlights how much young children know and can do mathematically (Johnson et al., 2019), the mathematics that emerges from play (Wager & Parks, 2014), and the potential to connect (in)justice issues to early childhood mathematics (Ward, 2017), she will explore lessons from her three-year-old, namely how her child led the way in revealing mathematics connected to social-emotional growth and as part of learning about fairness and solidarity (e.g., the mathematics that emerged out of a community circle of stuffed animals). As the presenter interrogates her privilege as a white woman and a mathematics teacher educator mother, she also wonders: How can we support parents and caregivers to feel affirmed that meaningful mathematics lives within activity that is not intentionally mathematical (and is something accessible to them and their participation); and how can schools shape mathematics teaching and learning with that in mind?

References

Jackson, K., &. Remillard, J. T. (2005). Rethinking parent involvement: African American mothers construct their roles in the mathematics education of their children. *School Community Journal, 15*(1), 51–73.

Johnson, N. C., Turrou, A. C., McMillan, B. G., Raygoza, M. C., & Franke, M. L. (2019). "Can you help me count these pennies?" Surfacing preschoolers' understanding of counting. *Mathematics Thinking and Learning, 21*(4), 237–264.

Knapp, A., Landers, R., Liang, S., & Jefferson, V. (2017). We all as a family are graduating tonight: A case for mathematical knowledge for parental involvement. *Educational Studies in Mathematics, 95*(1), 79–95.

Muir, T. (2012). Numeracy at home: Involving parents in mathematics education. *International Journal for Mathematics Teaching and Learning, January 25*, 1–13.

UNESCO. (2020). COVID-19 impact on education. *Education: From disruprtion to recovery.* http://en.unesco.org/COVID19/educationresponse

Wager, A., & Parks, A. N. (2014). Learning mathematics through play. *Handbook of play and learning in early childhood,* 216–227.

Ward, J. (2017). Early childhood mathematics through a social justice lens: An autoethnography.

Interrogating common-sense assumptions toward a more just mathematics education

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In this symposium participants will interrogate common sense assumptions that serve to (re)produce long standing inequities in mathematics education. "Common sense" is historically and culturally constructed, rather than a natural reflection of "the way things are." Therefore, surfacing the historical roots of common-sense assumptions provides powerful opportunities to reframe problems of access, oppression, and erasure. We invite participants to consider common sense assumptions behind problems of equity in their own contexts, trace their histories, and offer alternative narratives to reframe these problems. We hope to better understand how common-sense assumptions have shaped mathematics education across the globe, and use these insights to reimagine mathematics education.

Aims of the symposium

In the context of a school year in which a horrific global pandemic and widespread racist violence have dominated the global discourse, we come together as a collective to question and reimagine a different world for mathematics education. In the U.S., at least, mathematics education, rooted in assimilationist labor, military-industrial, and capitalist agendas, has continued to play a significant role in perpetuating white supremacy in schooling practices, (Vossoughi & Vakil, 2018). Across Europe and around the world, research has argued that the simultaneous rise of neoliberalism and nationalism has resulted in educational policies in mathematics sustained by raced, gendered, and classed ideas of mastery, ability, and autonomous selfhood (e.g., Ineson & Povey, 2020). In this symposium, we explore how the practices of mathematics education rest on a set of assumptions, or a "common sense" that is historically and culturally, and politically constructed. Surfacing these common-sense assumptions, we argue, will provide opportunities to reframe the persistent problems of access, oppression, and erasure in mathematics education. The aim of this symposium is to invite participants to share and discuss common sense assumptions in their own contexts,

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then consider alternative narratives that may serve to reframe the problem space of mathematics education, thereby providing opportunities to ask new, more productive research questions and to reimagine how we design and take action for equitable teaching and learning.

In the following, we discuss what we mean by common sense assumptions and how they operate in our mathematics education scholarship and practice. We end with a description of proposed symposium activities.

Relevance of the symposium

Activists, mathematicians, and other scholars have highlighted the ways in which certain assumptions around mathematics have been deployed to support systems of oppression around the world (e.g., Wong, 2020). Mathematics education, which plays an exaggerated role in mediating the educational experiences and possibilities of students across the globe, has also been implicated in these conversations (e.g., Valero et al., 2012). For example, the discourse of “learning loss” presumes a particular set of knowledge must necessarily be acquired in each year of a child’s life, and that any deviation from this progression will produce detrimental effects. In response to concerns around students’ “learning loss” as a result of the COVID-19 pandemic, a host of responses have been proposed in the US, including the implementation of impractical and demonstrably inequitable standardized tests to measure “learning loss” while the pandemic continues to rage, and an increase of school hours and days in the next year (Ewing, 2020).

However, the very idea of “learning loss” hinges on common sense assumptions that include valuing a standardized yet arbitrary trajectory of achievement, as well as the insistence that the only learning of value occurs in school (McKinney de Royston & Vossoughi, 2021). We take up Schutz’s (1953) argument that common sense, while critical for the accomplishment of everyday activity and social order, is, in fact, historically and culturally constructed (Garfinkel, 1967). What is considered common sense in one context (e.g., taking off one’s shoes before entering homes is a common cultural practice in many communities) may seem absurd, or even offensive in another (e.g., guests from other communities may find it overly intimate to reveal their socked or bare feet). Common sense understandings also change across local contexts, including institutional spaces (e.g., one might expect to go to the bathroom when the need arises; in many school settings permission must be granted first) and disciplinary domains (e.g., some disciplinary conventions lead students to expect right and wrong answers, whereas others are presented as more flexible; Schutz, 1953).

Further, common sense understandings and assumptions serve to reproduce the cultures from which they originate. Garfinkel (1967) explained, “Not only does common sense knowledge portray a real society for members, but in the manner of a self-fulfilling prophecy the features of the real society are produced by persons’ motivated compliance with these background expectancies…. Seen from the person’s point of view, his commitments to motivated compliance consist of his grasp of and subscription to the ‘natural facts of life in society'" (p. 53). It follows that the common-sense understandings on which the inequitable conditions of mathematics education have been built serve to reproduce these conditions. In our example of "learning loss," we see how the common-sense assumptions about where valuable learning occurs and standardized achievement expectations bolsters the discourse of "learning loss," which in turn perpetuates inequitable practices like standardized testing and reifies the importance of normative, factory style schooling practices.

We argue that illuminating the historical origins and cultural development of these common-sense assumptions may allow us to understand anew the problem space of mathematics education. As Schutz (1953) notes, "all cultural objects--tools, symbols, language systems, works of art, social institutions, etc.--point back by their very origin and meaning to the activities of human subjects [...]. This historicity is capable of being examined in its reference to human activities of which it is the sediment" (Schutz, 1953, p. 3). There is a relationship between these histories and how they have sedimented in contemporary society. For our learning loss example, we can trace some of this discourse in the U.S. to the history of standardization of public education. Tyack and Tobin (1994) described how sorting classrooms into age-determined grade levels was an invention which, while commonplace today, did not become so until the twentieth century. Inspired by specialization and division of labor practices in factories, reformers from universities and state departments of education wanted to increase both efficiency of and control over public education. In a related effort, school content was divided into narrowly defined subject areas with required sequences, and tests were administered to determine whether students could advance to the next grade. While fears over learning loss is sensible within this context, examination of the context itself reveals a commitment to the needs of university reformers and politicians over concerns about, for example, the wellbeing of children, their teachers, and their communities. Considering this (simplified, for the purposes of this proposal) history of standardized, graded schooling in the U.S. allows us to reframe "the problem" of learning loss as many different problems, such as grade level standardization, whose knowledge counts in curricular sequences, and the role of efficiency in policy decisions in education.

Format of the symposium

This symposium is designed to support participant discussion and collective learning. The co-authors of the symposium will begin by introducing the concept of common-sense assumptions and provide the example of learning loss described above. The remainder of the symposium will support small group discussions of "a problem" related to mathematics education in participants' local contexts. In discussion, groups will unpack the common-sense assumptions required for defining "the problem" in a particular manner, and delineate the political and social premises from which these assumptions operate. As a consequence, these investigations will transform the problem space, allowing small groups to articulate different problems that may be addressed, expanding possibilities for reimagining mathematics education.

References

Ewing, J. (2020, December 28). The ridiculousness of learning loss. *Forbes*. https://nyti.ms/2QkAYwo

Garfinkel, H. (1967). *Studies in ethnomethodology*. Polity Press.

Ineson, G., & Povey, H. (Eds.). (2020). *Debates in mathematics education*. Routledge.

McKinney de Royston, M., & Vossoughi, S. (2021, January 18). Fixating on pandemic "learning loss" undermines the need to transform education. *Truthout*. https://truthout.org/articles/fixating-on-pandemic-learning-loss-undermines-the-need-to-transform-education

Schutz, A. (1953). Common-sense and scientific interpretation of human action. *Philosophy and Phenomenological Research*, *14*(1), 1–38.

Tyack, D., & Tobin, W. (1994). The "grammar" of schooling: Why has it been so hard to change? *American Educational Research Journal*, *31*(3), 453–479.

Valero, P., García, G., Camelo, F., Mancera, G., & Romero, J. (2012). Mathematics education and the dignity of being. *Pythagoras*, *33*(2), Art. #171. http://dx.doi.org/10.4102/pythagoras.v33i2.171

Vossoughi, S., & Vakil, S. (2018). Toward what ends? A critical analysis of militarism, equity, and STEM education. In A. I. Ali & T. L. Buenavista (Eds.), *Education at war: The fight for students of color in America's public schools* (pp. 117–140). Fordham University Press.

Wong, T. A. (2020, January 31). Can mathematics be antiracist? *AMS Blogs: Inclusion/Exclusion*. https://blogs.ams.org/inclusionexclusion/2020/01/31/can-mathematics-be-antiracist

Diversity and inclusion in mathematics teacher education: Lessons from Chile and Sweden

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Based on the examination of Chilean and Swedish research, the symposium addresses the possibilities and challenges for researching diversity and inclusion in mathematics pre- and in-service teacher education. Departing form concrete localized research and its contextual, theoretical and methodological stances, larger reflections and implications for the education of mathematics teachers that may lead to an increased sensitivity towards students' diversities and their impact in inclusion of students and change of educational experiences in mathematics are drawn.

Motivation

Transnational reports point to the correlation between students' diversities, including gender, socio-economic disadvantages, racial and ethnic differences, immigration background, etc., and low-performance in mathematics (e.g., OECD, 2014). The sharp inequities in results seem to undermine students' opportunities to access higher education and to break the poverty circle in which they live. Therefore, the connection between students' diverse position of disadvantage and the access to quality mathematics education is a problem to tackle by research in the field of mathematics education. This is an issue in many countries, among those Chile and Sweden, which have taken in substantial number of immigrants and have explicit policies for promoting equitable access to education.

The Chilean educational system is characterized by a deeply rooted inequity. Recent reform policies have taken steps towards improving quality and access to education. The "Law of inclusion" (MINEDUC, 2015) disallows schools to select students which is likely to

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help disadvantaged students; and the primary mathematics curriculum was re-written with a greater emphasis on problem solving (MINEDUC, 2012), as a way to improve the quality of education and students' results. However, little is known in about racial and ethnic minority students' mathematics experiences and performance, and about the challenges in educating teachers to implement reform-based mathematics instruction in diverse school contexts in Chile. Few studies have explored the teachers' learning process to teach mathematics to diverse student populations. This research sheds light on how teachers' views of students in positions of disadvantage play a critical role in successfully implementing reform-based mathematics instruction (Darragh & Valoyes-Chávez, 2019). Overall, the teacher is left to cope with other aspects of teaching, such as dealing with students' diversity.

In Sweden, educational policies have responded to the increase of diversity in student population by guaranteeing school attendance and thereby opening opportunities for social mobility and integration. The Swedish Agency of Education has launched reforms emphasizing students' attainment (Skolverket, 2015). Thus, success is equated to mathematics scores in a hierarchical system of evaluation. Even though gender differences in achievement have diminished, particular gender stereotypes are reproduced in mathematics teaching (Sumpter, 2016). Immigrant students' lower results in mathematics persists. Studies have focused on the impact of the language of instruction and students' home languages as a barrier or resources for learning mathematics (Caligari et al., 2021). The results show that teaching practices generate exclusion, and that teachers' views on students' abilities given their gender, socio-economic status or ethnicity are influential in students' opportunities for learning.

Furthermore, teacher education — initial and in-service — is recognized as a key for addressing diversity and generating inclusion of students in disadvantaged positions (Darling-Hammond, 2017). Researchers contend that a different type of understanding is needed to teach mathematics in diverse school settings (e.g., Anderson & Stillman, 2013). Thus, the question remains of how to educate teachers to support the mathematics learning of diverse student populations, so that quality and inclusion can go hand in hand; as well as of how research may support such education.

Aim

This symposium addresses the general problem of the challenges that students' diversity and its impact on more equitable school results pose for mathematics teacher education research. By taking the case of Chile and Sweden as a point of departure, the symposium aims at: (1) Discussing experiences that led to challenging teachers' education through distinctive research approaches to notions of inclusion and diversity. (2) Identifying theoretical and methodological ways of reasoning and operate with inclusion and diversity in mathematics teacher education research. (3) Rethinking the complexity of inclusion and diversity and its implications and possibilities for mathematics teacher education practices in different social contexts.

Organization

The symposium has a series of thematic conversations among researchers from Chile and Sweden. Each conversation involves a statement by each participant (10 m each), a time for discussion among them (5 m) and a time for conversation with the audience to draw connections with other contexts and problems (15 m). The symposium convener (Valero) will moderate the symposium. It is planned to take for 180 m.

Opening concerns: An introduction (10 m)

The symposium starts with a motivation by the convener for the topic and a series of questions about the relevance of the discussion for research and teacher education.

Challenging policies of inclusion and diversity (40 m)

This theme unfolds experiences that led to challenge inclusion and diversity in mathematics teacher education. The intention is to build an understanding of how diversity has entered the scene of mathematics teacher education practices and institutions in different settings. We draw on an experience with a group of embroiderers in Mexico to discuss how culture-centric views of mathematics can be questioned from cultural diversity perspectives to account for historically marginalized social groups' relationship to knowledge and school (Solares et al., forthcoming) Drawing on a teachers' experience in multilingual classrooms, we address the collision of expectations promoted—and imposed—by Swedish curricular guidelines, teacher training programs and parents (Caligari et al., 2021).

Dealing with teacher education's paradoxes (40 min)

This theme explores theoretical tools to trouble the current (and historical) state of mathematics teacher education. The toolboxes invite to examine the double gestures that make visible the paradoxes on mathematics teacher education. We explore how mathematics teacher education in Chile has been historically structured around discourses of a desired mathematics teacher in a constant state of becoming and quasi-Darwinism of mathematics teachers (Montecino, 2019). And, with this notion of the *image of a desired teacher* used for an analysis of policy and practice in teacher education, we show the operation of images as desire-constructs imposing inclusion/exclusion in teacher education in Sweden (Österling, 2021).

Experiences of diversity and inclusion in the mathematics classroom (40 min)

This theme focuses on how notions of diversity and inclusion are unfolded in particular research projects and educational settings. We revisit experiences collected in different school contexts and their implications for research. We discuss the limits of the Chilean law of inclusion and examine the challenges of implementing it in the case of Black immigrant students (Valoyes-Chávez & Darragh, under review). Then, we explore Swedish mathematics teacher's practices, based on research on multilingualism, to point call teacher educators for the need of elaborating different ways—both in practice and research—to cope with the diverse body of student teachers (Norén, 2015).

Challenges to and opportunities (40 min)

The symposium closes identifying key issues for further research and the implications for teacher education. We argue on how Chilean educational policies have been historically delineated to fabricate a particular citizen, currently attracted in making the global Homo-economicus that lead to invisibilize diversity and, therefore, negate inclusion (Andrade-Molina, in press). We also discuss whether inclusive methodologies that involve teacher students may be a path forward to generate new opportunities for inclusion for teacher students and teacher educators (Skog, 2014).

References

Andrade-Molina, M. (In press). When numbers dictate common sense. Transnational's aspirations of a global curriculum. In W. Zhao et al. (Eds.) *Historicizing Curriculum Knowledge Translation on a Global Landscape*. Routledge.

Anderson, L. M., & Stillman, J. A. (2013). Student teaching's contribution to preservice teacher development. A review of research focused on the preparation of teachers for urban and high-needs contexts. *Review of Educ. Research, 83*(1), 3–69.

Caligari, L. et al. (2021). Multilingual students working with illustrated mathematical word problems as social praxis. In A. A. Essien & A. Msimanga (Eds.), *Multilingual Education Yearbook 2021: Policy and practice in STEM multilingual contexts*. Springer Nature.

Darling-Hammond, L. (2017). Teacher education around the world: What can we learn from international practice? *European Jrnl. of Teacher Education*, 40(3), 291-309.

Darragh, L. & Valoyes-Chávez, L. (2019). Blurred lines: producing the mathematics student through discourses of special education needs in the context of reform mathematics in Chile. *Educational Studies in Mathematics, 101*(3), 425-439.

MINEDUC (2012). *Bases curriculares 2012. Educación Básica Matemática*. Santiago: República de Chile. Retrieved from http://www.curriculumenlineamineduc.cl/

MINEDUC (2015). *Ley de inclusión escolar. Ley núm. 20.845.* Retrieved from https://www.leychile.cl/Navegar?idNorma=1078172.

Munoz, A. M. (2019). The professional mathematics teacher: fabricating and governing of teacher becoming. *Educação Unisinos*, *23*(1), 141-154.

Norén, E. (2015). Agency and positioning in a multilingual mathematics classroom. *Educational Studies in Mathematics 89*, 167–184.

OECD (2014). PISA 2012 Results in Focus. What 15-year-olds know and what they can do with what they know. Paris: OECD.

Skog, K. (2014). *Power, positionings and mathematics – discursive practices in mathematics teacher education: Climbing Lion's Head* [Doctoral dissertation, Stockholm University].

Solares, A. et al. (forthcoming). Challenging culturocentrism in school mathematics: The knowledge of women embroiderers from the Mezquital Valley.

Österling, L., (2021). *Desires for mathematics teachers and their knowledge. Practicum, practices and policy in mathematics teacher education* [Doctoral dissertation, Stockholm University].

Skolverket (2015). *Slutbetyg i grundskolan, våren 2015*. Dnr 5.1.1-2015:1103.

Sumpter, L. (2016). 'Boys press all the buttons and hope it will help': Upper secondary school teachers' gendered conceptions about students' mathematical reasoning. *International Journal of Science and Mathematics Education*, 14(8), 1535-1552.

Valoyes-Chávez, L. & Darragh, L. (under review). Interrogating the promise of equity and inclusion for Black immigrant students in mathematics education.

Publish or perish: Power and bias in peer review processes in mathematics education journals

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The "publish or perish" slogan represents a constant pressure to survive in academia and to be considered a competent professional. This symposium will open conversation among researchers, editors and reviewers to address issues of diversity, ethics and politics in the publication process. We will facilitate discussions with colleagues around the world to explore biases in the scholarly publication process to uncover the mechanisms and practices responsible for the underrepresentation of particular groups of researchers. The symposium will be a meeting point, to find and explore ways to address biases in the processes of peer review. We hope to contribute to the efforts to make the publishing process more transparent and accessible to researchers.

Rationale

The publication of research in scholarly journals is a critical goal for researchers in mathematics education and in every academic field. This goal not only relates to scientific interests such as expanding the extant knowledge, disseminating novel theories and methods and engaging in academic conversations in our field. Getting papers published in prestigious mathematics education journals is almost the exclusive path towards academic recognition, promotion and job stability for novice and early career researchers. It becomes an accountability system that most universities use to measure academic productivity. The "publish or perish" slogan represents the Damocles' sword hanging over the researchers' neck and constitutes a constant pressure to survive in academia.

Researchers are expected not only to continuously publish their work but also to do so in high-impact journals. This is because, as Andrade-Molina, Montecino and Aguilar (2020) argue, publishing in well-known journals in the field adds value to both researchers and their institutions of affiliation. As different ranking systems consolidate in mathematics education, publishing in journals indexed in, for instance, the Web of Science and Scopus, constitutes an indicator of the researchers' productivity, a measure of the quality of their work and a criterion for allocating resources (Andrade–Molina, Montecino & Aguilar, 2020). A hierarchical system of universities and researchers is then introduced, validated and

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sustained; in this sense, scholarly publication transcends academic purposes and becomes a contested arena where different political and economic interests emerge shaping the entire process. Moreover, the publication process shapes ways of normalizing and conducting the researchers' work by delineating what is considered theoretically and methodologically valuable for academia and therefore publishable. In this vein, it draws an aesthetics according to what is established as "good/desired" research.

The phenomenon of exclusion in the mathematics education system of practices has been widely discussed (e.g., Louie, 2017). Nevertheless, little is known about inequity and marginalization in the processes of scholarly publication (Meaney, 2013). Until just recently and within the context of strong anti-Black racism protests and the consolidation of the feminist movement worldwide, calls for unpacking mechanisms of exclusion that lead to the underrepresentation of racially and ethnically minoritized scholars and female researchers in scholarly publications have emerged (Wagner et al., 2020). In particular, questions about how the review process is conducted and handled by editors have been voiced in different contexts. It is argued that, although peer review contributes to move the scientific field forward by awarding high quality research, far from being an objective and rational process, it is shaped by issues of power that end up rendering invisible particular voices and epistemes in academia. Although peer review is either a single-blind or double-blind process to ensure objectivity, transparency, impartiality and fairness, different studies evidence that these apparent goals are not realized (Lee et al., 2012).

Lee et al. (2012) point to research in diverse fields, which uncovers the existence of different biases during the review process ranging from errors in assessing a submission's "true quality" to the social characteristics of the authors. For instance, dominant representations about what high-quality research looks like may lead the reviewers to fail to assess the real qualities of the proposed work. In mathematics education, for example, prominence is given to cognitive investigations while sociopolitical and critical studies are delegitimized by the "where is the math" question (Martin et al., 2010). Also, in our field there are clear disparities among countries and regions in terms of which research gets published (Mesa & Wagner, 2019). Social characteristics of the authors also seem to play a critical role in peer review. These biases result in the differential evaluation of an author's submission as a result of her/his perceived membership in a particular social category. As Lee et al. (2012) argue, "social bias challenges the thesis of impartiality by suggesting that reviewers do not evaluate submissions–their content and relationship to the literature–independently of the author's (perceived) identity" (p. 11). Thus, national origin, language, gender, content, racial and ethnic biases seem to shape the review process.

With these ideas in mind, this symposium has the potential to generate discussions around the following areas, which we hope the participants will engage and contribute:

- What are the main obstacles that mathematics education researchers from underrepresented groups face when trying to get their research published?

- How do the researchers' social identities shape the chances of getting a paper published in a prestigious journal in the field?
- What is considered as high-quality research in mathematics education?
- What and whose knowledge are valued in mathematics education?
- What and whose knowledge are ignored in mathematics education?
- What can editors do to mediate or control biases in the peer review process?
- What is the impact of these values and practices on dominant views of what mathematics education looks like and what its concerns are?
- What are the ethical responsibilities of reviewers and editors?
- What can scholars in the field do to help develop the diversity of published research?

Aim

In this symposium we attend Mesa and Wagner's (2019) call for open conversations among researchers, editors and reviewers to address issues of diversity, ethics and politics involved in the publication process. We call on mathematics education researchers worldwide to fully engage in discussing their experiences in getting their papers published in high-impact mathematics education journals. The purpose of this symposium is twofold. First, it is aimed at facilitating discussions with colleagues around the world to explore biases in the publication process in order to uncover the mechanisms and practices responsible for the underrepresentation of particular groups of scholars in scholarly publications. Second, the symposium is thought as a meeting point to find and explore ways to address biases in the processes of peer review. We hope to contribute to efforts to make publishing processes more accessible to researchers who identify with groups that have been marginalized, as well as, to unpack mechanisms of power that normalize and control us.

Planned Structure

The introduction to the symposium (5 minutes) will be followed by brief input in which presenters will draw on their own experiences as researchers, reviewers, editors and editorial members. Melissa Andrade-Molina will problematize the ranking system of journals that govern the publishing practices in mathematics education (5 minutes). Alex Montecino will discuss a pseudo aesthetic shaped in mathematics education research (5 minutes). Luz Valoyes-Chávez will discuss research about gender, racial, language and national biases in peer review (5 minutes). David Wagner will outline recent discussions pursuing anti-racism amongst editors of mathematics education journals (5 minutes). Symposium participants will then be asked to bring their own histories to these thought-pieces and questions in small group discussion and plenary report-backs (45 minutes). The voices of participants with underrepresented identities will be encouraged and promoted in this time. Finally, participants will propose and discuss possible pathways for action to address the challenges identified in the symposium (20 minutes).

References

Andrade-Molina, M., Montecino, A., & Aguilar, M.S. (2020). Beyond quality metrics: defying journal rankings as the philosopher's stone of mathematics education research. *Educational Studies in Mathematics, 103*(2), 359-374.

Lee, C., Sugimoto, C., Zhang, G., & Cronin, B. (2013). Bias in peer review. *Journal of the American Society for Information Science and Technology, 64*(1), 2–17.

Louie, N. (2017). The culture of exclusion in mathematics education and its persistence in equity-oriented teaching. *Journal for Research in Mathematics Education, 48*(5), 488-519.

Martin, D., Gholson, M., & Leonard, J. (2010). Mathematics as gatekeeper: power and privilege in the production of knowledge. *Journal of Urban Mathematics Education, 3*(2), 12-24.

Meaney, T. (2013). The privileging of English in mathematics education research, just a necessary evil? In M. Berger, K. Brodie, V. Frith & K. le Roux (Eds). *Proceedings of the 7th International Mathematics Education and Society Conference* (65–84). Mathematics Education and Society. Hansa Print.

Mesa, V., & Wagner, D. (2019). Behind the door: a critical look at the process of publication in Educational Studies in Mathematics. *Educational Studies in Mathematics, 101*(3), 301-324.

Wagner, D., Bakker, A., Meaney, T., Mesa, V., Prediger, S., & Van Dooren, W. (2020). What can we do against racism in mathematics education research? *Educational Studies in Mathematics, 104*(3), 299-311

Researching experiences of mathematics: Black/feminist and queer lenses

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This symposium will explore how critical methodologies in mathematics education shed light on the sociohistorical, political, and cultural role of mathematics in shaping experiences of learning mathematics. Drawing from Black/feminist and queer theories, panelists and participants will consider the subjective experiences of learners that tend to be obscured in research, and how critical shifts in methodology might bring such stories to light.

Aims of the symposium

In the last four decades, research in mathematics education has seen a shift toward an examination of the cultural practice of mathematics and of the identities and subjective experiences that unfold in and through this culture (Darragh, 2016). Decentering achievement, this research has sought instead to consider the role of mathematics—as a historical, political, and cultural institution—in engendering particular learner experiences in formal STEM spaces (e.g., Mendick, Berge, & Danielsson, 2017). As such work has been primarily concerned with surfacing as-yet untold stories of learners in mathematics, it has simultaneously fostered the development of a variety of critical methods in mathematics education research.

The Mathematics Education and Society conferences and the wider MES community have been at the heart of this shift. However, feminist, intersectional and queer perspectives have been less commonly taken up here than in the wider critical education research literature. This symposium will address this gap by offering three examples of these perspectives in action in order to open up discussion on what they offer for research into mathematics education.

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Relevance

Scholars within the MES community have championed the perspective that mathematics is a cultural and political practice and that mathematics education is governed by neoliberal and imperialistic interests (Gutiérrez, 2013; Pais & Valero, 2012). For example, Mendick, Berge, and Danielsson (2017) made sense of the identities of two students in a Swedish upper-secondary science program in terms of "active accomplishments, neither fixed nor singular but multiple and fractured, and as coming into being through talk, actions, and relationships" (p. 486). Through multi-modal micro-analysis we see how students positioned themselves in and are positioned by neoliberal discourses around equity in STEM. Sengupta-Irving and Vossoughi (2019) located their analysis in the view that equitable STEM educational policies are sustained by U.S. hegemonic agendas that shape the gendered and racialized experiences of minoritized girls. In spite of this, the authors found that participants were able to reclaim science with "ingenuity and humanity" (p. 495), refusing the prescription of science as primarily for U.S. hegemony.

Such studies surface important stories—both about learners and about STEM institutions—that have fallen by the wayside. Dotson (2014) argued that stories go untold due to our "inclination to understand oppression as we experience it and to extend our analysis of it beyond what we ourselves can see from our particular vantage point" (p. 57). Importantly, most "socioepistemic orientations towards oppression will illuminate as much as they obscure" (p. 45). Ahmed (2006) wrote similarly about orientation: "some things are relegated to the background in order to sustain a certain direction; in other words, in order to keep attention on what is faced. Perception involves such acts of relegation that are forgotten in the very preoccupation with what it is that is faced" (p. 31). That is, attending to something necessarily implies not attending to something else. Thus our work as researchers demands regular reflection and recalibration, which we hope to engage in at this symposium. We ask: what stories does our vantage as researchers render "theoretically invisible" (Dotson, 2014, p. 46)? And how might Black/feminist and queer perspectives bring these stories to light?

Format of the symposium

The symposium will begin with three provocations that present some possibilities of using Black/feminist and queer lenses on mathematics experiences.

Provocation 1

Maisie Gholson will offer new epistemological methods to resist deficit-perspectives on Black learners in mathematics education research. Expanding on earlier work in Gholson & Martin (2019), she centers performativity and pain to illuminate some of the ways in which mathematics education is "violent, painful, and dehumanizing" (p. 393) to Black learners. Through an examination of the repetitive and resistive identity-based performances and gestures of Black girls, Gholson is interested in methods that animate the oppressive structures that operate within the mathematics classroom. Adopting Dotson's (2014) theory of oppression as a "multistable" phenomenon — which renders subjective experiences of oppression theoretically invisible in research — she argues that intersectional phenomenological lenses are crucial to telling such stories, while also helping demonstrate the broader politics of mathematics as raced, classed, and gendered.

Provocation 2

Sarah Radke's perspective seeks to disrupt dominant notions of learning and identity development as spatially and temporally linear. It builds on her collaborative work in Ma, Kelton, Radke, & Della Volpe (2020), in which a cross-setting analysis of participation illuminated how the "relational practices" (p. 259) of one youth were not merely incidental to a moment of statistical learning, but rather instrumental to it. Drawing on Butler's conception of performativity as contextual and structured by powered relations (1990), Radke's analytic lens is, in part, a response to Langer-Osuna and McKinney de Royston's (2017) call for tools to study how power mediates learning and positioning at multiple time-scales. Drawing from—and complicating—Saxe's (1991) focus on shifts in form-function relations across interactions, this view attends to the cultural, political, and social construction of a repertoire of forms across space and over time. Radke inspires us to reconsider where and how we look for learning, opening up the possibility that moments of learning and identity development are dispersed across experiences and interactions.

Provocation 3

Heather Mendick will speak about a research project with Eva Silfver, Maria Berge and Andreas Ottemo about contemporary geek identities. The data will be the opening sequences of the 2008 film *Iron Man*. These illustrate how mathematical, scientific and technological expertise is being repositioned. Previously the dominant image of the math/science/tech genius was a physically-weak socially-awkward geeky white man. Tony Stark / Iron Man exemplifies a new entrepreneurial configuration of the genius. While still geeky, white and male, he is both physically strong and socially confident. A queer poststructural reading of this geek entrepreneurial masculinity illuminates how it legitimates gender, economic and neo-colonial power relations, and represents a shift in hegemonic masculinity in which previously marginalised 'geeky' traits carry new status.

Symposium Structure

Participants and panelists will reflect on these provocations, sharing their own critical orientations to and analyses of the data. Following an initial discussion, panelists will describe the potential of their particular conceptual lenses for mathematics education research. Finally, the group will be encouraged to explore what intersectional, feminist, and queer lenses bring.

References

Ahmed, S. (2006). *Queer phenomenology: Orientations, objects, others.* Duke University Press. https://doi.org/10.1215/9780822388074

Butler, J. (1990). *Gender trouble: Feminism and the subversion of identity.* Routledge. https://doi.org/10.4324/9780203824979

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics, 93*(1), 19–33. https://doi.org/10.1007/s10649-016-9696-5

Dotson, K. (2014). Making sense: The multistability of oppression and the importance of intersectionality. In M. M. O'Donovan, N. Goswami, & L. Yount, (Eds.), *Why race and gender still matter: An intersectional approach* (pp. 43–58). Routledge. https://doi.org/10.4324/9781315654270

Gutiérrez, R., (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68. https://doi.org/10.5951/jresematheduc.44.1.0037

Langer-Osuna, J. M., & McKinney de Royston, M. (2017). Understanding relations of power in the mathematics classroom: Explorations in positioning theory. In A. Chronaki (Ed.), *Mathematics education and life at times of crisis: Proceedings of the ninth International Mathematics Education and Society conference* (Vol. 2, pp. 645–653). Volos, Greece: MES9.

Ma, J.Y., Kelton, M., Radke, S., & Della Volpe, D. (2020). Toward Conceptualizing Resources for Learning Across Settings. In Gresalfi, M. and Horn, I. S. (Eds.), *The Interdisciplinarity of the Learning Sciences, 14th International Conference of the Learning Sciences (ICLS) 2020, Volume 1* (pp. 254–261). Nashville, Tennessee: International Society of the Learning Sciences.

Martin, D. B. (2009). Researching race in mathematics education. *Teachers College* Record*, 111(2),* 295–338.

Mendick, H., Berge, M., & Danielsson, A. (2017). A critique of the STEM pipeline: Young people's identities in Sweden and science education policy. *British Journal of Educational Studies, 65*(4), 481–497. https://doi.org/10.1080/00071005.2017.1300232

Pais, A., & Valero, P. (2012). Researching research: Mathematics education in the political. *Educational Studies in Mathematics, 80*(1), 9–24.

Saxe, G. B. (1991). Emergent goals in everyday practices: Studies in children's mathematics. In F. Furinghetti (Ed.*), Proceedings of the 15th Conference of the International Group for the Psychology of Mathematics Education* (Vol 3, pp. 230–237). PME.

Sengupta-Irving, T., & Vossoughi, S. (2019). Not in their name: Re-interpreting discourses of STEM learning through the subjective experiences of minoritized girls. *Race, Ethnicity and Education, 22*(4), 479–501. https://doi.org/10.1080/13613324.2019.1592835

Disrupting normativity in mathematics education: Meeting queer students at the intersection of their queer and mathematics identities

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In western mathematics and mathematics education, normative structures serve to reinforce hierarchies of oppression along lines of race, gender identity, class, dis/ability, and sexual orientation. This symposium aims to create a space in which participants can discuss and interrogate, from an intersectional perspective, the normative structures of mathematics and mathematics education, specifically those that reinforce heteronormativity and gender-normativity and to discuss ways of re/humanizing mathematics for LGBTQ+ people.

Aims of the symposium

This symposium aims to create a space in which participants can discuss and interrogate the hetero- and gender-normative structures of mathematics that serve to dehumanize mathematics for LGBTQ+ people. We will discuss research on queer identity in mathematics teaching and learning, queer theory as a means to re/humanize mathematics for LGBTQ+ students, and implications for mathematics pedagogy.

Rationale

With the dawn of the 21st century, the field of mathematics education began experiencing a shift in perspective—embracing the idea that the teaching and learning of mathematics, previously regarded as neutral, is influenced by political and sociocultural factors (Gutiérrez, 2013). This shift in perspective has led many scholars to investigate how race, sex assigned at birth, dis/ability and other identities impact mathematics teaching and learning. More often than not, the findings of such studies suggest that mathematics teaching and learning reproduce normative social structures that serve to sustain oppressive hierarchies (Leyva, 2017; Mendick, 2006). These normative structures often lead to dehumanizing mathematics experiences for students from traditionally marginalized groups (Goffney, Gutiérrez, & Boston, 2018; Tan et al., 2019).

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In Western schools, queer students are particularly vulnerable to dehumanizing experiences in educational spaces (Watson & Miller, 2012), given the politicization of their identities by religious groups and the experiences of harassment or assault that queer students report in K-12 and higher education institutions. Over the last decade, GLSEN has consistently reported that over 85% of LGBTQ+ students in U.S. grades K-12 have been harassed or assaulted at school (Kosciw et al., 2020). Such dehumanizing experiences continue for queer students in institutions of higher education, where they are significantly more likely than their non-queer peers to report experiences of harassment, discrimination, and feelings of being unsafe at their institution (Greathouse et al., 2018). Around the world, in the mathematics classroom, such experiences come in a variety of forms, from the complete erasure of LGBTQ+ people and experiences in textbook problems (Esmonde, 2011; Waid, 2020) to stereotypes pertaining to masculinity, femininity, and what it means to be "good at math" (Mendick, 2006). Such dehumanizing experiences may explain why, for example, queer students are less likely to complete Algebra II than those that do not identify as queer (Whipple, 2018).

At the time of writing this proposal (less than 70 days into the year), 2021 has seen the murder of least 10 transgender or gender nonconforming people (at least half of whom were Black transgender women) in the U.S. (Human Rights Campaign, 2021a) and there are 147 anti-LGBTQ+ measures (73 of which target transgender people specifically) being considered by U.S. state legislatures (Human Rights Campaign, 2021b). Considering the results of GLSEN's National Climate Surveys (e.g., Kosciw et al., 2020) and Greathouse et al.'s (2018) study of the experiences of queer people at institutions of higher education, it is clear to us that these hate-filled acts of violence against queer people are a symptom of a larger problem of homo- and transphobia that is reinforced by the normative structures of western society—normative structures that are reproduced in our mathematics classrooms. We believe, however, that this can change. Such change would require that mathematics educators begin to identify hetero- and gender-normative structures in mathematics education and develop tools to interrogate and disrupt those structures.

In this symposium we invite participants to envision a new, rehumanized form of mathematics, one that honors queer identity, knowledge, and experience. We will provide educators a lens through which to understand the intersection of queer and mathematical identity, namely through the use of the Queer Identity Intersection (QII) of Mathematics Education (Moore, 2020). The QII necessitates critiques of the ideologies of mathematics education and discussions of how they collide with students' subjectivities—gendered, sexualitied, and otherwise. Using this lens, we will then explore the following questions: What hetero- and gender-normative structures are present in mathematics education? What other normative structures exist in mathematics education? How do we interrogate and disrupt those normative structures in ways that honor and re/humanize mathematics (and the larger schooling environment) for queer students, educators, and students from queer families?

Symposium structure

The symposium will begin with a discussion of the mathematics classroom as a potentially dehumanizing space for queer students (building upon the literature presented earlier in this proposal). This will be followed by an exploration of the intersection between mathematical identity and queer identity, using Moore's (2020) Queer Identity Intersection (QII) of Mathematics Education. To begin thinking about how one might navigate the "road" of Queer Identity, participants will be invited to identify (in small groups) hetero- and gender-normative structures present in mathematics education. Presenters will then discuss how they have worked to enact critical/queer pedagogies in their own teaching to disrupt those normative structures and re/humanize mathematics for queer students in K-12 public and independent school settings in the US, as well as with graduate and undergraduate students pursuing degrees in mathematics and mathematics education.

References

Esmonde, I. (2011). Snippets and snails and puppy dogs; tails: Genderism and mathematics education. *For the Learning of Mathematics*, *31*(2), 27–31. https://doi.org/10.1016/s0140-6736(51)93464-2

Goffney, I., Gutiérrez, R., & Boston, M. (Eds.) (2018). *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students.* National Council of Teachers of Mathematics.

Greathouse, M., Brckalorenz, A., Hoban, M., Huesman, R., Rankin, S., & Stolzenberg, E. B. (2018). *Queer-spectrum and trans-spectrum student experiences in American higher education: The analyses of national survey findings.* Rutgers Tyler Clementi Center.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 37–68. https://doi.org/10.5951/jresematheduc.44.1.0037

Human Rights Campaign (2021). *Fatal violence against transgender and gender non-conforming community in 2021.* https://www.hrc.org/resources/fatal-violence-against-the-transgender-and-gender-non-conforming-community-in-2021

Human Rights Campaign (2021, March 11). *Breaking: First anti-trans bill of 2021 signed into law by Mississippi governor Tate Reeves* [Press release]. https://www.hrc.org/press-releases/breaking-first-anti-trans-bill-of-2021-signed-into-law-by-mississippi-governor-tate-reeves

Kosciw, J. G., Clark, C. M., Truong, N. L., & Zongrone, A. D. (2020). *The 2019 National School Climate Survey: The experiences of lesbian, gay, bisexual, transgender, and queer youth in our nation's schools.* GLSEN.

Leyva, L. A. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. *Journal for Research in Mathematics Education*, *48*(4), 397–433. https://doi.org/10.5951/jresematheduc.48.4.0397

Mendick, H. (2006). *Masculinities in mathematics. Educating boys, learning gender.* Open University Press.

Moore, A. S. (2020). Queer identity and theory intersections in mathematics education: A theoretical literature review. *Mathematics Education Research Journal.* Advance online publication. https://doi.org/10.1007/s13394-020-00354-7

Tan, P., Padilla, A., Mason, E. N., & Sheldon, J. (2018). *Humanizing disability in mathematics education.* National Council of Teachers of Mathematics.

Waid, B. E. (2020). Supporting LGBTQ+ students in K–12 mathematics. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(11), 874–884. https://doi.org/10.5951/MTLT.2019.0403

Watson, S., & Miller, T. (2012). LGBT oppression. *Multicultural Education, 19*(4), 2–7. https://files.eric.ed.gov/fulltext/EJ1014937.pdf

Whipple, K. S. (2018). *LGBTQ secondary mathematics educators: Their identities and their classrooms* [Doctoral dissertation, University of Minnesota].

Project Presentations

Teacher agency and professional learning: Narrative explorations

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There is considerable research on teachers' professional learning in mathematics, much of it written by those involved in leading 'development' activities and projects. In this paper, secondary mathematics teachers' accounts of learning provide a stimulus for an exploration of teacher agency and professional learning. Differing experiences and perspectives inform an evolving theoretical framing of mathematics teacher agency, an examination of how agency is achieved and restricted in professional learning.

Introduction

Much of the literature on teachers' professional learning in mathematics (and in other areas) is written by those involved in teacher professional 'development', with studies focussing on evaluations of specific professional development initiatives. Such studies contribute to knowledge of how these initiatives impact on teachers' beliefs and practices. However, such initiatives and programmes are frequently developed from a position that views teachers' skills, knowledge and practice as deficient in some way, with interventions taking the form of remediation. Where teachers' voices are heard, they are frequently restricted to their experience of a particular programme. What happens if we take a different starting point in a consideration of mathematics teacher learning, focussing on individuals' experiences and our own critical reflections?

In this paper I discuss an on-going exploration of teacher agency in relation to professional learning. I start from a fragmentary recollection of my own experience as a teacher, an account initially centred around learning through a one-day event. Other fragments are drawn from a life history study of mathematics teachers' experiences. These fragments provide snapshots of professional learning experiences from the early 1990s, when collective approaches to teacher learning and to curriculum development were beginning to be challenged by increasing state control, to a time of increasing national reform in the late 1990s, through to the early 2000s, when these reforms increasingly emphasised performativity.

Teacher agency

There is growing interest in teacher agency in relation to school reform (Pyhältö, Pietarinen, & Soini, 2014; Lasky, 2005) and teacher professional development (Insulander et al., 2019;

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Martinie et al., 2016). In England, continued challenges in recruiting and retaining secondary teachers in particular subjects, including mathematics, has prompted renewed focus on teachers' experience, particularly on their 'job satisfaction' (Worth & Van den Brande, 2020). In their study of retention and satisfaction in England, Worth & Van den Brande focus on autonomy, defined as a capacity to make informed decisions. The concepts of autonomy and agency are variously understood in the literature and often conflated. In this paper, I work towards a clarification of teacher agency through critical engagement with professional learning stories. I take as a starting point, Eteläpelto et al.'s (2013) definition of professional agency as 'exercised when professional subjects and/or communities influence, make choices and take stances on their work and professional identities' (p. 61). Their argument for a 'subject-centred socio-cultural and life-long learning perspective' (p. 60) facilitates a study of teachers' learning as they navigate their careers, changing roles and moving workplaces, engaging in identity work that encompasses professional and personal lives. An alternative framework advanced by Emirbayer and Mische (1998) enables the in-depth study of agency in action. They define human agency as 'a temporally embedded process of social engagement, informed by the past (in its habitual aspect), but also oriented toward the future (as a capacity to imagine alternative possibilities) and toward the present (as a capacity to contextualize past habits and future projects within the contingencies of the moment)' (Emirbayer & Mische 1998, p. 963).

Working with these temporal dimensions of agency, Priestley et al. (2015) develop a framework to aid enquiry into teacher agency. This encompasses teachers' life histories and their professional experiences (iterational element), social, cultural and structural aspects (practical-evaluative element) and long- and short-term aspirations (projective element) (Priestly et al., 2015, p. 30).

These conceptualisations of agency focus on the individual within a socio-cultural context. Relationships figure in these conceptualisations as workplace conditions. An alternative, perhaps complimentary theorisation of agency, relational agency, involves a focus on the recognition of others as a resource, acknowledging that 'work needs to be done to elicit, recognise and negotiate the use of that resource' (Edwards, 2005, p. 172). In shifting the focus from the individual, Edwards directs our attention to moral purposes of working together, highlighting possibilities of individual and collective benefits. Thus, relational agency is seen as an enhanced version of individual agency. Edwards draws attention to implications for professional learning, noting that the capacity for relational agency can be developed and that it may support teachers to recognise the value of working with others and negotiating meanings, rather than these actions being seen as evidence of a lack of skill or competence.

Building on the discussions above, I develop an analytic framework that enables exploration of the ways that policy might restrict or enable the achievement of teacher agency. Using vignettes drawn from life history studies, I consider policy effects across Emirbayer and Mische's temporal dimensions of agency (1998), focussing particularly on relational aspects.

Methodology

In this paper I revisit narratives co-constructed as part of my doctoral studies (Adams, 2013). These stories have remained significant to me, informing historical policy studies. Although they represent moments in time my understanding of them shifts as my knowledge, perspective and awareness shifts. Such revisiting of narrative research provides opportunities to 'explore the new and unfolding meanings' (Andrews, 2007, p. 5).

Vignettes or 'compact sketches' (Ely, Vinz, Downing, & Anzul, 1997, p. 70) are found in various forms in qualitative research; here they enable the introduction of characters, together with a glimpse of their experience. In addition to these vignettes, I begin with a reflexive account of my own experience as a secondary mathematics teacher, before working with documentary evidence to gain an alternative perspective on this experience. The account centres on recollections of one day in 1990, this day represents an impression of my experiences across several similar days across the first fifteen years of teaching. Much of my learning in that period was related to SMILE mathematics, a curriculum development project initiated by teachers in the 1970s (see Gibbons, 1975).

Vignettes drawn from the narrative study focus on two periods. The first, in the late 1990s, when changes initiated in the previous decade had gathered pace, the National Curriculum was undergoing its third revision and the focus on standards and accountability measures was growing. The second vignette is from the early 2000s, a time of global education reform, when the National Strategy was introduced in England.

Discussion

A focus on teachers' experiences of professional learning, albeit in the (short) form of vignettes drawn from a life history study, reveals how agency is achieved and constrained. The fragments provide starting points for an analysis of the complex factors influencing teachers' capacity for agency, broadening discussions of effective professional learning beyond specific opportunities provided to examine policy influences on individual and community. My reflections on learning through the SMILE project together with teachers accounts of learning in times of high accountability raise questions about scope for (and value of) collective learning activities, questions of learning what and why.

Accounts of complexity and variation are often missing from studies of mathematics teacher professional learning, yet such accounts can be productive, inviting us to reflect on our own experience. This initial work with selected vignettes will facilitate the development of an analytic framework to be utilized in further historic mathematics professional learning policy analyses.

References

Adams, G. (2013). *Is anyone listening? Women mathematics teachers' experiences of professional learning* [Doctoral dissertation, Open University].

Adams, G., & Povey, H. (2018). 'Now there's everything to stop you': Teacher autonomy in performative times. In M. Jurdak. & R. Vithal (Eds.), *Socio-political dimensions of mathematics education: Voices from margin to mainstream* (pp. 209–230). Springer.

Andrews, M. (2007). *Shaping history: Narratives of political change.* Cambridge, United Kingdom: Cambridge University Press.

Edwards, A. (2005). Relational agency: Learning to be a resourceful practitioner. *International Journal of Educational Research, 43*(3), 168–182.

Ely, M., Vinz, R., Downing, M., & Anzul, M. (1997). *On writing qualitative research: Living by words.* London: Falmer Press.

Emirbayer, M., & Mische, A. (1998). What is agency? *The American Journal of Sociology, 103*(4), 962–1023.

Eteläpelto, A., Vähäsantanen, K., Hökkä, P., & Paloniemi, S. (2013). What is agency? Conceptualizing professional agency at work. *Educational Research Review, 10*, 45–65.

Insulander, E., Brehmer, D., & Ryve, A. (2019). Teacher agency in professional development programmes: A case study of professional development material and collegial discussion. *Learning, Culture and Social Interaction, 23.* https://doi.org/10.1016/j.lcsi.2019.100330

Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education, 21*(8), 899–916.

Martinie, S. L., Kim, J., & Abernathy, D. (2016). "Better to be a pessimist": A narrative inquiry into mathematics teachers' experience of the transition to the Common Core. *Journal of Educational Research, 109*(6), 658–665.

Pyhältö, K., Pyhältö, K., Pietarinen, J., Pietarinen, J., Soini, T., & Soini, T. (2014). Comprehensive school teachers' professional agency in large-scale educational change. *Journal of Educational Change, 15*(3), 303–325.

Worth, J., & Van den Brande, J. (2020). *Teacher autonomy: How does it relate to job satisfaction and retention?* Slough: NFER.

A critical gaze on new digital technology: Answers from mathematics education?

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An increasing concern have been expressed in both academic and public debate that new digital technology might undermine democratic values and practices. This paper explores how studies in the field of Mathematics Education could present different answers to ramifications of new digital phenomena for both individuals and society. A transdisciplinary approach under a post structural theoretical framework is suggested, and two tentative studies are presented. One study will carry out a critical text analysis of Swedish educational steering documents and one will be a classroom action research study, where discourses will be analysed.

New digital phenomena in the world of information

Digital technology is an integral part of our lives. Information flows from the Internet to us. However, scholars within different research fields have also drawn attention to the flow in the other direction, from us to the Internet (Kosinsky 2013; Zuboff 2019). Following this, the aim of this paper is to suggest possible studies in mathematics education as answers to ramifications of new digital phenomena for both individuals and society. For heuristic reasons, a distinction is made upon the direction of the information flow. Techniques that make use of the flow *from* us – our digital trace – are called *reading* techniques since they can predict attributes never explicitly stated. Techniques that make use of the flow *to* us are called *writing* techniques since the can change our behaviour, i.e. rewrite us. A full understanding of the new digital phenomena comes with the realization that they are formed by a symbiosis of reading and writing techniques, like lichen are formed by a symbiosis of algae and fungus.

Techniques reading more about us than we tell

We leave digital traces when we are online. They show our history of preferences, and are used to predict what videos, music or books we might prefer when continuing using such services. Thus, our behavior is constantly being read. The increasing size of this information flow have made it possible to see more general patterns in the data. This enables prediction of information that we might be unwilling to provide if asked. With methods from linear algebra, it is possible to predict attributes such as sexual orientation, ethnicity, religious and political views, and to some degree even personality. This is possible by merely having access

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to information of users' interaction with the 'like' function on Facebook (Kosinsky 2013). The accuracy of the predictions increases with more information, and companies that sell advertisement have access to much more information than just 'likes'.

The economic incentive to read humans through digital traces has been proposed by Zuboff (2019) to create a new form of capitalism, called surveillance capitalism, which operates with our traces as raw materials, mathematical methods as means of production and behavior predictions as products. She warns that this mostly unregulated form of capitalism is prone to exploit the lack of users' awareness for profit, no matter the human cost. In fact, she underlines that people need to be kept unaware, else the extraction of predictive value from digital traces would not work.

Techniques writing our future

Both online and offline behavior can be changed by what we see online, and we do not always select independently what will be shown. Techniques that affect the selection of content shown may therefore 'write' our behavior. One example was demonstrated by Bond et al (2010), who showed that voter turnout in the 2010 US congressional mid-term election was increased by planting a message on social media. Increasing voter turnout is beneficial for democracy, but the opposite has also been tried. In the United States 2016 presidential election, the Trump campaign used citizens' digital traces in an attempt to deter them from voting. Sponsored messages on social media are routinely distributed to people based on what is known about them. In this case though, the 3.5 million-person list destined for messages deterring them from voting, consisted *only* of individuals identified as not likely to vote for Trump (Sabbagh 2020). It is debated to what extent such measures can change the outcome of elections. Nevertheless, the fact that they are being used changes the democratic process of election campaigns to now incorporate mathematical modelling of Big Data.

Other writing techniques are the algorithms in search engines and on social media. They select what the user will be shown. However, it can easily be demonstrated that they show different content depending on historical digital traces. Reading and writing are thus intertwined here, both feeding the other. Limitation of the users' agency must therefore be understood in relation to this *interaction* of reading and writing, and not by them separately. This is also true for the attempted voter suppression.

Foucaultian theory and a transdisciplinary holistic approach

To be able to propose studies in mathematics education answering to the new roles of mathematics in society, a Foucaultian (Foucault 1995) inspired approach is one way. As an example, I appropriate the concepts *discourse, subjectification* and *dispositive* as tools to analyze the interaction between the new digital phenomena, the individual and society. *Discourse* will be used close to Foucault's (1995) own work, meaning not only language but also norms, habits, artifacts, institutional praxis etc. *Subjectification* as I use it, describes how the self is developed in interaction with the flow of information partly steered by algorithms.

The reading-writing algorithms would here be viewed as reinforcing the normalization process that produce subjects in relation to a milieu. By trying to resist this, the mechanics of power in the system may become confirmed and reproduced rather than refuted. One example is the usage of programs that generate random cookies to blur the digital traces. They confirm and propel the notion of a relation between the ability to read our digital traces and production of power. *Dispositive* in my adaptation, is used to envisage how the digital phenomena are linked and constitute an integrated knowledge structure that exercise power in society. This is exemplified by surveillance capitalism.

Research in mathematics education concerning the relation between mathematical techniques and democracy, should take into consideration the limits of what mathematics can achieve. Some mathematical problems are not solvable if all contemporary views on equity are to be respected. Thus, it would then be wrong to accuse algorithms that attempt the impossible to be unjust. It would rather be the expectations on what is solvable under certain combinations of societal principles of justice that would be unfair. One example is algorithms using criminological data to predict risk of recidivism. Such algorithms are used to select whom to keep in jail until trial. When optimized for public safety, they will either give more false positive in some groups (discrimination) or treat groups according to different standards (e.g., depend explicitly on ethnicity, which may be illegal) (Corbett-Davies et al. 2017).

Therefore, understanding of the general problem depends not only on contributions from several disciplines, but also on how these contributions interact. I suggest a transdisciplinary holistic approach including results from data science, sociology, mathematics and mathematics education.

Critical text analysis of steering documents

A starting point in the overall aim of exploring answers could be in the already present. In a critical text analysis it will be investigated what discourses can be construed in relation to democracy, in present steering documents for Swedish Upper Secondary School. Of special interest is mathematics role in subjectification of democratic citizenship and the dispositive of mathematical practices.

The text analysis will include both general steering documents and mathematics specific documents to see what differences exists. Inclusion of the mandatory national tests in mathematics will add the important aspect of assessment. Any discrepancies between what is to be taught and what is to be assessed is part of understanding the present. The critical aspect of the text analysis will be investigating what is present and what is *not* present in the steering documents, both in terms of current internal consistency and also when relating them to the new digital phenomena. This provides a point of departure when exploring revision of curricula.

Action research

An important aspect of the exploration of answers is what happens when teachers and students discuss the new digital phenomena in a classroom setting. For ecological validity, this will be investigated in classrooms in Swedish Upper Secondary schools together with teachers that already have an interest in teaching this content. Through negotiations with the teachers about details in objectives, methods, roles, etc, it will be attempted to achieve a milieu that augment learning for students, teachers and researchers under the framework of action research. This process will be unpredictable and the exact form of the study cannot be known beforehand. However, some general principles can still be sketched.

A concern is that the mathematical methods used from linear algebra are too different from the content of any pre-university curriculum, especially in their complexity. However, drawing on Skovsmoses (1990) three notions of knowledge, mathematical knowledge, technological knowledge, and reflective knowledge, I would suggest that the aim here is not for students to be able to use the methods themselves. The objective is rather reflective knowledge, i.e. knowledge about the methods, their limitations etc.

Data in form of observations, video, and interviews will be analysed though a discursive Foucaultian theoretical lens. Discourses can be construed from several viewpoints; (1) finding possible obstacles, what differences exist between teachers and students, before, under and after teaching (2) how is the complexity of the digital phenomena expressed, e.g., the reading-writing dialectic, (3) relating the classroom to research and public debate, (4) role of mathematics, (5) subjectification in the digital context, (6) dispositive of digital technology in a sociological context.

Which exact route the analysis will take depends on the nature of the discourses construed from the data. Nevertheless, the outcome will be relevant to the overarching aim of exploring how mathematics education can answer to new digital phenomena.

References

Bond, R., Fariss C., Jones J., Kramer A., Marlow C., Settle J., & Fowler J. (2012). A 61-Million-Person Experiment in Social Influence and Political Mobilization. *Nature 489*, 295-298

Corbett-Davies, S., Pierson, E., Feller, A., Goel, S., & Huq, A. (2017). Algorithmic decision making and the cost of fairness. In *Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining,* (pp. 797-806). ACM.

Foucault, M., (1995). *Disipline & Punish.* Vintage Books.

Kosinski, M., Stillwell, D., & Graepel, T (2013). Private traits and attributes are predictable from digital records of human behavior. In K. Wachter (Ed.) *Proceedings of the National Academy of Sciences 110*(15), (pp. 5802-5805). PNAS.

Sabbagh, D. (2020, September 28) Trump 2016 campaign 'targeted 3.5m black Americans to deter them from voting' The Guardian https://www.theguardian.com/us-news/2020/sep/28/trump-2016-campaign-targeted-35m-black-americans-to-deter-them-from-voting

Skovsmose, O. (1990). Mathematical education and democracy. *Educational studies in mathematics, 21*(2), 109-128.

Zuboff, S. (2019). *The age of surveillance capitalism.* Public Affairs, Hachette Book Group.

Emerging teacher identities: Exploring the identity negotiation of early career teachers of mathematics

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In England, the recruitment and retention of secondary mathematics teachers is of continuing concern to governmental bodies. The teachers most likely to leave the profession are in their first five years of teaching, where their performance is subject to intense scrutiny, judgement, and measurement. Given this context, a better understanding of the identity negotiation of these teachers could improve teacher retention, as well as help prepare and support teachers at the start of their career. This presentation will focus on the early analysis of a pilot study which aims to understand how context and experiences with mathematics as a student can contribute to an emerging teacher identity.

Background

The manifestations of neoliberalism can be seen in all aspects of English education policy, including the way teachers' work is specified through detailed teacher standards and monitored and audited through performance management systems and mechanisms. The current context poses additional challenges for early career teachers (ECTs), defined as those with less than five years of experience, whose 'performance' is subject to even more intense scrutiny than that of other teachers (Hobson & Maxwell, 2017) and who are required to pass both their training course and first year of teaching against the same set of standards. In an attempt to assess the quality of teaching and learning, policy makers have often chosen lesson observations as the easiest way to capture the complexities of teachers' work. A result of such high-stakes assessment of teachers' practice is that many teachers discount pedagogical ideas and practices that do not work to their personal advantage in a performative system (Loh & Hu, 2014; Smagorinsky et al., 2004). ECTs in particular may find themselves having to make difficult choices between pleasing the cultural gatekeepers of the school and their own ideals and values.

This study focuses on the identity negotiation of ECTs of mathematics during the period of induction into their school. During socialisation into the school setting, the school culture coupled with workplace opportunities and constraints on pedagogical choices impact on an emerging professional identity. This research takes place at a time of significant policy change for the induction of ECTs with the launch of the Early Career Framework (ECF), which has been promoted as "the most significant reform to teaching in a generation"

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(Department for Education (DfE), 2019, p. 6). Introduced in a select number of areas from September 2020 and nationally from September 2021, the ECF entitles new teachers to a two-year induction, access to a trained mentor and a professional development programme delivered by one provider, chosen by the school from a small number of DfE approved providers. How each school chooses to interpret, enact and manage the framework means that the school context has never been of more importance when understanding the development of ECTs.

This research will provide an important insight into how the introduction of the ECF affects the identities of mathematics ECTs. This early stage in a teacher's career is crucial for researchers and government bodies to understand if the recruitment and retention of secondary school teachers is to improve. The recruitment of secondary mathematics teachers has been below target since 2014 (Worth & Van den Brande, 2019) and while ECTs are the teachers most likely to leave the profession, leaving rates for mathematics ECTs are amongst the highest of any subject (Worth & De Lazzari, 2017).

My ongoing research therefore seeks to answer the following questions: 1) what are the influences on ECTs of mathematics' developing identities over the two years of their induction to the school? 2) how does context shape the teacher identities of ECTs of mathematics? 3) how do teachers' experiences with mathematics influence the development of a teacher identity?

Theoretical framing

As ECTs are socialised into their schools, membership in communities of practice (Wenger, 1998) allow them to negotiate the meanings of their experiences and help build an identity. In these socially and culturally constructed realms, or figured worlds, ECTs negotiate their identities through participating in the "coproduction of activities, discourses, performances, and artifacts" (Holland et al., 1998, p. 51). Such situated theories, by focusing on the examples of participation that constitute learning, are useful for my study because they help gain insights in the emerging identities of teachers as ECTs are *becoming* a part of their community of practice.

Becoming a teacher demands significant personal investment, both of time and energy. It is also a period of intense identity negotiation. ECTs enter the profession with an identity which is strongly embedded (Flores & Day, 2006); they have a clear image and an ideal of what it means to be a teacher. This ideal, or designated identity, can influence ECTs' actions, and they may be disappointed if there is a perceived persistent gap between their current and ideal identities. This ideal may also be challenged by their workplace context, either positively or negatively, and ECTs therefore have to reconstruct their professional identities accordingly.

Establishing what impacts on a strongly embedded identity has prompted some studies to look explicitly at past experiences of school mathematics (de Freitas, 2008; Flores & Day, 2006) and conceptions of what it means to learn mathematics (Ma & Singer-Gabella, 2011). This recognises that mathematics teachers' experiences as learners and doers of mathematics is vital to understanding their teacher identity. By paying attention to ECTs' past experiences and beliefs, it becomes possible to consider whether their start to teaching is as they had imagined, and how their experiences in school and classroom contexts have shaped or challenged their beliefs.

My initial literature review shows that the majority of studies utilise a sociocultural perspective, thereby responding to Lerman's (2000) call for a theory which equates learning with developing an identity in communities of practice (Lave & Wenger, 1991; Wenger, 1998). While this perspective is useful for my own study, as it focuses on the impact of social practices and context, I still appreciate that an individual's inner world is of importance. The individual emphasis alongside the social context would require a psychosocial approach and is used by far fewer researchers (for example Boylan & Woolsey, 2015), but offers a potentially more balanced approach. I consider the applicability of this approach through reflections on the pilot study.

Proposed methodology

In order to explore the identities of ECTs of mathematics, teachers' stories of their experiences will be collected to help develop a narrative study. In doing so, I equate these stories themselves with the identities (Sfard & Prusak, 2005). In sharing their experiences, I hope to allow participants the opportunity to reflect as they interact with the people and norms of their school context. Narratives allow me to explore the interaction of factors which contribute to a teacher's identity and look for any features in the teachers' narratives which demonstrate identity negotiation.

Final reflections

Identity in mathematics education research is useful as a way of understanding both individuals' experiences of teaching and learning mathematics and wider issues of context, social interactions and power dynamics (Darragh, 2016). My study will collect the stories of ECTs, which provide rich data about the process of *becoming* a mathematics teacher in a particular context. These stories can provide the means to effectively support ECTs to critically reflect on their learning and promote teacher agency throughout their early professional learning. By exploring the emerging identities of ECTs of mathematics, my study will make an important contribution to understandings of mathematics teachers' negotiations at a time of significant policy shifts.

In this presentation I will discuss my emerging theoretical framework, reflections on method(s) used in the pilot study and consequences for the main study and any further research.

References

Boylan, M., & Woolsey, I. (2015). Teacher education for social justice: mapping identity spaces. Teaching and Teacher Education, 46, 62-71. https://doi.org/10.1016/j.tate.2014.10.007

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics, 93*(1), 19-33. https://doi.org/10.1007/s10649-016-9696-5

de Freitas, E. (2008). Troubling teacher identity: preparing mathematics teachers to teach for diversity. Teaching Education (Columbia, S.C.), 19(1), 43-55. https://doi.org/10.1080/10476210701860024

Department for Education. (2019). *Early Career Framework.* (DFE-00015-2019). https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/773705/Early-Career_Framework.pdf

Flores, M. A., & Day, C. (2006). Contexts which shape and reshape new teachers' identities: A multi-perspective study. *Teaching and Teacher Education, 22*(2), 219-232. https://doi.org/10.1016/j.tate.2005.09.002

Hobson, A. J., & Maxwell, B. (2017). Supporting and inhibiting the well-being of early career secondary school teachers: Extending self-determination theory. *British Educational Research Journal, 43*(1), 168-191. https://doi.org/10.1002/berj.3261

Holland, D., Skinner, D., Lachicotte, W., & Cain, C. (1998). *Identity and agency in cultural worlds.* Harvard University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation.* Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). Ablex Publishing.

Loh, J., & Hu, G. (2014). Subdued by the system: Neoliberalism and the beginning teacher. *Teaching and Teacher Education, 41*, 13-21. https://doi.org/10.1016/j.tate.2014.03.005

Ma, J. Y., & Singer-Gabella, M. (2011). Learning to teach in the figured world of reform mathematics: Negotiating new models of identity. *Journal of Teacher Education, 62*(1), 8-22. https://doi.org/10.1177/0022487110378851

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14-22. https://doi.org/10.3102/0013189X034004014

Smagorinsky, P., Cook, L. S., Moore, C., Jackson, A. Y., & Fry, P. G. (2004). Tensions in learning to teach. *Journal of Teacher Education, 55*(1), 8-24. https://doi.org/10.1177/0022487103260067

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity.* Cambridge University Press.

Worth, J., & De Lazzari, G. (2017). *Teacher retention and turnover research. Research update 1: Teacher retention by subject.* Slough: NFER. https://www.nfer.ac.uk/publications/nufs01/nufs01.pdf

Assumptions, agency, and authority: Mathematical modelling and students' socio-critical reasoning

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Data from the pandemic has afforded the use of real-world modelling problems in mathematics classes to consider socio-critical mathematics. From an ongoing project, we examine grade 9 students' written work while solving a modelling task to consider their agency and authority as mathematicians. We use this analysis to posit that the "making assumptions" stage of the modelling cycle is a key moment in modelling for students' agentic decision-making via making and revising assumptions as they construct a model. We discuss our emerging conception of students' modelling authority and agency and conjecture about the ways that teachers' instruction may afford or constrain these acts. We contribute to the emerging research on modelling instruction and students' socio-critical reasoning.

Mathematical modelling from a socio-critical perspective focuses on how mathematics can be used to critique and inform decision making within society (Barbosa, 2006). Mathematical modelling is a non-neutral process in this perspective; the interpretations of a problem context and the assumptions that follow shape the development of the model and resultant solution (Anhalt et al., 2018).

The practices involved in socio-critical modelling activities ask students to reason throughout real-world tasks such that they author mathematical and situational-relevant knowledge to read and critique their world. They also propose solution paths and logically connect their interpretations and decisions with their model and solution (Blum & Leiß, 2005). The modelling cycle stages of identifying the problem and making assumptions can be seen as critical for achieving these goals, as they offer explicit opportunities for students to act agentically (Anhalt et al., 2018). While goals for student engagement in complex, critical modelling tasks have recently gained traction in the literature (Kaiser, 2017), understanding how they might be actualized in instruction needs further study. Additionally, there is minimal literature on instructional strategies to support students' agency in complex modelling tasks (Elliott et al., 2019; Kaiser, 2017). In this paper, we explore how U.S. grade nine students engaged in a modelling task, which explicitly attended to making and reflecting on assumptions, and developed agency and authority via socio-critical reasoning.

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Theoretical background

Socio-critical mathematical modelling is an opportunity for productive disciplinary engagement that examines the relationships between *problematizing* and *resources,* and *authority* and *accountability*, while recognizing how power shapes contexts for learning (Agarwal & Sengupta-Irving, 2019; Engle, 2012). Specifically, students' identities and histories shape their modelling experiences, in part through the assumptions they make and the reasoning they provide. Socio-critical modelling tasks provide opportunities for students to consider contexts in which they problematize situations, assert their authority, and leverage resources to solve the problem. These tasks, which are often ill structured and complex, require students to identify conditions and assumptions central to the task (Blum & Leiß, 2005). In this study, we consider the resources students accessed to support their problematizing and resultant reasoning. Resources for modelling include those students access from previous modelling experiences as well as new resources they seek as they consider the problem. We also consider the balance between *authority* and *accountability* as students develop socio-critical modelling practices while engaged in a task. *Agency* can be seen when students are authorized to take a stance on a task or method, and this can develop into *authority* as mathematicians and thinkers as they get recognized for their ideas in a more public space. With authority comes the need for *accountability*, or the ability to articulate how and why one's ideas make sense. As students build authorship, they must also build accountability to disciplinary practices, mathematical logic, and to their community of learners and stakeholders. Through a socio-critical modelling task which leveraged student assumptions, we seek to explore the relationship between agency, authority, and accountability evident in students' reasoning.

Study context and methodology

Data for this paper draw from broader study of teachers' instructional tools for modelling. The students in this study had abruptly shifted to remote learning in March 2020. In this transition, their teacher posed the following scenario, *Toilet Paper (TP) Task* (Figure 1).

How many rolls of toilet paper will a household use during the COVID - 19 pandemic?

Please submit your solution, supporting work, justification, and reasoning. Be sure to state any assumptions (decisions you made about unknown quantities) you made while calculating.

Figure 1: Modelling task for grade 9 students.

The grade 9 teacher and students had solved modelling tasks prior to remote learning using *modelling instructional routines.* These routines scaffolded students' learning of specific modelling practices, such as making assumptions and iteratively justifying models (Elliott et al., 2019). For the *TP Task*, the teacher asked students "to be thinking about those [modelling practices] … as they went through the task and answering like what assumptions they made and how did that affect the final solution." Students were asked to independently complete the task as directed in Figure 1. The student work and the teacher's reflection on the use of the task were data for this paper.

We examined students' work for their agentic moves to author their own interpretations of the task, their assumptions about the scenario, and the solution paths they developed. We also looked for ways students held themselves accountable to different sources of authority in their socio-critical reasoning processes. After reading the student work independently, the research team met to discuss themes of *agency/authority* and *accountability* in student work. We coordinated these data with the teacher's reflections provided during an interview on modelling instruction. Looking at the themes and evidence across all of the data allowed us to notice patterns in the ways students acted agentically to construct solutions to a societal problem and supported their reasoning.

Findings and discussion

As students presented their models they attended to conditions and assumptions that would shape their models. Students' agentic moves were made apparent when they considered conditions such as students' own household size and the potential length of the pandemic. They used these conditions to make assumptions about variables and quantities needed for their models, including the average rate of TP use when quarantined at home, average household size, gendered composition of family, and length of the pandemic. Students' histories and identities shaped the nature of conditions and assumptions they considered. Several students weighed the benefits and limitations of using single- versus double-ply toilet paper; one student noted that they "refused to even think about 1-ply toilet paper" and supported their reasoning by citing average American incomes to justify that a typical budget would be able to sustain 2-ply paper. These assumptions capture students' underlying attention to socioeconomics and humor, while grounding their arguments using data.

Students' attention to sources of accountability to justify features of models varied. A quarter of the students provided websites or organizational references and most students drew upon their own authority and solved the task taking into account their own household data. Given our analysis occurred 12 months into the pandemic, we found students' attention to accountability and authority to assert the duration of the pandemic most interesting. One student held themselves accountable to the World Health Organization and the Center for Disease Control to articulate their model. Another referenced that fact that "vaccines are still being tested and thus the pandemic will likely be in effect 900 days." A third stated that "scientists" have suggested the pandemic will last 52 weeks. Others relied on their own authority to determine the duration of the pandemic, making explicit that they were assuming the particular time frame; some students claimed it could last anywhere from 25 days to seven months.

We found a majority of students made explicit their assumptions which, given the teacher's initial prompt, may seem unremarkable. However, modelling instructional research and the teacher's prior instructional experience would suggest that students' find these practices challenging (Anhalt et al., 2018; Blum & Leiß, 2005). The teacher shared that the students are "getting used to not being able to get all the same answers in the end … when they can figure out different ways they want to approach it and get into the problem, they see it more as an exciting challenge." The TP task afforded the students to leverage their experiences relevant to constructing a model, thus asserting their agency while also considering different sources of accountability to productively engage. Given the unprecedented nature of the pandemic, students' attention to varying sources of accountability offers insights on how they were learning to read and write their world with mathematics through a lens of their experience and identity (Agarwal & Sengupta-Irving, 2019).

Conclusion

This ongoing study explores the possibilities for supporting students' development of agency, authority, and accountability through socio-critical modelling tasks that require attention to assumptions. Utilizing the structure for reflecting on assumptions provided via a routine for modelling and the problematization of a relevant, open-ended task, we analyzed student work for evidence of their agency, authority, and accountability. These reflections show a willingness to construct arguments based on student-made assumptions and interpretations of the scenario. Further, students referenced outside authority as well as their own experiences to defend their mathematical ideas, developing reasoning strategies in line with disciplinary expectations of the logical process of decision-making in modelling activities.

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References

Agarwal, P., & Sengupta-Irving, T. (2019). Integrating power to advance the study of connective and productive disciplinary engagement in mathematics and science. *Cognition and Instruction*, *37*(3), 349–366.

Anhalt, C. O., Staats, S., Cortez, R., & Civil, M. (2018). Mathematical modeling and culturally relevant pedagogy. In Y. J. Dori, Z. R. Mevarech, & D. R. Baker (Eds.), *Cognition, metacognition, and culture in STEM education* (pp. 307–330). Springer. https://doi.org/10.1007/978-3-319-66659-4_5

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM Mathematics Education*, *38*(3), 293–301. https://doi.org/10.1007/BF02652812

Blum, W., & Leiß, D. (2005). "Filling up": The problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1623–1633).

Elliott, R., Stoddard, E., & Brunner, M. (2019). Iterative designs of modeling tools for instruction. In S. Otten, Z. G. de Araujo, C. Haines, & C. Munter (Eds.), *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 589–593). University of Missouri.

Engle, R. A. (2012). The productive disciplinary engagement framework: Origins, key concepts, and developments. In D. Y. Dai (Ed.), *Design research on learning and thinking in educational settings: Enhancing intellectual growth and functioning* (pp. 161–200). Routledge.

Kaiser, G. (2017). The teaching and learning of mathematical modeling. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 267–291). National Council of Teachers of Mathematics.

Inclusion and social justice: Possibilities of mathematics education in the context with immigrants

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This article presents a doctoral research project that relates to the theme of inclusion and social justice from the perspective of critical mathematics education. The research aims at reflections beyond the school environment by researching immigrants and mathematics teachers from São Paulo State. In a qualitative methodological approach, interviews will be conducted, and comments will be made through a diary of notes. It is hoped that this research can contribute to the construction of critical perspectives on how mathematics education can be structured towards the inclusion of immigrant students and help to promote anti-racist mathematics education.

The theoretical frameworks

This doctoral research starts from concerns around issues related to immigration by dealing with themes of inclusion and social justice. According to the United Nations (UN) International Migration Inventory, in 2019, around 3.5% of the population on the planet were people who lived in countries other than the countries where they were born. Considering this amount, one in seven international migrants (about 38 million) are under the age of 20, which represents many school-age people.

International migrants are often faced with borders that can be material or immaterial. Material borders can be the concrete walls themselves, for example, which are placed as barriers, such as restrictions on mobility for immigrants. But also, the anti-immigration policies that are the subject of speeches by governments in different countries. Immaterial borders, on the other hand, are barriers that are often imperceptible or disregarded, such as those that appear during displacements in which people are subjected to precarious and dangerous modes of transport in crossings between countries. At the destination location, language barriers, customs, local laws, cultural differences, barriers to documentation, longing for the country of origin, barriers in the relationship with the local population, xenophobic speeches (whether recreational in jokes or more explicitly), also characterize immaterial boundaries.

In Brazil, the historical constitution of the population is also marked by international migrations, whether they are voluntary or forced. Amid the myth of racial democracy and

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migratory receptivity, research, and media show that migratory receptivity is often selective, that is, it may be related to the physical characteristics or the country of origin of immigrants. The myth of racial democracy distorts the real reality that racism in Brazil is structural and encompasses the reality of immigrants in the country today (Almeida, 2019).

In this text, racism is understood as a historical and social construction. As a systematic form of discrimination that gives a specific ethnic group a place of dominance over others. In other words, it is a systematic way of producing disadvantages and privileges depending on the group to which people belong (Almeida, 2019). In the context of immigrants, racism works more broadly and goes beyond the model of racism based on the black / white binary. It can be based on discrimination based on phenotype (visible characteristics of people's bodies), accent, geographic origin, religion, customs, and immigration situations.

In the face of diversified environments generated by migration, it is important that anti-racist practices are present inside and outside the school context. Therefore, the connection between tolerance and social and racial justice is fundamental. When talking about social and racial justice, it is necessary to recognize the existence of racism, to consider reparation for inequalities of opportunity, to question arguments based on meritocracy and to reflect policies of racial equality (Martin, 2013).

According to Gutstein (2006), mathematics education has the role of helping structurally marginalized groups to investigate and question injustices, such as racism and other social inequalities. Therefore, it is important to guide students to engage in the quest to challenge and transform structures of oppression. For the author, teaching mathematics for social justice is also teaching mathematics for racial justice. "This includes providing students with opportunities to analyze whether and how racism is implicated in social phenomena and to understand different forms of racism." (Gutstein, 2016, p. 490). In this context, it also includes rethinking the context of international migration and reflecting differences in the classroom.

For Skovsmose (2016), critical mathematics education has concerns with different groups of students and is relevant to teaching and learning about issues of social injustice and oppression. These concerns contribute to the organization of environments that favor inclusion and tolerance. Thinking about these environments with immigrant students requires considering social justice and racial justice, and the encounter between differences.

According to the UN (1995), education policies and programs must contribute to an understanding of solidarity, tolerance among ethnic, social, cultural, religious, and linguistic groups between nations. Tolerance, as defended by Freire (2017), is not a favor from the tolerant to the tolerated, in the sense of class, race, gender superiority. Nevertheless, tolerance is a virtue of human coexistence, of living with the different and not with the inferior.

Thus, the question of this research is: *How is mathematics education structured for the inclusion of immigrant students?* This research seeks to understand the context of immigration in Brazil and around the world, to reflect the connection between racism and xenophobia and to discuss possibilities for an inclusive mathematical education in the context of immigrants.

Research planning

This investigation perceives the complexity of the social, historical, and political context when working with social phenomena and the subjects involved in them, and in this way, this investigative process uses a qualitative approach to research.

It is also based on the assumptions of the Critical Race Theory (CRT) (Davis & Jett, 2019). For this reason, it gives special importance to the voice of people who are racially minorized through their reports and narratives about their lived experiences. It considers various forms of injustice and explores differences within and between groups when considering the historical and political context together with an awareness of racial inequalities.

In this sense, the data production of this research will be developed in the following moments: Interview with immigrants in which the following points will be considered: about the context of the immigrant family; about inequality / about exclusion / about social and racial justice; situation as a student / school context; about racism and xenophobia. Interviews with mathematics teachers and school managers in which the following points will be considered: context of immigrant students / context of schools with immigrant students/ about inequality, exclusion and social and racial justice/ about the role of mathematics in the context of immigration /about and racism and xenophobia/ about possibilities in mathematics classes in the context of immigrant students.

According to the data produced, we will point out research results in the light of Critical Mathematical Education (Skovsmose, 2019). The interviews will be recorded in audio and video. These moments will be transcribed and supplemented with notes made by the researcher that will contain memories of the context of the speeches, the atmosphere of the discussions, the episodes of silence, elements of the interactions between the researcher and the research participant, constituting information for the understanding and interpretation of the theme. There will be a reflection on the interaction of the materials and attention to the contents as official documents, responses to the production instruments, and to the content as elements of the social life of the participants, context of the countries of immigrants, context of Brazil, among others.

Comments

This research aims at reflections beyond classroom environments. It is hoped that the realization of this research can contribute to debates on the teaching and learning of mathematics and describe possibilities for actions that seek to promote contributions to an anti-racist and inclusive mathematical education in the context of immigrants. More broadly, favor conditions to combat social inequalities and injustices.

References

Almeida, S. L. De. (2019). *Racismo estrutural.* São Paulo: Jandaíra.

Davis, J., & Jett, C. (2019). *Critical race theory in mathematics education.* Routledge.

Freire, P. (2017). *Pedagogia da tolerância.* São Paulo: Paz e Terra.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York, NY: Routledge.

Gutstein, E. (2016). "Our issues, our people, math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education, 47*(5), 454–504.

Martin, D. B. (2013). Race, racial projects, and mathematics education. *Journal for Research in Mathematics Education, 44*(1), 316–333.

UN. (1995). *World Declaration of Principles on Tolerance.* https://www.ohchr.org/EN/Issues/Education/Training/Compilation/Pages/13.DeclarationofPrinciplesonTolerance(1995).aspx.

Skovsmose, O. (2016). Politics of meaning in mathematics education. *Philosophy of Mathematics Education Journal, 31*, 1–15.

Skovsmose, O. (2019). Inclusions, meetings and landscapes. In D. Kollosche, R. Marcone, M. Knigge, M. G. Penteado, & O. Skovsmose (Eds.), *Inclusive mathematics education: State-of-the-art research from Brazil and Germany* (pp. 71–84). Springer.

Sustainable e-assessment in mathematics instruction

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This study aimed at moving beyond content mastery to help students develop sustainable, transferable skills such as self-regulated learning. Each Student conducted a self-inquiry on a selected topic in mathematics. They also formulated some multiple-choice questions, asked their peers to solve them and engaged in active discussions afterward. Collected data were analyzed in terms of Student-generated Questions' (SGQs) quality by two instructors independently. Findings showed while taking the responsibility of assessment is a promising strategy in developing self-regulated learning, it might not automatically lead to a higher-order learning (i.e., critical thinking). It is suggested that a combination of instructors' feedback and regular use of digital technologies can enhance students' questioning competence.[1]

Introduction

Assessment should meet both the specific goals of a course and equip students with necessary skills to undertake their own assessment activities, i.e., *judging the quality*, in the future professional workplace (Boud, 2000). However, the conventional assessment conducted in higher education mainly aims at measuring mastery of content knowledge at the end of a course (summative assessment) instead of developing sustainable competences in students (integrative or formative assessment). Mathematics is inherently an inquisitive discipline which evolves around questions and problems. In a typical classroom, teachers retain control of asking questions: the questions are initiated by teachers and students take their turn to answer. There are some opportunities when the teacher invites students to ask questions. However, when the teacher is the one who constructs the most interesting questions and problems, students become dependent upon the teacher to catalyze inquiry (Bowker, 2010). To facilitate development of mathematical competence, teachers should create effective learning environments and encourage students to ask relevant and scientifically sound questions (Foster, 2011).

[1] This paper is a short version of Caspari-Sadeghi, Forster-Heinlein, Maegdefrau, & Bachl (2021), where further explanations can be found.

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Objectives of the study

This study aimed at moving beyond content mastery to help students develop the sustainable, transferable skill of self-regulated learning or 'learning to learn' as one of the 'key competences for life-long learning' (European Commission, 2012). Generating well-crafted questions is a creative act, and at the heart of what doing science is all about (Chin and Osborn, 2008). SGQs open a window to the mind of the students: they indicate what counts as significant for the students, what they understood, misunderstood or missed altogether. In this empirical study, we tried to use SGQs as a technique to (a) foster a culture of inquisitiveness in mathematics learners, (b) involve students more critically and meaningfully with the content, and (c) make the students the owners of their own and peer assessment.

Context of the study

As higher education is forced to turn to online teaching and learning during the COVID-19 pandemic, the authors used this short-term crisis to reconceptualize what constitutes Student Learning Outcomes in an online mathematics instruction. This study was conducted in an applied mathematics course during COVID-19 pandemic (March–Sept. 2020). The course was run in online, synchronous mode (via ZOOM) with the further support of Learning Management System. The course was taught jointly by a professor and her teaching assistant who was recently graduated. The participants were Bachelor and Master students with no prior experience in online instruction.

Procedure

The students were required to select a related topic in mathematics, conduct an inquiry on it and present their summary and findings to the class. Self-directed learning was supported by the instructors, by recommending literature, answering questions, etc. The students also formulated some multiple-choice questions, which they asked their peers to solve and engaged in subsequent active discussions.

Results

Students' perception towards the value of the SGQ strategy was assessed through an online questionnaire. Findings revealed students' positive attitude towards the experience, with 88% of the participants reporting that it contributed greatly to their focused attention and engagement with content. The Quality of SGQs were analyzed based on a two-dimensional rubric, (a) the overall quality, i.e., content coverage, relevance/clarity, and plausibility of a question, and (b) cognitive demand involved in a question based on Bloom's Taxonomy (1956). Each dimension has several levels. The majority of questions generated by students (66%) were classified at the lowest category (remembering), 25% at level 2 (understanding), and less than 10% at level 3 (application) of Bloom's taxonomy. None of the SGQs was at the higher-order levels, such as analysis, synthesis, evaluation/creativity. No significant correlation could be established between SGQ quality and students' academic attainment in the final exam (for full discussion, see Caspari-Sadeghi, et al., 2021).

Although Bloom's taxonomy received some criticisms (Moore, 1989), and there are alternative frameworks, i.e., Anderson & Krathwohl (2001) or Illeris (2002), Bloom's Taxonomy was preferred due to its clear categories as well as its widespread use in education, which facilitated comparing our results with other available studies.

Bottomley and Denny (2011) suggested such results are to be expected, since this was likely the first time these students were asked to write their own questions systematically. The development of appropriately aligned multiple-choice questions is not an easy or trivial task. The instructors decided to use both human support and digital technology solutions, i.e., *PeerWise,* to improve the process in the next course. Authoring questions for self and peer-assessment is an effective strategy to develop self-regulated learning which can facilitate future life-long learning beyond academia.

References

Bloom, B. S., Engelhart, M. B., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals.* New York: Longmans Green.

Bottomley, S., & Denny, P. (2011). A participatory learning approach to biochemistry using student authored and evaluated multiple-choice questions. *Biochemistry and Molecular Biology Education, 39*(5), 352–361.

Boud, D. (2000). Sustainable assessment: Rethinking assessment for the learning society. *Studies in Continuing Education, 22*(2), 151–167.

Bowker, M. (2010). Teaching students to ask questions instead of answering them. *Thought & Action, 20*, 127–134.

Caspari-Sadeghi, S., Forster-Heinlein, B., Maegdefrau, J., & Bachl, L. (2021). Student-generated questions: Developing mathematical vompetence through online-assessment. *International Journal for the Scholarship of Teaching and Learning, 15*(1), Article 8. https://doi.org/10.20429/ijsotl.2021.150108

Chin, C., & Osborne, J. (2008). Students' questions: A potential resource for teaching and learning science. *Studies in Science Education, 44*(1), 1–39.

European Commission. (2012). Assessment of key competences in initial education and training: Policy guidance (Staff working document). *Accompanying the Communication from the Commission on Rethinking Education: Investing in skills for better socio-economic outcomes.* SWD 371.

Foster, C. (2011). Student-generated questions in mathematics teaching. *Mathematics Teacher, 105*(1), 26–31.

Supplement: Sample of SGQs

Wandle die Dezimalzahl 67,25 in das Oktalsystem (b = 8) um. Welche Antwort ist die korrekte Ziffernschreibweise?

- ○ a.34
- ○ 103.2
- ○ 1000011.01
- ○ 1003.1

Was war nicht Teil der Kettenreaktion, die durch den ungeschützten Cast losgetreten wurde?

- ○ Die SRIs schalteten sich nacheinander ab.
- ○ Der OBC interpretierte nichts aussagende Bit-Pattern als korrekte Messdaten.
- ○ Das Haupttriebwerk wurde kurzzeitig abgeschaltet
- ○ Eine Hardwareexception wurde ausgelöst.

In welchem Fall ist das Nash-Gleichgewicht eindeutig bestimmt?

- ○ F ist quasikonvex
- ○ F ist pseudomonoton
- ○ F ist gleichmäßig monoton

Gibt es im Spiel „Schere-Stein-Papier“ ein oder mehrere Nash-Gleichgewichte?

- ○ Ja, in den Punkten {Schere,Schere}, {Stein,Stein}, {Papier,Papier}.
- ○ Nein, es existiert kein Nash-Gleichgewicht.
- ○ Ja, in den Punkten {Schere,Stein}, {Stein,Papier}, {Papier,Schere}.

Statistical literacy to empower coexistence within Brazilian semiarid region

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The Brazilian semiarid region is characterized by high temperatures and evaporation, in which during periodical crises, political actions to combating drought proved to be corrupt and ineffective, placing the population in vulnerability. Coexistence within the semiarid is a political approach that supports public policies to recover respect for the cultural diversity of peoples and territories, sustainability, agroecology, and food sovereignty. This project aims to investigate possibilities for the development of statistical literacy in a collaborative context of continuous teacher education with participants who teach in the semiarid region, intending to develop pedagogical activities that contribute to re-signify understandings of the paradigm of coexistence.

Introduction

This project intends to approach this statistical literacy perspective with elementary school teachers who live and teach in the Brazilian semiarid, which is a geographic region characterized by high annual averages of temperature (27° C) and evaporation (2,000 mm), with rainfall up to 800 mm per year (Lima, Cavalcante & Perez-Marin, 2011). The Brazilian semiarid is comprised of multicultural activities and territories, as well as a unique biome called *Caatinga* (which in Tupi-Guarani language means White Forest). The Brazilian semiarid comprises 969,589.4 km² which is equivalent to 11% of national territory.

The occupation of semiarid by Europeans began in the 16th century. The colonizers introduced unsuitable cultural practices, such as agriculture based on deforestation on the banks of water sources, burning, and exotic crop plantation. These activities are still current in several areas. However, due to the economical practices being unsuitable for the semiarid climate and soil, the social situation for most populations of the semiarid becomes a serious issue during longer drought periods. During these crises, it is common to develop ineffective temporary governmental projects, but this kind of projects it is just as a political instrument of domination and alienation, as well as a promoter of corruption. This problem has historically been used for political purposes.

From the 1990s, social movements and non-governmental organizations began to develop an alternative political project which we called *coexistence within the semiarid* to reduce historical problems, such as disputes over water resources and monopolization of arable land.

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The perspective of *coexistence within the semiarid* gives value to the processes of understanding these sociocultural aspects to plan, elaborate and carry out actions in search of the good living together. Therefore, the interventions must aim to rescue respect for the cultural diversity of the various existing peoples and territories, building the sustainability of water resources and a relationship between nature and agroecological practices, coexistence technologies and food sovereignty.

Silva (2006) identifies five meanings for the *coexistence within the semiarid:*

Meanings	Descriptions
1. Coexistence with the Environment	Management and sustainable use of natural resources in the ecosystem, without making their reproduction unfeasible, considering the balance of the common space experienced.
2. Coexistence Economy	Capacity for sustainable use of natural and cultural potential in productive activities that are appropriate to the environment.
3. Coexistence with the quality of life	To be able to identify the satisfaction of fundamental needs as a condition for expanding human capacities and improving the quality of life, conceived as a reduction in inequalities, poverty, and misery.
4. Coexistence Culture	Valuing and rebuilding local population knowledge about the environment in which they live, its specificities, weaknesses, and potential.
5. Political Dimension	Mobilization of civil society, through networks of social movements and organizations, which promote the dissemination of social values of coexistence within Semiarid and fight for the improvement of its economic and sociocultural conditions.

Table 1: The five meanings of *coexistence within semiarid*

Considering the context of Brazilian semiarid and the possible relationship with mathematics and statistics as discussed by Skovsmose (2021), we might ask the following questions: How can statistical literacy strengthen the political project of coexistence within the Brazilian semiarid from a critical and liberating perspective? What aspects were historically used to reinforce stigma and prejudice towards this region and its people? What is the role of mathematics in this process? As much in the possibilities of transformation and change as in its uses as a tool of oppression?

The general objective of this doctoral research project is to investigate how to utilize the theoretical-methodological assumptions of statistical literacy to promote the problematization of sociopolitical aspects and thereby empower the political project of coexistence with the semiarid.

The roles of mathematics and statistics to empower the coexistence with the semiarid

Skovsmose (2021) argues that the world faces more and more large-scale crises such as environmental ones, which might include those which happen in the Brazilian semiarid. The author identifies at least three different types of relationships between mathematics and crises: mathematics *can picture a crisis*; *can constitute a crisis*; and *can format a crisis*. The political and economic interests play a decisive role to develop different crisis discourses about the reality, which can be based on mathematical and statistical arguments. For example, for over one century political actions to the semiarid emphasised *the combating of drought*, as it could benefit a hegemonic minority of wealthier groups dominating the semiarid. Then, in this case the statistical data is used to describe the extent of the drought, the resources people need, and somehow manipulate the reality. But also, Skovsmose considers that mathematics (and statistics) can play transformative roles in regards with crises if they support human-centred lines of arguing and actions. Statistics can support important social values, such as democracy and argument based on scientific evidence (Engel, 2019).

However, statistics can also be used to justify certain narratives by manipulating data representations that can emphasize or disguise some statistics in news (Monteiro & Ainley, 2010). In addition to that, there are disseminations of misinformation and malinformation associated with fabricated statistics, deliberately created to harm a people, social groups, organizations, or countries (Carmi, Yates, Lockley, & Pawluczuk, 2020).

Therefore, we understand that it is necessary to problematize the perceptions of how statistics can be used, either correctly and ethically, faithfully to the reality to be analysed, or in an unethical way, with the aim of pretending to validate distorted realities, favouring individual and antisocial interests, distant from the truths of the facts.

> The secret language of statistics, with so much appeal to our “fact-based” culture, is used to sensationalize, inflate, confuse, and oversimplify. Statistical methods and terms are needed to report data on social and economic trends, business conditions, “opinion”, research, censuses. But without writers who use the words honestly and comprehensively, and without readers who know what they mean, the result can only be semantic absurdity. (Huff, 1993, p. 8)

In a panorama of contradictions and pitfalls, in which the biased uses of statistics can have strong implications in the social, economic, cultural, political, and historical contexts, we point out that statistical literacy is particularly important to achieve critical elements that can provide conditions to understand, transform, reflect and act up these controversial contexts.

Statistical literacy is “a stand-alone complex competency with many unique elements, well beyond knowing statistics per se. Further, I argue that statistical literacy, and its many building blocks, are seldom or insufficiently addressed in regular statistics or mathematics instruction” (Gal, 2021, p. 26). This competency is causally related to people’s attitudes towards the countless statistical information that surrounds them daily and how they critically evaluate graphs, infographics, tables, charts, statistical data from journalistic, scientific, and informational texts. Gal (2002, p. 2) argues:

> the term “statistical literacy” refers broadly to two interrelated components, primarily (a) people’s ability to interpret and critically evaluate statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to discuss or communicate their reactions to such statistical information, such as their understanding of the meaning of the information, their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions.

Following the perspective of statistical literacy (Gal, 2002), our focus is on mobilizing among the participants the structural elements: elements of knowledge (literacy, statistics, mathematics, and context) and elements of disposition (beliefs and attitudes, and criticality).

In this context of ideological, political, cultural, economic dispute between political perspectives, narratives are constituted as background for the use of statistical elements for validation, confrontation, imposition, and refutation of the facts presented by each side. It is in this aspect that we set out to investigate possibilities to enhance the paradigm of living with the semiarid using the elements of statistical literacy.

Theoretical and methodological construction of research study

The theoretical-methodological construction process that we are proposing to carry out, seeks to articulate three interrelated perspectives, statistical literacy, *the paradigm of living with the semiarid* and the continuing teacher education.

We intend to propose a qualitative study based on a research-action type according to Tripp (2005). This type of research produces an analysis of reality while seeking to transform it through an intervention. Therefore, this project aims to contribute to the potentiation of *the coexistence within the semiarid*, using the elements of statistical literacy. For that reason, we decided to have an investigative structure as part of an activity of continuous teacher education which consisted of contextualized problem situations related to specificities of Brazilian semiarid. The referred continuing education will count on the participation of teachers and professors who teach statistics in basic education level in the semiarid territories.

The development of formative activities with teachers is based on three theoretical perspectives. Firstly, we dialogue with contextualized education for living with the semiarid region (ECSAB), a theoretical perspective that emerged from the context of the *paradigm of coexistence*, which carry out studies, pedagogical resources, and teacher education activities within the context of the Paradigm of Coexistence (Lima, 2008).

Another perspective was that of the investigative research cycle originally proposed by Wild and Pfannkuch (1999) and adapted by Guimarães and Gitirana (2013). These authors argue that an experience with engagement in statistical research, following the cycle and going through its stages in a critical and contextualized way, is a fundamental educational activity for the construction of statistical literacy.

A third perspective with which we dialog is proposed by Watson and Callingham (2003) who presents an analytical framework composed of six levels of positioning in relation to statistical literacy. It will help us to create the action research experiences, as well as to analyse whatever data produced through the project.

The participants will be six basic education teachers who teach statistics in public schools belonging to the semiarid territory. We called our initial approach with the participants as a *recognition step*, which consists of to know more about teachers' professional and academic backgrounds, as well as their knowledge and perspectives about semiarid socio-political aspects. This step will be developed by carrying out individual interviews.

A second stage is to propose to the teachers to participate in a cooperative group. The meetings will be based on a focus group approach in which will produce data about teachers' perceptions, opinions, attitudes, critical interpretations, and beliefs. We plan to develop at least four meetings with an estimated duration of 2:30 hours.

During the focus group meetings, we will discuss the five meanings for the *coexistence within semiarid* associated with statistical literacy aspects. The meeting will be activities in which are involved in semiarid socio-political statistical contexts, which include statistical data. Table 2 presents examples of data which might be used to problematize.

Some social indicators of the Brazilian semiarid region
Division of semiarid lands suitable for agriculture About 1.5 million farming families (28.82% of all Brazilian farming families) occupy only 4.2% of agricultural land in the semiarid region. While 1.3% of rural establishments with more than 1,000 hectares (called *latifundios*) hold 38% of semiarid territory.
The economic situation of population The majority of Brazilians (59.1%) in extreme poverty are in the Northeast where the semiarid is located. Most of this population (52.5%) live in rural areas, and 4 out of 10 extremely poor people are between 0 and 14 years old (IBGE, 2010).
The vast majority of semiarid municipalities (60.09%) of the, with more than nine million inhabitants, the Human Development Index (HDI) varies from Very Low to Low. The HDI considers indicators of longevity, education and income. All towns in the Semiarid region had a lower Municipal human development index (HDI-M) than in Brazilian national rate, which is 0.727.

Table 2: Social indicators of the Brazilian semiarid region

The information presented on Table 1 can be a starting point to interpret the semiarid reality based on statistical data. To begin the reflections, the researcher would ask questions, such as:

What do you think about this information?
Do you think it is reliable information?
Do you think that this information can stereotype people who live in the semiarid region?
What have you heard, during your life, about the semiarid region?
Do you believe in these opinions?
Have you ever used statistical data to describe the semiarid?
What kind of information have you used to describe the people who live in semiarid?
What did you want to argue?
What other data can we use to demonstrate the potential of this region?
Do you think the way this data is used by politicians leads to more inequality?
How can aspects of statistical literacy help in building a dignified understanding of this region and these people?

The researcher will mediate the group proposing reflections, reviewing understandings and possible stereotypes, perceptions living, existing and resisting in the Brazilian semiarid.

Provisional considerations

The articulation between the theoretical perspectives of contextualized education for coexistence within the Brazilian semiarid and statistical literacy, suggests a promising path for the potentializing coexistence paradigm. In this sense, we expect that it is possible to transform and reframe the understandings and practices of teachers and professors of Brazilian semiarid so that they become mobilizing agents of this perspective.

We believe in the proposal under construction and in its effectiveness, so that we can disarm, disturb the hegemonic discourses, and introduce provocations into the debate, problematizing and denaturing exclusionary and silencing practices, so that we can dismantle the accepted and naturalized imagery about Brazilian semiarid.

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References

Carmi, E., Yates, S. J., Lockley, E., & Pawluczuk, A. (2020). Data citizenship: Rethinking data literacy in the age of disinformation, misinformation, and malinformation. *Internet Policy Review*, *9*(2), 1–22.

Engel, J. (2019). Statistical literacy and society. In J. M. Contreras, M. M. Gea, M. M. López-Martín, & E. Molina-Portillo (Eds.), *Actas del Tercer Congreso Internacional Virtual de Educación Estadística* (pp. 1–17). www.ugr.es/local/fqm126/civeest.html

Gal, I. (2002). Adult statistical literacy: Meanings, components, responsibilities. *International Statistical Review*, *70*(1), 1–25.

Gal, I. (2021). Promoting statistical literacy: Challenges and reflections with a Brazilian perspective. In Monteiro, C. & Carvalho, L. (Eds.), *Temas emergentes em letramento estatístico* (pp. 24–38). Universitária UFPE.

Guimarães, G., & Gitirana, V. (2013). Estatística no Ensino Fundamental: a pesquisa como eixo estruturador. In Borba, R. & Monteiro, C., *Processos de ensino e aprendizagem em educação matemática*. Universitária UFPE, 2013.

Huff, D. (1993). *Como mentir com a estatística*. Financeiras.

IBGE [Instituto Brasileiro de Geografia e Estatística]. (2010). *Censo 2010*. IBGE.

Lima, E. S. (2008). *Formação continuada de professores no semi-árido: Valorizando experiências, reconstruindo valores e tecendo sonhos* [Master's thesis, Universidade Federal do Piauí]. https://ufpi.br/arquivos_download/arquivos/ppged/arquivos/files/dissertacao/2008/elmo_lima.pdf

Lima, R., Cavalcante, A., & Perez-Marin, A. M. (2011). *Desertificação e mudanças climáticas no semiárido brasileiro*. INSA-PB.

Monteiro, C., & Ainley, J. (2010). The interpretation of graphs: Reflecting on contextual aspects. *Revista de Educação em Ciência e Tecnologia*, *3*(2), 17–30.

Silva, R. M. A. (2006). *Entre o combate e a convivência com o semiárido: Transições paradigmáticas e sustentabilidade do desenvolvimento*. [Doctoral dissertation, Universidade de Brasília]. Repositório UNB. https://repositorio.unb.br/handle/10482/2309

Skovsmose, O. (2021). Mathematics and crises. *Educational Studies in Mathematics*. Advance online publication. https://doi.org/10.1007/s10649-021-10037-0

Tripp, D. (2005). Pesquisa-ação: Uma introdução metodológica. *Educação e Pesquisa*, *31*(3), 443–466.

Watson, J., & Callingham, A. R. (2003). Statistical literacy: A complex hierarchical construct. *Statistical Education Research Journal, 2*(2), 3–46

Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223–248. https://doi.org/10.1111/j.1751-5823.1999.tb00442.x

An Augustinian take: The loves of the mathematics education research community

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Psychoanalytic research in mathematics education focuses on the unconscious narratives in the field with a predominant focus on desire and its derivatives. Yet, over a millennium ago, Augustine of Hippo addressed similar themes with his concept of love and his definition of a community. In this project presentation, I argue that Augustine's notion of love as craving and his definition of a community can inform research efforts in mathematics education in an original way. Namely, I will use Augustine's work to help identify potential loves of the field and discuss how those loves subsequently shape the field and its research.

Introduction

There has been a recent uptake of psychoanalysis for research purposes in mathematics education (see the work of Alexandre Pais, Tony Brown, Tamara Bibby, etc.). In the introduction to the book *The Psychology of Mathematics Education: A Psychoanalytic Displacement*, Tony Brown, of Manchester Metropolitan University and the editor of the book, discusses how psychoanalysis entered the scene of mathematics education research. He describes how there was a shift away from traditional psychology towards "a psychology understood more through relations between people" (Brown, 2008, p. 1). Much of this initial work used Lacan's theories to promote "the shift from bio-scientific to narrative emphases in interpreting Freud's work" (p. 3) and is focused on what Lacan called "the truth of desire" (Lacan, 2019). Thus, psychoanalytic research in mathematics education attempts to identify subconscious or unconscious narratives in the field that often depend on desire.

Approximately 1,650 years ago, however, Augustine of Hippo discussed themes of desire and the role of desire in communities. Adjusting a definition from Cicero, Augustine (2009) wrote:

> [...] for example, if one should say, 'A people is the association of a multitude of rational beings united by a common agreement on the objects of their love,' then it follows that to observe the character of a particular people we must examine the objects of its love. (§ 19.24)

For Augustine, "love is indeed nothing else than to crave something for its own sake" and "a kind of motion, and all motion is toward something" (as cited in Arendt et al., p.9). This elevation of love (as craving, or desire) shares similarities with some of the work in psychoanalysis. Thus, while I do not intend to diminish the work of psychoanalysts in

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mathematics education research, I'd like to offer an Augustinian "take" on mathematics education research and suggest that this ancient North African bishop can inform our work in an original and profound fashion—meaning, I'd like to use Augustine's conception of love as an opportunity to address the mathematics education research community's loves. This presentation reports the initial findings of an ongoing project where I attempt to answer two questions: first, what are the field's loves? And second, how do those loves shape and form the field and its research?

Before continuing, I think it is important to note that I am not suggesting a theological analysis of the field. As Hannah Arendt demonstrated in the field of political theory, Augustine's work extends beyond theology and Christian doctrine. Arendt thought that, regardless of culture, religion, or worldview, it was relevant that humans are first and foremost lovers. Arendt (1996) even suggested that, "Strictly speaking, he who does not love and desire at all is a nobody" (p. 18).

Arendt felt so strongly about Augustine's conception of love, she wrote her doctoral dissertation on the subject (entitled *Love and Saint Augustine*) and spent the remainder of her life refining and editing the work (Arendt et al., 1996). The lifelong work on her dissertation reflects her love of Augustine, demonstrates his influence on her work, and highlights how highly she thought of her original research question regarding "the relevance of the neighbor" (p. x). She explored Augustine's conception of love in order to study the phenomena of "neighbor" and, as Arendt explains, "to understand in what sense our neighbor is loved in adhering to the commandment of neighborly love" (p. 7).

Augustine's concept of love

Augustine distinguishes between two different types of love: *caritas* and *cupiditas*. For Augustine, *cupiditas* is love with a wrong object and *caritas* is love with a right object. Augustine's definitions of "wrong" and "right" are, of course, grounded in Christianity. But, as Arendt (1996) points out, "both right and wrong love (*caritas* and *cupiditas*) have this in common – craving desire, that is, *appetitus*. Hence, Augustine warns, 'Love, but be careful what you love.'" (p. 17).

Augustine offers this warning because he believes a person or communities' loves are central to who they are. Arendt writes,

> Desire mediates between subject and object, and it annihilates the distance between them by transforming the subject into a lover and the object into the beloved. For the lover is never isolated from what he loves; he belongs to it. (p. 18)

This close intimacy – this binding – of the lover and the beloved is defining for the lover. The stronger the love, the more intimate the union with the beloved. Augustine (as cited in Arendt et al., 1996, p. 18) writes, "Such is each as is his love." Arendt (1996) describes how Augustine uses the word *inhaerere*, which is translated as "clinging to," to denote the closeness of lover and beloved. Commenting on this theme of Augustine, she writes, "Happiness is achieved only when the beloved becomes a permanently inherent element of one's own being" (p. 19).

Arendt (1996) also highlights that, "A thing is sought for its own sake (in *caritas* or *cupiditas*) if its possession puts desire to rest. Thus, nothing can be said to be "loved" which is sought for the sake of something else" (p. 32). Which begs the question: what are the mathematics education research community's loves?

Science, our first love

In this presentation of my ongoing attempt to operationalize Augustine's concept of love, I'd like to suggest that the scientific paradigm is a core love of the mathematics education research community and describe how it can warp our field. Ernest (1998) describes scientific research in mathematics education as being founded on

> rationalism and the scientific method as employed in the physical sciences, experimental psychology, etc. It is concerned with objectivity, prediction, replicability, and the discovery of scientific generalizations or laws describing the phenomena in question. (p. 77)

Ernest goes on to describe two other major research paradigms: the interpretive, which for Ernest (1998) has to do with "understanding, interpretation, intersubjectivity, [and] lived truth" (p. 77), and the critical-theoretic, which aims for social and institutional change through social critique. He argues that each respective paradigm plays a role in the mathematics education research community, that no paradigm should be put on a pedestal, and that antagonistic attacks against entire paradigms should be discouraged.

Yet the scientific paradigm's strong grip on the field as a whole has ramifications for the field per Augustine's notion of love. Prominent researchers (e.g., Cobb, 2007; von Glasersfeld & Steffe, 1991) that work in the interpretive paradigm still frame the field as a science. Consequently, while many of these researchers assent to pragmatist notions of truth and meaning, they function within remnants of the scientific paradigm. This continued worship of the scientific paradigm has led to a scientism that looms large over the field. Thus, I argue that science is the object of the majority of the field's love and is sought for its own sake.

Per Augustine, I argue that this love is *cupiditas* and must be critiqued. While Ernest (2012) recognizes the consequences of adopting certain theories, he seemingly overlooks the consequences of having the scientific paradigm as a core love of the field. Yet the scientific paradigm distorts the field as a whole and skews the field's conception of knowledge. Anzaldúa (1987) identifies a major consequence of scientism. She writes,

> In trying to become "objective," Western culture made "objects" of things and people when it distanced itself from them, thereby losing "touch" with them. (p. 59)

Anzaldúa's description of the consequences of the West's pursuit of objectivity is precisely what is occurring within mathematics education research community today. In Augustinian fashion, Anzaldúa explains that the pursuit of objectivity ultimately leads to dehumanization. While researchers within the interpretive tradition reject the idea that knowledge reflects an objective, ontological reality (Ulrich et al., 2014), they also embrace obvious expressions of scientism. For example, Campbell (2020) suggests that soon we will be able to equip students to wear caps that transmit "high spatial and temporal resolution

EEG or MEG signals" (p. 99) to a central console that can then be analysed by their teacher in real time. Yet, per Anzaldúa, is there anything more distancing and dehumanizing? Where are the affections in such a cold, imagined future? For a field predominantly concerned with human beings, the scientific paradigm and scientism should not be objects of the mathematics education research community's love.

Concluding thoughts

In this project presentation, I argued that Augustine's notion of love can help diagnose the ills of the mathematics education research community. I then briefly excavated and examined the field's love of the scientific paradigm. In the future, further loves need to be explored as psychoanalytic research has already identified the field's love of capital (Pais, 2014). Ultimately, the mathematics education research community needs an imaginative overhaul. The field remains captive to unworthy loves and these objects of love continue to warp the field as a whole.

References

Anzaldúa, G. (1987). *Borderlands/la frontera: The new mestiza.* Aunt Lute Book Company.

Arendt, H., Scott, J. V., & Stark, J. C. (1996). *Love and Saint Augustine.* University of Chicago Press.

Augustine, S. (2009). *The city of God.* Hendrickson.

Brown, T. (Ed.). (2008). *The psychology of mathematics education: A psychoanalytic displacement.* Sense.

Campbell, S. (2020). Educational neuroscience: Past, present, and future prospects. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), *Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 93–100). PME-NA. https://doi.org/10.51272/pmena.42.2020-4

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (Vol. 1, pp. 3–38). Information Age.

Ernest, P. (1998). A postmodern perspective on research in mathematics education. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 71–85). Springer.

Ernest, P. (2012). What is our first philosophy in mathematics education? *For the Learning of Mathe–matics, 32*(3), 8–14.

Lacan, J., Miller, J. A., & Fink, B. (2019). *Desire and its interpretation: The seminar of Jacques Lacan, Book VI.* Polity Press.

Pais, A. (2014). Economy: The absent centre of mathematics education. *ZDM Mathematics Education, 46*(7), 1085–1093. https://doi.org/10.1007/s11858-014-0625-8

Ulrich, C., Tillema, E. S., Hackenberg, A. J., & Norton, A. (2014). Constructivist model building: Empirical examples from mathematics education. *Constructivist Foundations, 9*(3), 328–339.

von Glasersfeld, E., & Steffe, L. P. (1991). Conceptual models in educational research and practice. *The Journal of Educational Thought, 25*(2), 91–103.

Promoting early arithmetical skills by using part-whole thinking as a way to guide joint learning for students of all abilities

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This paper presents preliminary findings of the large-scale project Counting with all children from the very beginning, (Mit allen Kindern von Anfang an rechnen) a project that has the aim of developing and testing joint teaching-learning situations in primary schools, based on part-whole thinking ('Teile-Ganzes-Denken') from children's first maths lessons. Using design research, five mainstream teachers volunteered to participate in the Beta-cycle over a ten-week period in First Grade. Data analysis revealed that not all the teachers managed to define coherent aims for their students' mathematical learning, especially in the instances of children who have poor mathematical skills and of children with special educational needs (SEN).

Preliminary remarks

International organisations such as UNICEF, UNESCO or the European Union, define 'the right to education' as a common ideal regarding inclusion of all people. With the passage of time, the Austrian education system, like that of many other countries, has made great progress as far as the issue of inclusion is concerned. Since 1885, there have been special educational classes for children with learning difficulties, initially called 'auxiliary classes' and subsequently changed in 1956 to 'special schools'. A wide range of special schools were established as the proper place for children and young people with disabilities. The number of different special schools reached a peak in the 1980s with 10 different types available. Austrians' first attempts at integrative education in mainstream primary schools were in the 1980s (Buchner & Proyer, 2020). With the ratification of the UN Convention on the Rights of Persons with Disabilities in 2008, Austria set out on a path towards an inclusive education system. Even today, Austria, like many other countries in the world, still focuses on a so-called two-track approach: Parents are free to decide whether to send their child to a special school or a mainstream school. In 2018-19, of the 578,417 school-age children approximately 5% (29,000) had SEN. 63% of the children with SEN were included in mainstream schools, while 37% attended a special school (Statistik Austria, 2019). Currently the main focus of inclusive education is to promote the quality of teaching-learning situations (Florian, 2008). There seems to be uncertainty, and not only in German-speaking countries, about how to teach inclusively and how to create inclusive teaching-learning situations, mainly in early

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arithmetical education (Korff, 2016). Thus, this project seeks to design teaching-learning situations for First Grade based on part-whole thinking, in order to support teachers in their challenging task.

The TIGER concept of part-whole thinking

It seems to be accepted in the relevant German literature that at the end of First Grade children should learn that numbers (up to 20) are composed of other numbers. It might be helpful if teachers first focus on the consolidation of part-whole thinking and then, based merely on that, initiate children's understanding of addition and subtraction (see also, for example, Gaidoschik, 2019). These goals seem to be the norm in international mathematics education also, with for example 'Number framework' (New Zealand Ministry of Education, NZME, 2008) or 'Mathematics programme of study' (UK Department of Education, 2013). Part-whole thinking is incorporated as early as possible in the school programme in these guidelines for teachers' actions. The TIGER (Teile im Ganzen Erkennen und damit Rechnen) concept, developed by Gaidoschik (2017), focuses on solid number concepts in early arithmetical teaching. The concept consists of three fundamental aspects. Firstly, counting with an understanding of numbers. This includes children learning how to work out the 'counting principles' of Gelman and Gallistel (1987). Secondly, comparing quantities and numbers. On the basis of one-to-one matching up, children should learn and/or consolidate that no counting is needed for pairing the items or for number comparison such as identifying that there is one more left over. The third area is that of subitizing or perceptual subitizing without counting (see also, for example, Clements & Sarama, 2009). Teachers should work with students in all fields more or less concurrently.

The research project, research method and research question

The 13 teaching-learning situations of the current research project are based on the ideas of the TIGER concept, with a balance between individual and mutual learning. To be able to examine the effectiveness of the learning environments, collective case studies in the framework of design-based research (Euler, 2014) have been conducted. Within the framework of an Alpha-cycle, conducted in school year 2019-2020 with 45 First Grade students in two Tyrolean mainstream classes, video-recorded conclusions were drawn for the implementation, analysis and further development of the teaching-learning situations. The results were processed in an additional cycle. The Beta-cycle, which is relevant for data analysis, was carried out in five Tyrolean primary school classes in school year 2020-2021 with 95 pupils. To generate insights into students learning, each lesson was video recorded. Thus, in this paper, I write about six and seven-year-old children of the Beta-cycle over a 10-week period in First Grade in inclusive mainstream classes in Austria. The questions guiding the study are: Can teaching-learning situations based on part-whole thinking be designed to foster learning processes in early arithmetical teaching of all students in primary school? How should such teaching-learning situations be designed? A combination of 'Abstraction in Context' (Dreyfus & Kidron, 2014) and 'Assessment of Teaching-Learning-Situations in Mathematics of the Early Grades' (Steinweg, 2010) is used for data analysis. AiC is a theoretical framework to analyze students' processes of constructing abstract mathematical knowledge. There are three epistemic actions: Recognizing (R), Building-with (B) and Constructing (C), hence the RBC-model. Steinweg's idea of dimensions focuses on the teachers' possibilities for action. The combination of both approaches gives indications for further development of teaching-learning situations.

Interim results

The following brief sequence of the teaching-learning situation Throwing Tiles focuses on six-year-old girls Anna and Rosa. They take turns throwing the 10 reversible tiles, which have a red side and a blue side. Then together they have to match one red with one blue tile, using one-to-one matching without counting (AiC - Recognizing). They create two rows to identify who has more tiles, and how many more there are of one colour than the other. Anna has six blue tiles, Rosa four red tiles. The teacher joins in:

Teacher: Great! Both of you have already thrown tiles and matched them in two rows. Each blue tile in the top row has a red tile in the bottom row. Rosa, how many tiles do you have less than Anna?

Rosa: I'd better count them again. *(Rosa takes her red tiles and starts again).*

Teacher: How many tiles do you have, Rosa? Did you have to count them again?

Rosa: I have four. *(Rosa is uncertain again).*

Teacher: *(The teacher takes the blue tiles and arranges them in a row).* Can you match a red one of yours with each blue tile?

Rosa: Yes. *(Rosa puts a red tile on each blue tile in a second row).*

Teacher: Great. How many tiles more does Anna have than you? *(The teacher points to both rows with her index finger).*

Rosa: Six? *(Rosa is unsure how to answer).*

Analyzing the transcript using AiC indicates that the ideal of learning processes is not achieved by all children. Regarding Steinweg's idea of dimensions, not all teachers were able to foster children's competence in one-to-one matching, as seen in Rosa's lack of understanding of the one-to-one matching up process. Particular attention should be paid to the mathematical thinking of underachieving children and to children with SEN, as in the case of Rosa mentioned above. In this sequence, the teacher did not realize that Rosa requires a solid understanding for one-to-one matching up (AiC - Recognizing) as a basis for further arithmetical strategies (AiC – Building with and Constructing). No further improvements need to be made to the teaching-learning situation Throwing Tiles. It is important to boost teachers' didactical knowledge and their knowledge of possibilities in the early arithmetical education of all children. In Austria, one third of children with SEN are integrated into mainstream primary school classes, but we can presume that their mathematical competence is not being continuously fostered, and neither are the abilities of children with poor mathematical skills. Most authors, at least in the relevant German mathematics education literature, seem to agree that these children do not need completely different concepts, but instead need targeted support from highly qualified class teachers and teaching assistants.

References

Buchner, T. & Proyer, M. (2019). From special to inclusive education policies in Austria – developments and implications for schools and teacher education. *European Journal of Teacher Education, 43*(1), 83-94. https://doi.org/10.1080/02619768.2019.1691992

Clements, D. H. & Sarama, J. (2009). *Learning and Teaching Early Math: The Learning Trajectories Approach.* Routledge.

Dreyfus, T., & Kidron, I. (2014). Introduction to abstraction in context (AiC). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of Theories as a Research Practice in Mathematics Education* (pp. 85-96). Springer.

Euler, D. (2014). Design Research – a paradigm under development. In D. Euler (Ed.), *Design-Based Research* (pp. 15-41). Steiner.

Florian, L. (2008). INCLUSION: Special or inclusive education: future trends. *British Journal of Special Education, 35*(4), 202-208. https://doi.org/10.1111/j.1467-8578.2008.00402.x

Gaidoschik, M. (2019). Didactics as a source and remedy of mathematics learning difficulties. In A. Fritz, V. Haase, & P. Räsänen (Eds.), *The International Handbook of Math Learning Difficulties: From the Lab to the Classroom* (pp. 73-89). Springer.

Gelman, R. & Gallistel, C.R. (1987). *The Child's Understanding of Number.* Harvard University Press.

Korff, N. (2016). *Inklusiver Mathematikunterricht in der Primarstufe: Erfahrungen, Perspektiven und Herausforderungen* (3. Aufl.). Schneider Hohengehren.

New Zealand Ministry of Education (2008). *Getting Started.* Crown.

UK Department of Education (2013). *Mathematics programme of study: key stages 1 and 2.* Retrieved from https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study

Statistik Austria (2018/19). *Schülerinnen und Schüler mit sonderpädagogischem Förderbedarf.* Retrieved from https://www.statistik.at/web_de/statistiken/menschen_und_gesellschaft/bildung/schulen/schulbesuch/029658.html

Steinweg, A.S. (2010). Einschätzung der Qualität von Lehr-Lernsituationen im mathematischen Anfangsunterricht – ein Vorschlag. *Journal für Mathematik-Didaktik, 32*(1), 1-26. https://doi.org/10.1007/s13138-010-0022-y

Disciplining bodies: Affect in mathematics education

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Identifying children's affective responses towards mathematics and putting efforts to change them are fairly common aspects of modern-day mathematics education. In this paper, I begin to explore how such efforts involve a disciplinary power that aims to maximize the economic utility of bodies and create a specific kind of citizen-worker.

Concerns about the consequences of how children feel about mathematics have often been expressed by researchers and educators alike (Boaler, 2009; McLeod, 1992). As a result, the identification and subsequent modification of affective responses such as beliefs, emotions, and attitudes have become integral to mathematics education. According to the OECD (2016), "teachers should be concerned about students' attitudes towards mathematics and should take steps to increase students' positive feelings, self-confidence and interest in mathematics when needed" (p. 77). Today, identifying and changing the way students feel about mathematics is aided by various resources – from teaching practices and strategies to even biosensors for monitoring children's emotional activity (Williamson, 2016). While these efforts often come across as 'innocent' pedagogical practices, "any attempt to promote subjectivity through governing thought is neither benign nor neutral" (Popkewitz, 2004, p. 27). In this paper, I argue that modulating affective responses involves a disciplinary power (Foucault, 1995) that aims to maximize the economic utility of bodies and create a certain kind of citizen-worker. I begin by examining a historically assumed motivation behind efforts to modulate the affective – the desire to increase test scores.

Affect and achievement

Affective responses include "a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (McLeod, 1992, p. 576). The association between affective responses and achievement became a naturalized belief due to considerable research in the latter half of the 20th century that posited attitudes and other affective responses as predictors of achievement. These statistically driven studies often established causal relationships between student achievement and a plethora of affective factors such as self-efficacy, confidence, and beliefs (McLeod, 1992). As these relationships became commonsensical, the belief that modulating affective responses will lead to higher test scores took hold.

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However, this belief has been challenged often – due to its lack of insights about the complex and unpredictable ways in which affect and achievement interact (McLeod, 1992), and the uncertainty of whether positive affective responses cause high test scores or the other way around. Still, the modulation of affective responses remains a significant aspect of 21st-century mathematics education. An additional impetus to the prevalence of this aspect has been the growing belief in a connection between mathematics competence and the economic steering of society (Valero, 2018). Thus, through achievement on tests, affective responses get linked to national economic progress. The usage of Likert-type questionnaires on student attitudes and beliefs in comparative global assessments like PISA validates this association.

However, increasing test scores does not seem to be the only motivation anymore for modulating children's affective responses towards mathematics. Several 'high-achieving' countries in the PISA mathematics test such as Korea and Singapore have continued to express concerns over low interest and self-confidence of students in mathematics (OECD, 2016), and made large-scale curricular revisions to improve student attitudes. It becomes imperative then to ask what else drives the efforts to change how students feel about mathematics, beyond mere desires for high test scores. In the next section, I explore the same.

Citizen-workers for modern-day economies

The scientific optimism that began following World War II intensified the popular belief that technological development was the solution to every problem (Skovsmose, 2005). Given the mathematical knowledge required to perform the supposedly essential functions of problem-solving and data-driven decision making in this technological landscape, mathematics has become indispensable. Consequently, mathematical knowledge – seen as a "human right in itself" (Valero, 2018, p. 108) – is now positioned as essential for any nation to advance its economy and technological competence. Anyone with a lack of this knowledge is perceived to be unprepared for 'an uncertain future' (Popkewitz, 2004) – a belief that has further amplified the 'STEM pipeline'.

Following WW II, a notion emerged – of human beings as citizen-workers who had to be trained and equipped with skills for the growth of a nation's economy (Valero, 2017). With the emergence of mathematical knowledge as vital during the same era, mathematics achievement became a way of assigning an economic value to a human being (Valero, 2018). This idea of human beings as bodies with economic exchange value placed the onus on school mathematics to produce the desired workforce for the technological future. In recent times, the economic potential of individuals is determined not just by their achievement, but through their "ability to apply knowledge to solve problems" and their propensity "to view mathematics as a useful tool that must constantly be sharpened" (National Research Council, 2001, p. 144). In other words, to be economically valuable in today's technological societies, one needs certain affective qualities too besides their cognitive skills in mathematics.

Due to such labor demands, it becomes necessary to socialize children to possess specific affective responses – turning mathematics classrooms into sites for inculcating these attitudes and dispositions. With the emergence of cognitive capitalism – in which the production of wealth takes place through not just material production but also through developing the relational, the affective and the cognitive capacities of labor (Moulier Boutang, 2011) – achieving high scores in mathematics tests is not enough anymore. Children now also need to feel a certain way about themselves and mathematics in order to be 'productive' as a modern-day citizen-worker. Governing the child's affective responses, thus, becomes a focus within mathematics education.

Disciplining bodies

Considering such workforce requirements, I argue that the efforts to modulate children's affective responses are ways of governing their feelings and emotions towards a norm that is valued by modern-day jobs. I use the lenses of *discipline* and *docile bodies* (Foucault, 1995) to elaborate on this. Foucault defined docile bodies as ones "that may be subjected, used, transformed and improved" (1995, p. 136). He saw discipline as a mechanism that ensures that the increased utility of a human body makes it more obedient to commands, and vice versa through a constant subjection of its forces. Discipline produces docile bodies by increasing "the forces of the body (in economic terms of utility) and diminishes the same forces (in political terms of obedience)" (p. 138). Disciplinary techniques initiate "a subtle coercion, of obtaining holds upon it [the body] at the level of the mechanism itself – movements, gestures, attitudes, rapidity: an infinitesimal power over the active body" (p. 137).

To achieve a cognitively and affectively standardized labor force for modern-day technologized societies, mathematics education becomes a site for disciplining bodies. Specifically, with respect to the affective qualities of such bodies, the act of disciplining involves attempts to make all children feel *positively* about mathematics. Those who do not are identified and categorized for subsequent intervention – broadening the notion of what needs *fixing* in mathematics education beyond just the cognitive. A child's body language, pitch, stress, and other embodied aspects are made available for scrutiny by humans, or in some cases, by technologies such as student sensor bracelets (Williamson, 2016). Such scrutiny marks bodies that need disciplining – whose affective qualities must be standardized for enhancing their economic value. In the case of mathematics, this disciplinary power attempts to ensure specific mental habits, ways of thinking, and self-regulated behaviors that align with a modern-day desire for 'scientific rationality' (Popkewitz, 2004). *Negative* affective responses towards mathematics are seen as a threat to progress and development. For example, in the case of the United States, Boaler (2009) states:

> Far too many students in America hate math and for many it is a source of anxiety and fear. American students do not achieve well and they do not choose to study mathematics beyond basic courses, a situation that presents serious risks to the future medical, scientific, and technological advancement of society. (p.10)

Seen another way, not feeling a certain way about mathematics diminishes an individual's mathematical exchange value, and in turn their human capital value (Valero, 2018). Moving away from their earlier motivations of increasing test scores, efforts to modulate children's affective responses towards mathematics have reorganized under a new goal – the creation of a productive technological workforce.

The aim of this paper is not to evaluate efforts to modulate affect as 'good' or 'bad'. My goal is to draw attention to the mechanisms of discipline entwined in these efforts which render a child's mind and thought governable towards a predetermined norm – one that emerges from the desires of neoliberal economies. Any attempt to standardize minds involves classifications based on idealized criteria and the subsequent exclusions of bodies. As a result, what I challenge through this paper is the standardization of the affective - the commonsensical idea that every child has to feel the same way about mathematics. In the current moment marked by the proliferation of STEM education and its associated exclusionary practices, any efforts that claim innocence in their work to change how students feel about mathematics require scrutiny and critical reflection.

References

Boaler, J. (2009). *What's math got to do with it? How parents and teachers can help children learn to love their least favorite subject.* New York: Penguin Books.

Foucault, M. (1995). *Discipline and punish: The birth of the prison* (2nd ed.). New York: Vintage Books.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575–596). New York, NY: Macmillan Publishing Co, Inc.

Moulier Boutang, Y. (2011). *Cognitive capitalism.* Cambridge: Polity Press.

National Research Council. (2001). *Adding it up: helping children learn mathematics.* Washington, DC: National Academy Press.

OECD (2016). *Ten questions for mathematics teachers… and how PISA can help answer them.* Paris: PISA, OECD Publishing.

Popkewitz, T. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Research Journal, 41*(1), 3–34.

Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility.* Rotterdam, The Netherlands: Sense Publishers.

Valero, P. (2017). Mathematics for all, economic growth, and the making of the citizen-worker. In T. S. Popkewitz, J. Diaz, & C. Kirchgasler (Eds.), *A political sociology of educational knowledge: Studies of exclusions and difference* (pp. 117–132). New York: Routledge.

Valero, P. (2018). Human capitals: School mathematics and the making of the homus oeconomicus. *Journal of Urban Mathematics Education, 11*(1), 103–117.

Williamson, B. (2016). Coding the biodigital child: the biopolitics and pedagogic strategies of educational data science. *Pedagogy, Culture & Society, 24*(3), 401-416.

CiviMatics: Mathematical modelling meets civic education

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While mathematical modelling generally has an important position in mathematics education, discussions and approaches about normative modelling are underrepresented. Modelling can be seen as normative, when models are not only used to describe reality, but value or even generate reality, such as mathematical models used to assign legitimate carbon footprints within climate change discussion. Normative modelling for the political discourse requires reflecting about assumptions, simplifications and relevant consequences coming with the models. Starting from the well-known modelling cycle we integrate civic education perspectives to develop a didactical framework around normative modelling, suggesting an enhanced normative modelling cycle that explicitly refers to the judgement formation.

(Normative) mathematical modelling

Mathematical modelling is an important part of mathematics. A mathematical model is a simplified representation of reality that takes into account only certain aspects that can be sufficiently objectified and to which mathematical methods can be applied in order to obtain mathematical results. Models are not unambiguous, because it is possible to make simplifications in different ways. Thus, models should not be considered right or wrong, but more or less useful. The usefulness of a model can only be assessed with regard to the problem to be worked on (Greefrath et al., 2013).

Modelling may have different purposes, pending on the intendent use of the model. For example, descriptive modelling mainly obtains to describe reality, such as physical relations e.g. the planetary orbits. At the same time we may use modelling to "specify or design objects or structures that are meant to inhabit some extra-mathematical domain whilst possessing (if possible) certain required or desired properties" (Niss, 2015, p. 69). This is what we call normative modelling, when reality is not just described, but prescribed as well. Common questions that would indicate normative modelling could be: "Where should a new power plant or a huge shopping centre be located? In what way should seats be apportioned amongst parties in parliamentary elections?" (ibid.). It is clear that normative modelling requires taking a stance that values certain qualities of a solution differently. Unlike many modelling problems in mathematics education, normative modelling cannot be validated.

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A recent and significant example for the relevance of normative modelling is climate change. It is one of the most striking problems that people and governments across the world are facing and it heavily relies on mathematical models both to understand the problem and to evaluate possible solutions. This issue has the potential to change the social and political order of the world (Beck, 2015, p. 75). The introduction of a CO2 tax or global CO2 quotas, the orientation towards the 1.5-degree target or the calculation of ecological footprints are only a few examples of themes, that are widely discussed within politics and society involving mathematical models.

The political dimension of normative modelling

Normative modelling is not only a special kind of modelling but it takes on an important role for political decision-making processes. The application of mathematical models for political decisions is hence in need for reflection. Central, for example, is the understanding that political judgements can never be obtained by calculation alone and that political actors can deliberately influence outcomes by making different assumptions. This brings mathematics education into the focus of education processes for democratic competences, especially for critical analysis skills. It is therefore of democratic importance, both from an individual and a societal perspective, to provide citizens with the competences to understand the role of mathematics in complex situations, such as climate change. In this context, "the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also perform the application of mathematics in society" (Niss, 1983, p. 248). This includes the application of models for political decision-making, but also the reflection on the principles and assumptions that shape the transformation of societal issues into mathematic models. We build on the principles of civic education aimed at providing competences for the critical reflection of democratic processes as well as the principles of a critical mathematics education. Our goal is not only to focus on normative models as an example for conjunction of political and mathematical processes, but also provide approaches to enable learners to critically reflect on said assumptions (Reinhardt, 2013, p. 101; Skovsmose, 1994, p. 58).

Teaching (normative) mathematical modelling

Mathematical modelling has found its way into (German) schools due to different aspects – but not in terms of normative modelling. The curricula focus on descriptive modelling, promoting real world applications of mathematics in problem solving and modelling (Kultusministerkonferenz, 2015). A very popular model used for teaching mathematical modelling is the modelling cycle. It can be used to illustrate the concept of modelling, taken as an aid for learners in the process of modelling, as well as diagnostic tool and basis for empirical studies (Greefrath et al., 2013). The modelling cycle involves seven steps to model a situation, which are displayed in blue in Figure 1 (see Greefrath et al., 2013, for further explanations).

Taking a closer look on steps two and three, the differences between normative and descriptive modelling become clear: For all modelling processes, the simplifications made in these steps, determine the results. However, in normative modelling, the results may have a political dimension that should be discussed. We can only critically reflect on the results, when we take the assumptions for the model into account. This suggests that the sub-processes of the modelling cycle may not be able to fully capture what happens in evaluating models arising from normative modelling (Niss, 2015, pp. 69–77). We thus aim for an enhanced modelling cycle starting with the following points: Normative modelling requires more, namely political analysis of the situation (in addition to mathematical analysis), discussion of political possibilities, and judgment formation as subsequent steps. At a minimum, the discussion of political possibilities requires that different possibilities emerge from the mathematical models or that they be considered from the very beginning. Political analysis also requires identifying interests of involved stakeholders.

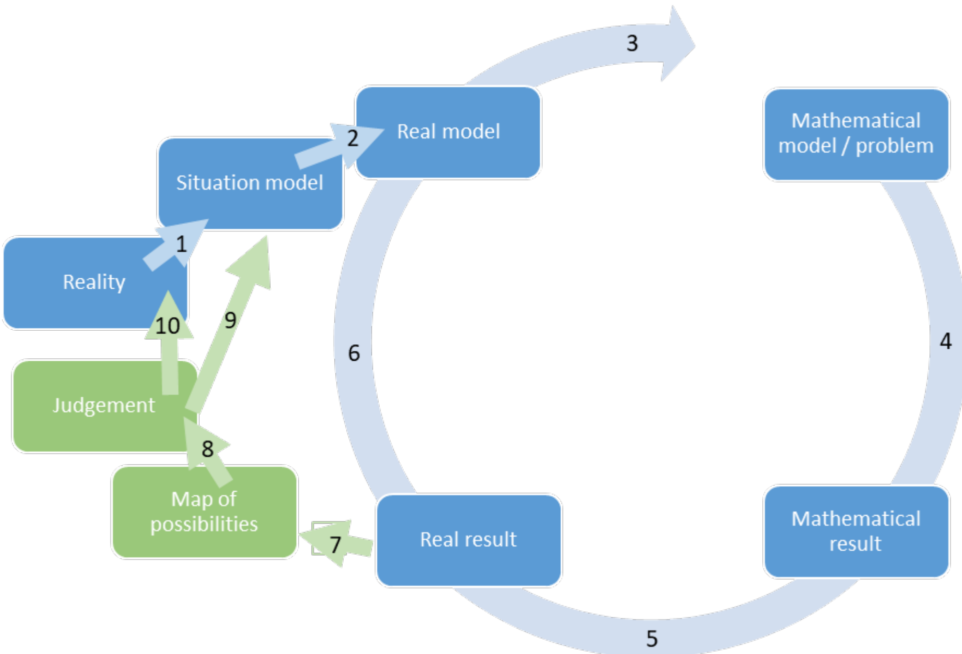

Figure 1: Our purpose of an adjusted normative modelling cycle

Developing an adjusted modelling

The modelling cycle provides indications of the steps in which alternatives and interests might be found. Understanding/Constructing (1.) at least does not involve conscious steps, but suggests that in normative modelling we may need to reconcile different conceptions of reality if we are to negotiate solutions in our societies. Simplifying (2.) is one of the most relevant steps. Which parts of the situation model are included at all and how interrelationships are simplified essentially determines the result. Here, alternatives have to be considered, their consequences for the model have to be estimated and they have to be classified with regard to political interests. Mathematization (3.) is in itself a technical step, provided that the real model is specified precisely enough. In practice, however, the real model is specified more concretely in this step, so that simplifications similar to those in (2.) are to be expected here as well. The mathematical work (4.) will rarely provide starting points for the political discussion. Although alternative actions exist here (e.g., obtaining solutions algebraically or numerically), the differences, if any, should be irrelevant. Interpreting (5.) should also be more of a technical step because it initially involves only the translation of mathematical variables, functions, etc. into reality. However, generalizations could be made at this step, concerning e.g. model assumptions or restrictions of variable ranges, etc. Moreover, the presentation of the results will very often suggest actions, at least implicitly. Such (normative) statements can never be the result of a mathematical calculation and should therefore always be outsourced to the further steps. First, the different possibilities and the different implications related to the stakeholders' interests should be noted through the reflection and critique of the modelling just described. We named that to build a "map of possibilities". After that, everyone should form their own judgment (8.) by weighing the interests. Finally, we should acknowledge that our decision might have an impact on the world as we assume at this moment (relating to our situation model; 9.) and as it is (reality; 10.).

In relation to climate change, the normative modeling cycle could be used, for example, to teach about different ways of intervening to reduce CO2 emissions. One possibility discussed is that of the meat tax or, more generally, the question of the CO2 emission of various foods that are, for example, transported a long way or are expensive to grow or cultivate (1). Based on these individual situation models (2), a model could be set up in which different foods are each assigned a specific CO2 emission for the production process (3). For the transformation into a mathematical model, relevant categories have to be identified and defined to describe the CO2 emission of different foods. Among others, production, transport and the consequences of land use come into question here. In the simplest mathematical model, mean values per kilogram of the foodstuff are researched and summed up for these categories (4). The mathematical result is an overview of the total CO2 emission for one kilogram of the respective food. The result would show that beef on pure meat herds has a relatively high CO2 emission according to this model, as well as chocolate. In contrast, rice and tomatoes for example, have a significantly lower one (5). Scaling to 1000 kilocalories per food instead of kilograms would show a slightly different result: tomatoes would actually show higher CO2 emissions than chocolate (Ritchie & Roser, 2020; Poore & Nemecek, 2018). It could also be discussed whether the preparation and composition (e.g. sugar vs. protein) of the food should be considered in further models (6). The various results could then be the starting point for a discussion, for example, on a meat tax and the associated effects, for example, of an economic or social nature (7). Students may take up different roles and may discuss who would be using which mathematical model to build argumentations and why. Following this, the students may develop a reasoned judgment about the best possible solution to the problem from their perspective and to reflect on the associated effects on the original real model (8). The judgment and the associated decision can then serve as a starting point for a new situation model (9) or guide the students' own political action outside the classroom (10).

Outlook

In July 2021, the first teaching materials on CO2 emissions from food as well as insect populations will be deployed and evaluated in a school context. First explorative results on student conceptions in the context of normative modelling will be collected. We hope to be able to discuss these with the scientific community, as well as our presented theoretical framework for normative modelling.

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References

Beck, U. (2015). Emancipatory catastrophism: What does it mean to climate change and risk society? *Current Sociology*, *63*(1), 75–88.

Greefrath, G., Kaiser, G., Blum, W., & Borromeo Ferri, R. (2013). Mathematisches Modellieren: Eine Einführung in theoretische und didaktische Grundlagen. In R. Borromeo Ferri, G. Greefrath, & G. Kaiser (Eds.), *Mathematisches Modellieren für Schule und Hochschule* (pp. 11–38). Springer.

Niss, M. (2015). Prescriptive modelling: Challenges and opportunities. In G. A. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *Mathematical modelling in education research and practice* (pp. 67–80). Springer.

Niss, M. (1983). Considerations and experiences concerning integrated courses in mathematics and other subjects. In M. J. Zweng, T. Green, J. Kilpatrick, H. O. Pollak & M. Suydam (Eds), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 247–249). Birkhäuser.

Kultusministerkonferenz (2012): Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife. Wolters Kluwer. http://www.kmk.org/fileadmin/Dateien/veroeffentlichungen beschluesse/2012/2012 10 18-Bildungsstandards-Mathe-Abi.pdf

Poore, J., & Nemecek, T. (2018). Reducing food's environmental impacts through producers and consumers. *Science, 360*(6392), 987–992.

Reinhardt, S. (2013). Teaching for democratic learning. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 99–109). Rotterdam: Sense.

Ritchie, H. & Roser, M. (2020). Environmental impacts of food production. https://ourworldindata.org/environmental-impacts-of-food

Skovsmose, O. (1994). Towards a critical mathematics education. *Educational Studies in Mathematics, 27,* 35–57.

Signs of power and dominance: Mathematics curricula in Indian boarding schools, 1879–1932

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This project examines an unexplored area of scholarship in the United States: mathematics education in Indian boarding schools, 1879–1932. The aims of this research include 1) contributing to historical analysis of the role of mathematics education in the U.S. colonial curricula and 2) increasing our understanding of the origins of contemporary mathematics education policy and practices. Theoretically, we argue that the systemic inequality and oppressive practices in mathematics education today are traced to U.S. assimilationist policies and thus rooted in the legacies of colonization and white supremacy. Using archival methods, we examine the history of mathematics instruction for Native youth in federal boarding schools. Finally, we discuss implications for contemporary mathematics education.

Description of the project

We recently launched a historiographic project that intersects mathematics education, educational history, and Native American studies. This interdisciplinary project examines an unexplored research area: mathematics education in Indian boarding schools (IBS) during 1879-1932 (assimilation era of federal Indian policies). U.S. historians have examined the assimilationist policies and practices of IBS; however, little work discusses the significance of mathematics education in these colonial institutions. Using archival methodology and interpretive approaches, our scholarship examines the history of mathematics instruction for Native youth in the federal boarding school system. We seek to develop a theoretical understanding of mathematics education as a colonial assimilatory project and its impact on the experiences of Native students attending IBS during that period. Exposing the powerful undercurrents of assimilation, colonization, and white supremacy running through the history of mathematics education in the U.S. and globally has implications for approaching future research on culturally sustaining pedagogies, specifically Native epistemologies and pedagogies focused on mathematics content.

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Research questions and significance of the project

- How did the mathematics curriculum in Indian boarding schools during 1879-1932 promote U.S federal assimilationist goals?
- How did the mathematics curriculum in Indian boarding schools institutionalize Eurocentric epistemologies and enforce U.S. citizenship and values?

In the United States, there is increased attention on Black and Latinx students in the mathematics-education literature. However, limited research exists for Native youth. As mathematics curricula "appear" unbiased, impartial, and apolitical (Bishop, 1990; d'Ambrosio, 1985), research on mathematics curricula as a colonial tool could illuminate how assimilationist discourses and practices continue for Native students and other under-represented groups in the United States.

Whereas previous scholarship has examined other curricular areas within U.S. boarding schools, such as art education (Lentis, 2017), little research has been done on the history of mathematics curricula. For example, in the *Handbook on the History of Mathematics Education* (Karp & Schubring, 2014), none of the American chapters mention mathematics curricula in IBS. This omission in the literature is startling since mathematics was always included as a priority in the global colonial project (Bishop, 1990). In the United States, the Civilization Fund Act of 1819 provided funding to Christian missions to "civilize" Native children by "teaching [them] reading, writing, and arithmetic," and mathematics content also appears in the 1901 publication, *Course of Study for Indian Schools*, which was acclaimed for unifying the curricula across different federal Indian schools (see below). Our research begins to fill this gap in the literature while building on global Indigenous scholarship (e.g., Meany, 2020; Meyer & Aikenhead, 2021; Nutti, 2013; Parra & Trinick, 2018).

Conceptual framework: U.S. colonization, White supremacy, and mathematics

Horsman (1981) argues that the "American Anglo-Saxon" conceived of themselves as "a separate and innately superior people who were destined to bring good government, commercial prosperity, and Christianity to the American continents and the world" (p. 2). Our project is informed by scholars arguing that the dominant models of mathematics education function as racial projects within white supremacist structures (e.g., Gutiérrez, 2019; Martin, 2013). Monica Miles and her colleagues (2019) observe: "In mathematics in particular, Eurocentrism—in both epistemology and pedagogy—dominates, requiring students to conform to White ways of knowing and learning" (p. 105). The application of the IBS mathematics curriculum shows the distinct ways schooling is a form of white supremacist power and human dominance.

Utilizing TribalCrit (Brayboy, 2005) and Safety Zone Theory (Lomawaima & McCarty, 2006), we conceptualize contemporary mathematics education as the ongoing legacy of the U.S. colonial project of forced assimilation. During the federal Indian assimilation policy period and our period of study (1879–1932), the metaphoric safety zone of "allowable cultural expression" (Lomawaima & McCarty, 2006, p. 5) had a diameter nearly at "0," in which "Indian-ness" was extracted and replaced with subservience to American citizenry. During the assimilation period, mathematics education at IBS did not include Indigenous knowledges (Holm et al., 2003) and instruction was for subservience in the manual labor market. The expectations for learning arithmetic are reflected in the following excerpt from the *Course of Study for Indian Schools*, published by the Office of Superintendent of Indian Schools in 1901:

> Let all problems be practical and so simple that the child has no difficulty in stating them before he performs the operation. Aim at only reasonable facility on the part of the child, but he must be accurate. All exercises in fractions and percent should be confined to small numbers and to subjects likely to come within the pupil's experience. Number work involving a labored [sic] process of reasoning as in "catch examples" should be discarded. (p. 42)

Policymakers and practitioners conceived of mathematics education imposed through the IBS to be innately superior as they attempted to extirpate "Indian-ness."

In the contemporary period, educational scholars have argued that liberal mainstream schooling for minoritized students functions as de facto assimilation (e.g., Paris & Alim, 2017), where the language and practices around academic learning employ English-only policy, reproduce cultural erasure, and inscribe dominant values and ideology rooted in whiteness and colonization. The de facto assimilation within contemporary schooling has not progressed far from the explicit assimilation policies enacted through the IBS. We hypothesize that both oppressive and "safe" policies and practices in contemporary mathematics education for Native students can be traced to early federal assimilationist policies. This period of mathematics education has implications also for math instruction for students of color as schooling and educational practices are also tied to this history.

History of boarding schools in the United States and their curriculum

In the late nineteenth century, the United States designed Indian boarding schools as a "civilization" project (Child, 1998). Their curricular model consisted of half a day of instruction in reading and mathematics and the remainder was manual labor. Previous research on IBS has focused on policies, sports and athleticism, and students' social and emotional experiences. The government developed a "Uniform Curriculum" in the early 1900s (Lomawaima & McCarty, 2006); however, research on the implemented mathematics curricula during 1879-1932 is non-existent. By the 1920s, teachers and school administrators developed local curricula while following federal guidelines. For example, Chilocco's curriculum differed from Haskell's because of their distinctive missions (agricultural vs. industrial training), but all integrated Christianity as part of their plan to "save" and "civilize" Native children. This project will contribute to the literature on Indian boarding schools by investigating the significance of mathematics in the assimilationist mission.

Archival methods: Analysis of IBS mathematics curricula

We have started a historiography of mathematics education in six Indian boarding schools, between 1879-1932: Carlisle Indian Industrial School, Pennsylvania (1879–1918); Chemawa Indian School, Oregon (1880–present); Chilocco Indian School, Oklahoma (1884–1980); Haskell Indian Industrial Training School, Kansas (1884–present); Phoenix Indian School, Arizona (1891–1990); and Sherman Institute, California (1892–present). This list was generated based on IBS historiography, geographical diversity, and well-documented secondary sources, which can enhance connections between our findings and previous scholarship. The U.S. Bureau of Indian Affairs operated dozens of off-reservation boarding schools between 1879–1932; we narrowed our project to six with the intent of expanding the list in the future. Specific analysis methods, examples of archival data (e.g., policy documents, photos of mathematics class, etc.), and preliminary findings will be provided in our presentation.

References

Bishop, A. (1990). Western mathematics. *Race & Class*, *32*(2), 51–65.

Brayboy, B. M. J. (2005). Toward a Tribal Critical Race Theory in education. *The Urban Review*, *37*(5), 425–446.

Child, B. J. (1998). *Boarding school seasons*. University of Nebraska Press.

d'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, *5*(1), 44–48.

Gutiérrez, R. (2019). Mathematx: Towards a way of being. In J. Subramanian & P. Basu (Eds.), *Proceedings of the 10th International Mathematics Education and Society Conference*. University of Hyderabad.

Holm, T., Pearson, J. D., & Chavis, B. (2003). Peoplehood: A model for the extension of sovereignty in American Indian Studies. *Wicazo Sa Review, 18*(1), 7–24.

Horsman, R. (1981). *Race and manifest destiny*. Harvard University Press.

Karp, A., & Schubring, G. (Eds.). (2014). *Handbook on the history of mathematics education*. Springer.

Lentis, M. (2017). *Colonized through art*. University of Nebraska Press.

Lomawaima, K. T., & McCarty, T. L. (2006). *"To remain an Indian": Lessons in democracy from a century of Native American education*. Teachers College Press.

Martin, D. B. (2013). Race, racial projects, and mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 316–333.

Meany, T. (2020). Indigenous students in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 369–373). Springer.

Meyer, S., & Aikenhead, G. (2021). Indigenous culture-based school mathematics in action. *The Mathematics Enthusiast*, *18*(1-2), 100–118.

Miles, M. L., Marshall, S. A., McGee, E. O., Buenrostro, P. M., & Adams, M. (2019). Cultivating racial solidarity among mathematics education scholars of color to resist white supremacy. *International Journal of Critical Pedagogy*, *10*(2), 97–125.

Nutti, Y. J. (2013). Indigenous teachers' experiences of the implementation of culture-based mathematics activities in Sámi school. *Mathematics Education Research Journal*, *25*, 57–72.

Paris, D., & Alim, H. S. (Eds.). (2017). *Culturally sustaining pedagogies*. Teachers College Press.

Parra, A., & Trinick, T. (2018). Multilingualism in indigenous mathematics education: An epistemic matter. *Mathematics Education Research Journal*, *30*, 233–253.

Involving students' perspectives in multilingual mathematics learning spaces

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In a small on-going participatory research project, we collaborate with mathematics teachers. The project has reached a point where we, both the researchers and the teachers, have come to recognize the need to involve students in the design of their mathematics learning spaces because their knowledges and experiences may provide a crucial contribution. Therefore, we investigate in what way the students, teachers and researchers may collaborate to design mathematics learning spaces to facilitate multilingual students' mathematics learning. Involving the students in designing socially just mathematics learning spaces necessitates particular methodological decisions and identifies challenges in need of careful attention.

Focus and significance of the project

Elev: *[...] jag menar vi pratar arabiska på lektionen. Vi säger så här för att vår lärare är arabisk så han låter oss skratta på arabiska mycket och vi skrattar och vi blandar arabiska och så.*

Student: *[...] I mean we speak Arabic during the lesson. We say this because our [mathematics] teacher is Arabic, so he lets us laugh in Arabic a lot and we laugh and mix Arabic [and Swedish] and so.*

(Alhadi Alhasani & Zaki, 2021, p. 38)

The quote above comes from an interview that two pre-service teachers conducted as part of writing their degree project. The student's mother-tongue is Arabic. She attends middle school in Sweden, where the language of instruction is Swedish. In the quote, she describes how her teacher promotes laughing in a mathematics class. That she could use her full range of language resources made us feel joy and sparked the hope for social justice in language-diverse mathematics classrooms. In an on-going small participatory research project with three in-service mathematics teachers from the south of Sweden, we have jointly decided to investigate in what way these teachers' students can be involved in the discussions on and design of their multilingual mathematics learning spaces.

Despite persevering efforts in Sweden to promote all students' first languages as a resource in mathematics learning, many students with other language backgrounds than Swedish cannot (or may not) use the full range of their language resources in school

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(Lundberg, 2019). Among other things, this influences the interaction in language-diverse mathematics classrooms (Ryan, 2019) and the multilingual students' foregrounds (Svensson, et.al., 2014). Previous studies have demonstrated in what ways teachers can design learning activities that involve the multilingual students' languages and cultures as resources for mathematics learning (Celedón-Pattichis, et.al., 2018; Planas & Setati-Phakeng, 2014), rather than viewing these languages and cultures as deficits, which often is the case (Gutiérrez, 2008; Källberg, 2018). Students are responsible, knowing, feeling, thinking and acting subjects. Students know about language diversity and mathematics learning. When taken seriously, their knowledges and experiences can make a crucial contribution to the establishment of learning spaces (Stith & Roth, 2006). This motivates us, both the researchers and the teachers, to involve the students in the design of their learning spaces. To develop the project along these lines, we aim to investigate how the students' perspectives can be incorporated in the design of learning spaces *by student participation* to enhance socially just mathematics learning opportunities for multilingual students. We recognize that external factors – for example, demands for summative assessment – position the students and teachers in ways that may limit the design of socially just mathematics learning spaces (Cabral & Baldino, 2019).

The participants in this project are the two authors, three mathematics teachers and their students. The teachers teach grade 4 and 8 (10- and 15-year-old students). Concept development and problem solving were identified by the participating teachers as areas in their teaching they wanted to develop, which, therefore, frames the project. The following two over-arching research questions guide the project:

1. How can the students, teachers and researchers collaborate to design mathematics learning spaces to facilitate multilingual students' opportunities to work with mathematical concepts and problem solving?
2. How do the participating students and teachers experience working together to design mathematics learning spaces to facilitate the students' work with mathematical concepts and problem solving?

Theoretical background

In the context of multilingualism, problem solving in mathematics class presents multidimensional challenges to students and teachers. For example, usually students need to read a text that mediates the problem to be solved. In other words, students usually approach problem solving as a matter of dealing with text-rich tasks. Word problems pose lexical and discursive challenges because of the use of concepts that range from subject-specific to everyday concepts. In addition, word problems in mathematics represent a very particular genre which students need to be familiar with to understand the problem posed. To add to this complexity, many mathematical problems are situated in the so-called everyday experiences and events, which suggests a cultural dimension to problem solving (Barwell, 2009). Language and culture are issues of, for example, communication, identity and epistemology because language and culture are matters of knowing and of knowledge (Ryan, 2019). Hence, to solve mathematical problems, multilingual students need to (re)produce ways of knowing mathematics and mathematical concepts, and mobilize lexical, discursive and cultural resources across all their languages and cultures to do so. The notion of language-as-a-resource holds various epistemological potentials for multilingual mathematics activities as suggested in Ryan, Källberg and Boistrup's (2021) epistemological framework for multilingual mathematics activities. For example, the 'lever' potential when actualised, can move students from informal mathematics talk in their first language to formal mathematics talk in the language of instruction. Hence, this potential functions as a *lever.* The 'one new whole' potential draws on the theoretical construct of translanguaging and, therefore, constitutes prerequisites to produce new ways of languaging and knowing mathematics (Ryan, et al., 2021). Our assumption is that various epistemological potentials make different kinds of learning spaces available which influence students' and teachers' agencies and identities when they engage in multilingual mathematics activities.

Methodology

With the aim to develop innovative ways of facilitating learning spaces for multilingual students' work with mathematical concepts and problem solving, the project builds on a close cooperation between the teachers, their students and the researchers. We draw on ethno-mathematical knowledge to envision our cooperative meetings as 'barters', a way of perceiving

> elements that other participants do not produce. Sometimes these elements are either exchanged or given as a gift [...] Every person involved in the barter returns home with something new gained in the barter. Roughly speaking, when people are engaged in a barter, tasks that only can be done with a joint effort are accomplished and the interdependence of the agents is emphasized, benefiting everyone. (Parra-Sanchez, 2017, p. 94)

To stage such 'barters', the students, teachers and researchers conduct workshops on themes that connect to the jointly identified problem solving in their multilingual classrooms. The workshops are designed to enable the participants to brainstorm new ideas and solutions in relation to the chosen theme (Alminde & Warming, 2019). In addition to the student-teacher-researcher workshops, the researchers and the teachers meet in seminars once a month. During the seminars, the researchers and the teachers discuss and reflect on the implementation of the outcomes of the barters in the classrooms.

We recognize that asymmetric power relations – for example, pertaining to authority, responsibilities and interests among students, teachers, and researchers – are embedded in our project. This necessitates being explicit about such asymmetries, making power relations visible and addressing them carefully. We look forward to learning from and discuss with the MES community how to handle asymmetric power relations ethically and responsibly in participatory research projects that involve students.

References

Alhadi Alhasani, H., & Zaki, S. (2021). *Arabisktalande elevers användning av språkliga resurser i matematikämnet.* Exam paper, Malmö university, Malmö.

Alminde, S., & Warming, H. (2020). Future workshops as a means to democratic, inclusive and empowering research with children, young people and others. *Qualitative Research, 20*(4), 432-448.

Barwell, R. (2009). Mathematical word problems and bilingual learners in England. In R. Barwell (Ed.), *Mutilingualism in mathematics classrooms: Global perspectives* (pp. 63-77). Bristol, Buffalo, Toronto: Multilingual Matters.

Cabral, T. C., & Baldino, R. R. (2019). The credit system and the summative assessment splitting moment. *Educational Studies in Mathematics, 102*(2), 275-288.

Celedón-Pattichis, S., Borden, L. L., Pape, S. J., Clements, D. H., Peters, S. A., Males, J. R., ... & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education, 49*(4), 373-389.

Gutiérrez, R. (2008). A "gap-gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education 39*(4), 357-364.

Källberg, P. S. (2018). *Immigrant students' opportunities to learn mathematics. In(ex)clusion in mathematics education* [Doctoral dissertation, Stockholm University].

Lundberg, A. (2018). Multilingual educational language policies in Switzerland and Sweden: A meta-analysis. *Language Problems and Language Planning, 42*(1), 45-69.

Parra-Sanchez, A. (2017). Ethnomathematical barters. In H. Straehler-Pohl, N. Bohlmann & A. Pais (Eds), *The disorder of mathematics education* (pp. 89-105). Switzerland: Springer.

Planas, N., & Setati-Phakeng, M. (2014). On the process of gaining language as a resource in mathematics education. *ZDM, 46*(6), 883-893.

Ryan, U. (2019). Mathematical preciseness and epistemological sanctions. *For the Learning of Mathematics, 39*(2), 25-29.

Ryan, U., Källberg, P. S., & Boistrup, L. B. (2021) 'Language as Resource' in multilingual mathematics activities - an epistemological framework. *For the Learning of Mathematics, 41*(2) 8-13.

Stith, I., & Roth, W. M. (2010). Teaching as mediation: The cogenerative dialogue and ethical understandings. *Teaching and Teacher Education, 26*(2), 363-370.

Svensson, P., Meaney, T., & Norén, E. (2014). Immigrant Students' Perceptions of Their Possibilities to Learn Mathematics: The Case of Homework. *For the Learning of Mathematics, 34*(3), 32-37.

Bridging mathematics students and the challenges of their learning disabilities

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Students without recognized levels of academic numeracy and literacy for university entrance are often diverted into a six-to-twelve month bridging or foundation programme where they must pass a mathematics course. However, a small number have struggled to learn mathematics from their earliest schooling and they again find themselves in an uncomfortably similar position. The students have known or unknown mathematical learning disabilities (MLDs) which may account for their own perceived lack of progress in mathematics, their frustrations and anxiety. An aim of the study is to find out how the mathematical learning needs of these students at a few New Zealand universities are being supported. In the early stages of this doctoral project, students with MLDs who failed at bridging mathematics share their experiences.

Introduction

Students entering degree programmes are expected to have sufficient levels of academic numeracy (and literacy) before being accepted. Most entrants arrive with Year 12 and 13 mathematics credits from the National Certificate of Educational Achievement (NCEA) as evidence of their readiness for university degree studies. Students without these levels and credits are often diverted into bridging, pre-degree or foundation programmes which may take from six months to a year to complete. Benseman and Russ (2003, p. 45) define bridging (or foundation) programmes as providing learners with "the requisite academic skills that will enable them to enrol in other tertiary programmes to which they would not otherwise have been able to gain entry." At one of the participant universities, each student on a bridging programme must pass at least one mathematics course to satisfy academic numeracy requirements. However, a small proportion of students on the programme have struggled to learn mathematics from their earliest educational experiences and often find themselves in an uncomfortably familiar situation when they must again face learning mathematics.

Some entrants have known or unknown mathematics learning disabilities (MLDs) which may account for their perceived lack of traction when learning mathematics, and the resultant frustrations and anxieties. This is exacerbated for those who fail their first semester bridging mathematics course, then find themselves once again having to take the same course in semester two. MLDs are described as "cognitive *deficits* in a student's processing of numerical information that lead to persistent and pervasive difficulties with mathematics"

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(Lewis, 2017, p. 321, original italics). However, labelling with a 'deficit' assumes that people without MLDs somehow occupy the high ground of normality while the diversity among human beings and their abilities is overlooked, and what mathematics they are actually being asked to do.

Positioning students with LDs is tenuous at best when describing what they contend with as some kind of medical condition and ignores their social identities (Lambert, 2018) and their working and learning environments. In terms of framing this study, Lambert's and Tan's (2017) disability studies in mathematics education (DSME) supports the privileging of student stories about learning and their reflections about the systemic trials they have encountered. Accordingly, the early part of this doctoral study documents some voices of students with MLDs who have shared their mathematical learning experiences, particularly those with MLDs who failed the bridging mathematics course in their first semester then went on to repeat the same course in their second semester to try to meet a challenging academic numeracy target.

Motivation for the study

The initial motivation for the study came from observing students' struggles in the years prior to the second semester repeat course being offered. Before 2015, even though students had failed the mathematics course in their first semester, they were still promoted into a more conceptually demanding second semester course to meet their academic numeracy requirements in the same calendar year. It is likely that these struggling mathematics students were given an even deeper hole within which to flounder and unsurprisingly several dropped out. In the 2011 cohort for instance, of the seventeen students who achieved D grades or less in the first semester course, and who then were invited to take the second semester course, seven recorded yet another D grade while the other ten finished with a Did Not Sit result.

With the repeat class being offered from 2015, although the same mathematical hurdles are still in place for failing students, the material is relatively recent. With there being a five-week mid-year break between the courses, these students have possibly maintained some learning momentum depending on what they understood in the first run. This understanding, however, is not always evident since several students with experiences of learning disabilities have inconsistencies in some fundamental mathematical ideas with which they were perhaps not supported through in their earlier learning. Their long histories of frustration with mathematics and the stories of their endurance provide a second motivation for this interpretative study.

Method

A small initial sample (n=7) of bridging mathematics students from a university bridging programme who failed their first semester maths course and then were asked to repeat it, were approached by a programme administrator to take part in an interview with the author. The group of student participants was spread across several cohort years from a large local university. The bridging programme they were on typically had a year-long schedule of eight papers with at least one being mathematical. With the COVID-19 situation, three participants chose to be interviewed via Zoom while the others opted for a face-to-face setting while the city was between lockdowns. In a semi-structured interview setting, participants responded to a set of questions about their previous experiences when learning mathematics at school and their transition to university bridging mathematics. The author also took field notes during some of the conversations and two participants wrote to aid their explanations. The Zoom audio-recording function enabled transcribing, before each transcript was returned to the participant for checking. Some preliminary themes emerged from the interviews and several of these are discussed in the next section.

Preliminary findings

There is not room to include all of the preliminary themes so only a few key participant themes that recur have been discussed below. Most of the learners recounted struggles with learning mathematics from their primary schooling. Julia explained how her ADHD impacted on how she was treated by teachers.

> I was always labelled as the problem kid, and a little too difficult to deal with. And I think a lot of the time, especially in primary school, if I was disrupting, or I wasn't paying attention, they'd think that I ... I don't know, they'd just pop me off to something like they wouldn't bother with trying to teach me and actually sit me down.

Often other family members or tutors would be engaged for support, but this assistance with mathematics would sometimes backfire. Sarah's mother is a teacher and they would often work side by side after school, but Sarah's memories were far from fond.

> We always did our homework together and that's when you know we she has a very short attention. I have a very short attention span. She y ... she's very impatient. So, you know, we always ended up fighting and I always ended up crying. And it was just, yeah. I think that's probably how it started. Just associating maths words with crying and anxiety.

Another recurring theme was how just about every participant ended up in the lowest stream in secondary school. So not only was there the struggle to understand mathematical concepts, but there was what could also be described as the twin losses of face by being withdrawn from their peers <u>and</u> shifted into learning mathematics alongside a younger cohort.

After leaving school some went to work before entertaining the idea of returning to studies at university. They often worked in poorly paid jobs to support themselves during or between school and university. Gracie was one of the earlier work starters and describes how exhausted she would be in classes, which also perhaps contributed to her learning struggles:

> Even during school … I started working when I was about 14. Um I was looking everywhere so I started working at Maccas … and then in the end a lot of fast food places. And so even when I came to University I was still working. So it was pretty hard to keep up and I that's why I was like (someone would ask) did you go to that lesson? I was tired all the time ...

Drawing on fundamental mathematics remains a challenge for some. Two participants who work in hospitality separately described how the need for speed and accuracy when cashing up the register or balancing the electronic or credit card transactions at the end of their shifts made them feel inadequate beside other staff members who could be 'trusted' to complete the task. While there was no lack of either wanting to be able to manage those tasks by themselves, the amount of time they would need to deliberately check and recheck the money tallies was, after an unspecified period of time, either not offered them, or they ended up not volunteering for it altogether.

There is a lingering question about what actually happens when these learners are trying to understand mathematics. Julia describes a kind of 'brain fog' enveloping her when confronted with learning, and her exasperation with combatting this.

> So cognitively, since I already struggle to focus. I mean, obviously, with years of practice, I've become better at that but my brain fog is not great. It's just like your brain doesn't work properly. It almost feels like your brain half shuts down. You're very slow to think; things that should come quickly to you that should be basic, don't. Just legitimately feels like your brain's kind of shut down and given up and it's very frustrating because you are aware of it and you try to force yourself ...

Participants explained how they had neither the confidence nor the wherewithal to develop authority in mathematical learning, specifically the concepts presented in any mathematics courses. In earlier schooling they would often find themselves being compared to others who were adjudged to be more capable, perhaps reinforcing the negative mould that was never far away from them with regard to this subject.

Looking ahead

Two other universities are being invited into the study. Later interviews will employ focus groups with university bridging mathematics teachers and Disability Support Services staff. It is hoped that their perspectives will provide institutional data with what is being done to support these students.

References

Benseman, J., & Russ, L. (2003) Mapping the territory: A survey of bridging education in New Zealand. *New Zealand Journal of Adult Learning, 31*(1), 43-62.

Lambert, R., & Tan, P. (2017). Conceptualizations of students with and without disabilities as mathematical problem solvers in educational research: A critical review. *Education Sciences, 7*(2), 51. https://doi.org/10.3390/educsci7020051

Lambert, R. (2018). "Indefensible, illogical and unsupported"; Countering deficit mythologies about the potential of students with learning disabilities in mathematics. *Education Sciences, 8*(72), 1–12.

Lewis, K. E. (2017). Designing a bridging discourse: Re-mediation of a mathematical learning disability. *Journal of the Learning Sciences, 26*, 320–365.

Abeng for multispecies' flourishing

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In this project/poster we present a collaborative poetic inquiry (Sameshima et al., 2017) that draws upon our overlapping sets of expertise as mathematics, science and quantitative literacy educators. We use as our prompt the metaphor of "breath in our bones" and start with the literal – how the atmosphere comes to life in our multispecies kin (Tsing et al., 2019). Shifting scales and working to bring light to the shadows that continue to be cast by plantation practices associated with the founding of the modern world economy—slavery, racism, genocide, and ecocide we attempt to signal that without multispecies' flourishing (Khan, 2020; Tran et al., 2020) the probability of widespread human flourishing is limited. We draw upon an analogy with the abeng as we present examples from our practice as teacher educators.

C de breath in dese bones

Who listens to the earth, to the species on it?
Who feels the shortness of their breath?
Whose job is it to give warning?
Most cultures have had elders or spiritualists to give the call,
to listen to the world,
to read and write the world and the word,
to blow the abeng,
to cause us to gather.
If not mathematicians, scientists, technologists and educators then who?

I think it needs to be more explicit.
It is the colonial cultures,
the 'more developed world',
the old and new Empires
that have contributed most of the carbon.
They have stolen the pathways
from "'lesser' developed" to "'more' developed."
Unjust as colonialist countries stole people, culture, resources,
they've stolen an 'easy' future,
they've stolen the breath in our bone

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by handcuffing those country's options for advancement.
All we have left are uneasy futures.

There is power in the Abeng
Gaia's call
The earth a breathing being, exhaling
inhaling...
Climate change as Earth's call...
the Abeng as humanity's call…?
Last call.

In what way might poetry inform the epistemic foundations of mathematics/science/technology?

Drawing upon the framing of ethnomathematics as mythopoetic curriculum (Khan, 2011) – a third approach in Curriculum Studies that establishes the imagination as central to curriculum work and brings together progressive and critical approaches in post-Industrialized contexts – we consciously and humbly draw upon an analogy with the *abeng*, a Ghanian word meaning an animal's 'horn.' The *abeng* is the archae[1]-texture anchoring our work. The blowing of the horn in the West Indies called slaves to the sugarcane plantations and allowed Maroon[2] armies to communicate among themselves (Cliff, 1984/1995). Today, 'New World' Africans blow the abeng symbolically as "a call to arm themselves…to stand up and defend their culture and traditions against extinction" (Abengcentral, n.d.).

As we ponder the breath in our bones, we wonder, what are changes in the composition of that atmosphere—the breath in our bones— due to rising anthropogenic carbon dioxide and human activity (e.g., logging/mining) doing to the bodies/bones of our multispecies kin? Our inquiry draws on recent scientific research into nutrient de-densification (Ebi & Loladze, 2019; Loladze et al., 2019; Zhang et al., 2020; Zhu et al., 2018) ocean acidification (Mekkes et al, 2021), changing seasonal patterns (Karrow, Khan & Fleener, 2018), biophonic 'desertification' (Krause, 2012), which we use as examples in our practice as teacher educators. Our poetic representations will be multimodal/synaesthetic, including poetic verse, visual data stories (graphs and infographics), photographs and soundscape. Some examples are included. Our work is a form of *symbiopoetics*, riffing off of Helmreich's (2009) *symbiopolitics*. In this sense, "A breath of our mouth becomes the portrait of the world" (Herder, quoted in Heidegger, 1971/1975, p. 139) and a poetic call to gather our marooned communities together. Our choice to use poetry is very much influenced by Feyerbend (2001).

Hearing the rhythm, pulse and pattern of data requires an attuning to its details – its silences and its rhythm - just as one listens to music. To many it would appear that the data

[1] Archaea are unicellular prokaryotes, obligate anaerobes, evolutionarily distinct from bacteria and other eukaryotes. The first species identified were known as extremophiles. They are important in oceans and in the microbiome of all species.

[2] Maroons were communities formed by slaves and Indigenous peoples who escaped plantations. These communities continue to exist today in the Caribbean.

representing climate change is akin to listening to Lou Reed's "Metal Machine Music" at full volume; incoherent, grating and without meaning or merit (to the casual listener the album sounds like a microphone buried in badly functioning industrial equipment occasionally being beaten with a sledgehammer). When one reads comments in news media posts about climate change one sees the raging voices of those to who the music of climate data seems to be just that (Bowen & Rodger, 2008). To a climate scientist that data is a rising orchestra of sound, an opera with more and more instruments and more and more singers joining in heading collectively towards a terrible climax, a crescendo followed by a sharp fall with a single keening, voice at the end slowly and plaintively fading away. Climate scientists don't want that opera, don't want that crescendo, don't want that outcome, do not want an opera in which everyone dies.

Quantitative literacies are important in rendering sensible (sense-able) the connections among multiple movements towards justice for example racial justice, ecological justice and economic justice. When combined with other multiliterate and multimodal practices, a focus on multispecies' flourishing and interdisciplinary practice decenters (but does not devalue) D'Ambrosio's (2010) ideas of mathematics as a means for human survival with dignity and as a means to more peaceful coexistence and extends it to our many multispecies partners on this planet and their impact on our mathematical attentions.

Figures

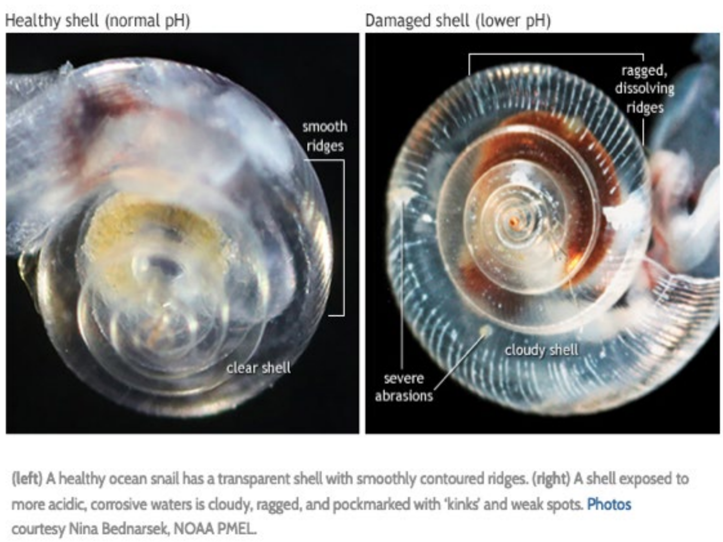

Figure 1: Scientific photograph of changes affecting shell integrity due to ocean acidification. (Image credit: National Oceanic and Atmospheric Administration)

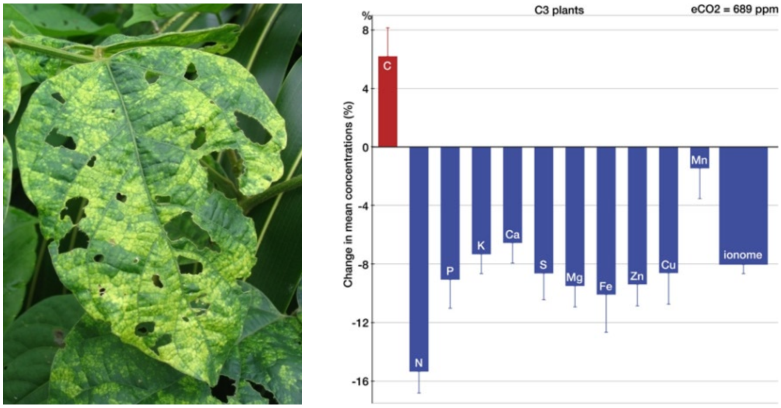

Figure 2: Representations of missing nutrients in plants under elevated CO2 levels. Left image S. Khan, Right image Irakli Lolzdze (2014)

Figure 3: Spring melt and tapping the sugar maples. (Images D. Karrow)

References

Abengcentral (n.d.) The abeng. https://abengcentral.wordpress.com/jamaica/#abeng

Bednaršek, N., Feely, R. A., Reum, J. C. P., Peterson, B., Menkel, J., Alin, S. R., & Hales, B. (2014). *Liminica helicina* shell dissolution as an indicator of declining habitat suitability owing to ocean acidification in the California current ecosystem. *Proceedings of the Royal Society B, 281*(1785), Article 20140123.

Bowen, G. M., & Rodger, V. (2008). Debating global warming in media discussion forums: Strategies enacted by persistent deniers. *Canadian Journal of Environmental Education, 13*(1), 89–106.

Cliff, M. (1984/1995). *Abeng (Ä·běng)*. New York: Plume.

D'Ambrosio, U. (2010) From Ea, through Pythagoras, to Avatar: Different settings for mathematics. In M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp 1–20). Belo Horizonte, Brazil: P11E.

Ebi, K. L., & Loladze, I. (2019). Elevated atmospheric CO2 concentration and climate change will affect our food's quality and quantity. *The Lancet, 3*(7), E283-E284. https://doi.org/10.1016/S2542-5196(19)30108-1

Feyerbend, P. (2001). *The conquest of abundance.* University of Chicago Press.

Heidegger, M. (1975). *Poetry, language, thought.* Harper & Row. (Original work published in 1971).

Karrow, D., Khan, S., & Fleener, M.-J. (2018). Our mathematical relationship with climate change: mathematics education's ethical imperative. *Philosophy of Mathematics Education Journal, 32.*

Khan, S. K. (2011). Ethnomathematics as mythopoetic curriculum. *For the Learning of Mathematics, 31*(3), 14–18.

Khan, S. K. (2020). After the M in STEM: Towards multispecies' flourishing. *Canadian Journal of Science, Mathematics and Technology Education, 20*, 230–245. https://doi.org/10.1007/s42330-020-00089-4

Krause, B. (2012). *The great animal orchestra: Finding the origins of music in the world's wild places.* Little Brown.

Mekkes, L., Renema, W., Bednaršek, N., Alin, S. R., Feely, R. A., Huisman, J., Roessingh, P., & Peijenburg, K. T. C. A. (2021). Pteropods make thinner shells in the upwelling region of the California Current ecosystem. *Scientific Reports*, Article 1731.

Loladze, I., Nolan, J. M., Ziska, L, H., & Knobbe, A. R. (2019). Rising atmospheric CO2 lowers concentrations of plant carotenoids essential to human health: A meta-analysis. *Molecular Nutrition and Food Research, 63*, Article 1801047.

Sameshima, P., Fidyk, A., James, K., & Leggo, C. (Eds.) (2017). *Poetic inquiry: Enchantment of place.* Vernon.

Tran, T. T., Khan, S. K., & LaFrance, S. (2020). Mathematics for multispecies' flourishing: Make kin with Vietnamese Bánh Chưng. *Journal of the Philosophy of Mathematics Education*, 36.

Tsing, A. L., Mathews, A. S., & Bubandt, N. (2019). Patchy anthropocene: Landscape structure, multispecies history, and the retooling of anthropology. *Current Anthropology, 60*(20), 186–197.

Zhang, X., Xu, D., Han, W., Wang, Y., Fan, X., Loladze, I., Gao, G., Zhang, Y., Tong, S., & Ye, N. (2020). Elevated CO2 affects kelp nutrient quality: A case study of *Saccharina japonica* from CO2-enriched coastal mesocosm systems. *Journal of Phycology, 57*(1), 379–391. https://doi.org/10.1111/jpy.1309

Zhu, C., Kobayashi, K., Lolzaze, I., Zhu, J., Jiang, Q., Xu, X., Liu, G. et al. (2018). Carbon dioxide levels this century will alter the protein, micronutrients and vitamin content of rich graisn with potential health consequences for the poorest rice-dependent countries. *Science Advances, 4*(5), Article eaaq1012. https://doi.org/10.1126/sciadv.aaq1012

Teaching middle school mathematics through global perspectives: An open online course

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Mathematical ideas are used across cultures to make sense of the surroundings, represent patterns, and predict future events. However, most students associate mathematics as a tool to manipulate numbers rather than a means to communicate complex ideas. To bridge this gap, an open online course was developed to introduce mathematics as a means to understand major social, political, and ecological issues in the world. The course is designed for middle school students (grades 6-8) and aims to build competence in statistics and data handling. The course content can be used to complement the existing mathematics curriculum and promote awareness about global issues. During the project presentation, we will showcase the open online course and the process of constructing the course.

Problem statement

We use mathematical ideas across cultures to make sense of our surroundings, represent patterns, and predict future events (Wagner, 2010). Some examples include (a) our representation of time through globally accepted 24 hour-days, or (b) our systems of evaluating the worth of goods and services using currency. Through the use of mathematics, we have often found ways to communicate our ideas and experiences with other people irrespective of geographical and linguistic boundaries (Orth, 2013). In fact, the scope of mathematics education to encourage active citizenship and help students understand global issues has been identified (Boylan & Coles, 2017; Renert, 2011).

However, contemporary school mathematics is designed to build a certain level of mathematical skill. Students are rarely exposed to the scope of applying their knowledge beyond classroom contexts. Even the application or story problems fit a specific template and students are taught to identify key words and perform relevant operations (Gerofsky, 2004). The emphasis on solving problems swiftly causes students to automate the process of calculation and desensitizes them to the context of the problem and meaning of the numbers (Wagner & Davis, 2010). As a result, most students associate mathematics as a tool to manipulate numbers (Davis, 2014) rather than a means to communicate complex ideas.

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Given this disconnect in the current system, there is a need to bridge the gap between school mathematics and the real-world. In response, we have developed an online course that introduces mathematics as a means to understand and communicate details about the major social, political, and ecological issues that our world faces today. This open online course aims to enable teachers and learners to complement the existing middle school mathematics curriculum with content that promotes awareness about global events and issues. Additionally, the project team will share the course development process which may be beneficial to other educators and researchers.

Significance

In 2015, the Sustainable Development Goals (SDGs)—a universal call to action to end poverty, protect the planet and ensure that all people enjoy peace and prosperity was adopted by the 193 member states of the United Nations. Of the 17 SDGs, SDG 4 focuses not only on providing 'quality, inclusive and equitable education' – but also an education that improves people's lives and seeks to build peaceful and sustainable societies. In order for SDG 4 to be achieved, the purpose of education needs to transform from a factory system of 'producing economically efficient beings', to 'learners empowered to question inequality, unsustainability, loss of common identity and violence'. In response to this need, the open online course is an initiative towards reframing the scope of mathematics education to promote awareness about critical global issues.

Outline of the course

The course[1] will build skills in statistics and data handling. Most curriculums (Common Core Standard, NCERT, Schola Europaea) around the world emphasize on teaching about data and data handling throughout middle school. Given that data handling is the process of data (a) collection, (b) organization, (c) analysis and (d) depiction of insights with the help of graphs or charts, the course consists of four modules targeting each step of the process. In each module, with the exception of the first module, the mathematical concepts are introduced in the context of a globally significant topic. A concise module-wise outline of the course is provided in Table 1.

Module	Context
1. Introduction	NA
2. Data Organization	Comparison across countries based on government system, levels of peace, and environmental sustainability
3. Data Analysis	Occurrence and impact of natural phenomena such as earthquakes, volcanoes, and lightning
4. Data Visualization	Water on earth and the impact of climate change on water related disasters.

Table 1: Module-wise outline of the open online course

[1] Link to the complete course: https://framerspace.com/course/BJaEOnDmI-sel-for-stem

Process of course building

Based on the mathematical focus of the module, suitable global data was chosen to highlight the key mathematical concepts. For example, module 3 focused on natural disasters, which were particularly relevant to the course due to their global ecological impacts. To discuss the concept of median, data related to deaths from volcanic explosions was used. Amidst the variety of data related to volcanic explosion, death rate could uniquely demonstrate the need for median due to the large variability within the data. Similarly, relevant data was selected from open sources for each module context (shown in Table 1).

The overall course construction process was based on the UDL planning process for instruction (Rao & Meo, 2016). Five pedagogical tools, namely, (i) storytelling; (ii) gamification; (iii) inquiry; (iv) reflection; and (v) dialogue were employed to design an engaging multisensory learning experience (Rautela & Singh, 2019). In line with the 7 design principles of UDL (Burgstahler, 2009), the following design considerations were used throughout the course to provide a conducive digital learning space.

Clear instructions and ease of Navigation

As the course was completely online, providing clear and simple instructions for students was key to sustain engagement. The course materials were colour coded to indicate the expected learner actions (as shown in Figure 1). Links to navigate through the course pages were made easily accessible through a course map.

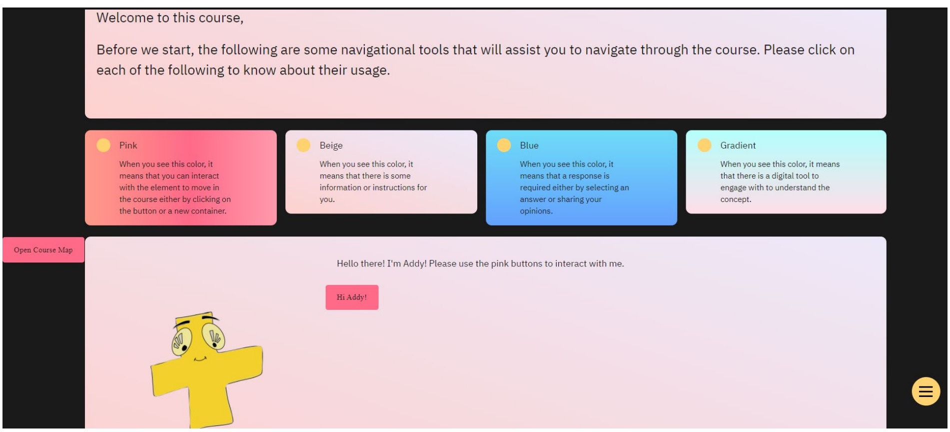

Figure 1: Example of a course page with instructions and a course map button.

Interactive and flexible delivery methods

The overall tone of the course was a dialogue with a peer rather than an instructor. The course had several mathematical characters, with the most prominent being Addy – a companion for students going through the course. The different modules within the course were designed as interactive conversations that the student could have with the various characters, offering them the flexibility to engage with the concept. Elements of storytelling were used to introduce, explain, and explore ideas to creating an imaginative and emotional experience for the learners.

Self-paced content exploration

Following an inquiry-based approach, each lesson started with an observation or question that Addy brings up. Students were encouraged to explore the question through a variety of activities and digital tools. They were provided the opportunity to choose their learning trajectory with links to review older content, seek alternative explanations or engage with external sources for additional information.

Gamified feedback and assessments

Games were used to help students reinforce new concepts. Gamification or using elements of games within activities was to motivate progress through the learning material and provide the learners with opportunities for reflection. Based on their responses, learners received feedback and suggestions to engage with certain learning materials. The course not only tolerated error, but also used error as a learning opportunity rather than learning assessment.

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References

Boylan, M., & Coles, A. T. (2017). Is another mathematics education possible? An introduction to a Special Issue on Mathematics Education and the Living World. *Philosophy of Mathematics Education Journal*, *32*.

Burgstahler, S. (2009). Universal design in education: Principles and applications. *DO-IT*.

Davis, B. (2014). Toward a more power-full school mathematics. *For the Learning of Mathematics*, *34*(1), 12–17.

Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problems as genre in mathematics education*. New York: Peter Lang.

Orth, B. (2013, May). *Mathematics orality and literacy*. Portland State University Library. https://pdxscholar.library.pdx.edu/cgi/viewcontent.cgi?article=1027&context=studentsymposium

Rautela, R., & Singh, N. C. (2019). Libre: Nourish the brain so the future can flourish. *Childhood Education*, *95*(5), 34–43.

Rao, K., & Meo, G. (2016). Using universal design for learning to design standards-based lessons. *Sage Open, 6*(4), 1–12.

Renert, M. (2011). Mathematics for life: Sustainable mathematics education. *For the Learning of Mathematics*, *31*(1), 20–26.

Wagner, D. (2010). If mathematics is a language, how do you swear in it. *The Best Writing on Mathematics, 6*(3), 449–457.

Wagner, D., & Davis, B. (2010). Feeling number: Grounding number sense in a sense of quantity. *Educational Studies in Mathematics*, *74*(1), 39–51.

Ledor project: The role of the *ledor* (reader) of visually impaired candidates in job and universities admission exams

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Our research project intends to understand the role of the ledor (reader) of the visually impaired candidate in tests of job and universities admission exams. Such research would broaden the academy's understanding of the ledores's role in interacting with candidates during their tests, as well as helping to build mechanisms that make the participation of the visually impaired person in such processes more inclusive. This is an anti-deficiencialist perspective, which seeks to remove psychophysiological barriers from society, allowing people with disabilities to participate in selection/evaluation processes on a fair basis. The expected result from the work is the creation of an action protocol for ledores (readers), which assist visually impaired candidates in job and universities admission exams.

Introduction

Since the 2000s, there are laws that guarantee the right of people with disabilities to have priority service in several facilities in Brazil. In 2004, Decree No. 5.296/2004 regulated a previous legislation, specifying the services to which people with disabilities are entitled in various situations, among them the taking of job and universities admission examinations. Therefore, there is a look out for improvements in the task of meeting this right. One of the means that has been used involves the participation of a person, called *ledor* (reader), to read tests for the candidate who requests this service at the time of registration.

However, little is known about the interference that this actor exerts on the performance of the candidate's test. Therefore, this research will seek to understand in more depth the characteristics of this interaction, allowing a reflection on ways to improve this process and proposing actions to assist on this task.

To this end, a partnership was set between UNIFESP (Federal University of São Paulo) - Campus Diadema, UNESP (São Paulo State University) – Campus Rio Claro and VUNESP (Foundation for the Vestibular (Entrance Exam) of the São Paulo State University), a foundation that conducts job and universities admission exams, and assessments throughout Brazil, including the entrance exams for all UNESP and some UNIFESP courses, such as the medical course for example.

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The partnership

The beginning of this partnership took place in December 2017, when Renato Marcone was invited to VUNESP by Rodrigo Bortolucci to assist in a specific task: to discuss ways of adapting mathematical tests for a blind student on a specific exam.

After completing the task of thinking about mathematical tests adapted for candidates with visual impairment in general, Renato Marcone proposed that the collaborative experience did not end there, that it should become a research partnership between the two institutions they represented. The idea was well received by the board of VUNESP, as well as by the board of UNIFESP – Campus Diadema.

Theoretical basis

To build a theoretical basis to which we will focus to justify the research that we are proposing, we bring Renato Marcone's PhD research (2015), which was an effort to think about the dynamics of the relationship between the people with disabilities and the teaching and learning processes of mathematics. Marcone (2015) states that, since he started working with inclusion, he was always convinced that teaching mathematics to excluded people was something that would contribute to the autonomy of the society, and not only for the people with disability. As Paulo Freire (1996) taught, autonomy only makes sense when it is collective.

These arguments also apply to admission mechanisms, such as job and universities admission examinations, in the same way that they apply to assessment and evaluation mechanisms. They are almost invariably built by "normal" people for "normal" people. When *differences* arise seeking to participate in these selection and evaluation systems, the gap that exists between the desire to include and the ability to perform this inclusion in a satisfactory manner is evident.

Given this perspective, Marcone brought an analogy, at first, between the way the West sees the East, based on Edward Said's book, "Orientalism: the invention of the East by the West", first published in 1978, and the way "Normal" people see, often inventing, the "Abnormal", an analogy that gradually became theorizing, culminating in the creation of the concept: *deficiencialism.*

Deficiencialism would be the set of practices and networks of stereotypes that build a distorted view of people with disabilities as incapable of tasks, extrapolating the real limitation that the disability can cause in the person. For example, a blind person, who is unable to fly a plane with the technology currently available, is also stereotyped as a person who, consequently, will not be able to learn mathematics, as this is a very visual science (Marcone, 2010). The extrapolation of limitations is at the heart of what Marcone calls *deficiencialism* (for more, see Marcone (2015). The following statement by Vygotsky (our translation) reinforces what Marcone has been saying:

> The whole apparatus of human culture (of the external form of behaviour) is adapted to the person's normal psychophysiological organization. Our entire culture is calculated for the person with certain organs - hand, eye, ear - and certain brain functions. All our instruments, all the technique, all the signs and symbols are calculated for a normal type of person. And from here comes that illusion of convergence, of natural passage from natural to cultural forms, which, in fact, is not possible due to the very nature of things and which we try to reveal in their true content (Vygotsky, 2011, p. 5).

Still in his PhD dissertation, Marcone proposes an *anti-deficiencialist* attitude towards barriers arising from the psychophysiological organization of society in general. Then, this research presents itself as an *anti-deficiencialism* attitude, seeking to interfere in the job and universities admission examinations in Brazil, while proposing to build mechanisms that could overcome such barriers instead of only criticizing them.

Research Aim

This research aims to understand how the role of the *ledor* (reader) influences candidates with visual impairment and based on this understanding, to think of alternatives that can improve the relationship of the *ledor* with the candidate with visual impairment. As a secondary objective, this research intends, in the end, to propose an action protocol for *ledores* who work in tests assisting visually impaired candidates, as well as thinking about a digital solution, such as a software or a special device.

Methodology and evolution so far

Because of the new coronavirus pandemic, our research changed its course drastically. Instead of face-to-face interviews and pilot assessments with visual impaired volunteers, we are analysing audio recordings of the candidates taking their tests aided by a *ledor*. VUNESP records, in audio, all the tests where there is the aid of a *ledor*, as a standard, and it is these recordings that we are having access and analysing, to understand the interferences of the *ledor*.

All the participants of this research are 18 or more in age and have agreed to giving us access to their recordings, by signing an Informed Consent Form (ICF), created by us, which is provided by VUNESP, online, during the registration process for any admission examination, through their own website. Also, our research has been approved by an independent Ethics Committee in Brazil.

Preliminary data provided by VUNESP in 2018 showed that they received an average number of 150 requests per year for a *ledor*, including all the assessments and job and universities admission examinations they organized from 2012 to 2017. We got the permission to access all the authorised audio recordings of 2020. Our expectation was to receive, at least, 10 authorizations, to perform a relevant analysis. The average time of each audio is 7 hours, because the visually impaired students have the right of an extended time to perform their exams in Brazil. However, early January 2021, we have got the consolidated data: until October 2020, we had 321 authorizations from candidates who signed the ICF.

Then, our first decision was to classify those candidates in two groups: job admission candidates and university admission candidates and we started looking into the first group.

The authorizations referring to job admission examinations are a total of 167. Of that number, 58 registrations were not confirmed (some due to the suspension of the admission exam because of the COVID-19 pandemic restrictions, others because some lack of documents, etc.), 24 were absent and 84 showed to take their test. Among the 84, we have 72 audios available (sometimes the candidate releases the *ledor* at the moment of the test and audio is not recorded). Then, we have, potentially, 500 hours of audio recordings to analyse.

At this point, we are facing the challenge of: how to treat all this data to reach our research goal, without giving up the qualitative character of the research?

Last comments

Once we saw the huge amount of data, our first reaction was joy, because this is a clear sign that students with disability are fighting their way towards a public job or a seat in a university.

After the first feeling of rejoice, we felt very worried. We have in front of us a unique opportunity to make some change on the exclusionist processes that were consolidated for so many years in Brazil, helping to create a more inclusive environment on the job and universities admission examinations.

Hence, we are at a place where we are seeking how to use those data in the wisest way, and for that, we count on the MES colleagues' support to design our next step.

References

Brasil Governo Federal. (2004, December). *Decreto Nº 5.296 de 2 de dezembro de 2004*: *Regulamenta as Leis nos 10.048, de 8 de novembro de 2000*. Brasil. https://legislacao.presidencia.gov.br/atos/?tipo=DEC&numero=5296&ano=2004&ato=e93UTVq5keRpWT529

Marcone, R. (2015). *Deficiencialismo: a invenção da deficiência pela normalidade* [Doctoral dissertation, São Paulo State University]. Repositório Institucional UNESP.

Marcone, R. (2010). *Educação matemática inclusiva no ensino superior: Aprendendo a partilhar experiências* [Master's thesis, São Paulo State University]. Repositório Institucional UNESP.

Freire, P. (1996). *Pedagogia da autonomia: Saberes necessários à prática educativa*. Paz e Terra.

Vygotsky, L. S. (2011). A defectologia e o estudo do desenvolvimento e da educação da criança anormal. *Educação em Pesquisa*, *37*(4), 863–869. (Original Work published 1995).

Possibilities for mathematics education in a university-school partnership

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This ongoing project is about initial and continuing teacher education of science and mathematics, more precisely at the university-school interface. In this perspective, the relationship with the school which participates in the project, as well as the development of research, emerge from the processes of action research. In this text, we present the actions we have developed about mathematics education with all the participants of the project. Teachers and educators problematized ideas from an investigative activity on numbers, based on landscapes of investigation (Skovsmose). From this, the participants start to criticize the didactic textbook used in the school, as mandatory, contributing to their critical development teaching professional. The way the participants have faced teaching during COVID-19 pandemic is also discussed.

The research project

The present project is located in the field of research on initial and continuing education of science and mathematics teachers, more precisely at the university and school interface. Both institutions are interrelated in search of improving the quality of teaching and, for consequently, the professional development of teachers, teacher educators and managers involved, and the quality of learning of undergraduate students in science and mathematics, and students from the participating school. The project is financed by a Brazilian development agency (Grant #2018/16585-1, Sao Paulo Research Foundation – FAPESP).

The relationship with the school, as well as the development of research that are part of the project, emerged as a movement, an action research process (Thiollent, 2004), in which the school, the teachers and the management group appeared simultaneously as subjects and objects of investigation. Researchers from two Brazilian public universities, management participants and teachers from a public school in the State of Sao Paulo meet periodically in forums constituted at the school and at universities. In this process, the motives of the various participants, as well as the objectives of the group's collective work, have been refined and articulated in the direction of building the partnership developed in this project.

More specifically, we seek to discuss possible answers to the following questions: "Which changes in teaching, in school management and in the teacher education of prospective

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teachers and school teachers are being leveraged by the work developed in the collaborative partnership? What elements enhance such changes?".

The current group of researchers is made up of a very heterogeneous group, whether in their graduation and PhD (pedagogy, philosophy, specialists in teaching and training teachers in the areas of science and mathematics, and specialist in educational public policies), or in their professional experiences and school levels practices.

According to Foerste (2004), the partnerships between universities and schools can assume different objectives, levels of involvement and integration of participants. In this sense, Jones *et al* (2016) state that the types of partnerships are defined by three different levels of integration of the subjects: *connective partnerships* are those in which exchange relationships are established and each of the involved parties offers something to exchange with the partners; *generative partnerships* occur when subjects are able to help each other, through the development of collective projects, which leads to changes in the structure of existing activities; and *transformative partnerships* are those in which there is active and collaborative involvement of all participants in a reflexive-critical practice, guided by the articulation between theory and practice, in order to develop all participating members. It is through this last partnership that the project seeks to develop.

In this multiplicity, the relationship between university and elementary school is expressed in a heterogeneous and diversified way, generating networks in which acceptance/denial, rupture/continuity/discontinuity are threads whose intertwining produces interesting processes. Such processes "give meaning to partnership movements, teacher professional training and the insertion of research as a constituent of training, in which we have constituted ourselves as actors in the process" (Almeida et al., 2000, p. 53). In this sense, the authors point out that recognizing the various roles that the university and the school assume, modifying or producing them, is fundamental to identify the borders, the overlaps and the specific areas of work. According to the authors, the outcomes of these partnerships provide a "meaningful panorama of the teacher education, considering the teachers as critical producers of their pedagogical work" (Almeida et al., 2000, p. 53).

Based on Fiorentini (2009), Nacarato (2016) and Oliveira (2015), we established discussions regarding partnerships in mathematics education. The authors point out that the partnerships between the university and the school enable conditions for teachers and prospective teachers to reflect on trends in mathematics education, as well as to discuss the mathematics curriculum at school, in order to face the difficulties that often are raised in the teaching practice.

With such a perspective, we hope to bring contributions both to the improvement of the teaching at the partner school, as well as the learning of teaching, the professional teacher development and the respective areas of research in these fields. Thus, we understand the relationship between learning of teaching and professional development as processes that emerge from the needs created in their practice and from the internship activity. In this way, we hope that the actions shared between prospective teachers, school teachers and university professors can contribute to the dialogue between different types of knowledge.

In this present text, the focus is to present and the discus one of the training actions developed in the project in order to contribute to the two research questions, related to the changes in the mathematics teaching and in the teacher education of the participants of the project, and to the elements developed in the collaborative partnership that leverage such changes.

Teacher education meetings on mathematics education

In 2019, we held action research meetings of the group. In monthly meetings, at the universities the researchers met for reflect on relevant topics related to the project's issues. After some of them, on inquiry-based science teaching, there were two meetings on mathematics education in the early years of elementary school. The theme was chosen and organized by the group. Critical Mathematics Education (Skovsmose, 2011) was the theoretical perspective chosen by the universities researchers to base the meeting. The school teachers intended to discuss students' difficulties in learning mathematics and to know different ways of teaching. The teacher educators' intentions were to understand how school teachers used to teach mathematics, and to learn from them about students' difficulties.

In order to achieve these aims, previously, for the first meeting, we requested that participants to read the text "Landscapes of Investigation", by Ole Skovsmose (2000). We have chosen that text for three reasons: 1 – the text is based on critical mathematics education and by adopting this theoretical framework to reflect on the mathematics teaching is a possibility to question what is given as true; 2 – the proposal presented in this text (the landscapes of investigation) departs from a well known context to the school teacher and close to their practice (the paradigm of exercise); 3 – the author presents the mathematics as a tool to interpret situations and make decisions, and argues that mathematical discoveries made by the students can enable different qualities of learning. We expected these ideas raise in the study meetings.

Thus, at that meeting, an investigative activity on numbers, proposed in this text, was developed. An investigative activity aims to enable students to make discoveries, to have an active participation in solving the activity and to be responsible for their learning process. Such discoveries do not concern genuine knowledge of mathematics, but rather new knowledge for those who carry out the action. Dialogue is the form of communication that predominates in this type of activity.

The work with the landscape of investigation was developed in small groups and encouraged participants to explore a table to discover some regularities and numerical patterns. It was possible to notice that the participants shared their findings, doubts and hypotheses with their colleagues. Each idea raised in the small groups was presented and discussed with everyone. Views were shared. The action of understanding what the other was saying was present, and this is an essential element both for dialogue in the learning process and for the development of teacher education.

Some investigative questions were raised regarding the patterns and regularities of the referred numeric table. Surprise, restlessness and suspicious smiles were expressed by the participants when carrying out the activity. The invitation to investigate had been accepted by the group. The curiosity to try to find out why there were certain patterns persisted throughout the meeting. We realized that the discussion about generalizations, so important in the algebra of the early years, was put into action, because, during the activity, legitimate mathematical knowledge was raised for that group and essential for the development of the proposed task.

We reflected on the similarity between the activity carried out and those developed in previous meetings on inquiry-based science teaching. Raising hypotheses, evaluating them, arguing, making discoveries, dialoguing with colleagues are some of the common actions that we perceived among the proposals discussed for the teaching of mathematics and science. These actions are also important elements which enables changes in the mathematics teaching in the early years.

The second meeting on mathematics education began resuming the investigative activity carried out at the previous meeting. We emphasize the importance of teachers provide moments to the school students make discoveries in mathematics classes, playing an active role in the activity developed. Important aspects of Skovsmose's text (2000) were raised and discussed by the participants. We noticed that there is a huge difference between investigative classes, such as the activity we developed, and those based on the paradigm of exercise, in which the teacher explains the content, shows some examples and the students reproduce, in a list of exercises, what they heard from the teacher.

Because of the investigative activity we run in the previous meeting involved generalizations, we also discussed about Algebra in the early years. The search for patterns in numerical and graphical sequences and in tables, such as the one we explored in that meeting, is an aspect of algebra that can be worked on in the early years.

The second part of this meeting was intended for the presentation, by the school teachers, of the didactic material "Mathematics Education for the Early Years" (EMAI), used in the state schools of Sao Paulo, pointing out what the material is and how it is used. It should be noted that the teachers had the opportunity to dedicate themselves, in a group, to read and discuss the material. In their presentation, the teachers verbalized that this reading allowed them to demystify how to use it. They realized that the material is proposed as a support and not as a manual to be used daily and in a tied way, as it was presented to them in many moments of continuing education, provided by the management team of state schools. In this way, the teachers started to characterize it as a collaborative project of continuing education among the teachers of each school. Outbursts, complaints and solutions were verbalized.

Subsequently, we analyzed the summary of the material to see how it was organized. We identified that the language of the EMAI is very imperative ("do", "distribute", "ask"), which restricts the possibilities for the teacher to decide how best to develop the activity proposed in the material. We also discussed the pressure that the author of these types of text experiences when producing didactic material.

The group started to question the possibilities of making the use of the didactic material more flexible. In order to connect the discussion on landscapes of investigation and the didactic material, one of the professors have raised the following question: Is it possible to create “landscapes of investigation” (Skovsmose, 2000) from the guidelines prescribed by the material? A deeply awareness and knowledge about the didatic material was one of the conditions considered by the group to be essential in order to proceed with the search for an answer to this question. The meeting ended at this time and with such a proposal. Considering the discussions played at these meeting we understand we were moving towards promoting changes both in the teaching of mathematics and in the teacher education.

The discussions and reflections, made by the school teachers about their teaching and didactic material, were important to their education, and also to highlight elements for thinking about and developing university-school partnership actions. For example, we realize that, in the beginning of the research project, the school teachers understood collaboration in an objective way to help them in some difficulty, and not as the possibility of a joint work that would contribute to thinking together about the development of a certain subject or a different approach to discuss with students. Thus, the discussions and verbalizations of the teachers show us how the partnership has potentialized the school’s activities in relation to changes in the teaching.

The discussions were interrupted due to the end of the year and the advent of the COVID-19 pandemic. With regard to the specific issues of mathematics education, the year 2020 was intended for the collaboration between school teachers and educators in organizing the remote classes held with the few students who had technical, psychological and family conditions to follow on-line teaching. The classes in the State of Sao Paulo were held and made available by a media center, which did not count on the participation of the teachers themselves in its production. In this way, what was revealed was a loss of their didactic choices and, consequently, a challenge for the continuity of our project.

Thus, the two mathematics education meetings raised important questions regarding the professional development of the teachers. Throughout the year 2020, such meetings reverberated in their speech in the project remote meetings that followed and in the interviews provided by them. We intend, as one of the next steps of the project with regard to mathematics education, to return to the records related to these meetings (audios, interviews and written conversations on whatsApp), looking for elements that contribute to the potentiality of the collaborative partnership.

Finally, we understand that the project plays a political role, by offering, through the interaction between the participants, subsidies so that answers and solutions can be found in order to promote transformative actions in the professional development of all those involved.

References

Almeida, C.M.C., Amorim, A. C. R., Pacheco, D., Silva, L. L. M., Galzerani, M. C. B., Bagnato, M. H. S., Ferreira, N. S. A. F., & Scorsi, R. A. (2000). Professores da universidade e da escola básica: Parceiros no ensino e na pesquisa. *Pro-posições*, *1*(4), 43–55.

Fiorentini, D. (2009). Educação matemática: Diálogos entre universidade e escola. *Anais do Encontro Gaúcho de Educação Matemática*, http://www.projetos.unijui.edu.br/matematica/cd_egem/fscommand/CO/CO1.pdf

Foerste, E. (2004). Parceria na formação de professores. *Revista Iberoamericana de Educación*, *34*(3), 1–12.

Jones, M., Hobbs, L., Kenny, J., Campbell, C., Chittleborough, G., Gilbert, A., Herbert, S., & Redman, C. (2016). Successful university-school partnerships: An interpretive framework to inform partnership practice. *Teaching and Teacher Education*, *60*, 108–120.

Nacarato, A. M. (2016). A parceria universidade-escola: Utopia ou possibilidade de formação continuada no âmbito das políticas públicas? *Revista Brasileira de Educação, 21*(66), 699–716.

Oliveira, N. Z. M. (2015). *Parceria entre universidade e escola em projetos de educação matemática* [Doctoral dissertation, Universidade Estadual Paulista].

Skovsmose, O. (2011). An invitation to critical mathematics education. Sense Publishers.

Skovsmose, O. (2000). Cenários para investigação. *Bolema: Boletim de Educação Matemática*, *13*(14), 66–91.

Thiollent, M. (2004). *Metodologia da pesquisa-ação*. São Paulo: Cortez.

Borders, gender, and performative contradictions in active learning

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We present some ongoing (under review) work to conceptualize gender in mathematics education using a radically different approach: though an intersection of Nail's border theory, performative new materialism, and elements of Hegelian philosophy (as set forth by Marx and Žižek). Through this approach, gender is conceptualized as a border between the masculine and feminine, and for some, this is a border to be crossed. In response to researchers' call to elucidate the meaning of gender (e.g., Damarin & Erchick, 2010), the approach discussed here claims that much of gender's influence is lost on researchers due to overlooking the reflex-category nature of the masculine and feminine in performance. Metaphors of immigration/emigration, power differentials, and performativity are discussed apropos of participation in mathematics.

A gender problem in mathematics education and active learning

Active learning continues to spread its grasp on education fields as a "panacea" for mathematics and STEM education, with more and more literature being published that posit its beneficial universality for students. For instance, Freeman et al. (2014) found that the active learning environment (ALE) leads to "increases in achievement [...] across all of the STEM disciplines and [these increases] occur in all class sizes, course types, and course levels" (p. 8412). In another study, Theobald et al. (2020) found that ALE is "disproportionately beneficial [...] for underrepresented minority students" (p. 6478). Similarly, Laursen et al. (2014) found that ALE "levels the playing field" for men and women in terms of achievement. These large, widely-published studies clearly herald the trend in mathematics education towards seeing ALE as beneficial for all students, and even perhaps more beneficial for those who may need it the most.

However, the last decade has concurrently seen a confusing emergence of literature claiming that ALE impacts different groups of students in different ways, not always resulting in content gains or increased achievement. For example, Johnson et al. (2020) found that men outperformed women in a type of ALE called Inquiry-Oriented Instruction. In their study, ALE had no negative effect on the women, but rather it had no effect at all. Meanwhile, the men experienced content gains as a result of the ALE intervention. Compared to national representative samples that show no gender difference in student performance (with,

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presumably, data obtained from a traditional instructional environment), the results of Johnson and colleagues' study lead to a troubling contradiction. Their work mirrors the findings of Bando and colleagues' (2019) large-scale study, which found that a gender performance gap was exacerbated by ALE interventions, and, alarmingly, *increased* the longer that the intervention was being employed.

Gender-as-border: Towards a new theory of performative identity

These contradictory results have led an increasing number of scholars, including us, to ask "why," and to question the claim that ALE is indeed a universal panacea. While it is possible that the studies were not conducted in the same way, their overall claims apropos of gender are universal ones, and thus we approach our work from a position of troubling that universality. This paper reports on a larger, in-process theoretical development that attempts to re-examine these contradictions through a theoretical intersection of borders, gender performativity, and Hegel-Marx reflex categories.

Borders

Thomas Nail's (2016) border theory has emerged recently as a promising new contribution to the field of philosophy. Nail's work centres on motion and movement, and as such, his work on border theory is also highly dynamic. Borders, as mechanisms of social regulation, divide spaces and create their own unique ontological presence (cf. an ontological lack) as the divider itself. He describes two types of borders: extensive and intensive. *Extensive* borders create an "absolute break—producing… discontinuous entities" (Nail, 2016, p. 3); *intensive* borders create "new path[s] […] qualitative[ly] chang[ing] […] the whole continuous system" (p. 3). Consider that the masculine and feminine are historically constructed as an extensive border system. They are seen as quantitatively different, exemplified by the typical options on standard forms to check a box next to "male" or "female," as if they are mutually exclusive categories. We argue that, because of this extensive historical construction, "gender" leads to contradictions of many types—e.g., the current conflicts over transgender ontology—that could potentially be addressed via an intensive conception. Of particular interest here is the implications this impasse creates apropos of participation in mathematics classrooms.

Performance

Within the new materialist tradition can be found several variants, one of which is performative (Gamble et al., 2019). In this paradigm, the material real is a stage on which performances are made by actors: as such, ontology and epistemology are "inherently co-implicated and mutually constituting [of each other]" (p. 122), becoming one understanding of an *ontoepistemology* that, further, "problematize[s] anthropocentric binaries" (p. 111) such as gender. A key consequence of this perspective is that "humans can therefore never observe the universe as though from outside it […] [and thus, being bound by the] material configuration of the world […] [necessarily] leads to a thoroughly 'performative' and relational materialism" (p. 123). This perspective allows gender to be seen as a performance of students as actors engaged in the performative role of producing themselves and their identities through the labour of their performance—a performance which, crucially, is political (cf. Butler, 1990; Moore, 2020). The politics of one's performance of gender is realized in the material real through the dynamics of Hegelian and Marxian power relations.

Hegel/Marx

Hegel (1807/1977), in one section of the *Phenomenology*, develops the notion of power dynamics between two self-consciousnesses in relation to each other by using the metaphor of lord and bondsman; the former is self-consciousness for itself and the latter is self-consciousness for another. The bondsman, in performing the lord's labour on the world of things (viz. the material world) reverses his subordinate position to become fully self-aware *through his labour.* The lord merely takes the enjoyment of the bondsman's labour *qua* the products of the world of things without becoming self-aware of his position as lord *vis-à-vis* recognition of the bondsman *as such.* The bondsman becomes ironically and symbolically free *vis-à-vis* the self-awareness he acquires in the act of labouring—labour imposed on him by the lord—that forces him to recognize the lord *as such.* Thus, Hegel shows that in being-for-another (viz. the bondsman), one transcends into a "more fully being-for-self" role. Hegel called this reflexive identity a reflex-category: the bondsman would not be a bondsman without the lord, and the lord would not be a lord without the bondsman, however neither is what the other sees himself as. This was later further developed by Marx: "[O]ne man is king only because other men stand in the relation of subjects to him. They, on the contrary, imagine that they are subjects because he is king" (Marx, 1867/1887, Footnote 22, p. 55, as cited in Žižek, 1989/2008, p. 20). Marx's symmetry here is homologous to Hegel's argument dialectically.

Consequences in the mathematics classroom

Combining all the above theory allows us to re-envision gender as a performative dimension in the classroom, a dimension which, crucially, conflicts with ALE. The Hegel-Marx axis gives us the power dynamics of masculine (lord) and feminine (bondsman), as co-constituted and reflexively determined, the source of our current gender impasse. Evidence of the extensive gender border, as the historical determination of gender's construction that we observe in modernity, can be found in such phenomena as women's suffrage, the gender pay gap, maternity leave and benefits, and the disproportionate ratio of women to men in professional roles. Nail's border theory describes how the division between them functions and describes the ontoepistemological "stage" over which students must perform their gender in the material real of the classroom. In both the active learning and lecture environments, the teacher takes the masculine position, and the students assume the feminine: "The teacher *knows for* the students, and the students—relegated to performing the teacher's labor—merely *work on it*" (Moore & Johnson, under review). However, in the ALE case, a split occurs: the teacher transfers some of her power onto the students in the form of

ALE tasks and expectations. When this occurs, the men and boys take the assertive voice, which is championed in ALE, whereas the women and girls are emburdened with the task of "emigrating"—leaving their feminine home and enacting a gender performance across the border to the masculine side—if they want to be successful. Crucially, the boys in lecture do not need to reciprocate this. Our goal with this project presentation is to discuss our theoretical development and its implications.

References

Bando, R., Näslund-Hadley, E., & Gertler, P. (2019). *Effect of inquiry and problem-based pedagogy on learning: Evidence from 10 field experiments in four countries* (Working Paper 26280). National Bureau of Economic Research. http://www.nber.org/papers/w26280

Butler, J. (1990). *Gender trouble: Feminism and the subversion of identity.* Routledge.

Damarin, S., & Erchick, D. B. (2010). Toward clarifying the meanings of "gender" in mathematics education research. *Journal for Research in Mathematics Education*, *41*(4), 310–323.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences, 111*(23), 8410–8415.

Gamble, C. N., Hanan, J. S., & Nail, T. (2019). What is new materialism? *Angelaki: Journal of the Theoretical Humanities*, *24*(6), 111–134.

Hegel, G. W. F. (1977). *Phenomenology of spirit.* (A. V. Miller, Trans.). Oxford University Press. (Original work published 1807)

Johnson, E., Andrews-Larson, C., Keene, K., Melhuish, K., Keller, R., & Fortune, N. (2020). Inquiry and gender inequity in the undergraduate mathematics classroom. *Journal for Research in Mathematics Education*, *51*(4), 504–516.

Laursen, S. L., Hassi, M.-L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, *45*(4), 406–418.

Marx, K. (1887). *Capital.* Progress Publishers. (Original work published 1867) https://www.marxists.org/archive/marx/works/download/pdf/Capital-Volume-I.pdf

Moore, A. S. (2020). Queer identity and theory intersections in mathematics education: A theoretical literature review. *Mathematics Education Research Journal.* Advance online publication. https://doi.org/10.1007/s13394-020-00354-7

Moore, A. S., & Johnson, E. (under review). *A border theorization of gendered STEM identity in active learning and lecture.*

Nail, T. (2016). *Theory of the border.* Oxford University Press.

Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Arroyo, E. N., Behling, S., Chambwe, N., Cintrón, D. L., Cooper, J. D., Dunster, G., Grummer, J. A., Hennessey, K., Hsiao, J., Iranon, N., Jones, L., II, Jordt, H., Keller, M., Lacey, M. E., Littlefield, C. E., … Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences*, *117*(12), 6476–6483.

Žižek, S. (2008). *The sublime object of ideology.* Verso. (Original work published 1989)

Unpacking field trips: The role of a teacher educator in post-field mathematics teacher education courses

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This paper reports on an ongoing research project to study how mathematics teacher educators 'unpack' the field experiences of prospective teachers. By viewing post-field practices through the lens of disruptive pedagogies, we aim to better understand the roles of mathematics teacher educators and to reconceptualise post-field possibilities in teacher education.

Introduction

Research in the area of teacher education theory-practice transitions has been extensive (Gainsburg, 2012), including transitions from university (theory) to field experience (practice), as well as transitions from the process of *becoming* a teacher (university) to the first few years of *being* a teacher in schools. Another key transition in teacher education programs is the under-researched transition from field experience back to university. As noted by Eriksen and Bjerke (2019), "little is known about the way in which teacher educators integrate prospective teachers' actual experiences when they return to university after fieldwork" (p. 9).

The 'unpacking' of field-back-to-university transitions is relevant to the community of teacher educators since teacher education programs, and corresponding field experiences, are frequently critiqued for being steeped in technical-rational approaches (Nolan & Tupper, 2020). Mathematics teacher educators (MTEs) struggle with the tensions implicit in these transitions, as they seek to disrupt dominant 'technique-oriented' discourses of school mathematics and becoming a teacher.

Research theory and design

We first acknowledge the difficult, but necessary, task of moving away from using the language of theory and practice to describe the transitions between university teacher education courses and school-based field experiences. To counter this false binary and hierarchy, where expertise is seen to rest primarily with academics, Zeichner (2010) proposes teacher education hybrid or third spaces that "bring practitioner and academic knowledge

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together in less hierarchical ways to create new learning opportunities for prospective teachers" (p. 92). Similarly, Rust (2019) calls for teacher educators and teacher education programs to "be situated at the nexus between universities and schools—the place where theory and practice can come together" (p. 524).

In our study, we propose a hybrid space of research where our focus is on disrupting and reimagining knowledge constructed in the movement from university to field and back to university. Within this movement, it is the post-field context that we focus our attention. By viewing MTEs' post-field practices through the lens of *disruptive pedagogies*, we aim to better understand the roles of MTEs and to reconceptualise post-field possibilities in teacher education.

We draw on Anderson and Justice (2015) in describing a pedagogy as disruptive if it "requires students to challenge or change their epistemologies and participation in their learning" (p. 400). As Schulz (2005) reminds us, "[i]f teacher educators want to change prevailing practices … they must provide frameworks that encourage different ways of thinking about teaching and learning about teaching" (pp. 149-150). This applies to both pre- and post-periods of field experience, and hence, it underlines the importance of drawing on prospective teachers' (PTs') field experiences in post-field university courses, where different theoretical and pedagogical tools have the potential to better understand and unpack the field.

In the research design, we review literature on university to field transitions in mathematics teacher education to construct a list of the barriers/challenges in transitions as identified across the research. We are specifically interested in knowing whether the challenges in university-to-field transitions also carry weight in field-back-to-university transitions and how/if MTEs address them in post-field courses. From this list of barriers/challenges, we construct several questions to ask MTEs to understand their practices as post-field course instructors. All of these questions emerge from the central question of this research study: *What are mathematics teacher educators' roles in unpacking field experiences?*

With the questions constructed, the research study's data collection is divided into two parts. Part 1, the primary focus of this paper, includes conversations between the two authors—a dialogue made possible through our own self-study reflections on the questions. In part 2, which moves beyond the content of this paper, we use the questions to interview 20 MTEs from teacher education programs across Canada and Norway to gain broader perspectives on the practices of MTEs in disrupting the field-back-to-university transitions through post-field courses.

Barriers/challenges in theory-practice transitions: Review of literature

Given the self-study context of Part 1 of this study, here we focus our brief review of research in the area of theory-practice transitions primarily on our own findings; the two authors (Bjerke & Nolan) have written extensively on the barriers/challenges encountered in theory-practice transitions, revealing the following (abbreviated) list:

PTs as visitors: The visitor 'stamp' prevents PTs from trying out new ideas (Nolan, 2012), focusing on unquestioning alignment with existing norms and plans, deferring to the mentor teachers' accountability for their pupils' progress (Solomon et al., 2017).

The different roles of the involved parties: A lack of understanding of the roles of cooperating/mentor teacher, PT, and university supervisor (Nolan, 2015).

The theory–practice divide: A reported disconnect between university and school methods/theories, often resulting in PTs favouring school placement (Eriksen & Bjerke, 2019) and expressing a need to be armed with a 'toolbox' in order to be aligned more closely with the school and performing the role of a teacher (Solomon et al., 2017).

The demands of reform teaching: Reform, or inquiry, approaches not taken up by PTs during field experience, for several reasons: Inadequate modelling by MTEs; lack of 'recipes' for implementing inquiry; inquiry-based lessons reported as taking too much time to plan and implement; PTs' lack of conviction (Eriksen & Bjerke, 2019; Nolan, 2012, 2015).

Questions for MTEs about the field-back-to-university transitions

Based on the barriers/challenges outlined above, and with the lens of disruptive pedagogy informing our interest in unpacking the post-field context, we have constructed 8 conversation/interview questions. Given space restrictions, we present only 4 of these questions here as illustrations: (1) As a MTE and course instructor, what are the most significant challenges you experience in your work with PTs upon their return from a field experience? How do the challenges relate to the list of theory-practice barriers/challenges above? (2) What pedagogical strategies do you draw on in your post-field courses that you think might (a) intentionally or unintentionally, further re-affirm a university-school divide between theory and practice, and (b) challenge and/or disrupt the division between university/theory and field/practice classrooms, and instead portray them as being more in relationship with each other? (3) What theoretical tools do you draw on in your post-field courses to 'unpack' the field? How and to what end do you draw on these tools to understand, disrupt and/or support PTs' thinking and growth? Describe successes and failures in these efforts. (4) What do you view as your primary role(s) as a MTE in the post-field context?

Part 1: Dialogue between Authors

As an illustration of the research process, we present the following snapshot of the authors' dialogue around one of these questions (#3 above):

Kathy: I have drawn on Bourdieu's social field theory in my post-field courses, through a basic introduction of the concepts of habitus, field and cultural capital to PTs. Introducing PTs to these concepts in the context of discussing unchanging pedagogical practices in schools was meant to illustrate how a person feels comfortable in a field where their habitus is a good fit with the logic and operation of that field. I had hoped that these discussions, drawing on Bourdieu's concepts, would aid in disrupting technical-rationality in teacher education by building PTs critical capacities for thinking *with* and *through* theory.

Annette: You 'had hoped'. Does this mean that it did not happen? My latest effort has been to introduce Biesta's virtue-based approach to education, and hence to mathematics teaching, discussing the PTs' experiences in relation to qualification, subjectification and socialisation. This has worked as a way to address both themselves as PTs, and as a way of talking about their experiences with different pupils.

Future directions and concluding thoughts

MTEs are called upon to make deliberate pedagogical choices toward "the disruption of practices which contribute to the reproduction of educational inequalities" (Beighton, 2017, p. 113). As this research focuses on disrupting and reimagining knowledge constructed in the movement from university to field and back to university, it is important ongoing work both for those teacher educators involved in our study (as a reflective self-study exercise) and for those reading about and relating to what we report. This brief introduction to our in-progress study highlights our approach to viewing MTEs' post-field practices through the lens of disruptive pedagogies.

References

Anderson, J., & Justice, J. (2015). Disruptive design in pre-service teacher education: Uptake, participation, and resistance. *Teaching Education, 26*(4), 400-421.

Beighton, C. (2017). Telling ghost stories with the voice of an ogre: Deleuze, identity, and disruptive pedagogies. *Issues in Teacher Education, 26*(3), 111-127.

Eriksen, E., & Bjerke, A. H. (2019). The fractal complexity of using theories in mathematics teacher education: Issues and debates, opportunities and limitations. In S. Llinares & O. Chapman (Eds.), *International Handbook of Mathematics Teacher Education: Volume 2* (pp. 255-284). Brill Sense.

Gainsburg, J. (2012). Why new mathematics teachers do or don't use practices emphasized in their credential program. *Journal of Mathematics Teacher Education, 15*(5), 359-379.

Nolan, K. (2012). Dispositions in the field: Viewing mathematics teacher education field experiences through the lens of Bourdieu's social field theory. *Educational Studies in Mathematics*, 80 (1/2), 201-215.

Nolan, K. (2015). Beyond tokenism in the field? On the learning of a Mathematics teacher educator and faculty supervisor. *Cogent Education, 2: 1065580.*

Nolan, K., & Tupper, J. (Editors) (2020). *Social Theory for Teacher Education Research: Beyond the Technical-Rational.* Bloomsbury.

Rust, F.O. (2019) Redesign in teacher education: The roles of teacher educators. *European Journal of Teacher Education, 42*(4), 523-533.

Schulz, R. (2005). The practicum: More than practice. *Canadian Journal of Education, 28*(1/2), 147-167.

Solomon, Y., Eriksen, E., Smestad, B., Rodal, C., & Bjerke, A. H. (2017). Prospective teachers navigating intersecting communities of practice: early school placement. *Journal of Mathematics Teacher Education, 20*(2), 141-158.

Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college- and university-based teacher education. *Journal of Teacher Education, 61*(1-2), 89-99.

Mathematics education in a context of climate change

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The overall goal of the research project presented here is to investigate and produce knowledge about how students and teachers are learning mathematics when presented with mathematics in the context of climate change. In order to address this question, some ethical issues will be taken into account. First, there is a brief overview of different ethical issues concerning the questions, "Is there is an ethical responsibility to bring the concept of climate change into the mathematical learning situation?". Secondly, I will present the school-based-research. A number of lessons will be created, and an overview of the content of those will be presented here. They will contain, ethical considerations but also mathematical models and different perspectives on climate change, for the students to discuss and engage in.

Ethical responsibility to bring climate change into the mathematical classroom

For this research to be fruitful, some ethical issues will be taken into account. Why should climate change be part of the mathematical curriculum is one of those. Is there even a moral obligation to address these issues, and if the answer is yes, how would this affect the teaching and learning? Abtahi et al. (2017) address the question "how incorporating issues of climate change into the teaching and learning of mathematics can be understood as a moral and ethical act.". They conclude that although including climate change in mathematics classrooms can be viewed as an ethical or moral responsibility of mathematics teachers, in their day-today practice their decisions about issues are complex. These ethical challenges relate to, for example, the degree of involvement and interest of students in the issue of climate change, the possible discomfort of students, the uncertainty of how to respond, the unclear path of any possible contributions of their actions to the wider society, and finally a more general sense of dealing with the unknown. Teachers face practical obstacles in incorporating issues related to climate change in their mathematics classrooms, such as "lack of resources, lack of sources of data related to their immediate community, lack of curriculum mandates, and lack of time" (p. 11).

Karrow, Khan & Fleener (2017) also discuss mathematics education's ethical relation with the climate change. They argue that mathematics education is skewed towards economic interests instead of ethical relations. According to them mathematics education must move away from preoccupation with economics to one found by virtue ethics. In reviewing a

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number of papers, they conclude that the current mathematics education is formed through the prism of economy, instead they suggest a mathematics education that that concerns the development of the individual in relation with our Planets Ecosystem. To do this they suggest moving mathematics more into the realm of nature. For instance, recursive mathematics which can describe the reproduction of rabbits or formation of flower leaves. Fractals, complex systems and uncertainty are other areas that can be found in this mathematics of nature. This suggests a new epistemology, were learning is not an acquisition and accumulation of knowledge but rather a process of engagement with an understanding that all knowledge is uncertain and situated. This is also in line with UNESCOs slogan "changing minds not the climate" UNESCO (n.d.).

Perspectives on climate change

The teaching and learning in a context of climate change also involves a personal viewpoint towards the issue of climate change which is elaborated in several papers. How do we view ourselves in relation to the world, climate change and the dystopian future that is presented to us? Drawing on the notion of the *Anthropocene* which signals the shift from solving a problem to learning to live with a problem, Coles (2017) suggests the concept of *habit* that allow us to conceptualize ourselves in the world around us. Moving away from a savior narrative, *habit* will help people to connect individual and global perspectives. This issue is also discussed by Latour (2018) where he suggests a perspective of the *critical zone* to deal with the climate change. He argues that the scientific notion of us being ON the blue planet, leads us to take the role of savior instead we must face the reality of the critical zone in which we all are in some sense part of and trapped in. (Mikulan & Sinclair, 2017) draws on the notion of the *Anthropocene* claiming that we need to think in a more non-human way were the human is not the center of everything. They also show in their paper why mathematics is well suited for this endeavor. The above-mentioned examples are all ethical considerations for a teacher going into the mathematical learning situation, they all based on different ethical assumptions, and these will be investigated and analyzed in the research.

School-based-research in the mathematical learning situation

In my coming school-based-research, I will create several lessons that will touch on the different aspects of climate change. In the classroom there will be discussions about ethical considerations concerning climate change. Different ethical standpoints and perspectives will be presented to the students, illustrating the complexity of the problem. Articles dealing with the ethical implications concerning climate change are multiple in the research community. Gardiner (2011) argues climate change plays a fundamental role due to decision making, which effect animal and future generations. They also point out some reasons for why it is hard to be ethically sound. Local emissions have a global effect. A decision in one place may have implications in a totally different place in the world. Raymond (2004) argues for an ethics of commons. The atmosphere is a global common good and he proposes that the emissions should be allocated between nations. It could be done by using the principle of equal burden. Meaning that nations should reduce their emissions based on the burden of this reduction. Another approach based on equal human rights would be to allow an emission level per capita. Shue (1999) concludes that “whatever needs to be done by wealthy industrialized states or by poor non-industrialized states about global environmental problems, the costs should initially be borne by the wealthy industrialized states” (p.111). He then describes the reasoning behind this conclusion and how proportional and progressive burden can be explained. All these suggestions, mentioned above, are based on different ethical assumptions and are examples of what can be presented and discussed in the classroom. They also deal with mathematical concepts, for instance proportionality and progression, that can be used as examples in the mathematical classroom.

When we say *climate change* what type of perspective and understanding are we referring to? Are there other models and perspectives that can be used to understand and explain the climate crisis and the world we live in? Huneman & Lemoine (2014) explore interesting distinctions of modeling. “Modeling can be seen in terms of representing a target system. Other epistemic functions, such as producing data or detecting phenomena, are at least as relevant” (p. 3). Other useful distinctions can be made, for instance between phenomenological and mechanistic models (p. 3). There are distinctions that can be applied to the different models of climate change as well. And relevant questions of which type of model to use in the classroom emerges. We may also ask ourselves if alternative models can bring light into the classroom concerning the climate crisis? Dutreuil (2014) investigates the epistemology of computational models that stem from an analysis of the Gaïa Hypothesis. The model has been criticized for being too abstract, describing fictive daisies on an imaginary planet, and trying to answer *what-if* questions, “how would a planet look like if life had no influence on it?”. For these reasons the model has been considered not testable and therefore not legitimate in science, and in any case not very interesting since it explores non-actual issues. “This criticism implicitly assumes that science should only be involved in the making of models that are *actual,* by opposition to *what-if*, and *specific*, by opposition to *abstract*.” Dutreuil (2014, p. 2). This research aims to challenge these criticisms and use the mathematical approach to illustrate how one can make a theoretical model first, without any empirical data and then see what understanding it will bring to the students. As an example, imaginary numbers were conceived long before there was any practical use for them. Several centuries later it was obvious that they were very practical tool to use in electrical engineering. A similar approach can be made to use an abstract mathematical model of the earth. And in this way remove a lot of variables, in doing so creating a new learning situation and a way to gain knowledge of how students learn mathematics in this context.

Analysis of data

In my analysis of the data from the classroom, I will use *actor-network-theory* (ANT). It is a methodological approach to study social phenomena where everything exits in a continuously changing network of relationships Latour (2005). As Latour (2005) describes, everything that happens in a social situation takes place on the same level. So, for instance humans as well as objects have agency, and both play a role in creating a social situation. In ANT there are two main concepts, *medidators* and *intermedieries*. *Mediators* "transform, translate, distort, and modify the meaning or the elements they are supposed to carry". (Latour, 2005, p. 39). *Intermedieries* on the other hand, is what transports meaning without transformation: "defining its inputs is enough to define its outputs" (p. 39). But how do we distinguish between mediators and intermediaries? Latour mentions that to learn ANT is nothing more than to "become sensitive to the differences in the literary, scientific, moral, political, and empirical dimensions of the two types of accounts" (p. 109).

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References

Abtahi, Y., Gøtze, P., Steffensen, L., Hauge, K. H., & Barwell, R. (2017). Teaching climate change in mathematics classroom: An ethical responsibility. *Philosophy of Mathematics Education Journal, 32.*

Coles, A. (2017). Habits and binds of mathematics education in the anthroposcene. *Philosophy of Mathematics Education Journal, 32.*

Dutreuil, S. (2014). What good are abstract and what-if models? Lessons from the Gaïa hypothesis. *History and Philosophy of the Life Sciences, 36*(1), 16–41.

Shue, H. (1999). Global environment and international inequality. *International Affairs (Royal Institute of International Affairs 1944-), 75*(3), 531–545. http://www.jstor.org/stable/2623635

Gardiner, S. M. (2011). *A perfect moral storm: The ethical tragedy of climate change.* Oxford University Press.

Huneman, P & Lemoine, M. (2014). Introduction: The plurality of modeling. *History and Philosophy of the Life Sciences, 36*(1), 5–15.

Karrow, D., Khan, S., & Fleener, J. (2017). Mathematics education's ethical relation with and response to climate change. *Philosophy of Mathematics Education Journal, 32.*

Latour, B. (2018). *Down to earth: Politics in the new climatic regime.* Polity Press.

Latour, B. (2005) *Re-assembling the social: An introduction to actor-network theory.* Oxford University Press.

Mikulan, P., & Sinclair, N. (2017). Thinking mathematics pedagogy stratigraphically in the anthropocene. *Philosophy of Mathematics Education Journal, 32.*

Raymond, L. (2004). Cutting the "Gordian knot" in climate change policy. *Energy Policy, 34*(6), 655–658.

UNESCO (n. d.) *Preparation of a declaration on ethical principles in relation to climate change.* www.unesco.org/new/en/social-and-human-sciences/themes/comest/ethical-principles

Reading and writing the world with mathematics: Exploring possibilities with socially vulnerable Brazilian students

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This article presents a doctoral research project related to reading and writing the world with mathematics from a Critical Mathematics Education perspective. From a qualitative approach, the research aims to identify potentials landscapes for investigation so that students can read and write the world with mathematics, through generative themes based on Paulo Freire's theory. As methodological procedures, it's intended to produce field reports, interviews, and alternative resources and it's expected that great possibilities and potential scenarios for reading and writing the world with mathematics within the school will emerge.

The research project and the theoretical framework

The topic of interest is Teaching and Learning Mathematics for Social Justice, in the settings of a Brazilian public school, with the aim of identifying possibilities and potential for students, promoting reading and writing the world with mathematics from generative themes.

The main theorists who have supported this research so far are Eric Gutstein, Ole Skovsmose and Paulo Freire. These authors stated education must be intrinsically motivated by concerns related to social, democratic, economic, cultural and political issues, aiming the transformation of society through education for social justice.

Paulo Freire, probably the most important reference in critical education in Brazil, who developed part of his work in non-formal educational environments, introduces concerns and manifestations about the relations of power and social, political and economic inequality. According to Freire (2001), education is a political act and an act of knowledge, centered on the dialogic act. Freire stresses the importance of the subject learning to read the world, understanding the text and the context, saying his word.

> The reading of the world precedes the reading of the word, hence the subsequent reading of the word cannot do without the continuity of reading the word. Language and reality are dynamically linked (Freire, 2001, p. 11).

When a person learns to read the world and, consequently, to say his words, with the language and reality involved, he/she starts to reinterpret his own reality, while he/she learns

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to understand themselves in the world. The person is placed in the world as a person of a historic struggle against oppressions and inequalities.

Ole Skovsmose argues that mathematics education is not neutral, that it is necessary to be concerned about the interests behind the subjects, to question what or for whom it serves, what are the knowledge-forming interests that are connected to the content (Skovsmose, 2001). In this sense, Skovsmose (2010) emphasizes that the critical approach needs to rely more on uncertainties, than certainties and ready answers, considering that "any approach that can be characterized as critical is left open. And, with such uncertainty, a critical approach can be built" (Skovsmose, 2010, p. 13).

For Gutstein, a mathematical educator, reading and writing the world with mathematics is to investigate and criticize structures and situations of oppression that are present in everyday life, especially those that involve social injustices, such as racism, feminicide, social inequality, etc. It also highlights this action for education, so that students learn mathematics and, at the same time, use it to study their social reality, in the search for understandings and transformations of this world (Gutstein, 2016a). Gutstein believes that teaching mathematics for social justice is synonymous of reading and writing the world with mathematics or teaching critical mathematics, stating that Ole Skovsmose and Paulo Freire have a wide influence on his thoughts, practices and research.

It is necessary to highlight that those three authors, mentioned above, assume in their understandings the dialogue as the main element so that an education for social justice occurs. Understanding dialogue in the same sense Faustino (2018), as the encounter of different world views that build new world views.

In this perspective of dialogue, Skovsmose (2010) brings the notion of *landscape of investigation*, which seeks the active participation of students in the teaching and learning process, inviting teachers and students to dialogue and walk through different learning environments. According to the author, "landscape of investigation opens up new possibilities for reflection. And the notion of reflection is important for any type of critical mathematical education" (Skovsmose, 2010, p. 13).

It is in this literature that seek to expand the understanding of critical mathematical education and social justice, within the school environment, in which the research project is being built.

The context of research project and methods

The research will be carried out in a public school located on the suburbs of a city of the State of São Paulo, in Brazil, in a context of social vulnerability in basic sanitation, food, work, income, rights, affectivity, etc.

Based on this context, this project intends to list and negotiating with students, through dialogue, some possible themes of interest to them, which will become generative themes, from Paulo Freire's perspective. It is expected that themes related to the context of the school and its surroundings will emerge, with social dimensions, and from that start a reflective dialogue of students with the researcher teacher for reading and writing the world with mathematics will begin. more details about the methodology and methodological procedures will be further defined, because the author is only in the first semester of the doctorate and the project is still in the initial phase.

In the book *Pedagogy of the Oppressed*, Paulo Freire brought generative themes as a teaching methodology in the context of literacy for peasants. Generative themes are concrete representations of the ideas, hopes, doubts, outlooks, fears, values, and challenges that arise from the thought-language of men and women in their relationship with the world (Freire, 2011). Noting "that the generative theme is not found in men isolated from reality, nor in reality separate from men. It can only be understood in human-world relations" (Freire, 2011, p. 136).

It is noticeable that he did not address ideas of mathematics in his work, but Guststein (2016a; 2016b) makes this connection between Paulo Freire's ideas and mathematics, bringing the concepts of reading and writing the world with mathematics. Gutstein (2016b) developed a work focused on mathematics education, that involved

> collaboration with students to discover their generative themes; creation of tasks based on these themes, so that students learn mathematics, at the same time that they prepare to read the world (Gutstein, 2016b, p. 462).

In this direction, in the research project, it is intended to use this methodology of generative themes in the context of mathematics, in a reality of periphery and social vulnerability in Brazil.

In the State of São Paulo, in Brazil, there is the Integral Education Program (PEI) for public schools, in which students spend most of their day at school. This program was created in 2012 for middle and high school and, since then, the number of schools that adhere to this proposal has been increasing. The author of this article teaches at one of these public schools, which is part of the PEI, and will conduct her research at her school, researching his own practice. The research will be developed with a group of students in an elective subject.

The elective subject was chosen for develop the research, since in this space the teacher is expected to promote the enrichment, expansion and diversification of the contents and themes. According to the Integral Education Program Guidelines (São Paulo, 2014), "elective subject occupies a central place with regard to the diversification of school experiences, offering a privileged space for experiment, interdisciplinary and further studies" (p. 29).

Thus, in the elective subject it is possible to work with mathematics covering different contexts, exploring interdisciplinary freely. Therefore, this project will not be developed in the mathematics subject which would have a limited space for the development of the proposal, due to the skills and programmatic content of the curriculum.

The elective subject also has a differential, which are the students who choose which elective they want to take in the semester. All teachers at the school offer subjects, disseminating the proposals, and students choose one that is close to their interest.

Considering the synergy of the ideas that have been explained so far, the reflections converge to the following guiding question: When working with generative themes in an elective subject, what are the potentials landscape of investigation for reading and writing the world with mathematics?

It is expected that potential landscapes of investigation for reading and writing in the world with mathematics for social justice will emerge, in the context of public school.

References

Faustino, A. C. (2018). *"Como você chegou a esse resultado?": O diálogo nas aulas de matemática dos anos iniciais do Ensino Fundamental* ("How did you get to that result?": The dialogue in the Matematics classes in the early years of Elementary School) [Doctoral dissertation, São Paulo State University]. https://repositorio.unesp.br/handle/11449/180358

Freire, P. (2001). *A importância do ato de ler: Em três artigos que se completam.* Cortez.

Freire, P. (2011). *Pedagogia do oprimido.* Paz e Terra.

Gutstein, E. (2016a). A letter to those who dare teach mathematics for social justice. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Helping all students understand important mathematics* (pp. 63-70). National Council of Teachers of Mathematics.

Gutstein, E. (2016b). "Our issues, our people–math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education, 47*(5), 454–504.

Skovsmose, O. (2001). *Educação matemática crítica: A questão da democracia.* Papirus.

Skovsmose, O. (2010). *Desafios da reflexão em educação matemática crítica.* Papirus.

São Paulo (2014). *Diretrizes do programa ensino integral.* Estado de São Paulo. http://www.educacao.sp.gov.br/a2sitebox/arquivos/documentos/342.pdf

Interactions in mathematics classrooms over different timescales

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I here introduce a classroom study that takes a dialogical approach on mathematics teaching and learning to investigate in what ways interactions between the participants in the classroom are interconnected over different timescales when communicating mathematical reasoning. The study is part of my ongoing PhD-work with an aim to further understand the connections between in the moment interactions and longer patterns of interactions in mathematics classrooms in upper secondary school in Norway.

Context

To understand teaching, we need to think about how students take part in activities in the classroom and in education research look at what learning opportunities students are given and what systems they are part of (Staples, 2008). Franke, Kazemi and Battey (2007) show how teachers' choices of actions and activities in teaching influence the conditions for interactions in the classroom, creating different opportunities for students to engage with mathematics and with other students. Equally, students' reactions or expectations and attitudes towards mathematics influence interactions, how they engage and what meaning knowledge is given. In this flow of interactions between participants in the classroom there is also a negotiation for what is allowed or expected to be done in the mathematics classroom (Cobb, 1999). Depending on responses from students, the teacher makes new choices that have impact not only in the moment (Bishop, 2008) but affect what happens in the next lesson or several lessons to come, creating an ongoing change in conditions for teaching and learning.

Curriculum documents in Europe highlight developing a mathematical competence (OECD, 2018) as a central goal for teaching mathematics, but the translation of such competence differ between education systems and are hence used in different ways in research depending on the context. In the ongoing renewal of the curriculum in Norway (Kunnskapsdepartementet, 2020), the concept of competence is described in terms of basic communication skills (to speak, to read, to write, to count and digital skills) and core elements, such as reasoning and argumentation, that teaching should address in all school subjects and on all educational levels. In mathematics, such competence involves students' ability using mathematical concepts in different situations, their capability of engaging in posing and answering questions related to mathematics and their use of mathematical

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language. Such aspects of students becoming active members of the mathematical discourse in the classroom are also present in education research, for example how participating in conversations about mathematics promotes students understanding of mathematical concepts or development of a formal mathematical language (Schleppegrell, 2007; Barwell, 2016).

Changes in learning conditions influencing teaching in longer timescales than a lesson implies that research on teaching needs to include studies that tie together how students approach or understand mathematical content during parts of individual lessons with how a teacher organizes the classroom also for longer learning processes (Klette, 2007). Although there are several studies in education research that address how time can alter ones understanding of mathematics, there have been more studies on shorter social processes such as those taking place during classroom lessons than on processes that lasts days or weeks (Lemke, 2000). To look at a sequence of lessons in different timescales could affect how classroom activities and interactions are analysed and classified in research (Dalland et al., 2020) and to investigate connections between interactions from different parts of such a sequence could give a new contribution to the understanding of teaching and learning mathematics.

Purpose and aim

The project I present here is part of my ongoing PhD-work with a research interest in the interactions that take place in the classrooms of novice teachers when communicating mathematical reasoning. The study is to be conducted in upper secondary school in Norway and a novice teacher has in this context up to three years of experience teaching mathematics since graduating from teacher education. Three questions guide my research: 1) What kind of patterns of interactions are established in the classroom when the new teacher and students are communicating mathematical reasoning? 2) What are the relationships between the mathematical content and patterns of interactions in the classroom? 3) How do patterns in classroom interactions connect to, or influence each other, over different timescales?

Focus for this project presentation is the third research question, with the aim to further understand how individual events in the mathematics classroom can be interconnected with longer teaching and learning processes when looking at more than just a single lesson. My study proposes to examine the shifts in learning conditions and students' opportunities to participate in the classroom by following interactions over a sequence of lessons. I use 'patterns in classroom interactions' to address "identifiable types of exchanges" (Lemke, 2000, p. 276) that evolve or reoccur over time.

Theoretical aspects of the research

With an interest in classroom interactions and communication I choose a theoretical starting point looking at learning and teaching as social practice and that humans are interdependent of others (Linell, 2009). The meaning of actions and knowledge are created together by the participants in a context that also influence the communication and negotiation of that meaning. In mathematics such meaning-making could be about a mathematical concept or to understand what counts as an acceptable mathematical explanation and justification in the classroom (Cobb, 1999).

One theoretical framework that highlights interactions, language and the influence of contexts is dialogism (Linell, 2009). In dialogism meaning-making is described as being multi-voiced and interactive and a dialogical research approach puts all the participants in the mathematics classroom as contributors to the dialogues and the meaning of mathematics (Barwell, 2016). Using the theoretical framework of dialogism as presented by Linell (2009) enables dialogues to be about all kind of human sense making and enables patterns in classroom discussions to be about social actions where participants interact with others depending on the situated linguistic practice. Participating in communication can then be a sign of learning and this classroom study seeks to develop understanding for what this participation looks like rather than for how or why interactions take place.

To address how interactions can be connected over time I will use the concept of different timescales presented by Lemke (2000). Timescales are then divided by powers of ten, for example utterance and the exchange of them being at the scale 1-10^2 seconds and a school day and units of them at the scale 10^5-10^6 seconds. According to Lemke it is especially interactions in the scale directly under and over the one in focus of an observation that is of interest to study. Lemke also uses semiotic artifacts to visualise how interactions on different timescales are connected in time and space through physical objects or utterances that reoccur in different situations. To follow dialogues over time, in a fairly limited context, could enable findings of interaction that form patterns both in language and other interactions with the environment that are not just accidental expressions and acts of routine but are baring some kind of meaning or reason (Linell, 2009).

Methodology

To address my research question about how events and interactions in mathematics classroom can be connected over time I plan to do classroom observations and visit three to four classrooms, and by that three to four different teachers. Observations and recorded video data from a sequence of lessons would enable studies of dialogues at micro and macro levels (Lemke, 2000) and by that identify interactions on different timescales. I intend to use Lemke's timescale of 10-12 days as a unit for the sequence of mathematics lessons to follow and then look at interactions within lessons, between lessons and over the entire period's timescale. Recordings make it possible to capture actions and things that can be analysed to connect different timescales, for example gestures, use of physical objects in different ways or words or utterances that reoccur over time. Patterns in interactions could also be connected to the level of responses given by students, the use of activities or the use of time and space in the classroom. Analysis of video recordings would make it possible to find thematic patterns in interactions, although there are challenges in finding those patterns and connecting interactions to a relevant timescale.

Challenges and final remarks

To conclude, previous research results have put into focus the importance of more detailed studies of how interactions affect the conditions for teaching and learning and I find the aspects of meaning-making, dialogues and timescales being highly relevant to further understand mathematics teaching and interactions in the classroom. Although video recordings make it possible to capture different interactions, finding those patterns in the recorded data pose a challenge not only in relation to the amount of data but in finding timescales and identifying relevant artifacts and aspects of dialogues to follow. I find this conference to be a good opportunity to meet with other colleagues of the community and discuss this study, its possibilities, and challenges.

References

Barwell, R. (2016). Formal and informal mathematical discourses: Bakhtin and Vygotsky, dialogue and dialectic. *Educational Studies in Mathematics, 92*(3), 331–345. https://doi.org/10.1007/s10649-015-9641-z

Bishop, A. J. (2008). Decision-making, the intervening variable. In P. Clarkson & N. C. Presmeg (Eds.) *Critical issues in mathematics education* (pp. 29–35). Springer.

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning*, *1*(1), 5–43. https://doi.org/10.1207/s15327833mtl0101_1

Dalland, C. P., Klette, K., & Svenkerud, S. (2020). Video studies and the challenge of selecting time scales. *International Journal of Research & Method in Education*, *43*(1), 53–66. https://doi.org/10.1080/1743727X.2018.1563062

Klette, K. (2007). Trends in research on teaching and learning in schools: Didactics meets classroom studies. *European Educational Research Journal*, *6*(2), 147–160. https://doi.org/10.2304/eerj.2007.6.2.147

Kunnskapsdepartementet. (2020, June 16). *Nye læreplaner for 16 valgfag er klare.* https://www.regjeringen.no/no/aktuelt/nye-fagplaner/id2709476

Lemke, J. L. (2000). Across the scales of time: Artifacts, activities, and meanings in ecosocial systems. *Mind, Culture, and Activity*, *7*(4), 273–290. https://doi.org/10.1207/S15327884MCA0704_03

Linell, P. (2009). *Rethinking language, mind, and world dialogically: Interactional and contextual theories of human sense-making*. Information Age.

OECD. (2018). *PISA: Programme for International Student Assessment*. http://www.oecd.org/pisa

Schleppegrell, M. J. (2007). The Linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139–159. https://doi.org/10.1080/10573560601158461

Staples, M. E. (2008). Promoting student collaboration in a detracked, heterogeneous secondary mathematics classroom. *Journal of Mathematics Teacher Education*, *11*(5), 349–371. https://doi.org/10.1007/s10857-008-9078-8

Developing mathematics education promoting equity and inclusion: Is it possible?

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This paper is a discussion of the possibility to develop an inclusive and equitable mathematics education in primary school based on success factors found in prior research. The overall goal is to develop a model for education and to develop an approach where sociopolitical and pedagogical issues are core. The study contributes to this important and challenging task by building on earlier research from different fields of relevance, generating a model for sustainable development of mathematics education, and at the same time, deriving from and anchoring the model in teachers and students' experiences of everyday life in the mathematics classroom.

Introduction

The Swedish school and mathematics education's ideal of a school for all has been heavily challenged in the last decade as segregation and inequalities have been enhanced at the same time as knowledge has decreased in the subject of mathematics. A struggle for both society, research and practice is to include an agenda of equity and inclusion in mathematics education (Roos, 2019; Bagger, 2017), even though these issues are central for every student learning in mathematics (Atweh, 2011). The struggle has been shown in research as complexity and multiple issues regarding equity and inclusion have been explained (Kollosche et al., 2019). Hence, the explanations to better understand inclusion and equity are found in very different fields. Examples are research on socioeconomics (e.g., Thien, 2016), gender (e.g., Leder & Forgasz, 2008), ethnicity (e.g., Martin, 2019) cultural background (e.g., Meaney, Edmonds-Wathen, McMurchy-Pilkington & Trinick, 2016), language (e.g., Planas, Morgan & Schütte, 2018), disability (e.g., Tan, Lambert, Padilla & Wieman, 2019), ability (e.g., Leikin, 2011), curriculum (e.g., Askew, 2015), educational approaches (e.g., Kolloshe et al., 2019), and assessment (e.g., Bagger, 2017). All these fields aim at creating a mathematics education for optimal opportunities to learn, though rather separate from each other. Out of this separate and multitudinous base of knowledge, success factors regarding sociopolitical and pedagogical issues can be retrieved. Thereafter these issues can be applied to develop an approach in mathematics education that promotes equity and inclusion. Following, this paper aims to investigate the possibility of a project focusing on promoting equity and inclusion in mathematics education in primary school in a Swedish context. This

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is done by presenting the core ideas behind a practice-based project with the aim to support the development of mathematics education which promotes equity and inclusion in the classroom. The result will be two-fold: to develop a model for education and to develop an approach where sociopolitical and pedagogical issues are core. The tentative research questions of the project are:

- What are already identified success factors on societal and classroom levels for equity and inclusion in education?
- How can this theoretically build a foundation for developing a model for inclusive mathematics education?
- How can criteria for equity and inclusion be developed in collaboration with teachers and students?
- How is the relation between this inclusive mathematics education in primary school and student's view of equity and inclusion?

Equity and inclusion in mathematics education

The notions of equity and inclusion are both complex and not always particularly defined in mathematics education research (Bagger, 2017; Roos, 2019). Although, they are frequently used in order to highlight the importance to consider every student's learning on both individual- and societal level (see for instance Askew, 2015). We define inclusion as processes of participation in learning and teaching in mathematics (Roos, 2019). We draw on Cobb and Hodge (2007) in our understanding of equity as something that "contributes to student empowerment, development, and in turn, their ability and agency to learn" (p. 71, Bagger, 2017). Consequently, we claim that inclusive and equitable mathematics education is an education that strives for every student's opportunity to participate in learning processes and develop the ability and agency to learn. Hence, education that considers equity and inclusion simultaneously. Research regarding educational quality and equity is complex and questions of who has access, or possibilities to get access are highlighted (e.g., Askew, 2015). Here, not only issues of what is happening within the mathematics classroom are at stake, but also issues of power and democracy in society (Halai, Mushaffar & Valero, 2016). This makes the learning of the individual student influenced by structures in society and continuous processes of in(ex)clusion is present (Halai, Mushaffar & Valero, 2016).

Bridging equity, inclusion and mathematics teaching – a reflection

In this future study both mathematics teaching and learning as well as students' own view of equity and inclusion needs to be studied at an individual and classroom level. Though, it also has to be understood and developed from a societal perspective foregrounding issues of power and democracy. To be able to, firstly develop a mathematics education based on success factors both on societal and classroom level, and secondly, investigate the relationship between success factors for equal and inclusive mathematics education and student's view of equity and inclusion, there needs to be a bridge between classroom and societal issues. Also, there is a need to be attentive to power relations in the recontextualisation, acquisition and transmission of knowledge between research and practice (see Bernstein, 2003). Therefore, we build our framework on Bernstein's (1999) theories of the pedagogical device and vertical and horizontal discourses in education. Bernstein states that:

> The shift in equity from equality ('of opportunity') to recognition of diversity (of voice) may well be responsible for the colonisation of vertical discourse [knowledge retrieved from research] or the appropriation by vertical discourse of horizontal discourse [knowledge retrieved from practice] (Bernstein, 1999, p. 169).

Challenges for the future study

One challenge for this future study will be to explicitly find and investigate both research regarding societal issues on equity and inclusion, as well as classroom issues within different research paradigms of relevance. Another challenge is to be able to create a collaboration with teachers and students generating data consisting of observations and interviews with teachers and students as well as quantitative measures of knowledge development. In order to not colonise horizontal discourse (see Bernstein, 1999) while investigating prior research and researching with teachers and students important questions to ask are: What factor is important to start with? How (if possible) do different factors connect? How do the factors work in relation to specific cultures, contexts, and students? Is it possible to reconsider all factors identified when creating inclusive mathematics education?

The overall aim and hope for this future study is to contribute with a model for supporting the development of mathematics education that promotes equity and inclusion. The study will take both sociopolitical and pedagogical aspects into consideration and aims at shedding light not only on successful mathematics teaching but also on the students' part and perspective in such teaching.

References

Askew, M. (2015). Diversity, inclusion and equity in mathematics classrooms: From individual problems to collective possibility. In A. Bishop, H. Tan, & T. N. Barkatsas (Eds.), *Diversity in mathematics education: Towards inclusive practices* (pp. 129–145). Cham: Springer.

Atweh, B. (2011). Quality and equity in mathematics education as ethical issues. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds), *Mapping equity and quality in mathematics education* (pp. 63–75). New York, NY: Springer.

Bagger, A. (2017). Quality and equity in the era of national testing: The case of Sweden. In J. Allan & A. Artiles (Eds.), *The Routledge Yearbook of Education 2017: Assessment Inequalities* (pp. 68–88). London: Routledge.

Bernstein, B. (1999). Vertical and horizontal discourse: An essay. *British Journal of Sociology of Education, 20*(2), 157–173.

Bernstein, B. (2003). *Class, codes and control: The structuring of pedagogic discourse*. London: Routledge.

Cobb, P., & Hodge, L. L. (2007). Culture, identity, and equity in the mathematics classroom. In S. N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 159–164). New York, NY: Teachers College Press.

Halai, A., Muzaffar, I., & Valero, P. (2016). Research rationalities and the construction of the deficient multilingual learner. In R. Barwell et al. (Eds.), *Mathematics education and language diversity*. New ICME Study Series. Cham, Switzerland: Springer.

Kolloshe, D., Marcone, R., Knigge, M., Gody Penteado, M., & Skovsmose, O. (2019). *Inclusive mathematics education: State-of-the-art research from Brazil and Germany*. Cham: Springer.

Leder, G., Forgasz, H. (2008). Mathematics education: New perspectives on gender. *ZDM Mathematics Education, 40*, 513–518.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast, 8*(1), 167–188.

Martin, D. B. (2019) Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education, 22*(4), 459–478.

Meaney, T., Edmonds-Wathen, C., McMurchy-Pilkington, C., Trinick, T. (2016). Distribution, recognition and representation: Mathematics education and indigenous students. In: K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in Mathematics Education in Australalasia 2012-2015*. Singapore: Springer.

Planas, N., Morgan, C. & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 196–210). New York: Routledge.

Roos, H. (2019). Inclusion in mathematics education: An ideology, a way of teaching, or both? *Educational Studies in Mathematics Education, 100*(1), 25–41.

Tan, P., Lamber, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiry and practice. *The Journal of Mathematical Behavior, 54*(June), 100672.

Thien, L. M. (2016). Malaysian students' performance in mathematics literacy in PISA from gender and socioeconomic status perspectives. *Asia-Pacific Education Researcher, 25*(4), 657–666.

A socio-critical perspective in mathematics education: Doing interviews

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This paper aims to present a seminar proposal, to be developed during the summer semester of 2021 at Freie Universität Berlin, Germany, based on an on-going thesis. This seminar does reference to concepts like dreams, being more, background and foreground, from Critical Mathematics Education and Freire's Pedagogy, and aims to promote the study on the Interview research procedure from a socio-critical perspective in Mathematics Education. In this text, we briefly explore the thesis and seminar's theoretical framework, as well as present the implementation, highlighting the topics, the interview transcript, and some reflections that we can make about these works so far.

Introduction

This paper is about an ongoing seminar, given by the first author, based on interrelated concepts such as dreams, being more, background and foreground (Freire, 1983, 2000; Skovsmose, 2011, 2014).

For Paulo Freire (1983), human beings know themselves as unfinished, and therefore, they have hopes and dreams. They have hopes because of the human nature, that put themselves in a search movement, trying to be more. And human beings dream because they want to have experiences of humanization and freedom.

Inspired by this educator, we could say that dreams have a political perspective, related to the social, political, economic, and cultural contexts of human beings. They also have a subjective perspective, related to personal life experiences. It means that theses dreams are historical and represent an important connection with human beings' backgrounds. Although these dreams make reference to the past, they are not defined by them because, as a part of the humanity's search movement, they point to the future, and generate foregrounds.

Foreground is a terminology named by Ole Skovsmose (2014), which refers to future perspectives, such as dreams, desires, hopes, obstacles, fairs and frustrations. These foregrounds can be seen as landscapes of the future, those influenced by the past, but not determined by it.

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The seminar proposal was inspired by these concepts, as they are part of the first author's doctoral thesis (Soares, in progress). In her work, the research is being carried out based on Freire and Skovsmose's concepts, among others. The aim is to understand how teenagers from a social oppression context dream, what we can say about their life stories, and how they see the role of school and, specially, mathematics classes in helping them to develop their dreams. In the thesis, she works with teenagers from two schools: one in Sao Paulo, Brazil, and one in Bogota, Colombia. She refers to the case study and the life history (Goldemberg, 2004, Nogueira et al., 2017) as inspirations for and uses the interview as the most important data production procedure.

For the referred seminar, participants will be invited to produce data with students from public schools in Berlin, from diverse backgrounds. We will encourage them to be open to students from social oppression context, as was done in the thesis.

We understand that the inclusion of schools outside the Latin American context, in this case, from a German context, which is known as a more privileged region, will bring great gains for the scope of the research and for its impact. The inclusion of this new context will help to identify the dreams of young students from public schools in a global context, and expand the possibilities of mathematics classes in different classrooms around the world, about the understanding of these rooms as spaces that foster dreams and social transformation.

The seminar

The seminar has been prepared to be part of Freie Universität Berlin's course catalogue during the summer semester of 2021, as a result of the first author's work during her stay at this university, as part of a doctorate exchange period, and supervised by the second one. The seminar proposal was built pursuing the following general aim: to promote the study of an Interview research procedure, from a socio-critical perspective in Mathematics Education, based on the Case Study and Life History methodologies (Goldemberg, 2004, Nogueira et al., 2017). Another important approach for this seminar was the multicultural perspective in Mathematics Education (McLaren, 2000). The seminar will be arranged intending to develop a theoretical study in a dialogical way, and creating conditions for the participants to experience doing interviews in practice, going into the field to investigate the following guiding question: how Berlin teenagers dream, and what they think about school and Mathematics classes?

During the referred seminar, which will be held in English, participants (mainly bachelor's degree students) will be invited to produce data that will consist of semi-structured interviews, as in the thesis, with teenager students. At the end of the seminar, everyone will present their produced data and we, among the participants, will think about an initial interpretation of these data.

The interview script, that was also part of the first author' thesis, based on the theoretical framework and on the original one's (naturally, in Portuguese), is as follows:

Life:

1. Who are you?
2. How is your family?
3. How was your childhood?
4. And your adolescence so far, how was it?

Schools:

1. How were the schools that you have studied so far?
2. How do you relate to school, from childhood to today?
3. Who have been your math teachers since childhood?
   Tell me a little about them and your relationship with them.
4. What do you think about mathematics classes?

Dreams:

1. Do you identify injustices with you, or in your environment, or in the world? Which ones?
2. What makes you move on, or drives you?
3. What are your dreams? Why do you have these dreams?
4. How do you imagine yourself 4 years from now? And in 15 years?
5. If you were to choose a profession, what would it be?
   (Say the 1st, 2nd, and 3rd place, and justify).
6. Would you change something in your life or in the world? What and why?

All these questions seek to respond the aim's thesis. Briefly, we can say that the first four questions were inspired by the life history methodology and the concept of background; the next four questions were also inspired by the life history methodology and include concerns about mathematics education, and the last six questions were inspired by the concepts of foreground and dreams, including concerns about social and political aspects.

The goals to be achieved with this seminar proposal were categorized into ten seminars which include the following topics: socio-Critical perspective in mathematics education; multicultural perspective in mathematics education; case study and life history - understanding the methodology; "how to make an interview?" – theoretical framework and presenting the semi-structured model; transcription and textualization of interviews - understanding the differences; and data production presentation and interpreting results based on the theorical framework.

The classes will take place during the summer semester in 2021, between April and July. The topics, naturally, are related to the theoretical perspectives addressed in the thesis on which the seminar is based, except for the multicultural perspective. It was added due to the multiethnic and religious context in which many students in Berlin find themselves.

Last considerations

The thesis is at an advanced stage, and we could present in this text some of its results. For instance, we could say that families have a big impact on students' dreams, that their past experiences influence their foregrounds a lot, but not only 'negatively', and that many social oppressed students reveal, through their dreams, their desires for social justice. However, we think this is not the right place to share some of these initial conclusions because we tried here to focus on the seminar and, with this, we intend to look further, covering diverse backgrounds that go beyond.

Finally, in addition to the objectives linked to the thesis, we understand that the purpose of this referred seminar is that the participants learn in the field (in a practical way) how to conduct qualitative interviews in the socio-critical area, having a bibliographic basis in this regard, and start to learn how presenting research results and interpreting them.

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References

Freire, P. (1983). *Educação e mudança*. Paz e Terra.

Freire, P. (2000). *Pedagogy of freedom: Ethics, democracy, and civil courage*. Rowman and Littlefield.

Goldenberg, M. (2004). *A arte de pesquisar: Como fazer pesquisa qualitativa em Ciências Sociais*. Rio de Janeiro: Record.

McLaren, P. (200) *Multiculturalismo crítico*. São Paulo: Cortez.

Nogueira, M. L. M, Barros, V. A. de, Pimenta, D. A. O. (2017). O método de história de vida: A exigência de um encontro em tempos de aceleração. In: *Pesquisas e Práticas Psicossociais, 12*(2).

Skovsmose, O. (2011) *An invitation to critical mathematics education*. Rotterdam: Sense.

Skovsmose, O. (2014). *Foregrounds: Opaque stories about learning*. Sense.

Soares, D. A. (in progress). *Sonhos de adolescentes e reflexões sobre as aulas de matemática* [Doctoral dissertation, Universidade Estadual Paulista].

Timescales of transgressive teaching in social justice mathematics

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This project presentation reports on a conversation amongst in-service secondary mathematics teachers who had just participated in a social justice mathematics professional development session. As they discussed how they might incorporate the social justice activity into their classes, their conversation invoked a wide range of timescales. Timescales, as presented by Lemke, are categories of actions that are somewhat predictable, periodic and that either constrain or enact agency. Collectively, the teachers mentioned 20 timescales that imagine forms of agency that unfold over a range of time frames, from a moment to nearly a century. Timescale analysis of this conversation reveals the intensively hegemonic conditions which teachers must consider when preparing to teach transgressively.

Introduction

This paper analyses a conversation amongst several experienced, in-service secondary mathematics teachers sharing their thinking about how one should teach a social justice mathematics activity. The teachers had just participated in a professional development presentation of a mathematics activity on gender inclusivity that would require them to explain concepts of transgender identity and gender-neutral forms of speaking (Whipple, Staats, & Harrison, 2020). During a subsequent focus group interview on attitudes towards equity teaching, two teachers, one with a progressive and the other with a conservative political stance, commented that they had particularly enjoyed the presentation and wanted to present it in their class. However, their initial thinking about how to do this was divergent, and created a lively argument about how one should teach social justice mathematics topics. The conservative teacher described in detail techniques of maintaining mathematical neutrality. The progressive teacher emphasized alliance with a university professional development program as a technique for teaching as she wished to teach.

While imagining themselves teaching a topic that would be received as controversial by some school stakeholders, teachers mentioned an extreme range of timescales. Timescales are activities or processes that unfold over a relatively predictable period of time and that repeat periodically, such as the time it takes to complete a routine calculation, teaching over a class day or year, using a particular textbook or curriculum for several years, and so on

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(Lemke, 2000). Timescales are sometimes hierarchical. For example, a yearlong curriculum might influence the activities in class each day, and each day in class might be composed of multiple cycles of routine calculation, or components of inquiry learning, or other repeated activities specified by the curriculum. After some years, a new curriculum might be adopted with new nested cycles of activities.

Timescales can be seen as modes of action, but also, because they are somewhat predictable, speakers can also refer to them or invoke them in a conversation. Referring to a timescale can be a socially powerful act, a speaker's bid to frame the relevant scope or basis of interpretation for the ensuing conversation. This project presentation treats references to timescales as a way to understand teachers' imagination of the landscape of power that surrounds them should they embark upon teaching social justice mathematics.

Timescales and agency

Discourse analysis using timescales requires attention to the ways in which forms of linguistic referencing specify elements of context — near or far in space or in time — and thereby establish the "scope of understandability" (Blommaert et al., 2015, p. 119). In mathematics education research, timescales have been used to clarify details of positioning theory (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015); propose new, multilevel research methods (Noyes, 2013); and to understand processes of linguistically-mediated social stratification (Barwell, 2020).

Timescales invoke agency because they imply particular types of action carried out over a stretch of time, forms of agency that are somewhat predictable by virtue of being periodic, but never fully so. The hierarchical nature of timescales requires consideration of how different kinds of agencies relate to each other. Timescale analysis is interested in the interplay of agentic constraint, because longer duration timescales such as adopted curriculum tend to require particular kinds of actions at shorter timescales. However, actions at shorter timescales are the constituents of longer timescales, and through this, disruptive agency to change is always possible (Lemke, 2000). For example, what a teacher talks about in one class day might be partly constrained or formatted by the curriculum chosen by the school for use over several years. However, if the curriculum does not allow teachers to teach, and to speak, as they wish, these shorter timescales might provoke teachers to take action towards changing the curriculum. In this way, timescale analysis can give insight into teachers' experiences of power, and potentially, into their means of enacting power in their educational system.

Methods of timescale analysis

Although the timescale construct has been productively critiqued as being less-well-defined than Lemke's 2000 presentation might suggest (Blommaert et al., 2015), here, timescales usefully describe the political range and intensity of the teachers' conversation. For teachers from different schools to share teaching perspectives, they need to refer to situations that are widely-experienced and that have partly-predictable responses. Teachers need to discuss the "scopes of understandability" of predictable events across school stakeholders. In this paper, we focus on identifying references to timescales in the conversation as a way to describe teachers' envisioning of the power dimensions of their work. A later paper will analyse the interactional processes of scale-jumping through which participants enacted power over each other during the emergent conversation (Barwell, 2020).

For this project, each sentence in each conversational turn was analysed for any reference to a social, physical or historical event that predictably repeats over some period of time. Shorter scale examples include how one should speak in class during a politically situated mathematics activity and how students, school administrators, and families might respond. Middle scale examples included references to political protests that were increasingly frequent at the time of the conversation in 2017, such as high-profile athletes' Black Lives Matter protests, the increasingly public presence of White supremacist racism and anti-Semitism, and student-led school activism towards gender diversity. Longer scale examples included references to the United States' periodic military actions so that a student's response to a social justice mathematics lesson might be conditioned by their relationships with multiple generations of military veterans in their families. We arranged these timescales in a roughly hierarchical manner in terms of the duration, that is, the time it takes to complete the process. Due to space constraints, only some of the 20 timescales are given in Table 1.

Timescale	Approximate duration
Multiple generations in a family: References to Brother, Dad, and Grandpa	Eight decades
The family life cycle: The 14-17 years it takes to raise a child to near-adulthood	1.5 decades
Passage into middle age	A decade
The cycle of the U.S. going to war periodically	Every few years
Types of contemporary political protest in the U.S.: Black Lives Matter; footballers protesting the National Anthem; right-wing marches	Monthly or weekly
The academic year shared by a teacher and a class of students	A year
Getting a pay check from your school	Every two weeks
Students' political expression during a day at school	A day
School administration, parental, community responses to student political expression	A day
Teaching a topic in a class	An hour
Introduce a conversational topic	A moment
Choose a response to a conversational topic	A moment

Table 1: Timescales relevant to social justice mathematics teaching.

Conclusion

Timescales mattered greatly to these teachers as they described how they might teach a social justice mathematics activity. As Table 1 indicates, teachers' discussion of whether mathematics should be, can be or must be taught neutrally was permeated with references to the periodic events from daily school routines, from the communities that the schools serve, and from contemporary and historical political actions. Analysis of this conversation suggests that as teachers consider transgressive teaching, they weigh matters of dramatically wide scale and scope: how to choose one's words, how hold one's face, how to maintain employment, how to respond to the spectrum of current political activism, how to address the displeasure of a family implicated in nearly a century of U.S. military action. The conservative teacher's vision of teaching in a neutral manner references timescales across a tightly regimented hegemonic system that strongly resists change. All the teachers in this conversation seemed to acknowledge that Table 1 lists a range of issues that they might need to address with local stakeholders when teaching social justice mathematics.

Future effective professional development towards social justice mathematics teaching may need to spend substantial time on uncovering these concerns, analysing with teachers which timescales afford them greater agency, and how to build discursive skill in re-framing discussions regarding the broader timescales. Beyond learning to teach a social justice mathematics lesson, with its associated techniques of engaging students' mathematical learning and socio-political consciousness, teachers may need explicit preparation to enact and protect their agency across a range of temporal events far beyond the classroom.

References

Barwell, R. (2020). The flows and scales of language when doing explanations in (second language) mathematics classrooms. In J. Ingram, K. Erath, F. Rønning, A. Schüler-Meyer, & A. Chesnais (Eds.), *Seventh ERME Topic Conference on Language in the Mathematics Classroom* (pp. 23-30). ERME. https://hal.archives-ouvertes.fr/hal-02970546/

Blommaert, J., Westinen, E., & Leppänen, S. (2015). Further notes on sociolinguistic scales. *Intercultural Pragmatics, 12*(1), 119–127.

Herbel-Eisenmann, B., Wagner, D., Johnson, K., Suh, H., & Figueras, H. (2015) Positioning theory in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics, 89*, 185–204.

Lemke, J. L. (2000). Across the scales of time: Artifacts, activities, and meanings in ecosocial systems. *Mind, culture, and activity, 7*(4), 273–290.

Noyes, A. (2013) Scale in education research: Towards a multi-scale methodology. *International Journal of Research and Method in Education, 36*(2), 101–116.

Whipple, K, Staats, S., & Harrison, K.C. (2020). Mathematical modeling activity: The spread of gender inclusive pronouns. S. Wooley & L. Airton (Eds), *How to teach about gender diversity: Teacher-tested lesson plans for K-12 classrooms* (pp. 271–276). Canadian Scholars Press.

Embodied and emplaced mathematical literacy: A refugee family's funds of knowledge toward regenerative farming

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In this paper, we present our preliminary findings from our ongoing ethnographic study on out-of-school mathematics learning for refugee families. Our paper provides a glimpse of embodied and emplaced mathematical literacy exercised by a Syrian refugee family engaging in intergenerational, small-scale farming practices, during the pandemic. Aligned with the funds of knowledge framework, we depicted a sketch of mathematical literacy that the family, including young learners, competently engaged. Our analyses call for the discussion on mathematical literacy that could challenge the hegemonic and normative relationships between body and place, and could lead us to the liberating interanimated relationships between body and place.

Conceptualizing embodied and emplaced mathematical literacy

Previous studies on non-dominant (im)migrant families' out-of-school practices have demonstrated funds of knowledge, which is "historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being" (Moll et al., 1992, p. 133). Funds of knowledge relevant to mathematical literacy include sophisticated geometric thinking in the practice of sewing (Gonzalez et al., 2001), the multiplicative thinking exhibited in gardening (Civil, 2007), and the proportional reasoning in calculating international currency conversions (Takeuchi, 2018). The funds of knowledge perspective have challenged the deficit views toward non-dominant, working-class families and demonstrated the possibility of transforming the school practice and curriculum, where teachers maximize the bodies of knowledge and skills that are embedded in family practices.

Such funds of knowledge are simultaneously embodied and emplaced. Interanimated relationships between a place and learners as agents are key in our inquiring into the family's knowing that is inextricable with the land that they are cultivating. In the recent scholarship (as seen in Krishnamoorthy & Ma, 2021; Marin et al., 2020; Takeuchi & Aquino Ishihara,

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2021), emplacement and embodiment have been synthetically analyzed. Learners are actively making places as they imagine new mobilities of bodies in the places salient to them (Marin et al., 2020). Embodied and emplaced mathematical literacy can thus be intertwined with social changes that the learners desire to envision (Takeuchi & Aquino Ishihara, 2021).

Methodology

Our ethnographic study focuses on a Syrian refugee family who has been engaging in small-scale farming after their resettlement to Canada in 2016, without the use of any pesticide or herbicide. The family used to engage in traditional farming in Deer al-Fardees village near the city of Hama in Syria. As the attacks on civilians in the city of Hama and Deer al-Fardees village escalated, the family evacuated to Lebanon and then moved to Canada as refugees in 2016. The family participants include three of the five siblings in early elementary years (age 6 years old to 9 years old at the time of the study in 2020), Aisha (9 years old), Rabih (8 years old), and Abir (6 years old), and the mother (Nahima) and father (Mohamed) of these children. The family lived in an inner city and commuted daily to a land located approximately 15 km away. Our ethnographic fieldwork was accompanied by video recordings to allow us to engage in the repeated and collective viewing of video data. We also used photographs and drawings that were produced with the children to understand the lived experiences on the farm. We conducted verbal interviews to understand the histories behind their farming practices. We also collected policy and media documents relevant to urban farming practices by this family. Emplacing our dialogues, together with the family participants, we engaged in a *shared* walk that "walkers have a particular way of being together that is more than just co-presence because it has sociability as the basis for bodily movement" (Lee & Ingold, 2006, p. 83). By walking together on the farm with each member of the family, we came to understand emplaced and embodied knowing of Science, Technology, Engineering, and Mathematics (STEM) enabled through their physical interactions with soil, plants, and animals on the farm.

Data and analysis

The data analyzed for this article includes video/audio recorded interactions collected over seven visits to the farm (each visit lasted 60 to 150 minutes) in the summer and fall of 2020. This ethnographic study is still ongoing for analysis of longitudinal development and program design for bridging informal and formal STEM epistemologies. Our analysis in this article focused on the embodied and emplaced mathematical literacy unveiled through the process of shared walks. For the analysis, we first created content logs of all the video/audio data and completed analytic memos for each data. For the parts of the interview conducted in Arabic, translations to English were completed by Author 2 (Raneem). Based on analytical memos, we inductively coded data (video/audio data) focusing on the participants' emplaced and embodied knowing of mathematics.

Findings

As we walked around on the land, the conversation went into the differences and similarities between the farm they had in Syria and the farm in Canada. Mohamed said they grew the same variety of plants that we saw on the farm: kouza, fava beans, parsley, chickpeas, beets, carrots, and so forth. Mohamed explained:

> The difference is… here is a short summer season while in Syria it is a longer summer season. The summer season is 4 months from when we plant till the end of the season. But we cannot forget that the daylight is twice as long here during the summer season (compared to Syria) and the sun is closer to us therefore the plants will grow faster. For example, Zucchini, we pick it every 3 days in Syria while here we have to pick it every day and sometimes twice a day because it is very quick to grow.

Then Mohamed grabbed soil from the ground and touched it with his hands to show us how much the soil can contain moisture. He added commentary about soil as follows:

> The land in Syria is a little hard to work with because there is no snow, therefore we are planting and planting every month of the year. We always need to add stabilizers to the soil in Syria. On the other hand, here we have about 6 months of snow/cold, which adds moisture to the soil which benefits us when we start planting in the summer. The soil in Canada generates around 200% more produces in the summer.

These excerpts from our dialogues demonstrate the proportional reasoning based on the relationships among the length of daylight, rate of plant growth, and length of a summer season between Syria and Canada. They came to notice that the same plant (kouza) produced "around 200% more produce" during the summer season in the Canadian city they were in, because of longer daylight. Based on such proportional reasoning, the family rationalized that shorter summer seasons in Canada would not be a disadvantage in the harvesting of produce.

During the pandemic, this urban farm attracted racialized families in an inner-city, especially those who live in the area of the city where many racialized immigrant and refugee people live. These communities are currently deprived of communal green spaces that served as a safer gathering space during the pandemic. This urban farm provided vegetables grown without the use of pesticides or herbicides with affordable prices or as donations for racialized immigrant and refugee families in need. Nahima explained, "I love the idea of how people come and pick vegetables by hand. Especially during the pandemic, we want people to be provided with fresh produce."

In the process of calculating the prices of vegetables, children, as early as 6 years old, were engaging in multiplicative thinking. As we walked on the farm together, we had conversations with Abir, Rabih, and Aisha about the quantity and weight of vegetables and their estimate of prices.

> Aisha said she'd take Author 1 and 2 to show the field of fava beans and we all walked together. Author 1 pointed at fava bean plants and asked how much it would be if you had a customer to sell. Aisha responded saying "1 kg is 5 dollars." Author 1 asked "okay, then what about if a customer takes 3 kg?" Aisha said, "20? No, 5, 10, 15."

A similar conversation happened when Author 1 asked Abir the price of 5kg of zucchini. Using skip counting, Abir said, "5, 10, 15, 20, 25. 25." Estimation of how much 1 kg of each vegetable would be and engaging in multiplicative thinking to calculate the price of vegetables were a layer of mathematical literacy that the children engaged in from the early years. Based on such experiences, Aisha surprisingly shared her observation on the affordability of fresh produces in the city, "do you know how much beets cost if you buy at a Superstore? So expensive!"

Discussion

Our paper provides a glimpse of embodied and emplaced mathematical literacy exercised by a Syrian refugee family engaging in regenerative farming practices. Aligned with the funds of knowledge framework, we depicted a sketch of mathematical literacy that the family, including young learners, competently engaged. Such embodied and emplaced mathematical literacy was "essential for household or individual functioning and well-being" (Moll et al., 1992, p. 133). However, the scope of this family's engagement in regenerative farming during the pandemic goes beyond the functioning and well-being of an individual household. The family was actively making a place for the collective good, by providing green spaces and making fresh produce free from pesticides and herbicides affordable to racialized refugee and immigrant communities in the city. Our preliminary analyses call for the discussion on mathematical literacy that could challenge the hegemonic and normative relationships between body and place in the discipline of mathematics.

References

Civil, M. (2016). STEM learning research through a funds of knowledge lens. *Cultural Studies of Science Education*, *11*(1), 41–59.

Gonzalez, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, *6*(1), 115–132.

Takeuchi, M. (2018). Conversions for life: Transnational families' mathematical funds of knowledge. In T. G. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 127–143). Springer.

Takeuchi, M.A. & Aquino Ishihara, V. (2021). Learning to assemble the hidden bodies: Embodied and emplaced mathematical literacy in transnational migrant activism. *Journal of the Learning Sciences, 21*(1), 103–124. https://doi.org/10.1080/10508406.2020.1820341

Krishnamoorthy, R., & Ma, J. (2021). *Memory cells are like the soldiers: Settler colonialism and the immune system* [Paper presentation]. Canadian Society for the Study of Education, Ottawa, Canada.

Lee, J., & Ingold, T. (2006). Fieldwork on foot: Perceiving, routing, socializing. In S. Coleman & P. Collins (Eds.), *Locating the field: Space, place and context in anthropology* (pp. 67–86). Berg.

Marin, A., Taylor, K. H., Shapiro, B. R., & Hall, R. (2020). Why learning on the move: Intersecting research pathways for mobility, learning and teaching. *Cognition and Instruction*, *38*(3), 265–280. https://doi.org/10.1080/07370008.2020.1769100

Moll, L., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice*, *31*(2), 132–141.

Exploring academic motherhood in mathematics education

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In this project presentation we describe an ongoing inquiry into the lived experiences of Academic Mothers in Mathematics Education (AM-ME) and the unique insights AM-ME gain while parenting, educating, and doing mathematics with their school-aged children. We focus on the rationale and significance for this inquiry and provide an overview of our project. During our session we will also discuss findings from a collaborative self-study, currently underway, that is part of our larger project on academic motherhood in mathematics education.

Rationale and significance

The ‘academic gender gap’ persists and disproportionately affects women with children (e.g., Huang et al., 2020; Thun, 2020) especially those who had children early in their career (e.g., Antecol et al., 2018; Mason et al., 2013). Women who are successful in combining full-time academic work with motherhood continue to face challenges in terms of working hours, stress levels, and work/family conflict, risking long-term health issues in the process (Ollianen, 2019). This reflects an historical bias against motherhood in academia, which has remained persistent despite more visibility of female faculty mothers (Mirick & Wladkowski, 2018), and has promoted a culture of silence around issues of academic motherhood (Pasque, 2015).

As we continue to live through a pandemic, during which inequities across the board are exacerbated, academic mothers may be the ones affected most in higher education institutions (Hermann & Neale-McFall, 2020) and risk suffering yet another ‘motherhood penalty’ (e.g., Baker 2012), as evident in journal submission data (e.g., Murdie, 2020; Staniskuaski, 2020), anecdotal reports from peers, and our own experiences as academic mothers of young children. Although academic fathers are not immune to the impacts of the pandemic, academic mothers have taken a greater hit (Langin, 2021). During the pandemic, academic mothers whose expertise lies in PK-12 education in particular became the default ‘teacher’ at home and continue to spend a significant amount of time supporting their children’s’ remote education, while focusing their academic work on serving their students and academic programs, rather than conducting research. In this way, the ‘invisible work’

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that affects both the career advancement and overall well-being of academic mothers, has exponentially increased during the pandemic (Minello, 2020).

Working closely with their school-aged children during the pandemic, however, AM-ME in particular have been gaining unique insights into mathematics teaching and learning and are reimagining educational opportunities during this time of crisis (Vomvoridi-Ivanovic & Ward, 2021). This has positioned us and other AM-ME with school-aged children to become immersed in the teaching and learning of mathematics and draw expertise from mathematics education in novel ways. However, we find that it is these AM-ME who are less likely to find the time and energy to disseminate these insights to the broader mathematics education community. Further, AM-ME may not even consider these insights worthy of dissemination since doing mathematics with our children, and learning from this activity, is not typically considered publishable work. We are concerned that unique AM-ME insights that may contribute to advancing mathematics education will remain invisible, just like much of the 'invisible work' (Ahn et al., 2017) academic mothers do at home (Offer, 2014) and in the workplace (Guarino & Borden, 2017).

Project overview

Our project seeks to generate a content-specific counternarrative to the existing 'narrative of constraint' (Ward and Wolf-Wendel, 2012, p. 28) of what it means to be an academic mother in mathematics education by exploring AMs'-ME lived experiences during the pandemic and beyond, and identifying and naming the unique and multiple insights AM-ME bring to the professoriate while parenting, educating, and doing mathematics with their school-aged children. Our goal is to shift the discourse around academic motherhood to one that recognizes motherhood as an asset, rather than a deficit, through disseminating AMs'-ME unique contributions in mathematics (teacher) education scholarship.

Drawing on gendered views of work (e.g., Valian, 2005), post-structural feminism (Weedon, 1997), relational cultural theory (Miller & Stiver, 1997), and a funds of knowledge conceptual framework (e.g., Gonzalez & Moll, 2002), we seek to address the following overarching research questions: In what ways do AM-ME describe successes and challenges navigating their roles as academics, mothers, and mathematics (teacher) educators, during the COVID-19 pandemic and beyond? In what ways do these mothers perceive bi-directional influence between their roles? Subsections include:

1. What are the sources of these successes and challenges and how do AM-ME cope with the latter?
2. What are the funds of knowledge AM-ME with school-aged children gain while parenting, educating, and doing mathematics with their children, and in what ways do these funds of knowledge inform their work in the field of mathematics education and the teaching and learning of mathematics more broadly?

We, the three participant-authors in this study, are all academic mothers in mathematics education in the US. We have 7 children among us whose ages range from 4 to 14. In this session we will discuss findings from a collaborative self-study, currently underway, whose purpose is to address the above questions as well as pilot data collection procedures for our larger project on academic motherhood in mathematics education. Our use of a collaborative self-study is purposeful in that we are aiming to focus on the nature of our work being self-initiated, improvement-aimed, interactive, inclusive of qualitative data, and trustworthy (LaBoskey, 2004). With our shared positions as AM-ME, we aim to guide each other towards sharing both our personal and professional growth and the ways in which we navigate our roles. It is from our collaboration that we begin to identify and illuminate our experiences in ways that we would not be able to individually (Baskerville & Goldblatt, 2009).

We engaged in narrative interviews (Jovchelovitch & Bauer, 2000), interviewed each other twice and offered probing questions until sufficient details were obtained, and transcribed each interview. Given the exploratory nature of the study, we used the principles of grounded theory to guide our coding and data analysis (Corbin & Strauss, 2015). We holistically read the interview transcripts and identified broad themes and categories. Then we used a combination of open coding and a-priori codes based on the lenses of our theoretical framework (Poststructural Feminism / Relational Cultural Theory / Funds of Knowledge). We engaged in multiple rounds of discussion about the codes. We used thematic analysis to identify, organize, and provide insight into the shared meanings and experiences of the participants (Braun & Clarke, 2012). Through the data analysis process, we named the specific funds of knowledge that we identified as leveraging across our roles as academics and mothers.

References

Antecol, H., Bedard, K., & Stearns, J. (2018). Equal but inequitable: Who benefits from gender neutral tenure clock stopping policies? *American Economic Review*, *108*(9), 2420–2441.

Baker, M. (2012). Gendered families, academic work and the 'motherhood penalty'. *Women's Studies Journal, 26*(1), 11–24.

Baskerville, D., & Goldblatt, H. (2009). Learning to be a critical friend: From professional indifference through challenge to unguarded conversations. *Cambridge Journal of Education,* 39, 205–221.

Braun, V., & Clarke, V. (2012). Thematic analysis. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology: Research designs – quantitative, qualitative, neuropsychological, and biological* (pp. 57–71). American Psychological Association.

Gonzalez, N., & Moll, L. (2002). Cruzando el Puente: Building bridges to funds of knowledge. *Educational Policy, 16*(4), 623–641.

Hermann, M. A., & Neale-McFall, C. (2020). *COVID-19, Academic Mothers, and Opportunities for the Academy*. https://www.aaup.org/article/COVID-19-academic-mothers-and-opportunities-academy#.YBiW6C2cbOR

Huang, J., Gates, A. J., Sinatra, R., & Barabási, A. L. (2020). Historical comparison of gender inequality in scientific careers across countries and disciplines. *Proceedings of the National Academy of Sciences*, *117*(9), 4609–4616.

Jovchelovitch, S.; Bauer, M. W. (2000). Narrative interviewing. *London: LSE Research Online.*

Laboskey, V. K. (2004). The methodology of self-study and its theoretical underpinnings. In J. J. Loughran, M. L. Hamilton, V. K. LaBoskey, & T. Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 817–867). Dordrecht: Kluwer.

Langin, K. (2021). *Pandemic hit academic mothers especially hard, new data confirm.* https://www.sciencemag.org/careers/2021/02/pandemic-hit-academic-mothers-especially-hard-new-data-confirm

Mason, M. A., Wolfinger, N. H., & Goulden, M. (2013). *Do babies matter? Gender and family in the ivory tower.* Rutgers University Press.

Miller, J. B., & Stiver, I. (1997). *The healing connection: How women form relationships in therapy and in life.* Boston, MA: Beacon Press.

Minello, A. (2020, April 17). *The pandemic and the female academic.* Nature. https://www.nature.com/articles/d41586-020-01135-9

Mirick, R. G., & Wladkowski, S. P. (2018). Pregnancy, motherhood, and academic career goals: Doctoral students' perspectives. *Affilia, 33*(2), 253–269.

Murdie, A. (2020). *Journal submissions in times of COVID-19: Is there a gender gap?* https://duckofminerva.com/2020/05/journal-submissions-in-times-of-COVID-19-is-there-a-gender-gap.html

Ollilainen, M. (2019). Academic mothers as ideal workers in the USA and Finland. *Equality Diversity and Inclusion*, *38*(4), 417–429.

Pasque, P. A. (2015). *Disrupting the culture of silence: Confronting gender inequality and making change in higher education.* Stylus Publishing, LLC.

Staniscuaski, F., et al. (2020). Impact of COVID-19 on academic mothers. *Science, 368*(6492), 724.

Thun, C. (2020). Excellent and gender equal? Academic motherhood and 'gender blindness' in Norwegian academia. *Gender, Work & Organization, 27*(2), 166–180.

Valian, V. (2005). Beyond gender schemas: Improving the advancement of women in academia. *Hypatia, 20*(3), 198–213.

Vomvoridi-Ivanovic, E. & Ward, J. K. (2021). Academic motherhood in mathematics education during COVID-19: Breaking the silence and shifting the discourse. *REDIMAT: Journal of Research in Mathematics Education, 10*(1), 41–61.

Weedon, C. (1997). *Feminist practice and post-structuralist theory* (2nd ed.). Malden, MS: Blackwell.

Posters Descriptions

Intercultural dialogue in school mathematics: Ethics of school-free data collection

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Our project aims to promote intercultural dialogue in school mathematics, by exploring the experiences of migrant students and teachers in mathematics classrooms. Although data collection was initially designed to be conducted in person, some ethical issues have emerged as it moved online, due to the COVID-19 pandemic. This poster explores a series of 4 ethical considerations: a) the "shielding" role of school making some injustices invisible, b) the mediating role of school around the relationships that researchers need to create with the families, c) the consideration of the value of different mathematical knowledges, and d) some, practical issues.

We endeavour to study the impact of student migration on mathematics learning from the perspectives of students and teachers. There is an existing and growing body of research focusing on challenges in relation to migration in mathematics classrooms (Barwell, 2016). We aim to examine the potential richness afforded by students' multiple cultural, linguistic and educational experiences of mathematics, including different ways of doing, learning and thinking mathematics. This poster description is organised as follows: we first describe the project and its goals, we then elaborate on the data collection methods and how they changed due to the COVID-19 pandemic, and we finally discuss four ethical issues which emerged in relation to this transition.

A few words about the project: Migration in mathematics classrooms

More than 500,000 children in Canada come from migrant backgrounds (Statistics Canada, 2011) and all must study mathematics. For this study, we define migrant students as those who, as a result of a change of residence, experience differences of culture and language in

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school. Thus, migrant students include those whose families have moved for economic or employment reasons, those whose families are escaping war or other disasters as refugees and asylum-seekers, and indigenous students whose families have moved between a rural indigenous setting and an urban school. How does migration affect students' and teachers' experiences of school mathematics? Research suggests that many students from immigrant backgrounds face challenges in mathematics and underachieve. We conjecture that the mathematics teachers of migrant students often have little understanding of what their students are experiencing or what mathematics they already know. Migrant students can find the culture of teaching and learning mathematics quite alien, may bring novel ways of doing mathematics, and may encounter new ways of thinking about mathematics. Cultural differences in mathematics classrooms lead teachers to feel uncertain and unprepared. In an extention to the research on migration in mathematics classrooms, we aim to a. understand the experiences of students and teachers of learning and teaching mathematics in the context of migration b. promote dialogue between students and teachers in relation to these experiences and observe the impact of this dialogue on teachers' practice. The study inspires to stimulate multivocal, intercultural dialogue between students and teachers of mathematics, and helps us understand the impact of migration on the learning and teaching of mathematics. In the context of increasing mobility and superdiverse mathematics classrooms (Barwell, 2016), the study aims to create conversational relationship among families, children, teachers and school systems, in relation to learning and teaching of mathematics, as well as to highlight "unfamiliar" maths for teachers and for students.

Data collection: Pre-COVID and post-COVID

Originally, participants were being recruited from ten different schools and data collection to be in schools. After schools closed as a precautionary measure for COVID-19, this was no longer possible. Instead, we are recruiting participants through organisations which serve migrant populations and data collection is conducted online.

The initial pre-COVID design of the data collection

In the initial design of this study, there were two phases of data collection: the first phase would involve students and the second phase would involve teachers. In the first phase, our group would visit 10 schools and work with 5 students in each school. These students, would take the role of co-researchers, gathering accounts of their own prior and current experiences of mathematics, from their parents, and in their mathematics classes. Towards the end of our meetings, students would synthesize their collective experiences in the form of collages, in order to communicate their experiences with teachers. At the end of the group meetings, each student would be interviewed to gain their individual perspective. The second phase would focus on teachers' experiences. We would share each collage with teachers from the school, to understand teachers' responses to students' experiences of migration. Teachers, as co-researchers, would collect observations of their own practice, noting shifts arising from their new understanding of their migrant students' experiences. At the end of this phase, teachers from the 10 participating classes would compare the collages created in their schools, their responses to them, and the resulting shifts in their practice in focus groups.

The post-COVID emerging data collection practice

As the study moved online due to the pandemic, the two data collection phases (one for students and one for teachers) are maintained but their character has changed. In the first phase, we hold ten virtual math clubs with six children in each club. Each group will meet virtually for five sesions and engage in both synchronous tasks as well as “take home” activities to work on with parents/siblings and to share at the next session. At the end of each math club, we hold interviews with each student and – if they want – a family member. The second phase, teachers’ experiences, will include six virtual groups of mathematics teachers who will meet twice. We will share and discuss the student participant profiles with teachers as well as examples illustrating children’s cultural-historical repertoires with respect to mathematics. Teachers will be asked to reflect on information shared, their reactions, and implications for practice.

Ethical considerations emerging

The transition to online data collection made visible to us a series of ethical considerations. As we are currently in the first phase of data collection, working with students, these ethical considerations are related to data collection with students.

The location of data collection changes from school to home

As the location of data collection changes from school to the virtual space (as students and researchers are physically at their home), aspects of the role of school are made visible to us. School conceals differences among students; the four walls of the school appear to be shielding students from these differences and to offer a sense of security. Inspired by Fasheh’s (1998) question “Which is more fundamental? Outward peace or being true to our humanity?”, we ask: What sort of an “outward peace” is created by schools and what becomes invisible by this portrayal of peace? What deeply rooted inequalities are given permission to be ignored in an “equal school”? For example, to identify potential participants, we visited richer and poorer parts of the city. This process made visible differences in the living spaces among students, which are concealed in school. We do not intend to evaluate this school function as “good” or “bad”; instead, we try to think about its complex social implications.

Relationships with children and their families are no longer mediated by school.

To conduct any research projects, relationships need to be built between researchers and participants, while in projects related to the concerns of the MES community, we hope that these relationships extend beyond whatever is necessary to obtain data from students or communities. The transition of data collection location made visible to us the school’s role in facilitating the building of these relations. If we had been able to continue with research in schools – and given that at least one of the schools were participating based on established relationships – the schools would mediate the relationship between researchers and students and their families. Due to the pandemic, we are attempting an online approach that reaches out to families directly using the connections we have in the group. When school stops mediating the relationship between researchers and children/families, questions arise: What is the nature of school's mediating role in building relationships between researchers and families? If the school is not there, for example at the time of COVID-19, how can/should researchers manage relationships with families and children?

Different mathematical knowledges are valued differently

Whose mathematical knowledge is highlighted? Our data collection methods entail doing mathematical activities with students with a migration background. While designing, conducting, and reflecting on these mathematical exchanges, a tension emerges about whose mathematics is highlighted. We aknowledge that these tensions cannot be resolved. We all bring our ideas and experiences of mathematics – our repertoires – to the project, and we 'see' participants' repertoires through our own. But we ask: whose knowledge counts as mathematical knowledge? (Abtahi, 2019). Extending the above line of thought, we are also thinking about what mathematical knowledge is valued. Walkerdine's "commodity" perspective draws our attention to how knowing the mathematics of the curriculum not only worths much more than other kinds of mathematics, but knowing "other kinds of mathematics" is often considered to be deficient and of lower level (Walkerdine, 1990).

Practicalities of online data collection

Finally, ethical concerns also emerge in relation to the practicalities of data collection. As many students' activities have moved online and students spend a lot of time in front of the screen, a question emerges about the ethical implications of asking students to spend more time online. Furthermore, issues emerge related to who has access to and who has comfort with technology, as well as the implications of video-recording students while they are physically at their homes.

Concluding thoughts

Through this poster, we present some ethical issues and dilemmas that have emerged in relation to our study's data collection transitioning from in-person to online. We hope that the conversations with the MES community will not only help us reflect on the ethical dilemmas that we are facing, but will also open up the discussion about ethical issues related to data collection in the cyber space and beyond.

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References

Abtahi, Y. (2019). Epistemological dialogue: Other ways of knowing and our blind spots. *Journal of Mathematics and Culture, 13*(1), 1–19.

Barwell, R. (2016). Mathematics education, language and superdiversity. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education* (pp. 25–39). Sense.

Fasheh, M. (1998). Which is more fundamental? Outward peace or being true to our humanity? *ZDM Mathematics Education, 30*, 78–81.

Walkerdine, V. (1990). Difference, cognition and mathematics education. *For the Learning of Mathematics, 10*(3), 51–55.

Teaching functions to 21st-century mathematics learners through a real-life problem

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Over a few decades now, educators all over the world are contemplating "How teaching-learning of mathematics should be?". In the 21st century, mathematics holds a special place due to the increased demands of skills that mathematics impart. Three major guiding ideas of 21st-century mathematics teaching are (a) Learning in a context, (b) Learning for the 21st century, and (c) Reducing the gap between school math and real life. This poster aims to provide a way to design a problem-based learning environment by posing a real-life problem in a high school classroom, specifically discussing the Vehicle routing problem designed around the day-to-day environment of students. This context of the problem will initiate the discussion among students and the facilitator on mathematical ideas around functions. This approach will provide a way for learners to understand the underlying mathematics of the real-life scenario under investigation.

Introduction

The Organization for Economic Co-operation and Development (OECD) in 2003 argued due to huge social changes and an explosion of knowledge in the 21st-century there is a dire need to change the fundamental units of education. The concept of formal education in the 21st century is characterized by the idea of the knowledge economy and globalization processes. In this century learners are expected to work towards generating new ideas and solve complex problems. According to OECD, these independent learners must acquire core competencies known as 21st-century skills to make them ready for future challenges of the 21st-century.

Mathematical competencies have been given a central position among the skills that are required by learners to thrive in the 21st century. A study (Gravemeijer et al., 2017) on future mathematical competencies required in a workplace explores what mathematics education prepares students for the future. The study suggests that modeling and applications should be one of the major goals for mathematics education, where learners can gain experience to devise solutions to an authentic real-life problem hence understanding the underlying mathematics.

Another research by the Advisory Committee on Mathematics Education (ACME) in 2011 shows how themes of mathematical modeling, costing, calculating risk, and quality control

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processes make up a huge portion of today's workplace. Through one on one interviews with employees (25 companies in the United Kingdom) across the sectors of employment, ACME has concluded that in a workplace more than the application of mathematics one should be able to solve problems within a context and communicate it effectively. They also recommend identified themes to be incorporated into the mathematics curriculum of the country.

This made us question:

1. What are the guiding ideas of 21st-century mathematics teaching? and
2. How can we design learning in a mathematical classroom that makes learners ready for the 21st century?

Taking a hint from aforementioned studies, we framed major guiding ideas for 21st-century mathematics teaching as (a) Learning for the 21st century, (b) Learning in a context, and (c) Making school mathematics workplace-ready i.e reducing the gap between school math and real life.

One of the possible solutions for our second question is provided by the pedagogy of problem-based learning (PBL) which has been gaining momentum lately. It is a blend of various past approaches (Hmelo-Silver et al., 2006) that had their influences on PBL during its development phases. Educators believe that problem-based learning has drawn inspiration majorly from (a) Constructivist Approach to Learning, (b) Critical Theory/ Critical Pedagogy, and (c) Pragmatism (Hirschman et al., 2018; National Council of Educational Research and Training, 2005; Savin-Baden et al., 2004).

PBL is a learner-centered approach that believes: a learner constructs knowledge while resolving a real-world ill-structured problem. PBL promotes the importance of peer-learning by enabling learners to work in collaboration. Throughout the process of resolving a problem a learner plays different roles (Savin-Baden et al., 2004, pp. 81–92) such as real-world problem solver, critical thinker, communicator and self-directed learners.

Keeping in mind the pedagogy of PBL and suggestions given in "The Role of Contexts in the Mathematics Classroom: Do They Make Mathematics More 'Real'?" (Boaler, 1993), the author has designed an activity. The objective of this project was to develop a lesson plan for the PBL environment to introduce the concept of functions to Indian high school students.

About the lesson plan

The lesson plan was developed to introduce the concept of functions to high school students. The topic skills that were taken into consideration before designing a problem statement are (a) Identifying independent and dependent variables, (b) Identifying the mathematical relationship between variables, (c) Designing a mathematical function from mathematical relation, (d) Domain and Range of a function, (e) Graph of the function, (f) Nature of graph of function and (g) Finding optimal solutions graphically.

The expected time to fulfill the objectives of the plan was kept 1.5 weeks. Understanding of the cartesian plane is the prerequisite knowledge required for this activity.

Choice of problem

Boaler, J., 1993 identifies contexts as general motivators to students which keep their interest going. But sometimes if a problem is an interpretation of a "real-life" scenario that expects students to enter a fantasy world they tend to think of it as some other textbook math problem. She also states "Using the real world, local community, and even individualized examples which students may analyze and interpret is thought to present mathematics as a means with which to understand reality. This allows students to become involved with mathematics and to break down their perceptions of a remote body of knowledge" (Boaler, 1993, p. 13).

On the other hand, Mauffette et al. (2004, pp. 11–25) explored the connection between the problem and the motivation of the students in a problem-based learning environment. Their findings suggest that initially for an introductory PBL level the facilitator should clearly identify and summarize the problem instead of putting it in a wider context which is the next level and is suitable for senior PBL learners. They also recommend that background information should be drawn from one source of data and information about the settings should be complete without omitting the details.

Optimization is one of the concepts we use often. In our day-to-day life from choosing a path to our destination or ordering food either we maximize or minimize certain quantities. Indian National Curriculum Framework 2005 put forward connecting knowledge to the outside world as one of the visions of mathematics education in the country. Choosing a problem that is deeply connected with the learners' life where the topic skills can be mapped was chosen, for the context Vehicle routing problem was finalized.

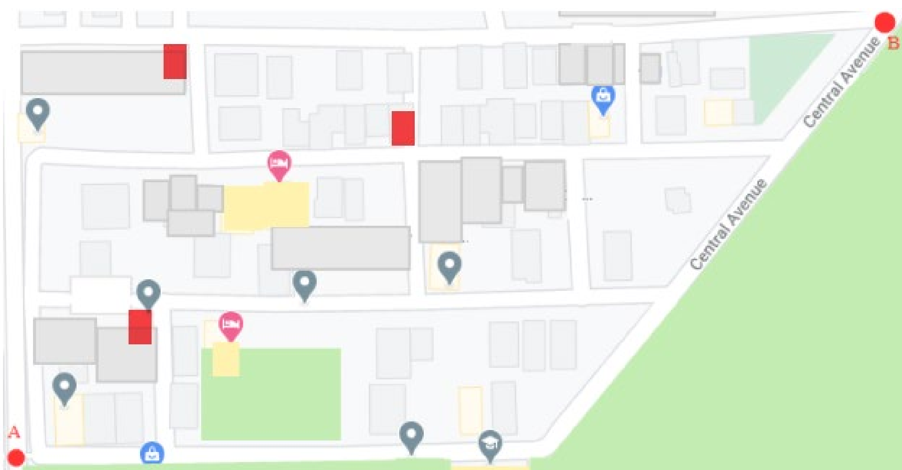

Figure 1: Map followed during the activity.
Note: The map was based on the locality students are familiar with.

Problem statement

"Using the map, find the best routes to reach point A to B in such a way you have to spend minimum time while covering all the highlighted places at least once."

In this, students are expected to answer: (a) Variables in the context, (b)Variables affecting the time, (c) Developing the understanding of their time function, (d) Representing their models on a cartesian plane, (e) Justifying their calculations, and (f) Finding accuracy in comparison with the estimated time given in Google Maps (see Fig. 1).

Critical discussions during the activity:

1. While finding variables both dependent and independent, the relationship between the mode of covering the distance (walking, running, cycling etc.), traffic encountered, and time spent will be discussed.
2. Interpreting mode of covering the distance in numerical terms.
3. Dependending upon students' approach/approaches different data sets will be obtained hence resulting in different functions.
4. Finding optimal solutions from the datasets obtained.

The rationale to the conference theme

Using problem-based learning for introducing concepts will help teachers to make mathematical learning relatable to learners and they will see how mathematics is a part of their life. Using PBL they will also help their learners to develop 21st-century skills. In the present time all over the world, there is a need for resources for mathematical teaching and learning which help learners to understand the underlying mathematics while interacting with real-life problems. When it comes to PBL, not many resources are available for teachers to implement in their classrooms. This presentation provides one way of designing a PBL environment in fulfilling the objective of mathematics education in the 21st century. It can be placed in the theme 'Sociology of Mathematics Education' of the conference as this presentation will talk about a new approach to teaching functions.

References

Advisory Committee on Mathematics Education. (2011, June). Mathematical Needs – Mathematics in the workplace and in Higher Education.

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more "real"? *For the Learning of Mathematics, 13*(2), 12–17.

Eitel, K., Hougham, J., Miller, B., Schon, J., & Lapaglia, K. (2013) Upload, download: Empowering students through technology-enabled problem-based learning, *Science Scope, 36*(7), 32–39.

OECD. (2008). *21st century learning: Research, innovation and policy directions from recent OECD analyses.* Paris: OECD.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education, 15*(S1), 105–123.

Hmelo-Silver, C. E., & Barrows, H. S. (2006). Goals and strategies of a problem-based learning facilitator. *Interdisciplinary Journal of Problem-Based Learning, 1*(1).

Hirschman, K., & Wood, B. (2018). 21st century learners: Changing conceptions of knowledge, learning and the child. *New Zealand Annual Review of Education, 23,* 20–35.

Moallem, M., Hung, W., & Dabbagh, N. (Eds.). (2019). *The Wiley handbook of problem-based learning.* John Wiley & Sons.

National Council of Educational Research and Training. (2005). *NCF 2005 position paper on teaching of mathematics.* New Delhi.

Savin-Baden, M., & Wilkie, K. (Eds.). (2004). *Challenging research in problem-based learning*. Open University Press.

Savin-Baden, M., & Major, C. H. (Eds.). (2004). *Foundations of problem-based learning*. Open University Press.

Tan, O. S. (2003). *Problem-based learning innovation.* Cengage Learning Asia.

Taylor, P. (2018). Teach the mathematics of mathematicians. *Education Sciences, 8*(2), Article 56.

Juxtaposing cases of delegating versus withholding authority of mathematical ideation in early algebra classrooms

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We will illustrate two cases of teachers implementing the same early algebra lesson exhibiting diametric forms of a teacher practice previously identified in the research literature as a means toward achieving equity—the delegation of mathematical authority. The teacher with greater student achievement gains on an early algebra assessment had greater occurrences of teacher moves that positioned students as having the power to rely on an intellectual authority situated internally or within the community of peers' "taken-as-shared" knowledge. On the contrary, the teacher with lower gains exhibited greater occurrences of teacher moves that, in effect, resulted in withholding authority from students to form mathematical justifications. Illustrative excerpts of both cases are shared.

Introduction

Non-dominate student populations are subjected to subtle detriments such as having teachers who assume deficits in students, have lower expectations, and hold biases that reinforce stereotypes about who belongs and excels at mathematics (Delprit, 1992; Flores, 2007; Oakes, 1990; Varelas, Martin, & Kane, 2013). Little is known about how day-to-day teacher instruction either conveys and reinforce or counteracts these messages. One way to counteract these messages is to position students as competent and capable of developing their own mathematical authority (Gresalfi & Cobb, 1996).

Research questions

We explore the ways in which elementary teachers of mathematics confer mathematical authority by asking the following:

How is the power to form and justify mathematical ideas either maintained by the teacher or delegated to students? How does the teacher's talk moves, uptake of student's ideas, participation structures, and general academic expectations of students constitute this delegation of mathematical authority? And, finally, how is the degree of delegation of mathematical authority associated with student academic outcome on an early algebra assessment?

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Methods

To study these questions, we use a comparative case study to offer a description of interactions in two third grade classrooms teaching an early algebra lesson in an urban, Title 1 school. Videos of two classroom observations are the in the process of analysis using open coding in two passes. The first pass we coded for classroom wide instances in which the power to form mathematical decisions is largely positioned within the teacher's power or students'. The second pass is ongoing and is an analysis of teacher's uptake of student's ideas, talk moves, and participation structures.

In order to measure each teacher's students' academic achievement, the gain or difference between a pre- and post-assessment scores of early algebra were used.

Preliminary findings

In order to illustrate the diametrically opposed implementation of delegation of mathematical authority, we will present two episodes extracted from two teachers teaching the same lesson within the same school. Teacher A had a student gain that was almost twice of that of Teacher B. Both excerpts are from the two classrooms as students consider the response to a warm up prompt: "If $a < b$ and $b < c$, how would you describe the relationship between a and c?"

Teacher A, episode of delegated mathematical authority

Teacher A reviews the problem after having students work on it within small groups.

Teacher: Who has something they want to present and defend? Sebastian, what were you and Jayden saying?
Student 1: If *a* is less than *b* and *c* is bigger than *b*, then it's bigger than all of them.
Student 2: What's bigger than all of them?
Teacher: Yes, which letter is the largest?
Student 1: The *c*-- the *c* is bigger than both the *b* and *a*.
Student 3: I don't get it.
Student 1: Look *(standing up, walking to the white board at the front of class)*, we can draw a picture to show it.

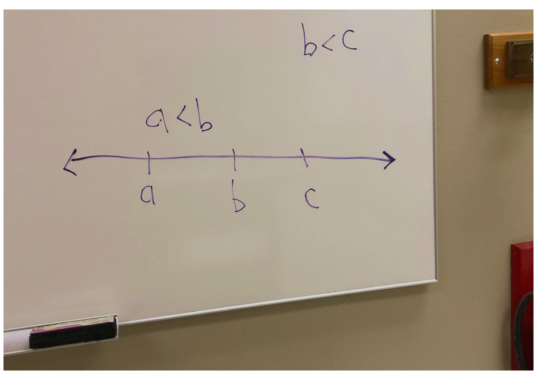

Figure 1: Student generated representation of the relationship between variables.

The two students presenting then draw a number line exhibiting the relationship $a < b < c$ for the class (see Figure 1). At the end of their presentation, the two students turn to their classmates and ask if others agree.

Teacher B, episode of withheld mathematical authority

Teacher B introduces the problem by having students read the problem out loud. There was no time given for students to consider the task within partners or small groups.

Teacher: Okay, so I'm going to write down two things... *(writes $a < c$ and $a > c$). (Turns to class)* Raise your hand if you agree with the first *(gesturing to $a < c$).* Okay, I count 5 votes. *(Writes 4 check marks).* And what about this one...a is more than c? *(gesturing to $a > c$)?* I see some maybes... one vote?

Student: I just guessed!

Teacher: Yeah? Well, it has to be the first one - a is less than c, because it is less than b and you know b is less than c.

The teacher moves on to the main activity of the lesson without further discussion.

Discussion and conclusion

The practices of Teacher A exhibit delegated mathematical authority in several ways. Firstly, students were given time to consider the task at hand suggesting that the teacher believes it is possible for the students to be successful with the problem and hold such authority. Secondly, the teacher asks students to present and defend their claims and justifications, thus situating mathematical ownership within her students. Lastly, it is evident that a classroom norm that students check for understanding and agreement with their peers. This evidences an established practice of the teacher delegating authority to the students as a whole.

The verbal and non-verbal actions of Teacher B largely withhold the authority of mathematical ideation from the students in that classroom. While the teacher does solicit votes from the students, there is a lack of opportunities for students to make sense of the relationship both in terms of time and peer-to-peer collaboration.

While these two interactions are brief, we maintain they are powerful in contributing to the ongoing conversations about equitable teaching practices in mathematics.

References

Delpit, L. D. (1992). Education in a multicultural society: Our future's greatest challenge. *Journal of Negro Education*, *61*(3), 237–249.

Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap? *High School Journal, 91*(1), 29–42. https://doi.org/10.1353/hsj.2007.0022

Gresalfi, M., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal, 1*(1), 49–57.

Oakes, J. (1990). Opportunities, achievement, and choice: Women and minority students in science and mathematics. *Review of Research in Education, 16*, 153–222.

Varelas, M., Martin, D. B., & Kane, J. M. (2013). Content learning and identity construction: A framework to strengthen African American students' mathematics and science learning in urban elementary schools. *Human Development, 55,* 319–339.

'There is no America without inequality': Imagining social justice writing in a calculus class

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Social justice mathematics pedagogies envision students "writing the world" with mathematics, in ways that often involve literal rather than metaphorical writing. However, neither the textual form nor the pedagogical processes of developing these persuasive mathematical compositions are envisioned clearly in current mathematics education research. In this poster, we present several samples of social justice mathematics writing responding to an idealized model of the COVID-19 epidemic. One was written by an undergraduate calculus student and others are "creative writing" by professors who are answering the classroom task as if they were students. Through this poster, we hope to create conversation about how students can demonstrate both mathematical knowledge and social awareness through a written text.

Overview

Social justice mathematics pedagogies envision students "writing the world" with mathematics, that is, using mathematical learning to change their communities (Freire, 2005; Gutstein, 2016; Skovsmose, 2020). Quite often, this metaphorical writing involves literal writing, for example, when a curricular lesson concludes with students writing a reflection on their work or writing a letter to a powerful stakeholder (Berry III et al., 2020). This conceptualization, however, seems to be weakly envisioned in current mathematics education research, that students will learn mathematics in relation to issues of justice, compose a written statement that unites mathematics and their critical vision in some fashion, and deploy this writing into the world to some positive effect. Even a brief consideration will generate a complex constellation of composition decisions: How much mathematical knowledge must the writing demonstrate to the teacher? Will students explain how mathematics works in public statements, or merely present the outcomes of their analysis? How does one write persuasively across the epistemological divides of

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mathematical reasoning and moral critique? How do student groups negotiate the process of producing this collective statement?

This poster aims to create conversation among researchers on what they value in social justice mathematics writing. We present several samples of social justice mathematics writing in response to a calculus activity modelling the spread of COVID-19. One sample was written by an undergraduate student in a calculus class, Ijeoma, just as the pandemic began to spread through her city of Minneapolis, Minnesota, U.S.A. The other samples are “creative writing” provided by professor co-authors, written as if they were students in a similar calculus class. We do not present the professors’ writing as ideal or perfect examples of social justice mathematics writing, but instead, as possibilities for how students might undertake the difficult task of calling attention to the appalling inequalities exacerbated by the COVID-19 pandemic while also demonstrating early-stage calculus knowledge. In effect, we have identified a lacuna in social justice mathematics research, and we take it as professors’ responsibility to imagine a range of possible student responses.

Layout of the poster

The poster will contain the following sections: a statement of the research problem; a brief literature review; the focal epidemiology task with the writing prompt; four brief writing samples; and a collective reflection that outlines priorities for future research on social justice mathematics writing.

An idealized COVID-19 model

The focal task is a generalized Susceptible-Infected model of an epidemic which has many unrealistic features, but is useful pedagogically for discussing calculus topics such as derivative graphs, changing slopes, concavity and inflection points. Early-stage calculus students can use Euler’s numerical method to generate graphs of monthly and of cumulative incidence of COVID-19 in an idealized community of 500 people:

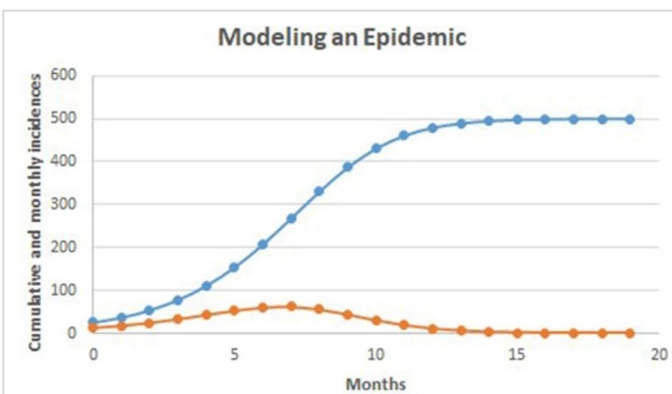

Figure 1: Modeling an epidemic

The poster will contain the following sections: a statement of the research problem; a brief literature review; the focal epidemiology task with the writing prompt; four brief writing samples; and a collective reflection that outlines priorities for future research on social justice mathematics writing. The task and the teaching process will be presented more fully on the poster.

The writing prompt, modified slightly for international professors' creative writing, is:

> Reflect on an issue of unfairness associated with the COVID-19 pandemic that affected any part of the world. Assume that eventually, the COVID-19 graphs in this place will look like the above graphs, except that they will have different scales on the axes and the "cumulative cases graph" will level out at a value or carrying capacity that is lower than the entire population.
>
> Explain carefully and in detail how COVID-19 raises issues of unfairness and social justice. Use the shapes of your graphs in your answer. For example, you could try to explain where the people you are writing about would show up in the graphs, or how they would be affected if governmental policies or changing social behaviours modified the various rates of change in the graphs.
>
> Your social justice writing should demonstrate your knowledge of calculus.

Envisioning social justice mathematics writing

In this proposal, due to space constraints, we present brief summaries and excerpts from a few of the social justice mathematics writing samples. The poster layout will present additional and longer texts. The layout will use graphic "callout" images to draw attention to writing techniques employed by each writer, particularly, demonstrating mathematical knowledge in order to criticize the dreadful impact of COVID-19 on exploited communities.

Ijeoma's real answer

> There is no America without inequality [...] The coronavirus is dangerously highlighting the inequality that lower class and working-class marginalized black and brown people face. [...] When referring back to the graph above, the people who are hit by this virus before the inflection point will consist of more working-class people of color. Where we see the graph leveling out will most likely be when people in privileged places of power take this situation seriously. It's heartbreaking to know that the numbers we calculated in class translate to real-life tragedies that lower-class black and brown communities will face.

Anna's fictional answer

Anna responded to the writing prompt as if she were an international student attending university in Minnesota, whose studies corresponded with both the experience of the pandemic there and the Black Lives Matter protests against the racist police killing of George Floyd, an African American man. Her concern was that the inflection point of the graph of cumulative cases is both an optimistic target to encourage social distancing but also, that it could also become a tool of governmental control against necessary political activism.

Swati's fictional answer

Swati's answer compared the classroom graph of "new cases per month" to public daily case data from India through the lens of national policies and seasonal events. She highlighted inequalities associated with social class. The coronavirus initially reached India through the air travel of middle- and upper-class people, whose ability to social distance caused unemployment among household service providers. She reads the growth and decline stages in the real data against governmental lockdown policies, seasonal movement for agricultural distribution, and festival gatherings. She highlighted continuities across the classroom and realistic graphs, but noted the significant difference that in the real data, the "new cases per day" approaches a positive, non-zero constant, so that cumulative cases will probably continue to rise.

The poster callout graphics on the poster will us notice writers' strategies for "writing the world" through mathematical knowledge. Some of these strategies include restoring a sense of racial and class-based difference to the mathematically uniform variable of "cases," interpreting growth and decline periods in terms of public policy and cultural activities, and reading the graphs in terms of their potential for public communication or manipulation.

Collective reflection

Our collective reflection will consider dilemmas in constructing texts that are intended to "write the world" with mathematics: How mathematically precise and explicit should this writing be? Must the student "teach" mathematics to the audience, or only present results of their mathematical investigation? What writing strategies manage to unify mathematical and political or moral positions? What kinds of writing prompts will encourage particular kinds of social justice mathematics writing?

We hope to point out varied strategies for writing that bridge mathematical insights and social, political or moral perspectives, and to notice different ways of valuing this kind of mathematical writing (Barwell, 2018). This conversation may heighten awareness of the complexity of social justice mathematics writing and the pressing need for wider pedagogical guidance towards it.

References

Barwell, R. (2018). Writing in mathematics classrooms. In A. L. Bailey, C. Maher & L, Wilkinson (Eds.), *Critical language, literacy, and learning in the STEM disciplines* (pp. 101–114). Routledge.

Berry III, R. Q., Conway IV, B. M., Lawler, B. R., & Staley, J. W. (2020). *High school mathematics lessons to explore, understand, and respond to social injustice.* Corwin Press.

Freire, P. (2005). *Pedagogy of the oppressed.* Continuum.

Gutstein, E. R. (2016). "Our issues, our people—math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education, 47*(5), 454–504.

Skovsmose O. (2020) Critical mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 154–159). Springer.

Research Papers

“Communicate, argue, share your ideas”: Values in talking and values in silence

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Many mathematics curricula emphasise the importance of communication in mathematics classrooms, supported by an extensive body of research. In much of the world, this emphasis promotes specific communicational practices as necessary or desirable in order to support mathematical meaning-making and effective learning of mathematics. In this paper, we seek to question the implied universality of this approach: in Arendt’s terms, we seek to judge, in order to provoke questioning and rethinking. Our examination draws on the idea of social norms. We consider what happens when children in mathematics classrooms orient to different norms from those assumed to be beneficial for learning mathematics and how could the epistemological effect of such orientation be conceptualised.

Communication has come to occupy an important place in efforts to develop forms of teaching and learning mathematics that result in conceptual understanding, rather than an exclusive focus on procedural fluency. This emphasis is apparent in curriculum documents in many parts of the world. The recommendations or requirements of such documents often reflect what is known as ‘reform’ mathematics approaches, in which students are expected to play an active part in mathematics classroom activity, through posing and solving problems, often in small groups, as well as participating in whole-class interaction. Among other practices, students are expected to pose questions to their peers and respond to such questions, as well as explain their thinking to the class. These curricular guidelines are supported, often explicitly, by much research in mathematics education (e.g., Cobb & Bauersfeld, 1995).

While such work is valuable, and the goal of conceptual understanding is significant, our work with students from diverse cultural and linguistic backgrounds leads us to trouble its perhaps unintended and implied universality. In our experiences in mathematics classrooms with children from Indigenous backgrounds or immigrant backgrounds we have come to wonder if the reform approach to mathematics classroom interaction unjustifiably assumes or even imposes supposedly universal values. We also wonder and attempt to judge what the impact of these unstated assumptions might be.

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Who is to judge?

In her book of *Responsibility and Judgement*, Arendt (2003) analyzed the relation among thinking (and not thinking), responsibility, and the capacity for developing of moral judgement. As the foundation for thoughts and analysis, she uses events in the history (such as human interactions in the World War II) to relate judgment with human dignity. She explains human dignity could be claimed neither through valorising history nor through denying history's significance, but rather through judging. She views judging as an activity that recognises history but goes even further to deny the histories right to be the ultimate judge. That is, being historically "there", doesn't mean that it should be there! Arendt's view of judgment is particularly important because in different curricula, the endorsement of communication in mathematics classes is there, but the effects of is belonging for students with different epistemological roots (Abtahi, 2019) is not questioned. Here, we took the liberty to judge and we extend the invitation to others. While judging, Arendt explains, we are able to look back at, reflect upon, and begin to make sense of the affairs that are related to our communities, and to form individual standpoints. The more people's take stands and the more standpoints being present [...] while we are ponder a given issue, "the better I [we] can imagine how I [we] would feel and think if I [we] were in their place, the stronger will be our capacity for representative thinking" (Arendt, 1993, p. 241).

Given how communication is endorsed in different curricula and given how values are therefore perpetuated or ignored, we seek to judge the significance of specific forms of commnunication as crucial foundations of the learning and teaching of mathematics. Communication is, of course, a part of human social life. So the issue is not whether there should be communication in mathematics classrooms (or in the curriculum) but how it is presented and practiced? And who decides? And how should we, as mathematics educators, judge its normalisation?

The origins of reform approaches: Sociomathematical norms

Ways of interacting in small groups, in large groups, among children or between children and adults vary in different societies or among different groups within societies. Heath's (1983) classic work on literacies, for example, showed clearly that children growing up in middle class White households, working class Black households and working class White households, in southern USA, all developed different repertoires of literacy practices, including different ways of talking and interacting around texts. For example, the uses of stories and ways of telling them varied, as did ways of interacting around a recently received letter.

The significance of Heath's study for education was that the repertoires of the middle class White children in her study aligned most closely with literacy practices in school. Those of the working class children did not align and were often seen as deficient. The example of Heath's work could be interpreted as an advantage of middle class White children, for whom parents have time and resources to educate them in line with what they, themselves, have learnt as children – hence reproducing the same repertoires. Judging more closely, we see that the norms of literacy practices in school are also aligned with the reproduction of middle class White social norms. For children from the other two backgrounds, there was a misalignment. Such misalignments may not only be due to class: children of many cultural backgrounds whose parents have time and resources to help their children will develop different repertoires of literacy practices, would therefore experience a misalignment of norms and hence be perceived as having some kind of deficit. A study by Street, Baker and Tomlin (2005) focused on numeracy practices showed exactly this.

In a now classic paper, Yackel and Cobb (1996) proposed a social perspective on mathematics classroom interaction. This approach drew on social constructivism, social interactionism and ethnomethodological principles. This last influence was apparent in Yackel and Cobb’s adoption of the idea of norms. They proposed two kinds of norm that were of significance in mathematics classrooms: social norms, and sociomathematical norms. It is important to note that from an ethnomethodological perspective, as Yackel and Cobb note, norms are not pre-ordained sets of rules governing human interaction. There is a reflexive relationship between behaviour and norms, through which interaction reflects prevailing norms, but also contributes to their interpretation, so that over time, norms will change. Here is how Yackel and Cobb explain social and sociomathematical norms:

> Our prior research has included analyzing the process by which teachers initiate and guide the development of social norms that sustain classroom microcultures characterized by explanation, justification, and argumentation. Norms of this type are, however, general classroom social norms that apply to any subject matter area and are not unique to mathematics. For example, ideally students should challenge others’ thinking and justify their own interpretations in science or literature classes as well as in mathematics. In this paper we extend our previous work on general classroom norms by focusing on normative aspects of mathematics discussions specific to students’ mathematical activity. To clarify this distinction, we will speak of *sociomathematical* norms rather than social norms. For example, normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms. Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm. (pp. 460–461)

Yackel and Cobb acknowledge that social and sociomathematical norms will vary in different classrooms. Nevertheless, their paper clearly favours particular norms that they see as productive of mathematical meaning-making and learning, such as their focus on explanation, justification and argumentation. The paper appears to assume that certain practices are ‘normal’ in classrooms and that the teacher plays a strong socializing role in guiding the development of desirable norms. These ideas can be seen in many mathematics curriculum documents. To illustrate this point, in the next section we look at two mathematics curricula with which we are familiar.

Two mathematics curricula

Theories of social reproduction, such as those articulated by Bourdieu and Passeron (1977) and by Bernstein (1975, 1996), explain the fact that curricula play roles in reproducing social qualities and values by constructing and presenting particular types of knowledge as legitimate. Apple (1979) emphasises the idea that the ideologically-based contents of curricula legitimate some kinds of values and make them appear natural and consistent with common sense. That is, in mathematics curricula, some actions and interactions are valued and legitimatised as norms and natural. For example, in the new Norwegian mathematics curriculum, "fagfornyelsen" (Kunnskapsdepartementet, 2020), the broad concept of competence is translated into basic communication skills (to speak, to read, to write, to count and digital skills). All these competencies are also strongly linked to basic communication skills. There is a specific emphasis on dialogue and participation (Skolforskningsinstitutet, 2018). The curriculum includes the following:

> Dialogue is crucial in social learning, and the school must teach the value and importance of a listening dialogue to deal with opposition. When interacting with their pupils, the teachers must promote communication and collaboration that will give the pupils the confidence and courage to express their own opinions and to point out issues on the behalf of others. To learn to listen to others and also argue for one's own views will give the pupils the platform for dealing with disagreements and conflicts, and for seeking solutions together. Everyone must learn to cooperate, function together with others and develop the ability to participate and take responsibility. The pupils and their homes are also responsible for contributing to a good environment and sense of belonging. Just as each pupil contributes to the environment in school, so will this environment contribute to the individual's well-being, development and learning. (Ministry of Education of Norway, 2020)

The newly revised Ontario Mathematics Curriculum (OMC) for elementary schools also gives explicit attention to communication, and highlights "understanding local and global perspectives and societal and cultural contexts, and using a variety of media appropriately, responsibly, safely". The curriculum documents include the following:

> Communication is an essential process in learning mathematics. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, the whole class, a community member, or their family. They may use oral, visual, written, or gestural communication. Communication also involves active and respectful listening. Teachers provide differentiated opportunities for all students to acquire the language of mathematics, developing their communication skills, which include expressing, understanding, and using appropriate mathematical terminology, symbols, conventions, and models.
>
> For example, teachers can ask students to:
>
> 1. share and clarify their ideas, understandings, and solutions;
> 2. create and defend mathematical arguments;
> 3. provide meaningful descriptive feedback to peers; and
> 4. pose and ask relevant questions.
>
> Effective classroom communication requires a supportive, safe, and respectful environment in which all members of the class feel comfortable and valued when they speak and when they question, react to, and elaborate on the statements of their peers and the teacher. (Ontario Ministry of Education, 2020)

These extracts raise many questions for us. What are the assumptions about communication (talking, interacting, social participation, social roles, etc.) revealed by these guidelines? One assumption seems to be that children should talk about their ideas. "Defending mathematical arguments" or "pointing out issues" might assume situations where someone has to convince others that they are right or criticise the ideas of classmates. Asking questions, active participation and communication seem to be idealised as signatures of a successful and engaging mathematics classroom.

We depart from Yackel and Cobb view of social norms and sociomathematical norms in our examination of the endorsement of communication in the mathematics classrooms, as highlighted in Norway (2020) and Ontario (2020) Mathematics Curricula. As mentioned by Yackel and Cobb, interaction reflects dominant norms, but at the same time, interactions contribute to the interpretation of the norms, making the norms change over time. We agree with such a view that norms and sociomathematical norms are reflexively produced and are changed in interactions and over time. We do not wish to be concerned with the soundness of this position but with what it means to a knower. Knowers are ones who grew up and were socialised into various norms (and sociomathematical norms) different from presented by the dominant classroom culture and of mathematics itself. We challenge Yackel and Cobb's perspective to examine epistemological assumptions such as: How do children come to know these norms? How are the epistemological effects of these norms conceptualised? And how do the norms of a group of people interact with and cause harm to the norms of another group of people? We think about these questions with a confined focus on the endorsement of communication. We attempt to judge the given value to 'communication' in mathematics classrooms. We judge to form a standpoint, and in the collection of many other standpoints, to contribute to a more just teaching and learning of mathematics.

These are mandates of the curricula, which by legitimising certain types of knowledge, and certain ways of acquiring that knowledge reproduce certain social qualities and values. Curricula, of course, are themselves embedded in and reflect particular societal norms and assumptions. Hence our fundamental questions become: how should cultural and, more importantly, epistemological sensitivity be conceptualised? And who is responsible to judge?

Speech, silence and communication in mathematics classrooms

Speech is silver and silence is gold – says a Filipino saying. Stay silent and reflect so that your thoughts become selected and strong speeches (or something like it) – says a Persian saying. One of the central instructions to many children worldwide is to practice quietness, listen, and speak only if one knows the full meaning of what one says.

Don't talk too often … Don't talk too long … Don't talk about those matters you know nothing about. Johnston (1990) explains that "Were a person to restrict his discourse, and measure his speech, and govern his talk by what he knew, he would earn the trust and respect of his [or her] listeners … people would want to hear the speaker again and by so doing would bestow upon the speaker the opportunity to speak, for ultimately it is the people who confer the right of speech by their audience." (p. 210). For many cultures, words are not objects to be wasted. They represent the accumulated knowledge, cultural values, the vision of entire peoples. Armstrong, a Cree philosopher, explains:

> It is said that you cannot call your words back once they are uttered, and so you are responsible for all which results from your words. It is said that, for those reasons, it is best to prepare very seriously and carefully to make public contributions. (1999, p. 90)

Not once but many times both in Norway (2020) and in Ontario (2020), Mathematics Curricula, communication is encouraged for multiple reasons. For example, communication is valued as it gives "the pupils the confidence and courage to express their own opinions" (Ministry of Education of Norway, 2020). Pupils are inspired to "argue for one's own views" to create a "platform for dealing with disagreements and conflicts and seeking solutions together" (Ministry of Education of Norway, 2020). Similarly, in OMC, communication is highlighted as "an essential process in learning mathematics". In OMC, developing communication skills is directly related to "expressing" and "understanding mathematical terminology, symbols, conventions, and models" (OMC).

Following the line of arguments in Norway (2020) and in Ontario (2020) Mathematics Curricula, in emphasis on the benefit of having communications and expression of an idea in the mathematics classroom, specifically to better understand mathematics, raised concerns for us. Could such a view of communication imply that silence is the opposite of knowing? That a student who is silent and does not "express" or "argue" her position (because she has learnt the fundamental values of being silence) now is considered as not understanding "mathematical terminology, symbols, conventions, and models"? This consideration - of not understanding - is precisely the point in which epistemological sensitivity is vital. Without such sensitivity to the ways in which students with different social and sociomathematical norm become to be normalised in the dominant norms (i.e. those of the curricula), who thrives, who survives and who get disappeared? If according to Norway (2020) competence translates into basic communication then silence becomes a marginalised capacity, showing lack of competency.

Interpreting students' knowing and participation

The following extract is from an interview conducted in 2010 with a Canadian elementary school teacher. It was conducted at the end of the school year, in which she had taught a class composed for most of the year of Cree children (see Barwell, 2020, for information about the study). In this extract, she is reflecting on some of the children's participation.

> I would say [Curtis] is the strongest one in the whole class with regards to math skills and confidence (.) he is willing to take risks and doesn't care if he is wrong as he will learn from it very quickly (.)
>
> um the other kids [...] um they are a lot less likely to take those risks (.) much more hesitant for a number of reasons I would say (.) one (.) they are not at school regularly (.) they are not here to hear everything like a concept or a lesson half way through and they have no idea what has been taught before or what is coming so (.) they have gaps of learning (.) they're not they're

> unsure and they just don’t really want to participate (.) they don’t see the importance in what they are doing (.)
> then you have someone kind of in the middle like Kevin (.) who wants to participate who wants to ask questions who wants to do well and try and things like that (.) but is so unsure (.) his skills like (.) he is so unsure of himself that he doesn’t want to risk being embarrassed or things like that sometimes (.) sometimes he is okay but sometimes I know he is nervous about being wrong (.) and I think that is one of the biggest things that we have overcome.

The teacher’s observations reflect the kinds of mathematics classroom norms we have been discussing. She refers positively to risk-taking, participating and asking questions. She refers to hesitancy, or being unsure as more problematic. But what if her interpretations, based on the implicitly adopted and reproduced norms, as found in many curriculum documents, are a misreading of behaviours of children from a cultural background quite different from her own and largely unfamiliar to her?

Abtahi (2019) warns us about the danger of an over-emphasis on cultural difference and neglecting the elements underlying the cultural differences, which are differences in ways of becoming to know, in mathematics classrooms. Similarly, for us, what is worrisome is beyond the cultural differences or even the diversity of social norms and sociomathematical norms. Instead, we are concerned with the epistemological differences and particularly are perturbed by the effect that such differences play in Curtis’ or Kevin’s mathematics learning. For example, in the above quote, the possibility of students’ orientation to silence and humility as a cultural norm is ignored. This legitimate form of participation leads to the interpretation of Kevin’s action as hesitancy, nervousness or a lack of willingness to take risks, leading to a (possibly harmful) epistemological assumption that Kavin “is someone kind of in the middle” and his sorts of behaviour “is one of the biggest things that we have overcome”.

We are not attempting to force an interpretation on the data here. Instead, in our judgment of the content of the curriculum, concerning the significance of communication and the endorsement of silence in different cultures, we are highlighting possibilities of epistemological diversities that, if ignored, could be harmful. We note also that teachers judge their students’ participation in mathematics – it can be difficult not to – and that sometimes these judgements will be harmful to the students.

Discussion

In many parts of the world’s mathematical curricula, communicational practices are desirable to support better understanding and learning of mathematics. In many parts of the world, silence is desirable to support deeper understanding and reflection on any phenomena. Knowing the value of silence in many cultures around the world, knowing that human movement from places to places and knowing the significance of communication in mathematics classrooms, in this paper, we set out to judge the relationships among these three assumptions and highlighted possible epistemological implications. We examined the content of Canada (2020) and Norway (2020) mathematics curricula to better understand their framing of the significance of communication in mathematics classrooms. We noticed not only that argumentation, expression and communication are highly endorsed in both curricula, but also that the lack of such is abilities is viewed as not understanding mathematical ideas. Following Arendt's view on judgment, we judged how the emphasis on communication, contradicts with social and sociomathematcial norms of some communities leading to not just cultural but epistemological ignorance. We finish this paper with an open question of how epistemological mindfulness could promote more just mathematics learning and teaching and how epistemological mindfulness could look to different teachers?

References

Abtahi, Y. (2019). Epistemological dialogue: Other ways of knowing and our blind spots. *Journal of Mathematics and Culture. 13*(1), 1–19.

Arendt, H. (2003). *Responsibility and judgement.* New York: Schocken Books.

Armstrong, J. (1995). Looking at the words of our people: First nations analysis of literature [Review]. *University of Toronto Quarterly*, *65*(1), 206.

Barwell, R. (2020). Learning mathematics in a second language: Language positive and language neutral classrooms. *Journal for Research in Mathematics Education, 51*(2), 150–178.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures.* Psychology Press.

Johnston, B. (1990). Summer holidays in Spanish. In T. King (Ed.), *All my relations: An anthology of contemporary Canadian native fiction* (pp. 201–210). Toronto: McClelland and Stewart

Kunnskapsdepartementet. (2020). *Nye læreplaner for 16 valgfag er klare.* Retrieved from https://www.regjeringen.no/no/aktuelt/nye-fagplaner/id2709476

Ontario Ministry of Education. (2020). Mathematics. Retrevied from https://www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics/context/the-mathematical-processes

Future School. (2020). Norway curriculum. Retrieved from https://www.futureschool.com/norway-curriculum

Skolforskningsinstitutet. (2018). Klassrumsdialog i matematikundervisningen: Matematiska samtal i helklass i grundskolan. Skolforskningsinstitutet. https://www.skolfi.se/forskningssammanstallningar/publicerade-systematiska-oversikter/klassrumsdialog-i-matematikundervisningen/

Street, B. V., Baker, D., & Tomlin, A. (2005). *Navigating numeracies: Home/school numeracy practices.* Springer.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.

Criticizing epistemic injustice: Rewarding effort to compensate for epistemic exclusion

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This is a theoretical paper prompted by reflection on practice. In the first part we criticize the notion of epistemic injustice in favor of epistemic exclusion and suggest rewarding effort to compensate for exclusion. In the second part, we briefly describe how we are rewarding effort to understand in our calculus courses. We ground the evaluation of this effort on firm theoretical basis, clearly distinguishing teaching, understanding, and learning. We frame the epistemology of twentieth century mathematics into an attempt to quilt signified and significant together through arbitrary language conventions called definitions. As teacher-researchers, we take our own classrooms as our object of study and ground theory on practice.

Epistemic injustice and mathematics education

In TWG-21 of Pre-CERME12, we reported our experience with calculus courses in the second semester of 2020. We received the suggestion to refer to "Epistemic injustice in mathematics education" (Tanswell & Rittberg, 2020). This we now do.

Epistemic injustice is a concept that made its *debut* in philosophy by the seminal work of Fricker (2007) who also coined the term.[1] Speaking of "injustice" unavoidably brings "victims" to mind. "Injustice" opens the issue of social victimization, discussed in Dolar (2017) and uttered by Žižek[2] about the METOO movement. Mladen Dolar remarks that the political correctness concern about social victimization is careful in defining a list of victimized minorities such as racial, sexual, religious, etc., but "curiously, the working class, the basis of capitalist exploitation, doesn't usually feature on this list" (p. 72). Indeed, he remarks, "the cultural and identity struggles inspire so much more passion and engagement than the political and the economic issues" (p. 72). However, this is not a mere curiosity. The apparent critical intentions of concerns about social victimization do not touch on the economic basis of the object of its critique. We go one step further and argue that this is the symptom of what we call *victimization ideology*.

[1] https://en.wikipedia.org/wiki/Epistemic_injustice

[2] https://youtu.be/ai_UAPaoEW4

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For the victimization ideology to be operational, the victim has to play its role. As Dolar (2017) remarks, a revolted Arab immigrant is no longer a victim, he becomes a terrorist. A conniving victim is no longer a victim, she becomes an opportunist. Therefore, repression and the Law have to be evoked to keep the victims attached to their roles, so that victimization ideology may display a humanist and charitable face. Here is our point: *victimization ideology functions to hide the fact that economic exclusion continuously produces "victims."* Consideration of economic exclusion would thwart the relief of culpability that victimization ideology is perceived to produce; it would reveal the true source of culpability. We will show how this cover-up of the economy happens in mathematics education.

Justice is not a natural law. Hegel (1991) shows that it is as old as humanity itself and stems from the exchange of commodities. For Marx (1962), Justice belongs to the ideological superstructure of society which, in its turn, depends on the economic infrastructure. Today, the economic infrastructure determines the dominant class almost directly as being the owners of capital. Finally, Althusser (1970) stresses that the dominant ideology is always the ideology of the dominant class. Along this line, we must conclude that an appeal for Justice and Right is an appeal for the reinforcement of the economic laws and the laws that ultimately ensure each of the myriad exclusion factors dealt with as epistemic injustice. In summary, *epistemic injustice is a call for more of what produces it.* This is the reason for its success.

The internalist view of mathematics

Epistemic injustice makes its entrance in sociology of mathematics via Rittberg, Tanswell and Bendegem (2020). Its fruits are surveyed in Kidd, Medina and Pohlhaus (2017). (How many trees had to be cut down to produce the cellulose for this edition?). It makes its entrance in mathematics education via Tanswell and Rittberg (2020), who endorse what they call the *apprenticeship model* of Dawkins and Weber (2017). Let us examine what this model brings into mathematics education.

Dawkins and Weber (2017) strongly advocate for teaching proofs in mathematics classrooms; they do not specify levels: "we maintain that proof instruction – as an apprenticeship in mathematical practice – should not shift away from proving practice itself" (p. 138). They address the recommendation that "enculturating students into classroom proving practice involves an enculturation into mathematicians' values" (p. 126). Clearly, their proposition amounts to aligning the mathematics classroom to the need of reproducing the mathematicians' qualified labor power (Baldino & Cabral, 2020), supported by the internalist view that mathematicians hold on their own practice.

Let us consider the epistemological rationale of Dawkins and Weber (2017). "By *proofs*, we are referring to the written artifacts that mathematicians call proofs" (p. 124). However, they do not say who mathematicians are. From the article, we infer that mathematicians are the members of a community of proof producers. Thus, they start from the circular definition: *proofs are the production of proof producers.* Hegel would say that there is nothing wrong with circular definitions; they may express the result of a dialectical process (Hegel, 1969).

As written artifacts, proofs are a form of speech. Cabral and Baldino (2021) show that the dialectics of history has generated a community of speech, together with a special form of speech of which proofs are a particular case. They call this community “mathematics” and its members, “mathematicians”. In this sense, the circularity of the expression “mathematics is what mathematicians do”, perfectly expresses the genesis of mathematics. We will see more of this below.

However, in the enunciation of that statement by Dawkins and Weber (2017), the dialectic character of the definition of proof is lost, in favor of a frozen image of proofs as construed by the internalist view of 20^{th}-century mathematics. As spokesmen for the present state of their community, they seek to “conceptualize proof in terms of ‘values and norms’” (p. 124). They need new definitions: “values represent a community’s shared orientations that underlie shared activities” (p. 125) while the term norm “refers to the expectations on practice accepted by the scientific community to uphold a value” (p. 126).

Tanswell and Rittberg (2020) invoke Kant’s philosophy to justify two of their values: “mathematical knowledge is justified by *a priori* arguments” and “mathematical knowledge and justifications should be a-contextual and specifically be independent of time and author” (p. 128). An effort in abstraction is necessary to make sense of these statements.

Let us consider “one plus one equal two”, a mathematical knowledge which certainly the authors would “perceive as being held by the mathematical community” (p. 128). For indigenous populations of Brazil, one plus one may be one, as in the case of a piton and a duck. It may also be zero, if one misses two shots and loses two arrows; or it may be three, as in the case of a father and a mother (Ferreira, 1997). Is this knowledge independent of context? Can it be justified *a priori*? It can only seem so after a great effort of abstraction: we are not talking about ducks and boas. We could ask them: what are you talking about, precisely? They only know that it is something that has norms and values.

However, how did it happen that “one plus one equals two” has become an *a priori* knowledge to common sense? This is what Hegel’s Logic is perceived to explain, but Hegel’s criticism of Kant is precisely what Dawkins and Weber (2017) must ignore to uphold their internalist view of mathematics (Pais, 2016). They do not seem to realize that, ultimately, the adequacy of norms and values to conceptualize proofs must be subjected to the judgement of the same community of proof producers.

As spokesmen for the community, the authors express the difficulty of establishing pedagogic conditions that will put students in the mood of proving, but they do not regret castoffs. For their purpose, it does not matter that only a few students endure enculturation into proof writing, as they are careful not to fail too many. During summative assessment, they always put some easy questions to the majority and one or two difficult quizzes to the “good ones”. This is what their internalist view calls mathematics education.

From injustice to exclusion

In Dawkins and Weber (2017) the words “justice” and “injustice” do not appear. This absence did not keep Tanswell and Rittberg (2020) from choosing this paper to support the entrance of *epistemic injustice* into mathematics education. They open their case by stating two axioms: “mathematics practices are governed by norms and values” and “classroom mathematics should refer to professional mathematics practices” (p. 1199). They justify these statements with what they call the *apprenticeship model* of mathematics education, which, on page 1201, they erroneously attribute to Dawkins and Weber (2017).

Next, they observe that the norms and values of mathematics practices do not coincide with the cultural background values of students, a difficulty that they call the *research gap* (p. 1199). For ethical reasons, they say, it is necessary to “negotiate and adjust” (p. 1199) these values. For this negotiation, they “introduce *epistemic injustice* as a philosophical framework that helps reveal the ethical imperative to traverse the research gap” (p. 1199); they hope that “epistemic injustice in the mathematics classroom may be avoided” (p. 1200). They point out that the language used in written proofs that students should learn to reproduce may be a source of epistemic injustice, since it is “devoid of many features of natural language” (p. 1202). The authors conclude that “the apprenticeship model is in need of refinement” (p. 1209).

For Tanswell and Rittberg (2020), negotiation is also necessary due to the possible clash between ethical orders stemming from mathematics practice and ethical values present in students’ cultural background. “The ethics we are subject to depends heavily on the role we are currently occupying” (p. 1205). The challenge to negotiating ethical values while seeking to traverse the research gap and avoid epistemic injustice is that “the classroom is a space in which multiple roles and their associated ethical orders are present and may come into conflict” (p. 1206).

Tanswell and Rittberg (2020) argue that “the framework of ethical orders also helps to identify the many roles that are inhabited in the mathematics classroom” (p. 1206). The reader expects that they will not miss two paramount teacher roles: their *tutoring* and their *summative assessment functions*. Summative assessment can put teachers into dramatic ethical dilemmas that Cabral and Baldino (2019a, 2019b) call the *splitting moment*. However, the reader’s expectation is frustrated:

> Adopting the framework of ethical orders also helps to identify (…) the array of values, norms and responsibilities these entail. For instance, there are the distinct roles for the student and teacher, which come with very different perspectives, but beyond this they are both engaging with the ethical order of mathematics (Tanswell & Rittberg, p. 1206).

By looking for ethical values and responsibilities in the classroom, these authors are led back to the vague “ethical order of mathematics”. This is astonishing: how is it possible to *look at* a classroom in search of roles that could lead to epistemic injustice and fail to *see* summative assessment?[3]. The mandatory universal summative assessment practice is the moment when a symbol is written beside the name of each student, meaning that a decision of passing/failing has been taken. This practice materializes epistemic exclusion in school, no matter its euphemistic disguises.

[3] “The officer paused with his glance on the space where Aureliano Segundo and Santa Sofía de la Piedad were still seeing José Arcadio Segundo and the latter also realized that the soldier was looking at him without seeing him” (Márquez, 317).

Summative assessment decides whether each student will get credit for the course, that is, whether she may or may not increase the value of her labor power by adding to it the work done by her, the teacher, and the staff during the course. Value is human labor crystalized in commodities, says Marx (1962). Upon graduating, the student will be the owner of a qualified labor power of higher value, which she expects to sell for a higher salary. Therefore, school is an economic enterprise and failing a course has economic implications (Pais, 2014; Baldino & Cabral, 2015, 2020).

As human work, value is substance and, as such, it does not disappear. In case the student fails, whither will the value already produced by the student go? It will be collected by her colleagues who passed, as a prize for having won a competition. *The work done by all is appropriated by those who pass.* This conclusion of Baldino (1998) is bound to raise objections. However, there is no success without failure. Even if it happens that everybody passes, the mere possibility of failure establishes the perspective of competition. Thus, the truth of epistemic injustice is *economic exclusion*. Epistemic injustice is a feature of the victimization ideology whose aim is to cover up and ensure the permanence of economic exclusion. This explains why Tanswell and Rittberg (2020) must avoid summative assessment when looking for epistemic injustice.

This point where the signified "exclusion" is attached to the signifier "injustice" is what Lacan (1971) calls a *quilting point*, borrowing the name from the mattress industry.[4] From this point on, Tanswell and Rittberg (2020) may be read back to reveal its political effect. The imposition of the "apprenticeship model" to enculturate students into the mathematicians' professional proving practice is actually intended to open the "research gap" that allowed the authors to present themselves as *defenders of their own victims.* The "apprenticeship model" assigns to teachers the impossible mission of teaching all students to "write proofs", under the sole argument that this is what mathematicians do. The qualified labor power of the students who succeed in this mission acquires a high sign value as the prize for winning a competition with so many losers. For these, it is necessary to "negotiate ethical orders" under the constraint of avoiding massive failure in the course. Thereby, *unspoken non-mathematical subsidiary criteria* are introduced into the summative assessment. This is the true ethical question that was kept in the shadows by the "ethics of mathematics", an abstraction ensuring that economic exclusion remain out of sight at adequate level.

Grounding theory on practice

To say it all at once: we too adopt non-mathematical, economically excluding, summative assessment criteria, but there is nothing in the shadows: *we reward effort to understand, instead of evaluating final learning.* To unfold this aphorism and justify our assessment criteria, we need three definitions, grounded on Lacan's psychoanalytic theory (Lacan, 1981).

[4] The quilting point is the stitch that transforms a sack into a cushion.

Evaluating learning or evaluate understanding?

Teaching is to ask questions and return the meaning of her answers to the student. The teacher assumes the position of the demanding Other: *what does he want?,* the student asks herself, an interrogation that Lacan writes in Italian *che vuoi?* to avoid the gender implication. In opposition to "conveying meaning," teaching is listening and showing understanding of what is heard.

Learning is to represent oneself as a subject by new signifiers. The student assumes the position of speaker and ventures a new representation into the field of the Other, where she must constitute herself as a desiring being. Learning is speaking, in the broad sense, including gestures, hesitations and silences. It is a restructuring of the subject's jouissance. It does not occur *in vitro*. It depends on a myriad of factors such as cultural background, gender, ethnicity, etc. These factors remain at work overnight.

Understanding is the student's ability to sustain her subjective representation by a signifier. In understanding, student and teacher are publicly interacting in the *agora*. Understanding is an objective process, it can be registered in video; it is a being-there (*Dasein*) and as such *has a measure* (Hegel, 1985, p. 330[5]); thus, at least in principle, understanding can provide a precise ground for summative assessment.

In summary, *one teaches by listening and learns by speaking*. Traditionally, one teaches by showing dominance over knowledge (*séance magistrale*. In this system, summative assessment is designed to give free course to the myriad of cultural biasing factors that work overnight to boost/hamper learning. One may say that this is a true epistemic exclusion of those who somehow have not incorporated (learned) the imponderable values of the ruling class present in the whole summative assessment situation.

To compensate for epistemic exclusion, we seek to develop a reliable way to evaluate the *effort to understand mathematics*. To unfold this statement, we must make precise what we mean by "effort" and by "mathematics".

What do we call mathematics?

It is useless to ask mathematicians. They do not know what mathematics is; they only say that mathematics is about this and that. They do not know what a proof is, either; they only say that it is "conceptualized in terms of values and norms". In Cabral and Baldino (2021), we describe the underlying historical process that started in Ancient Greece and has generated a special kind of speech, along with a *community of speech* that upholds it as epistemologically valid. These speeches we call *quilted speeches*.[6] Inside the community of speech, they came to be called proofs. The essence of a quilted speech is the *attempt to stop the sliding of the signified under the signifier*, that is, the possibility of justifying each inference from one

[5] *Alles, was da ist, hat ein Maaß*. Alles Daseyn hat ein Größe.

[6] Quilted speeches emerged in Ancient Greece, together with coinage; they were socially necessary to avoid intrafamily clashes between the economically broken landowner progenitor and his rich brothers who were free to go into commerce. These were indeed very special social circumstances.

statement to the next by an explicit consensual argument; these inferences are the *quilting points*. The community, with its statute of validity, is what can be properly called mathematics; its members are the mathematicians. An interesting description of the statute of this community, specifying what counts as a valid argument, may be found in Rittberg, Tanswell and Bendegem (2020).

To make clear what we mean by quilted speech, we present an episode of our current freshmen calculus course. The students had received a worksheet with the proof of the derivative of t^2 and had been required to justify each quilting point, represented by the numbered equality signs in the excerpt shown below. In the previous class, they had engaged in a lively discussion about “infinitesimals” and “limits,” based on what they had found in books and online. Is the limit indeed reached? What is the threshold below which a number becomes infinitesimal? In the present class we were evaluating how they would justify the following quilting points.[7]

$$\boxed{f(t) \approx \frac{F(t)-F(t-dt)}{dt}} \overset{(0)}{\Rightarrow} \frac{t^2-(t-dt)^2}{\Delta t} \overset{(1)}{=} \frac{t^2-(t^2-2t\ dt+(dt)^2)}{dt} \overset{(2)}{=}$$

$$= \frac{2t\ dt-(dt)^2}{dt} \overset{(3)}{=} \frac{(2t-dt)dt}{dt} \overset{(4)}{=} 2t-dt \overset{(5)}{\approx} 2t \overset{(6)}{=} f(t) \overset{(7)}{=} \frac{dF}{dt} \overset{(8)}{=} F'(t)$$

At quilting point five we received three students’ collaborations in a row (emphasis added):

Antônio: It’s an *infinitesimal*, so it’s only approximating 2t.

Cesar: You can divide by it and that’s not wrong. It’s not as if I were *dividing by zero*. But here, when I subtract, then it’s like it were *really zero*.

Erick: It’s like Antonio said, we have an infinitesimal number. So, we are practically *creating a limit* there.

We concluded:

Baldino: This whole discussion you’ve been struggling with since the previous class, and which mathematicians struggled with for 200 years, was resolved in 1824, when an important mathematician called Cauchy said this: “on dit que” – or “it is said that”. It is said that the derivative is the name of 2t. The whole discussion you had was reduced to one name: “derivative” is the name given to 2t by *language convention*.[8]

[7] *F* and *f* respectively refer to the totalized and the daily deaths graphs due to COVID19 as popularized by the media; these graphs had been object of previous classes. We started with stereotyped graphs of a disease outbreak that lasted for 60 days. Once the students were able to express the *moving average of deaths based on one day*, we asked them to express this in the case of an endemic disease that lasted 60 years. Using the 4000-times zoom of CorelDraw, the students were naturally led to express the duration of one day as *dt*, “a little bit of”, as in Thompson (1914).

[8] See the video in https://cabraldinos.mat.br/the-grounding-quilted-point-or-mathematics-of-the-20th-century/. The students authorized the publication of their names.

Point 5 is the grounding quilting point of twentieth century mathematics. It marks the moment when mathematics broke apart from philosophy and started determining its objects by arbitrary definitions solely aiming at the coherence of its global structure of quilting points. Hilbert hoped for absolute coherence of such language conventions. Gödel showed that this was impossible.

Quilted speeches make possible the historical development of the mathematics community and those quilting points make the *minimal units* of proofs. Therefore, *teaching proofs presupposes teaching the function of quilting points*. If students miss this point, they may continue looking for unquilted justifications, like the mathematicians of the past, and errors are bound to come up.

Evaluating effort

In our classroom practice, students are required to individually justify randomly assigned quilting points or sets of quilting points in previously assigned WS, thereby entering into a dialog with the teachers. *How much effort the student made to prepare herself for this moment becomes evident to all.* Close eye-to-eye conversation eliminates questions about the authenticity of answers. No written homework is required.

Studying a WS is an effort that can be demanded from any student of whatever cultural background. Contrary to what happens in evaluation of learning, this form of rewarding happens at the very moment and situation of understanding; there are no imponderable factors working overnight. Insufficient effort may lead to failure, and failure has an unavoidable economical implication. Thus, competition may still be present, but its criteria are explicit and measurable; thanks to systematic video records, grades may be contested by the students. If group heterogeneity becomes evident, it can be dealt with by establishing handicapped subgroups. Of course, we also evaluate learning for our own sake, not as a criterion for promotion. After a certain level of learning is achieved, it is not necessary to distinguish students with grades.

Does our strategy of rewarding effort ensure learning? It certainly ensures a certain network of learning, but this network must be evaluated by longitudinal and comparative mathematics education research. However, such research may reserve unpleasant surprises to those who support evaluating learning instead of effort, as Thompson (1994) showed.

References

Althusser, L. (1970). *Idéologie et appareils idéologiques d'état*. Paris: Éditions Seuil.

Baldino, R. R. (1998). School and surplus-value: contribution from a Third-World country. In P. Gates (Ed.), *Proceedings of the First International Mathematics Education and Society Conference*, (pp. 73–81).

Baldino, R. R., Cabral, T. C. B. (2015). The profitability of qualified labor power production. *Journal for Critical Education Policy Studies, 13*(1), 67–82.

Baldino, R. R., & Cabral, T. C. B. (2020). The productivity of students' schoolwork: An exercise on Marxist rigor. *Philosophy of Mathematics Education Journal, 36*. (Original work published 2013, *The Journal for Critical Educational Policy Studies, 11*(4), 70–84.)

Cabral, T. C. B., & Baldino, R. R. (2019a). The social turn and its big enemy: A leap forward. In J. Subramanian (Ed.), *Proceedings of the Tenth International Mathematical Education and Society Conference* (pp. 32–46). https://www.mescommunity.info/proceedings/MES10.pdf

Cabral, T. C. B., & Baldino, R. R. (2019b) The credit system and the summative assessment splitting moment. *Educational Studies in Mathematics, 102*, 275–288. https://doi.org/10.1007/s10649-019-09907-5

Cabral, T. C. B., & Baldino, R. R. (2021). "Mathematics"? What do you mean? — Don't play the fool; everybody knows. Unpublished manuscript.

Dolar, M. (2017). Who is the Victim? In J. Krecicby (Ed.), *The final countdown: Europe, refugees and the left*. Ljubljana and Vienna: IRWIN.

Dawkins, P. C., & Weber, K. (2017). Values and norms of proof for mathematicians and students. *Educational Studies in Mathematics, 95*, 123–142. https://doi.org/10.1007/s10649-016-9740-5

Ferreira, M. K. L. (1997). When $1 + 1 \neq 2$: Making mathematics in central Brazil. *American Ethnologist, 24*(1), 137–147.

Fricker, M. (2007). *Epistemic injustice: Power and the ethics of knowing*. Oxford: Oxford University Press.

Hegel, G. W. F. (1969). *Hegel's science of logic*. New York: Humanity Books.

Hegel, G. W. F. (1985). *Wissenschaft der Logik*. Hamburg: Felix Meiner.

Hegel, G. W. F. (1991). *Elements of the philosophy of right*. Cambridge University Press.

Huillet D. (2014) Teacher as researcher in mathematics education. In S. Lerman (Ed.) *Encyclopedia of mathematics education*. Dordrecht: Springer. https://doi.org/10.1007/978-94-007-4978-8_148

Kidd, I. J., Medina, J., & Pohlhaus, J. G. (Eds.) (2017). *The Routledge handbook of epistemic injustice*. New York: Taylor & Francis.

Kincheloe, J. L. (1991). *Teachers as researchers*. London: Routledge.

Lacan, J. (1971). *Écrits II*. Paris: Éditions du Seuil.

Lacan, J. (1981). *Le séminaire de Jacques Lacan: Livre II. Les psychoses*. Paris: Éditions du Seuil.

Márquez, G. G. (2014) *One hundred years of solitude*. Penguin.

Marx, K. (1962). *Das Kapital* [Capital], Vol 1. Karl Marx - Friedrich Engels - Werke, Band 23, S. 11-802, Dietz Verlag, Berlin/DDR 1962. Available in Marxists' International Archive: http://www.mlwerke.de/me/me23/me23_000.htm Accessed Jan 26, 2012.

Mills, J., & Earl Rinehart, K. (2019). Teachers as researchers. *Teachers and Curriculum*. https://doi.org/10.15663/tandc.v19i1.338

Pais, A. (2014). Economy: The absent center of mathematics, *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-014-0625-8

Pais, A. (2016). At the intersection between the subject and the political: a contribution to an ongoing discussion. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-016-9687-6

Rittberg, J. C., Tanswell, F. S., & Bendegem, J. P. V. (2020). Epistemic injustice in mathematics. *Synthese*. https://doi.org/10.1007/s11229-018-1981-1

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in mathematics*. https://doi.org/10.1007/BF01273664

Thompson, S. P. (1914). *Calculus Made Easy*. New York: The Macmillan Company.

Tanswell, F. S., & Rittberg, C. J. (2020). Epistemic injustice in mathematics education. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-020-01174-6

Critical mathematics education and social movements: Possibilities in an LGBT+ host house

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The LGBT+ community has historically been excluded from different spaces and has suffered with the prejudice and violence in Brazil and worldwide. The school has a fundamental role in the struggle for a less prejudiced society that respects and values differences. Through a study carried out in an LGBT+ host house, we will discuss in this article how spaces like this can become spaces for transformation and that bringing knowledge produced in these places to school and, specifically for mathematics classes, is a way to contribute to the struggle for a more just society.

Introduction

In the book *Pedagogia da Indignação*, more specifically in the chapter "Alfabetização e Miséria", Paulo Freire (2016) talks about an experience in Recife/PE, where he was walking with an educator friend in a slum in the city where many families lived from what they collected from a local garbage dump. The main question discussed was: "What to do in a context where many people are denied their humanity?" This is a recurring reflection of/in this book in which the author places/identifies indignation as essential for mobilizations to happen and changes to take effect.

In another chapter of the same book, entitled "Do assassinato de Galdino Jesus dos Santos – índio Pataxó", Freire reflects on what happened in the early hours of April 20, 1997, when a chief was murdered in Brasília. The indigenous leader, who was resting at a bus stop, was found by five young men who arrived by car and set him on fire. When they were accused of this inhumane act, they justified it by saying they believed he was a person on the street. Freire presents his indignation about this dehumanizing fact:

> What a strange thing, playing killing Indian, killing people. I keep thinking here, plunged into the abyss of a profound perplexity, amazed at the intolerable perversity of these young men disfiguring themselves, in the environment in which they have decreased instead of growing (Freire, 2016, p. 75).

This type of violence, suffered mainly by groups that we call underrepresented, is unfortunately common in our country. In this work, we will highlight the LGBT+ community (Lesbians, Gays, Bisexuals, Transsexuals, Transvestites, Transgenders and more) that has faced several struggles in our country. The reports of the Gay Group of Bahia carried out annually registered 329 deaths of LGBT+ people due to hate crimes in the year 2019 (Grupo

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Gay Bahia, 2019). In other words, several LGBT+ people are murdered in Brazil on account of their gender identity or sexual orientation.

These statistical data and the situations presented by Freire are examples of oppressions that underrepresented groups have faced in Brazil. When we look at the participation of these groups in decision-making spaces, such as political spaces, it has been very low, which can lead to the difficulty of public policies being designed and implemented in order to alleviate these situations of violence.

In Brazil, the National Congress is the national institution that exercises legislative power in Brazil and is divided into a chamber of deputies and a federal senate, where the chamber of deputies is understood as representatives of the people and has representatives in proportion to the number of voters in a given state and the senate has 3 representatives from each Brazilian state.

According to research by the National Congress, in the four-year term that began in 2019, of the 513 representatives of the people at the national congress, only 77 were female, 125 black and only one admitted to belonging to the LGBT+ community. These data do not portray the reality of the Brazilian population, because, for example, the Superior Electoral Court (TSE) points out that women constitute 51.3% of the total voters. As this data shows, we can think about representativity in the political space. However, when we discuss participation in formal or informal spaces, including the media, universities and the work market, we are also talking about representativity

As a result, underrepresented groups have been participating in social and collective movements in the search for a more just society. Butler (2018) highlights the importance of the struggle of social movements and emphasizes that not not all social movements are necessarily democratic, therefore, they need to be looked at carefully.

Keeping this argument in mind, we emphasize the importance of the role of social movements in the struggle for a more just society, promoting attitudes that favour the conquest of spaces by underrepresented groups to assist in the search to overcome stigmas and break barriers. The work developed in these spaces can also be understood as a promoter of learning, so the struggle is pedagogical (Gutstein, 2018). As an example, we can mention the work developed by some LGBT+ host houses in Brazil that are spaces that offer support to people who have been expelled from their homes on account of their sexual orientation or gender identity. The doctoral research (Barros, in progress), with which this article is linked, was conducted in one of these spaces. Information about the research will be presented below.

Conducting the research

The research reported in this article was carried out in a host house for LGBT+ people called Casassa in the city of Presidente Prudente/SP. The production of data took place in the months of August and September 2019 and the researcher sought to get involved in all the activities promoted, in addition to offering conversation circles.

Casassa is not for profit and everyone who works there is a volunteer. Its creation took place at the II Diversity Week that took place in the city of Presidente Prudente/SP in 2017. At this event there were several moments of reflection on gender and sexuality and in one of the activities the discussion arose about the need to have a permanent space so that welcoming work could be done to provide for LGBT+ people who were expelled from home and furthermore, that it was a space to hold discussions on relevant topics.

Casassa, not having any type of financing, carries out its work using money from donations and events that are promoted by volunteers. Every month there is a bazaar, called Bazarsasso, at which items donated to the house are sold and all the money goes to the local expenses. Educational and cultural events such as parties, English courses and workshops have already been held and promoted by the Casassa team.

Volunteers' work is organized by working groups. This work involves: reception, dissemination, donations, education, financial, legal and events. Casassa has two main purposes: welcoming LGBT+ people and carrying out cultural and educational activities. Thus, the word "welcoming" has two possible understandings, being a space for temporary housing of people who were expelled from home on account of their sexuality or gender identity and also a meeting place for discussions and interactions focused on the LGBT+ movement.

In August 2019, the researcher had his first face-to-face contact with the house, participating in meetings of the volunteer team, of which seven were interviewed throughout the month. The purpose of the interviews was to understand how Casassa originated, how work has been developed, relations with the community, the question of educational activities and the place of mathematics in this space. Participants were informed about the purpose and nature of the interviews and signed the Free and Informed Consent Form. The interviews were recorded in audio.

In September, two circles of thematic conversations were held. The first promoted discussions on representativity, focusing on the presence of LGBT+ people in politics. The second conversation circle dealt with the issue of visibility and stereotype thinking about how LGBT+ people are portrayed in diverse media. Casassa volunteers participated in the conversation circles, welcomed and also generated interested people from the community. In this stage, audio recordings were made during the activities and shortly after finishing the activity, the researcher made notes on important facts that happened.

The format of the conversation circles was based on Paulo Freire's (1987) cultural circles. These collective moments of dialogue sought to respect the knowledge of each participant and promote a democratic environment to reflect on reality inherent to the LGBT+ community. Critical Mathematical Education was used to help us think about the themes of representativity, visibility and stereotype thinking in the conversation circles. Looking at the mathematics that is involved in these processes helped us to compose the electoral system in the political space and to reflect on possibilities for social change. The theory from Critical Mathematics Education used to support the design of activities is the reading and writing of the world with Mathematics (Gutstein, 2006), where mathematics is used to promote discussions that help in the understanding of situations in society and also helps in the realization of transformations.

Gutstein (2006) makes use of investigative activities that make a movement that starts from issues for the community and goes to the classroom. We sought to be inspired by this movement to bring issues that were relevant to the LGBT+ community to be topics of conversation circles. In addition, the construction of proposals and choice of mathematical tools and strategies had as protagonists the participants who reflect and put mathematics into action.

At this point in the research, the transcripts were having been completed, both of the interviews and of the conversation circles. The records have been organized and relationships are being established with the researcher's notes. This is a complex process that is not linear. The analysis has been carried out taking into account the research question, the objectives and the aspects that stood out in the data.

A close look has been started in order to understand what possibilities of reading and writing the world with mathematics can emerge from a social movement. Based on these reflections, I bring below some notes that help us to reflect on how Casassa (as a social movement) collaborates with the community in which it is inserted.

A host house changing a community

Casassa's proposal is to be a welcoming home, but this goes beyond the idea of housing. José brings this perspective by talking about how his friend Larissa, who is a transvestite, has experienced society as not safe for LGBT+ people*:*

> This whole neighbourhood is a place and having a transvestite living two blocks from here, there is Cláudio who will live two blocks from here too, because having Casassa makes these people more comfortable in that space. I remember when Larissa, this transvestite friend from the neighbourhood lived in another place, a neighbourhood far away and I went to visit her one day in the morning, we went from her house to the bakery which is two blocks away and just walking this way I came back tired, because there were a lot of whispered looks and comments with us passing by. They were simply looking, they didn't need to speak or shout anything, but I realized that Larissa was attentive all the time. That is why I hope that the whole community can be here at Casassa and that they get to know the LGBT + people to make the whole neighbourhood more welcoming, changing the structure of the place.

One of Casassa's goals is to create a host and support network that goes beyond the physical location restricted to the host home. In this quest, making a simple exercise like going to the bakery safer for LGBT+ people are a necessary exercise. However, for this to happen structural changes are necessary for society to understand the importance of respecting differences.

Another Casassa volunteer, Davi, sees the host house as a motivator for discussions on the reality of the LGBT+ community, after all it leads people to think about this, even though sometimes the discussions are initially marked by stereotypes:

> I think it influences in a positive and negative way. We have seen both things happen. Because it is an LGBT+ host house, it has the common sense that it is a place that is related to sex, so there are a lot of people who look here with a crooked eye and do not have the courage to come and ask what is happening and are judging. The positive impact is that people in need of care have a hope of knowing that there is a space like this.

This discomfort highlighted by Davi helps to get people out of their comfort zone and makes them think. In this reflection on the discussions on the reality of the LGBT+ community, it is possible that there will be an appreciation of Casassa and consequently of the LGBT+ community. Corroborating José's observations, Davi also reinforces the importance of the existence of host houses so that LGBT+ people can feel more secure in the process of developing their sexuality or gender identity.

Another interviewee, Olga, reports that the volunteers are beginning to understand the impacts of Casassa on society and that the host house represents hope for those who are expelled from home, in addition to being a space that promotes necessary discussions on LGBT+ reality:

> It has a significant impact in many ways. First the impact of the service provided, let's say that having this welcome is a refuge for these people, not only for those who are expelled, but those who were there without having a reference and did not understand their sexual orientation and gender identity and, suddenly, to see people thinking "Ah, I'm like them". It is an impact of visibility, because it started to be talked about this subject in a more open way. We do not know very well if it reached certain layers of society and if we arrived in many places, but we had contact with some businesses that offered support to us. Universities too and social movements themselves. I think the main thing was visibility as a whole. I see in my own family, when I say that I am working on this project, people start asking me, "What is Casassa?", "Do you accept it?", "Do you have families that expel children from home for this?". Have! So, it was this opening to a reality that sometimes these people did not know. We need to understand that there are realities different from ours and that something needs to be done.

A common response in the interviews was that the interviewees did not want a host house for LGBT+ people to be needed, after all, nobody should be expelled from their home on account of their sexuality or gender. However, we must emphasize that it is important to have host houses such as Casassa in the current society.

About the presence of mathematics in the daily life of Casassa, the volunteers had a little difficulty in saying about aspects that could be seen in their activities. It was evidenced by the interviewees in the financial issues of managing the house and the bazaar that is held to raise money. However, at no time was mathematics seen as a promoter of social and political discussions. The conversation circles sought to change this perspective.

When conducting the conversation circles, it was possible to put mathematics into practice, with mathematics providing a reading and writing of the world. In this sense, themes related to representativity, visibility and stereotypes were discussed. These are pertinent issues that reflect on how society views the LGBT+ community and the participation of this group in in a range of spaces. During the first conversation circle, for example, we reflected on how underrepresented groups participate in the Chamber of Deputies, which is one of the houses of the Brazilian legislative power.

We used concepts such as percentage to compare the presence of underrepresented groups in these political spaces and in the community, and we find that there is a large discrepancy. We reflected on the reasons for this to happen from a critical perspective. These groups have historically been excluded from electoral processes, as many political parties have been concerned with issues that address the exclusive demands of a majority of the population.

In addition, we reflected on the electoral process, after all, the elections in Brazil are made by proportional votes by party. So, the votes destined for a certain candidate do not necessarily go to them, but these votes can help to elect another person from the same party. This proportional process could be a problem in the search for representativity.

Research participants showed themselves to be engaged during the discussions. The reality data on the representation of minorities surprised the participants in the conversation circle. Furthermore, mathematics helped to look to the future and think in an ideal context.

The need for greater participation by underrepresented groups in political and other spaces was evidenced so that, through public policies, transformations can be made showing a more inclusive society with more respect. Providing this type of debate in a space like Casassa proved to be necessary so that through struggle we can increasingly learn about human diversity. These were the conclusions of the participants in the conversation circles, who came to see mathematics as a possibility to discuss social and political issues.

At this point, we can see the importance of using mathematics from a critical perspective so that people who participated in that activity could reflect on the reality of the LGBT+ community and the participation of this group in politics.

These important discussions promoted in a space like Casassa can and should be taken to schools. However, this is not a simple task, since the advancement of conservatism has made it difficult for discussions about gender and sexuality to be held in Brazilian schools. However, teachers must take on this important task of promoting these discussions in their classrooms.

Conclusions

For change to happen, we cannot be unaware of the world's problems. We must be indignant about injustices, as proposed by Freire (2016) and in order to collaborate with the struggle of the oppressed, we must promote a pedagogy of resistance, where one learns to fight, fighting. Involvement with social movements in this sense is shown to be powerful, after all it is in these spaces that underrepresented groups organize themselves and carry out mobilizations seeking a more just society.

Through the realization of this research, we see that a space like Casassa is a pedagogical space. The engagement in the struggle of the LGBT+ movement can take place in several ways, but it is essential for us to overcome the prejudices that this community has suffered.

We as teachers are invited to be in these spaces participating in these discussions, but it is also important that we take these debates to our classes.

Mathematics showed its potential to collaborate with the reading and writing of the world when used from a critical perspective. The activities used in the circles helped us to think about ways to transform the reality of the LGBT+ community, which has still been largely excluded from society.

We fight for a less prejudiced society in which we will no longer have, for example, people being expelled from their homes on account on their sexuality or gender identity. In this sense, we believe that mathematics has the potential to support much needed discussions about diversity and difference.

References

Barros, D. D. (in progress). *Lendo e Escrevendo o mundo com a matemática em um movimento social: Discussões sobre representatividade e as lutas da comunidade LGBT+* [Doctoral dissertation]. Universidade Estadual Paulista, Rio Claro, Brazil.

Brasil. (2019). Câmara Federal dos Deputados. Parlamentares em exercício. https://www.congressonacional.leg.br/parlamentares/em-exercicio

Brasil. Tribunal Superior Eleitoral (TSE) (2019). Distribuição por gênero e faixa etária. https://www.tse.jus.br/eleitor/estatisticas-de-eleitorado/estatistica-do-eleitorado-por-sexo-e-faixa-etaria

Butler, J. (2018). Corpos em aliança e a política das ruas: Notas para uma teoria performativa de assembleia. Rio de Janeiro: Civilização Brasileira.

Correio Brasiliense. (2018). *Morte do índio Galdino, em Brasília, completa 21 anos hoje.* https://www.correiobraziliense.com.br/app/noticia/cidades/2018/04/20/interna_cidadesdf,675182/morte-do-indio-galdino-em-brasilia-completa-21-anos-hoje.shtml

Freire, P. (1987). *Pedagogia do oprimido.* Rio de Janeiro: Paz e Terra.

Freire, P. (2016). *Pedagogia da indignação: Cartas pedagógicas e outros escritos.* São Paulo: Paz e Terra.

Grupo Gay Bahia. (2019). *Relatório 2019: População LGBT morta no Brasil.* https://grupogaydabahia.com.br/relatorios-anuais-de-morte-de-lgbti

Gutstein. E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York: Routledge.

Gutstein. E. (2018). The struggle is pedagogical: Learning to teach critical mathematics. In P. Ernest (Ed.) *The philosophy of mathematics education today* (pp. 131–143). Springer.

Engagement and resistance in an equity-focused professional development: Toward caring with awareness

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Teacher development toward equity should support teachers in seeing the ways in which their practice is part of larger social and political histories and structures so that they can then disrupt oppressive systems influencing students' opportunities to learn mathematics. We examine one teacher's engagement with an equity-focused professional development designed with this need in mind and explore what her experiences suggest about transitions from caring to caring with awareness, explicitly considering race, culture, gender, and power in relation to academic achievement.

Efforts in mathematics teacher education have long emphasized the need to prepare teachers for teaching diverse populations of students in ways that provide equitable opportunities to learn mathematics. However, mathematics education is positioned as part of a larger, neoliberal narrative disconnected from the experiences and realities of students from non-dominant communities (Martin, 2013). Neoliberalism reflects a shift in education from a social to a market concern and contradicts a focus on students' communities, beliefs, and points of view (Lipman, 2012). Teacher development toward equity, then, should support mathematics teachers in seeing the ways in which practice is part of larger social and political structures so that they can then disrupt oppressive systems influencing students' opportunities to learn mathematics.

The Access, Agency and Allies in Mathematical Systems (A3IMS) research project aimed to address these needs, designing and studying a professional development (PD) around five strands: mathematics, discourse, privilege & oppression, culture & community, and action research. Here, we examine one teacher's – Julie's – response to these PD efforts. On one hand, Julie was engaged in the PD and positioned herself as having good relationships with her students and putting students first, always. On the other hand, Julie's engagement was frequently marked with emotive language that suggested dislike and resistance to some PD

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efforts. In short, Julie was a complex participant who challenged us as PD facilitators, showing signs of being a caring teacher but not yet demonstrating caring with awareness (Bartell, 2011), or caring that includes explicit focus on racial, cultural, and political dimensions affecting relationships and mathematics education. We explore the question: *What does Julie's experience with the PD suggest about the transition from caring to caring with awareness in mathematics teaching?*

Background Literature

One long-accepted characteristic of an effective teacher is the ability to cultivate and maintain strong interpersonal relationships with students, often described as caring (Noddings, 1992). Caring is a process; it is something teachers do rather than something teachers feel. Caring relationships between teachers and students lead to higher levels of student engagement and achievement (Pianta, 1999). Mathematical Caring Relationships include teacher's choosing appropriate problems to pose based on students previously demonstrated mathematical reasoning, supporting students' mathematical learning (Hackenberg, 2005). Caring relationships in mathematics classrooms must also reflect *caring with awareness* (Bartell, 2011). Such caring relationships "acknowledge racial identity, culture, racism, and racial privilege as factors that shape and colour experience" (Thompson, 2004, p. 26). For teachers of mathematics, awareness includes an understanding of how mathematics has been used to socially partition individuals by identity markers such as race, gender, and ability. Caring mathematics teachers value and support Students of Colour' status, identity, and prior knowledge and centre issues of race and ethnicity in relationships with students (Rolon-Dow, 2005). Teachers demonstrating *caring with awareness* explicitly reject deficit perspectives of students and their communities. They do not attach mathematical (and other) "failure" to a student, but to themselves, searching within to find a more effective way to reach students (Collier, 2005). They explore students' mathematical thinking and in- and out-of-school experiences to inform changes in classroom practice. These relationships are also political, where mathematics teachers take up "active political stands in solidarity with students and their communities about issues that matter" (Gutstein, 2006, pp. 132-133) and engage students in using and learning mathematics while understanding the world. Teachers demonstrating *caring with awareness* work to ensure that the curriculum reflects the lived experiences of their students, recognize existing oppression in their students' lives, and seek to use their own status to encourage students (and themselves) to understand and undermine those oppressive conditions (Beauboeuf-Lafontant, 2005).

Teachers, and particularly White teachers, often resist discussions of race, culture, and power with respect to teaching and learning. Such resistance can take many forms. Hytten and Warren (2003) identified rhetorical strategies their study participants used in discussions of systemic oppression, including ignoring colour, focusing on progress, victim blaming, and focusing on culture rather than race. For example, people might make statements that suggest people of colour are racist, too, or complain in ways that position White people as victims in interactions around race. Or people might argue that they "do not see colour," fuelled by a belief that by ignoring colour, racism is minimized. However, such colour-blind stances allow people to deny that "race, especially skin colour has consequences for a person's status and well-being" (Rosenberg, 2004, p. 257). It allows White people to dismiss their complicity in racial oppression. This dismissal of complicity is echoed in Annamma et al.'s (2017) expansion of colour-blind ideology where they argue that "colour-blind" is inadequate because it suggests passivity, hides the social construction of race and disability, and ignores the entanglement of these two in oppression. They instead define colour-evasiveness to call attention to the fact that avoidance of discussion about race is deliberate obliteration of the experiences of People of Colour.

It is important to note that many White women have been socialized toward resistance. Women tend to be socialized to avoid conflict (Gillespie et al., 2002). For White women, this avoidance of conflict exists within a social norm that talking about colour or difference is impolite (Mazzei, 2004). Thus, White female teachers [live within] ideologies that expect them to be complicit in the oppression of people of colour because of the expectation of gendered conflict-avoidance and deference. Instantiations of this ideology of the feminine interact with the ideology of colour-evasiveness in a manner that reifies the dominance of White male power and rests it in a space of academic achievement measured in abled ways. Teaching was historically viewed as women's moral obligation as mothers to raise a child as a productive member of the society (Grument, 1983). Relations of power and authority have historically been formed as patriarchal where teaching is considered both a feminine occupation and a kind of domestic service conducted by women, reifying a "caring teacher is a good teacher" narrative (Apple, 1983).

Method

Thirteen teachers from a small urban district in Granite City, located in the Midwestern U.S., signed up to participate in this PD. All the teachers were women; nine were White and four were African American. Julie is a White, middle-class, heterosexual woman who had taught in Granite City schools for 12 years. She grew up in Granite City but did not attend Granite City Community Schools (GCCS); rather, she attended schools also located in Granite City that she described as "straight down the street." Julie began teaching in GCCS as a graduate student; her undergraduate degree is in computer science and her graduate degree is in curriculum and secondary education. Most of her 12 years of teaching in GCCS has been at the high school level. At the time of this study, Julie was a math intervention teacher in a school that served grades 7-12.

The professional development

The PD was designed around five interrelated strands as noted above. The *mathematics* strand engaged teachers in mathematics tasks and in adapting curriculum to engage students in inquiry-based tasks and social justice activities. The *discourse* strand highlighted cultural assumptions embedded in expected mathematical discourse and built understanding related to how these discourse practices contribute to students' positioning and developing identities (Herbel-Eisenmann et al., 2013). The *privilege & oppression* strand gave teachers an opportunity to explore their own racial, gender, class, etc. identities and consider the systemic influence of oppression on mathematics teaching and learning. The *culture & community* strand focused on developing teachers' recognition of the resources for mathematics learning that students bring from their community and cultural experiences. Finally, the *action research* strand emphasized the process of systematically examining, reflecting on, and making important changes to mathematics teaching practices that teachers identified that they wanted to improve. Each year of the PD began with a week-long institute followed by twice-monthly PD meetings throughout the academic year. The first four strands were woven together such that individual activities often reflected the goals of more than one strand (see https://olemiss.edu/a3ims). These four strands were the focus of the institute and Fall PD sessions to inform teachers' action research projects in the spring.

Data Collection & Analysis

Any activity that had goals related to the *privilege & oppression* strand or the *culture & community* strand in the first two years of the PD were selected for analysis, representing 81% of PD activities in those two years. All whole group interaction was video recorded and small group discussion was audio recorded. Interviews were conducted with Julie prior to the start of the PD, at the end of the first year, and at the end of the second year. The first interview collected demographic information, teaching history, and asked Julie what she thought equity meant and to describe successes and challenges specific to equity in the mathematics classroom. The final two interviews asked for Julie's reflections on aspects of the PD she found useful, her conceptions of equity and good mathematics teaching, and about tensions that she might be feeling with the dual focus on mathematics and equity in the PD. Finally, at two points during the first year, Julie's teaching was observed, and video recorded and a follow-up noticing interview took place where Julie watched a researcher-selected clip of teaching or student engagement and probed for Julie's thinking about the clip.

To analyse the data, three researchers read through the data set independently and identified three broad codes to look at more closely: Julie's positioning of self, students, and families and communities. We compared our coding and refined codes, resulting in these code descriptions: *Self* focused on Julie explicitly, consciously positioning herself in relation to mathematics, various communities and groups of people or other teachers. *Students* included Julie positioning herself in relation to a particular student as well as to students in her classroom, school, Granite City, and writ large. *Families and Communities* included references to the Granite City community, other cities and school districts, her personal family, and the PD community. The first author then analysed the text within these three broad codes, grouping similar concepts (e.g., Julie talking about an effective teaching practice) into themes. Transcripts were read and re-read for confirming and disconfirming evidence of each theme.

Findings

In our pre-PD interview with Julie, she noted that she enrolled in this PD largely for financial reasons, as she had just purchased a car. At the same time, she was oriented toward this PD as an opportunity to "get involved, to better my practice, to better myself, and to provide for my students." She had not attended much to the PD goals and aims, perhaps evidenced by her response that what equitable mathematics instruction meant to her was "like an investment, but I don't know what you mean by equitable." A moment later, when reflecting on challenges in supporting equitable teaching, Julie said that accountability was key:

> If a teacher makes a recommendation, it needs to be followed. If it's not followed, then the person that didn't make the change happen needs to be held accountable. Because we're hurting the student. It's not about us. It's not about me…it's about getting what's best for that child.

Julie's assertion that a key feature of her teaching is building strong relationships with students and prioritizing the needs of students was a consistent PD theme.

Strong Relationships, All About the Kids

Julie regularly shared statements across the PD that suggested that she believed she had strong relationships with her students. Prior to the start of the PD, when asked to elaborate on what she meant by getting to know her students, Julie described:

> It allows us to have empathy for our students. There may be a student that comes in and is sleeping in class. You don't know what they went through. You don't know if they were in the hospital all night. You don't know if they had to watch their baby brother because Mom had to go to work. When you have those kinds of things happening in your classroom and the administrator comes in and says, "Why is their head down?" you have an explanation because you're able to show that empathy and understand what the kid is going through. There are times when you may not be doing the curriculum. You may have to stop and talk about those recent shootings in those areas. What if it happens in Granite City?

During the first summer PD, Julie shared stories about students reciprocating care by calling her to see how her mom was doing when her mom was ill and she suggested that she had stronger relationships with her Black students than her White students:

> I feel like my Black students will ask more questions than my White students. I don't know if that is a cultural thing because they feel comfortable talking at home with other people. I mean, I relate better to Black people than I think I do with White people.

During Year 2, after attending a talk with Dr. Jeff Duncan-Andrade, Julie reflected, "I do that on a regular basis. I have that relationship with my students…I'm empathetic. I know that I feel what my students feel. If they're cryin', I'm cryin." At the end of Year 2, Julie positioned treating students with empathy in relation to culture and race:

> I understand that we have cultural issues. We have differences in our lives, and I get that. Every person is different, but just because you are a specific colour, you're a specific gender, that does not matter. It doesn't matter because I believe everybody can learn. I don't think our society believes that, and I think that's why we talk about social injustice so much, and for me it makes

> me angry because instead of talking about it, why aren't we doing something about it. That's what I do in my classroom. I do something about it. I don't look at them because they're Black. I don't look at you because you're Asian. You're a person. I treat you with decency…with humanity, compassion, empathy.

Julie's narrative about having strong relationships with her students was often voiced through her assertion that teaching is "all about the kids." During the first summer, teachers were asked to bring a cultural artifact with which to introduce themselves. Julie shared that she forgot to bring something, but that she knew her artifact was her students. She demonstrated her passionate stance and love for her students with tears:

> Since I was in 9th grade, I've struggled with depression…and it wasn't until a few years ago that I ended up going to a therapist…and it wasn't until that, those couple of years ago, that I realized what my purpose in life is. I have such a love for my students (begins to cry), they saved me. They have taught me so much.

Julie also demonstrated a passionate stance about teaching being all about the kids when she shared stories of speaking back to administrators, justifying her teaching and "standing up for what we believe in." During Year 1, for example, Julie expressed a tension between being "pleasers of the government because they pay our paycheck" and education needing to be all about the kids. She elaborated:

> It's about what the kids need, about what the kids want, and far too often we push kids on and push kids on and push them on, and they are not where they are supposed to be. It's a growing problem that I don't think our government recognizes. We're limited with you've gotta do this curriculum…you've gotta have this on the board. It's very difficult and frustrating because I just want to come and teach. I wanna be here for the kids. I wanna support the kids. If it takes me two days to go over something because they need that time, then I need to have that freedom, and I don't feel we have that.

In Year 2, Julie continued to stress the importance of instruction being all about the kids, even as she acknowledged that this could be exhausting and demanding: "we may be so burnt out, but we still have a responsibility to those kids…we're supposed to be a friend, confidant, mom, dad…the kids are what is supposed to be our main focus." Julie positioned this stance as standing up to those in power:

> We need to stand up for what we believe in. I understand that we are supposed to follow rules and do this and that. I get that, and I'm gonna do that. I'm gonna make sure I get there on time. I'm gonna make sure that I'm teaching my students…If [district mandated curriculum] is not providing my students with the best education, that's where I have to use my professional judgment to modify, to provide those kids with the education they want. I'm not going to provide them a crappy education…it's time we take back our schools, and we take back our classrooms, and we do what we know is best for our kids, no ifs, ands, or buts.

Throughout both years of the PD, Julie asserted teaching as her "mission field" or the place "where I am supposed to be." Julie seems to position her mission not so much about saving students, but about being the best teacher she can be:

> I have always felt that, since I've been involved in my teaching, that this is my mission field. I wanna be the best I can be for these kids, and I can't do that by myself. I need others to support me, to encourage me, to discipline me…to become a better teacher.

This notion of continually growing and working to know her students to "be the best teacher she can be" occurred throughout the PD. At the end of Year 1, Julie reflected:

> I don't feel that I know my students as well as I would like to know them. I would like to know what kinds of things they're going through…I feel like they're my children. I feel like I'm like a mom to them. I love them. I care about them. I don't want anything to happen to them. Far too often we don't know our students like we think we do… We had a student who died in a house fire. Did not know he had a sister. Did not know he had a stepbrother here in school. I didn't know that. Here I'm thinking I know the kid, but I really don't.

In Fall of Year 2, Julie again voiced that she doesn't "always know everything they're interested in" and in the Spring of Year 2 that she doesn't:

> Have the experiences that they have. I've not grown up in the same kind of culture they've grown up in, so what I think might be relevant may not be relevant for those students. I think there's that big cultural difference. I'm still learning.

Anger Toward and/or Dislike of PD Activities

In some instances, Julie seemed to use the defence of instruction being all about the kids as reason not to engage in politics or assertions of Whiteness. For example, during the first summer she stated:

> I'd say about 60 percent of the time, the parents, without finding out the facts, side with the child. Because it's automatic you're out to get them for some reason. I feel like, as a White person, I have that stereotype that I'm going to try to take advantage of a person's colour. That's hurtful to me. Because I love my kids…I will do anything for them.

Later in Year 1, Julie lamented that "people are so political about everything that they forget to think about the kids." She went on to say:

> Even though the district may require something, you can still make it about the kids. It's frustrating to hear how colleagues feel. They're worried about that student evaluation. That's not what you're here for. You're here for these kids. If you didn't meet the expectations…what are you gonna do to improve it? That's how I feel we have to look at it. I think I'm different from how most teachers feel and think, because I want what's best for the kids. It's not about me, it's not about the job, it's about the kids.

Julie also expressed frustration or dislike with PD activities throughout the two years, even if those activities, from our perspective as facilitators, put students first or had the potential to strengthen her relationship with students. For example, in the first summer when provided a handout that listed types of oppression (e.g., racism, elitism, ableism) in Column 1 and then non-target groups (e.g., White people formally educated, temporarily "able-bodied") and target groups (e.g., People of Colour, informally education, people with disabilities) in the next two columns, Julie immediately voiced:

> I can tell you right now I don't like this activity…What I feel when I look at this is division. I feel like, in our country right now, we are dividing ourselves so much already that –I don't look

> at these individual – I look at the person as a whole. I don't necessarily relate with everything on here. In racism. I don't necessarily relate better with White people…it makes me angry because I don't feel that we're looking at unifying as a whole.

Similarly, Julie's discussions of race were often points of resistance to the PD activities. For example, at the end of Year 2 Julie noted:

> I don't look at their colour, their gender…Yes, they're present, but I don't focus on them because they're a human being that is just trying to find their place in this world. Listening to them I think is the best thing that I can do for them, rather than trying to judge them, or tell them they can do this, or they can do that…[later] I understand that we have cultural issues. We have differences in our lives…but just because you are a specific colour, you're a specific gender, that doesn't matter. It does not matter because I believe everybody can learn. I don't think that our society believes that, and I think that's why we talk about social injustice so much, and for me it makes me angry because instead of talking about it, why aren't we doing something about it?

Julie's resistance was not limited to colour-evasiveness. For instance, in Year 2 after engaging in an activity adapting problem contexts to be more authentic to students' lived experiences, Julie expressed that it "gave me anxiety because our students, I don't focus on the same things our students focus on in culture." She went on to say:

> I do see the value in what we're trying to do, but I also see that it is a problem. … because if we look at how tests are presented to our students, and I'm not saying we need to teach to the test, but if they are not exposed [they'll have more trouble].

This desire of wanting the students to succeed academically as a source of resistance to the PD was inconsistent; while here, during Year 2, she expressed resistance to the activity, at the end of Year 1, Julie noted that although she became frustrated with some of the first-year activities, they were a source of growth/learning:

> What I've been learning about myself this year is some of those things that frustrate me the most are the things that are what is making me a better teacher. That frustration turns into memorable moments, powerful moments that help me become the individual that I am. Not necessarily just as a teacher, but as an individual as well.

Discussion

Julie's descriptions of her teaching indicated a caring for her students rooted in ethics and reflected some components of caring with awareness. She is working toward knowing her students culturally, acknowledging difference and race, and recognizing that knowing her students means knowing them beyond the school's walls. Her anger about demands on teachers that she saw as counter to teaching, such as district-mandated pacing, were expressed as care that considered the well-being of the students and could be interpreted as awareness of power structures; she was recognizing that in this realm she can exercise a type of specialized resistance (Collins, 1990). At the same time, in working to know her students beyond school walls, it is not clear that Julie has gotten past "dominant stock stories" (Rolon-Dow, 2005) or that she wanted to know her students well racially. Yet, she also was sometimes reflective and glad for her growth related to learning about racialized experiences and students' cultures.

Applying a colour-evasive framework (Annamma et al., 2017), we see that the ideology Julie has taken up about race constructs race as an objective difference residing in the individual and does not include an understanding of how race has been socially constructed. Nor does it include the impact of the social and material consequences of racism on her students in their role of student or the educational structures that perpetuate racism. Julie's desire to ignore difference essentializes her students and neglects their complete humanity in favour of only specific parts of their humanity. Julie's belief of herself as a teacher who cares seemed to be a strong part of her identity as a teacher, and so it is possible that facilitating her toward growing the caring relationships to be more aware was felt by her as an attack on an identity she cherished, or even an attack on her students (and their opportunities to learn mathematics). Of note, Julie's assertion that she feels "like a mom" to her students reveals gendered assumptions in her role as teacher, and assumptions about what her students need: the unfit mother is a common stereotype of Black urban mothers in the United States (Gholson, 2016) and so Julie's assertion of taking up that role reifies this storyline. Julie's inconsistent colour-evasiveness made it difficult to know how to support her development from a caring teacher to one who cares with awareness.

Julie's responses in the PD align with documented acts of resistance when White people are confronted with conversations about systemic racism. But her inconsistent gender/colour-evasiveness also suggests that PD could meet her at her points of development and reflection and differentiate activities and experiences to target Julie's particular needs. In Julie's case, perhaps we would have leveraged differently her relationship with her students, having her listen to her students' stories of lived, racialized experiences and using her own students' voices as counter-narratives to her idea that talking about race is divisive and antithetical to caring. Or, instead of focusing on a multitude of intersectional identities, her experiences at some point should have been more narrowly focused on systemic racism, gendered uptake of roles, and their connection to students' opportunities to learn mathematics.

We wonder what it might mean to differentiate PD not only across participants and multiple intersectional identities, but also within groups of participants that share some characteristics (e.g., race and gender) but are at different places in their understanding of the role oppressive systems play in mathematics education. This differentiation likely requires PD facilitators to know their teachers, teachers' contexts, and even teachers' students to support each individual participant toward caring with awareness.

References

Annamma, S., Jackson, D., & Morrison, D. (2017). Conceptualizing color-evasiveness: Using dis/ability critical race theory to expand a color-blind racial ideology in education and society. *Race Ethnicity and Education, 20*(2), 147–162.

Apple, M. W. (1983). Word, gender and teaching. *Teachers College Record, 84*(3), 611–627.

Bartell, T. G. (2011). Caring, race, culture, and power: A research synthesis toward supporting mathematics teachers in caring with awareness. *Journal of Urban Mathematics Education, 4*(1), 50–74.

Beauboeuf-Lafontant, T. (2005). Womanist lessons for reinventing teaching. *Journal of Teacher Education, 56*, 436–445.

Collier, M. (2005). An ethic of caring: The fuel for high teacher efficacy. *The Urban Review, 37*, 351–359.

Collins, P. H. (1990). *Black feminist thought: Knowledge, consciousness, and the politics of empowerment.* Harper-Collins.

Gholson, M. L. (2016). Why we can't wait: (Re)examining the opportunities and challenges for Black women and girls in education. *Journal of Negro Education, 85*(3), 290–301.

Grumet, M. (1981). Pedagogy for patriarchy: The feminization of teaching. *New Political Science, 2*(3), 91–112.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* Routledge.

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education, 41*, 236–273.

Herbel-Eisenmann, B., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator, 1*(2), 181–196.

Hytten, K., & Warren, J. T. (2003). Engaging whiteness: How racial power gets reified in education. *International Journal of Qualitative Studies in Education, 16,* 65–89.

Lipman, P. (2012). Neoliberal urbanism, race, and equity in mathematics education. *Journal of Urban Mathematics Education, 5*(2), 6–17.

Martin, D. B. (2013). Race, racial projects, and mathematics education. *Journal for Research in Mathematics Education, 44*(1), 316–333.

Pianta, R.C. (1999). *Enhancing relationships between children and teachers.* American Psychological Association.

Rolon-Dow, R. (2005). Critical care: A color(ful) analysis of care narratives in the schooling experiences of Puerto Rican girls. *American Educational Research Journal, 42*, 77–111.

Thompson, G. L. (2004). Caring and colortalk. In V. Siddle Walker & J. Snarey (Eds.), *Race-ing moral formation: African American perspectives on care and justice* (pp. 23–37). Teachers College Press.

Building agency in children through mathematics: Applying Conscious Full Spectrum Response for developing skills, competencies and inner capacities

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Sustainable, equitable and enduring solutions to the complex problems of the world require not only technical solutions, but also shifts in structural and social norms of society grounded in responsibility and interconnectedness. How do we as teachers look at these aspects? In this action research we develop a perspective through the framework of a Conscious Full Spectrum Response (CFSR) model to develop not only academic and technical skills in Mathematics, but also competencies – using skills to shift systems and culture and inner capacities - self-awareness, self-regulation, responsibility and courage to create. These together build agency – the ability to act and transform based on what I deeply care about. We review case studies of the work of children both academic as well as real life projects with this framework.

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Context

C3STREAM Land (C3 is *C*onscious for Self, *C*onscious for Others, *C*onscious for Environment, STREAM = STEAM + R (Research), henceforth referred to as C3SL) are rural STEAM centres in Tamil Nadu in India. STEM education can become "technology for the sake of technology" and miss out in addressing social, cultural and structural biases and disparities, it can also ignore the development of inner capacities of children. C3SL strives to address these as the deeper purpose of education.

Radical transformational leadership

Radical Transformational Leadership, the book, describes how we can generate equitable and enduring results using a unique response model based on extensive application world-wide in many sectors, themes and topics – the conscious full-spectrum response model. This model is designed for sourcing our inner capacities and wisdom to manifest change that embodies universal values of dignity, compassion and fairness, and simultaneously transform unworkable systems and norms in order to solve problems.

While each of us have an accountability of teaching Mathematics we are also trained through RTL workshops on the distinctions, design templates and tools. The distinctions allow each of us to work out of what I stand for (universal values I deeply care about) rather than out of socialized fear, the design templates allow for alignment of universal values, system and cultural shifts and actions when we design projects; the tools formalize processes that are cognitively coherent with the distinctions and templates.

Introduction to the Conscious Full Spectrum Response model

The Conscious Full Spectrum Response model is used to generate results at scale and addresses complex problems across domains while allowing for alignment in:
Technical Solutions to solve immediate problems, e.g., employment, education.
Shifting patterns and unworkable system and societal norms required for sustainability of the technical solutions, e.g., policies, casteism, race, gender.
Underlying factors of what we deeply care about – why we want these shifts and how we act when we embody these universal values, e.g., dignity, equity.

We give an example of C3SL in Figure 1 mapped to the CFSR model and also set the context of this work. The outer circle of universal values of C3SL are responsibility, equality and the courage to create. We want to see these values in the children we work with, in ourselves and in what we do.

The middle circle addresses systemic and cultural causes here we work in rural schools we address not only the digital divide, but also use the work on STEAM to interrupt the common ISMs like genderism, class and casteism in rural India. We work equally with both boys and girls in STEAM education. We also move from mediocracy to excellence with the older children taking responsibility of learning to plan and set their goals for the week. They choose to do this individually, in pairs, in groups and in consultation with facilitators. They are also provided RTL training for children to move from dependence on teachers to independence to interdependence and creating a learning community.

In the inner circle of technical solutions, the children have access to Mathematics materials, strategy games, puzzles that help them engage with Mathematics and play games. They have access to computers where they program in Scratch, Geogebra and Alice and also 3D modelling and printing. They also have access to electronics, Makey-Makey and other materials interacting with engineers who work in the industry. These help children not only address their curriculum, but also create projects that demonstrate their mastery on topics learned. With younger children we work on making Mathematics with real life (Education by Design) EBDs projects that they can do on themes they care about.

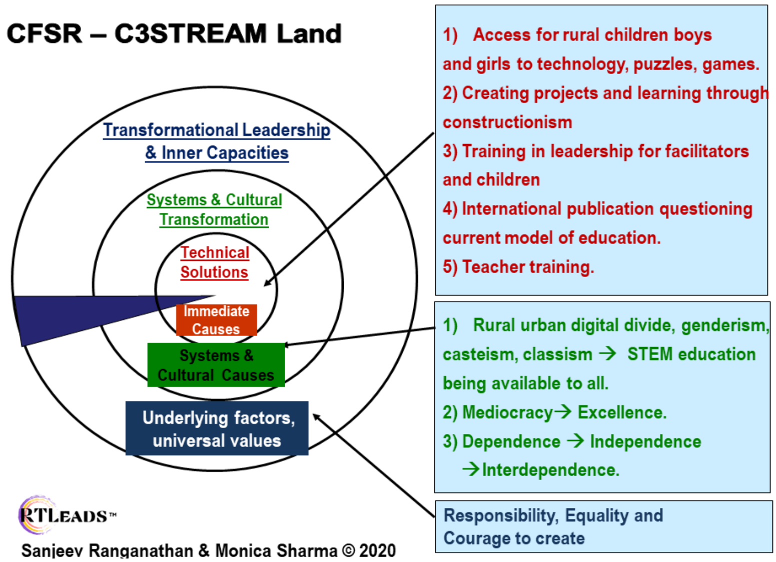

Figure 1: C3SL as an example of application of CFSR model.

All these move the children from mediocracy to excellence. The details of the activities of C3SL (formerly known as STEM Land) are documented in detail elsewhere (Ranganathan et al., 2018, pp. 294–302).

This research is conducted in two outreach schools of Auroville – Udavi School and Isai Ambalam School. The children attending come from villages surrounding Auroville. Udavi School follows the state board syllabus and we worked with 80 children from 6th to 10th grades intensively for 5 hrs/week for all their Mathematics classes. Isai Ambalam School follows the central board syllabus and we work with 71 children from 3rd to 8th grades intensively for 6 hrs/week as well as during Saturday activities and sleep overs for Mathematics as well as Environmental Sciences (EVS). In demographics, the primary occupation of parents in both schools is in unorganized labour, e.g., masons, painters, agricultural labours and schemes providing rural employment. The predominant community accessing Udavi School is MBC (Most Backward Caste) and that accessing Isai Ambalam School is SC (Scheduled Caste).

Philosophies underlying C3SL

The philosophy underlying the approach for C3SL is based on the principles of progressive and constructivist thinkers like Jerome Bruner, Seymour Papert in the United States, Sri Aurobindo in India. The philosophy of Sri Aurobindo of the integral development of the child (Aurobindo, 1921, pp. 1–8) emphasizes self-knowledge and assumes an important relevance in the recent National Education Policy (Government of India, 2020, p. 12) that is based on his work and states that "knowledge is a deep-seated treasure and education helps in its manifestation as the perfection which is already within an individual." The philosophy creates guiding principles as teachers and in how we engage with children. The three principles of true education by Sri Aurobindo are:

- Nothing can be taught
- The mind needs to be consulted in its own growth
- From near to far

The first principle can be linked to the constructivist theory that knowledge cannot be forced into the mind of a child. The role of a teacher is not to mould or hammer a child into the form desired by the adult. The teacher is a guide, or mentor that supports and encourages a child in the process of learning, enabling them to evolve towards perfection. Our engagement with children follows this principle.

The second principle indicates that the child needs to be consulted in his/her learning. This is done at C3SL as the elder children plan what they want to work on and how they want to organize themselves to do it with the broad ground rules of respecting themselves, others and the materials. With younger children this aspect was put in practice in the co-creation of challenges along with them.

The third principle is to work from near to far. To work from what is tangible and accessible to children to what is abstract to them. The children work on projects they care about in the environment they engage with and as they grow older move towards more abstract ideas. This paper will present projects both in the physical world and also in the abstract world.

Self-awareness and personal transformation are necessary, but not sufficient for social transformation. In this paper we take up a theoretical framework for social transformation that is aligned with values. This paper focuses on the application of the Conscious Full Spectrum Response capacity building framework that we use to review what we may be accomplishing through Mathematics and EVS.

Theoretical framework of CFSR for capacity development

We use the framework of a CFSR (Conscious Full Spectrum Response) capacity development (Monica, 2017, p. 236) as shown in Figure 2.

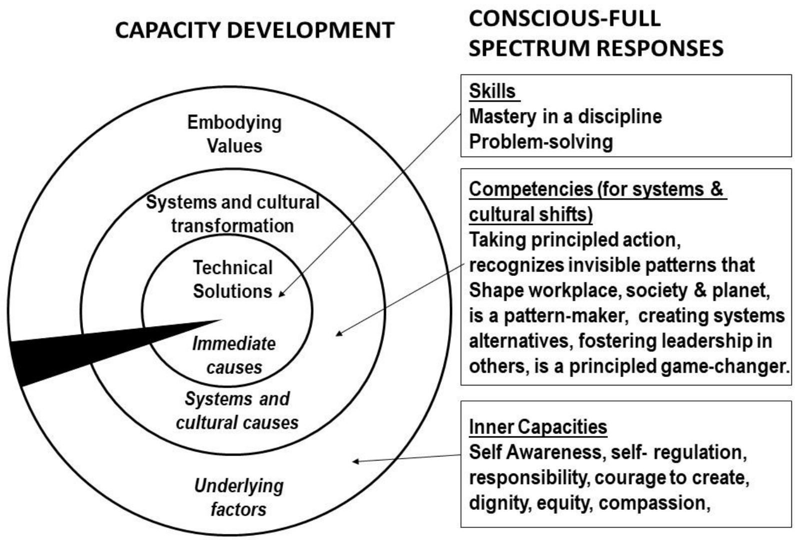

Figure 2: Capacity Development for Sustainable Results

A CSFR based capacity development simultaneously addresses:

1. Skills (inner circle) to generate technical solutions to address immediate causes. We look at mastery of the concepts as well as problem solving.
2. Competencies for systems cultural transformation (middle circle). We look at the ability of applying these skills in different contexts, pattern/system thinking, as well using skills to build healthy patterns in how children interact and learn.
3. Inner capacities to embody universal values (outer circle). We look what we noticed about children's responsibility, care and courage to create alternatives

Methodology

The topic/project and how the children went about creating them are described in each section. The skills are listed by analysis of the final product by the teachers. The competencies were observed by the teachers in the duration of the project and in conversations with the children. Inner capacities are not measured, but reflected. Opportunities were created for children to reflect on these and what were noted are derived through conversations on their reflections.

We will take a few case studies and deep dive into one of them to look at how these aspects are both supported and observed.

Case studies and observations

We first look at academic challenges and then at real world challenges. Can learning Set theory and algebraic identities be transformational?

Sets

A few children from 10th grade had built a physical game with a chart and materials with Sets. This physical game inspired Diva, a 9th grader the built a game in Scratch (a visual programming language) on sets shown in Figure 3. Each circle represents a hidden rule of either shape or colour. The player needs to determine the rules by guessing where the pieces fit. Programming helps children learn concepts because they need to break it down into simple instructions while improving problem solving, logical reasoning. The use of programming to develop mathematical thinking at C3SL has been documented before (Ranganathan, 2015, pp. 339–346). In this case, Diva realized that in order for the computer to understand which region of the Venn Diagram was being sensed by a new token (sprite) he needed to divide the Venn diagram into different regions (A-B, B-A, A∩B and U-AUB) helping him learn these better. He first made the game with a fixed rules and later generalized it for the computer to randomly pick the rules so it would be a challenge for him too. At C3SL we have sessions where children share their projects. Diva presented his project at one of these sessions. His presentation got the 8th graders, who were not expected to learn about sets, to learn about sets.

C3SL also organizes courses to learn programming to that gets children to make smaller projects and learn through interactions with peers and facilitators. After one such course the following project was created.

Algebraic identities

A few children from 8th grade created projects on algebraic identities. For example, Jan made a program that drew $(a+b+c)^2$ as three squares, i.e. a^2, b^2, c^2, and $2ab, 2bc, 2ca$ as areas of rectangles. Images such as these were also available in the text book, were static, but when children created them in scratch they were able modify their programs to use variable lengths for a, b, c and see for themselves that even with the different lengths the identity still held.

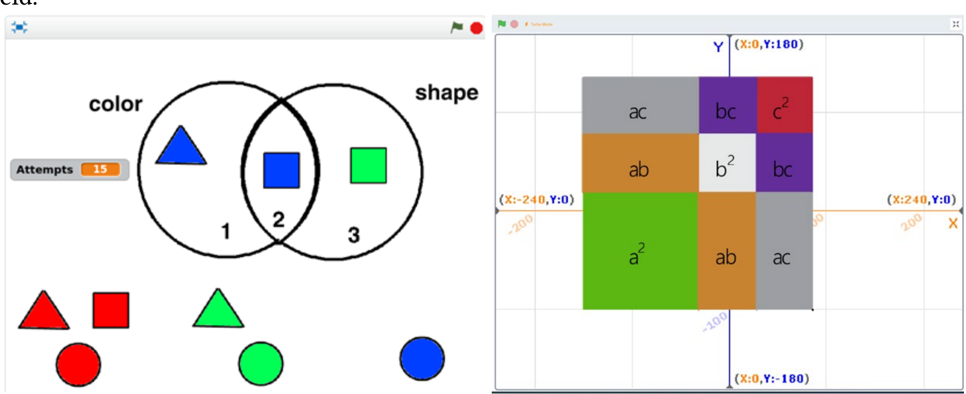

Figure 3 & 4: The two 2-sets game in scratch, algebraic identity in scratch

Observed skills, competencies and inner capacities

In the two virtual activities above through the CFSR framework for capacity development we see:

Skills: In academic skills they learned the different sections of overlap of Venn diagrams with 3 sets (A∩B∩C, A∩B-C, etc.), understanding that rules (descriptive form) can be used to define sets, deriving derive descriptive form from elements, algebraic identities. In programming, they learned interactive queries (sensing), drawing different shapes (pen), functions in scratch, variables, for loops, if, repeat.

Competencies: Ability to create projects to share their ideas, break down a problem into smaller components, move from specific implementation to generalization, shifting from dependency on teacher to independently working on projects to interdependent learning from peers and supporting peers learn through sharing projects.

Inner Capacities: Care – sharing knowledge & Courage to create.

Needful things co-operative (shop) project

We will now take an example of a real-life challenge and describe the methodology we follow in the guiding process as well as reviewing through CFSR model in practice.

In Isai Ambalam School the 7th and 8th graders had difficulty understanding profit and loss. Such skills (inner circle) could have been addressed by theoretical problems and even a mock market within the grade.

With most topics as teachers, we attempt to create opportunities for children to explore and understand the world around them and asked them to research what and where they buy the things they commonly use.

On looking at the prices of stationary in the shops they found that the price for the same product varied and the local shops in the village which were charging too much. The children began to wonder what is the ‘real’ price of a product. The children also noticed that it was not always easy for the young children to have access to shops for small items they needed like pencils, erasers, scales, notebooks that their parents were not always able to provide at the required time. Sometimes such explorations only support understanding, but the children felt a need to act and create a system that addressed this dependence on parents for time for purchases move towards independence of children and interdependence within the school. They decided to open a small makeshift store within their classroom at breaktime. This is the middle circle of looking at patterns and wanting to shift them.

Before starting the shop for a couple of weeks teachers organized group discussions on various topics, e.g., what are the needs of children, items that could practically be stored, investment required. The children surveyed and found preferred items that they would need to have in stock at the store. Practically, none of them had a background to fund the amount required. In conversation with their teachers the children felt that since it was a collective initiative it should not have distributed funding. They broke the amount down into 40 investors including the children themselves, volunteers and well-wishers of the school. For this they created a small kit for investors highlighting what they were attempting to do, the benefits it will give children, a period for which the funds would be locked and a small return that the investor could get.

Once the finances were raised and the items were purchased by the teachers in bulk from wholesale shops. The next set of discussions the teachers had with the children were on how things will be priced to meet all expenses including travel expenses, how it would be advertised, location of the shop, timings, roles and responsibilities.

Planning and Accountabilities: Children came up with several criteria for their shop including for investors, accounts, team work, being fair, following 5S system (Sort, Set in order, Shine, Standardise, Sustain) in their shop. They also came up with marketing strategies - cheap and best and rules of their shop - No borrowing, Fixed price and No bargaining.

Children divided their accountabilities among themselves − an accountant who collects all the cash and gives a bill, two helpers to sort the stationaries and arrange them in the appropriate place, a shopkeeper who gives the items a customer needs and one person to check the stocks at the end of the day. They exchanged their roles while maintaining the rigor of practice including those that included keeping the place clean. When keeping the shop space clean where they interrupted genderism when cleaners at the school initially objected to a boy doing a 'girl's job' the children stood for equality.

Addressing real life challenges also allowed them to demonstrate a variety of skills that academic classroom didn't and that we had not perceived in them in an academic classroom. The children ran the shop till the end of the schooling year and also realized that there are many other costs like electricity, rent, labour that they had waives for them to be able to make the products available significantly cheaper at the school.

Skills: Children learned to keep stock, write receipts, handle accounts and pricing, understand profit and loss, proportions, ratios and scaling, e.g., an individual item from a packet. Conversion from inches to milli meters, different angles, measurements, marketing strategies (by advertisements and attractive offers).

Competencies: The children noticed the patterns of how shops sell and noticed gaps in both the quality and pricing in local shops. They demonstrated the competencies to enrol others raise the investment for their initiative, to work as a team, allocate accountabilities among themselves and interrupt genderism. They moved from dependence on parents to have to find time to purchase stationary to interdependence and were able to handle real-life issues which helped us notice our own biases in children's capacity that was based on academic interactions.

Inner Capacity: We found that the children took responsibility and stood for well-being and care for children, demonstrated the courage to create an alternative. Further they found something each of them excelled at and felt more confident about themselves.

Pond Repair

The second case study is in Isai Ambalam School with real life projects. Taking responsibility for their school and surroundings, such as the water issue. The children had created a pond (Iyyanarappan et al., 2019, pp. 894−898). However, within a year the pond developed cracks due to roots from trees nearby. The children felt that they did not want all the work that they have done to go in vain so children wanted to create a stronger structure that would last.

Figures 5 & 6: Building mesh structure

The children supported by the facilitators built a frame in the shape of the pond and through this they learnt to bend metal rods (6mm and 12mm) at specific angles such as 90°, 45° etc. They also learnt unit conversion from inches to cm for buying the appropriate rods and to cut them in right dimensions. Once the frame was done, they mixed Reinforced Cement Concrete (ratio 1:2:3; cement : granite gypsum : sand) and poured into the structure filling all the rods and finally got some adult help to smoothen it.

Through this process they learnt angles and frames as well as ratios and proportions with more than two quantities. We observed children who are less engaged in academic classes are enthusiastic in building with their hands. In this example we have looked at building technology as a way for children to learn.

Skills: Children learnt conversion from inches to milli meters while building the mesh structure of the pond, angles such as 90°, 45° while bending the rods, they learned to measure length, calculated the circumference and how they wanted to mesh the pond. They learned to mix in the right proportions for the RCC mix and of course the practical skills of creating structure meshes and preparing the reinforced cement.

Competencies: The children took responsibility for what they have built, noticed the gap in what had been missed, worked as teams and shared learning and knowledge with each other, faced real-life problems and got the support they needed by enrolling partners.

Inner Capacities: Responsibility, self-awareness about what they cared about in the environment they wanted, Care – sharing knowledge, Courage - ability to create projects.

C3SL initiatives to support collaborative learning

As mentioned at C3SL we have sessions for children to share projects and conduct programming courses for children. We also work on initiatives across the schools we work with, e.g., a Rubik's cube tournament. The goal of the tournament was not so much to find the fastest solver, but to encourage people to learn to solve the cube. This included sessions at the tournament to learn the cube and teachers at the schools who were inspired by the children also learned to solve it from the children. This interrupted ageism where even teachers not part of C3SL were willing to learn from children.

We created open challenges for children to create videos for children to teach what they had learned visually with materials or drawings, e.g., integers. Children looked at different ways of demonstrating with materials integer addition, subtraction, multiplication and division. We used these videos across grades to encourage children to learn from each other.

Conclusions

In this paper we discuss the Conscious Full Spectrum Response model both in terms of a design template as well for capacity development that is needed for enduring and sustainable changes in the world in line with universal values. We give examples of the use of this model as a template of design for C3SL as well as how we used it observe what we are accomplishing with children beyond academic and technical skills.

Such cognitively coherent framework allowed us to step beyond the comfort of our primary accountability as Mathematics teachers and assume the responsibility of global citizens and community leaders. It requires us to work on technical skills needed to solve immediate problems, competencies of using skills to shift culture and systems by noticing systems and patterns and learning how to work together towards interrupting disempowering ISMs, while being aligned with universal values such as responsibility, equality and courage to create.

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References

Aurobindo, Sri. (1921). *The Human Mind, A system of National Education*, Tagore & Co. Madras (pp. 1-8).

Government of India. Ministry of Education (2020) *National Education Policy* [pdf]. Retrieved from: https://www.education.gov.in/sites/upload_files/mhrd/files/NEP_Final_English_0.pdf

Iyyanarappan A., Anand B., Saminathan L., Kumar N., Ganesan P., Patchaiyappan P., Perumal R., Ranganathan S., Bharathi S., & Kothandaraman S. (2019). STEM Land (Science Technology Engineering Mathematics Land): Deep Learning of Mathematical Concepts Through EBD and by Using Materials, *Proceedings of Tenth International Mathematics Education and Society Conference* (pp. 894-898).

Monica. S. (2017). *Radical Transformational Leadership: Strategic Action for Change*, North Atlantic Publishing, at Berkeley, California (pp. 236).

Ranganathan S., Iyyanarappan A., Anand B., Kumar N., Patchaiyappan P., et al. (2018) STEM (Science Technology Engineering Mathematics) Land: Fostering Responsibility for Learning in Rural Schools, *Proceedings of epiSTEME 7* (pp. 294-302).

Ranganathan S., Anand B., Kothandaraman S. & Gunasekar V. (2015). Using Programming with Rural Children for Learning to Think Mathematically, *Proceedings of epiSTEME 6*. pp. 339-346.

Mathematics in vocational education: An epistemic framework

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In this methodological paper, we present a framework, which was developed in an action research project where mathematics teachers and vocational teachers collaborated with a researcher. With this epistemic framework we challenge the view where mathematics is taken-for-granted as the theoretical knowing which is applied to the practical vocational knowing. The purpose of the framework is to aid teachers and researchers to capture theoretical and practical aspects of mathematical and *vocational knowing, both when the subject areas are separate, and when they are intertwined, in collaborative teaching. This way, power relations between different teaching contents may be reconstructed, which is helpful for collaborative teaching, and for students' learning in both mathematics and vocational teaching content areas.*

Ways of investigating mathematics in relation to vocational contexts

In the literature, mathematics in relation to vocational contexts in educational settings is conceptualised in various ways. We will describe examples of these here in the introduction, and then we will put the emphasis on presenting and elaborating on an epistemic framework addressing theoretical and practical aspects of mathematical *and* vocational knowing. The framework has been presented elsewhere with a focus on general design theoretical aspects (Boistrup & Hällback, in press), and here we focus mainly on epistemic aspects of mathematics in relation to vocational knowing. We discuss the framework in relation to how mathematics can be made relevant to students who, as a group, often experience obstacles in learning mathematics within vocational education.

One perspective adopted in the literature is Bernstein's theory of pedagogic practice (2000), which has informed research on, for example, how mathematics is recontextualised in different workplace activities. One example is FitzSimons (2015), who adopted the concept of recontextualisation by Bernstein when investigating the vocational mathematics taking place within the workers' own industry workplace. In this project, workers could identify unsuspected ways of the mathematics they already knew being transformed into authentic workplace activities, while at the same time appreciating their own roles within the overall setting of the workplace. Another theoretical perspective adopted in the literature is activity system theory by Engeström (e.g., 2001). A recent example is Frejd and Muhrman (2020) who

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investigated the learning space available for vocational mathematics education when carried out by teams with one mathematics teacher and one vocational teacher teaching collaboratively, as in this paper. The authors adopted the Engeström model in order to investigate notions like tools, norms, the division of labour and the community.

In critical mathematics education research with an interest in the characteristics of mathematical content in teaching, Chevallard's (e.g., 2006) Anthropological Theory of the Didactic (ATD), including the concept of praxeology, is quite often adopted (see, e.g., Straehler-Pohl & Gellert, 2013). Praxeology addresses two, albeit connected, dimensions of mathematical knowledge where praxis is *know-how* and logos is *know-why* (described below). Even though this model is mainly adopted in the literature of mathematics education, it is possible to adopt ATD also in other disciplines. This is described by Ladage, Achiam, and Marandino (2019), where one example is the didactic work in a museum. Another example is Quéré (2017), who investigated "mathematics in the workplace" in the context of engineering work in France. One outcome was that mathematics should not only be considered as a "tool" because engineers sometimes need to have an accurate understanding of the actual tools they use. Castela (2015) discussed the opportunities of adopting ATD in research on mathematics in connection to other knowledge areas, such as dress-making. The following quote highlights our rationale for choosing ATD as the framework we present:

> This anthropology of the mathematics should investigate social practices without too narrow restrictions on what is an interesting object. That is why I consider the praxeological model as previously presented as an interesting tool. It highlights dimensions of the institutional cognition that would be neglected otherwise, especially when the reference to acknowledged mathematics is too strong. (p. 18)

Background of the project

Boistrup, Bellander, and Blaesild (2018), in a study by the first author of this paper together with two teachers, drew on Bernstein's concept of recontextualisation to identify how mathematics was relocated and transformed (i.e., recontextualised) in vocational education (construction work). One conclusion, roughly speaking, was that there are at least two distinct ways to recontextualise mathematics in vocational activities. One is the explicit use of mathematics in, for example, problem solving, and another is mathematical concepts and methods being integrated –more implicitly – into the vocational activity. This project was helpful in finding ways to describe the interfaces between two teaching content areas, mathematics and construction work. Not surprisingly, in a survey with open questions, the students described that they found mathematics more accessible when the mathematics teacher and the construction work teacher taught together and discussed the different content areas' relationships. Since mathematics is the teaching subject which causes most problems for students in the Swedish upper secondary school to pass, this was of course good news. A surprise to the teachers and researcher in the project was that the students also expressed that the involvement of mathematics helped them in learning the vocational content.

In a subsequent action research project, in which the authors of this paper took part, new steps were taken in order to go into more detail on how epistemic aspects could be understood and interpreted when mathematics is taught in connection to vocational teaching content. The reason was that when the mathematics teacher and the vocational teacher taught collaboratively for one class per week, they wanted to move beyond a focus on practical matters and on which tasks to bring into the teaching. Instead they aimed at making mathematics relevant for the students in relation to the vocational teaching content, and to focus on explanations and reasoning in both teaching content areas. The researcher's choice then was to turn to Chevallard (2006) and his model of praxeology, both because praxeology consists of concepts which articulate epistemic aspects (described below), and because it is possible to adopt praxeology for analysis in a broad range of disciplines, including vocational knowing.

Why this epistemic framework?

The purpose of the framework is to aid teachers and researchers to capture theoretical and practical aspects of mathematical and vocational knowing, both when the subject areas are separate, but also when they are intertwined, as in collaborative teaching. This way, the power relations between the different teaching content areas may be reconstructed, which is helpful for collaborative teaching, and for students' learning in both mathematics and vocational teaching content.

We challenge the dichotomous conception where mathematics is taken as the theoretical knowing as opposed to the vocational knowing which is taken as purely practical. Rather, we view theoretical work as being developed by means of a variety of resources (such as body movements, artefacts, speech, and the like), maintained and changing over time in human practices (Selander, 2006) such as vocations and mathematics, as in the case of this paper. Rosvall, Hjelmér, and Lappalainen (2017) point to a connected tension between workplace and so-called academic knowing in vocational education in Sweden and Finland. This tension, as Rosvall et al. write, is exacerbated through the idea of a vocational learner as being practically oriented; using their hands instead of their heads and positioned as being in need. Such ideas constitute institutional norms, affecting the setting in which the teaching is designed. Through the framework presented in this paper it is possible, in research and teaching, to move beyond such ideas, and to strive to actively identify theoretical and practical aspects of different teaching content areas (in the case of this paper, mathematics and hair & makeup styling).

In Sweden, a large part of vocational education is included as study programs within upper secondary school, alongside study programs aiming for university studies. In these programs, the curriculum covers both the knowledge specified for the vocation in question, including periods of practicum, and general knowledge areas, for example English and Mathematics. In some schools, the vocational teacher and the mathematics teacher teach one lesson together each week, at least with the 1st year students. At one such school, a mathematics teacher (Hällback) was given the responsibility to take the lead in a process of developing collaborative teaching between mathematics and vocational teachers, affording possibilities of learning for the students. As part of this, Hällback made contact with a researcher, Boistrup, and a joint action research project was initiated with four teachers at the school and one researcher.

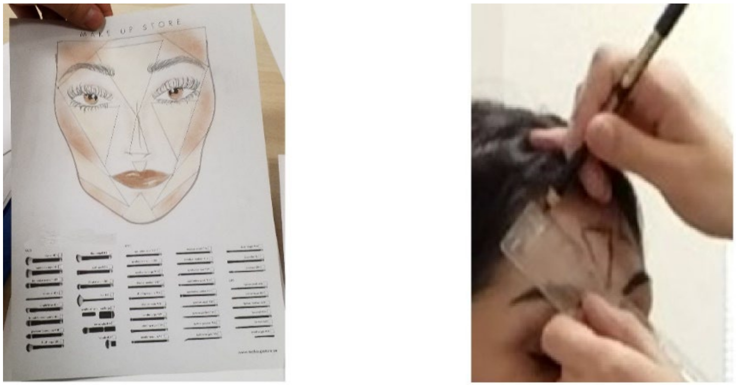

Figure 1: Student's face chart (left) and a student drawing triangles (right).

The framework of this paper was developed as part of the project and was helpful for the teachers in each team (one mathematics teacher and one vocational teacher). The examples in this paper derive from a lesson in the vocational knowledge area of styling, where the focus was on how to use triangles when doing facial makeup (see Figure 1). The lesson was video recorded, and documents and artefacts were photographed.

Theoretical underpinnings

In our action research project, we had a great interest in understanding vocational knowing aspects in relation to mathematics and, as mentioned above, we chose to draw on the model of praxeology. Praxeology is a model addressing the characteristics of knowing, and is part of Chevallard's (2006) ATD. Castela (2015) described ATD as being "interested in the processes and products of what we may consider as the institutional cognition, that is to say, in how institutions develop their socially acknowledged capitals of practices and knowing" (p. 8).

According to Chevallard (2006), praxeology is constituted by praxis and logos and offers us a foundation for addressing the practical and theoretical aspects in, and the connections between knowledge areas such as mathematics and vocational knowing. *Praxis* (know-how) concerns *tasks* (types of assignments) and *techniques* (procedures with which the task type can be carried out). An example of a vocational task in the project was curling hair with three different kinds of curls, while the mathematical task was to identify the angles of 45, 90, and 180 degrees, respectively, between the loop of hair and the surface of the skull. When curling the hair, aspects of the vocational technique concerned for example how to capture a loop of hair with the curling iron in a functional way. A mathematical technique could be to direct the loop of hair in the proper direction for the angle to be the intended one. *Logos* (know-why) concerns *technologies* (why a procedure works in the way it does) and *theories* (overarching structures on a general level). An example of a technology connected mainly to styling was why the curling iron needs to be handled in a certain way in relation to how it affects the hair, while a mathematical technology was the explanation of why a direction of a hair loop creates a 90-degree angle and not 180 degrees. The main function of theory is then to provide a basis for the technology (Bosch & Gascón, 2014). This basis may be constituted by axioms, traditions, research findings, or theoretical assumptions. An example of a theory connected to the vocational knowing from the data was the overarching knowledge about hair styles, where curling all hair loops with the same angle creates for example a hair style similar to what Marilyn Monroe had. Examples of theoretical aspects more connected to mathematics were what constitutes the concept of an angle including the correct mathematical terminology.

We also drew on a theoretical perspective, based on social semiotics and institutional theory: Designs for Learning (DfL) (Selander, 2021). In DfL, the setting is always part of the analysis of teaching events, incorporating the resources available and the institutional norms which may restrict and/or provide opportunities for the teaching. Furthermore, in DfL the multimodal character of all communication is emphasized, with an interest in how knowing is transformed between and within modes (for example, speech, text, images, symbols) in all communication. These modes hold affordances for students' learning.

The framework

Through the project, we identified the praxis and logos of both vocational and mathematical knowing. The four Ts in the model — task, technique, technology, and theory — are intertwined and constituted by each other. Analytically, we identified them in the data from the collaborative teaching in styling and mathematics. For the analysis of knowing aspects, we developed a model (Figure 2), which reflects the epistemic framework.

	Styling				Mathematics
Task					
Technique					
Technology					
Theory					

Figure 2: A framework for capturing a continuum of aspects of knowing within styling and mathematics, and the intertwinement of both.

The framework in Figure 2 provides the opportunity to identify epistemic aspects that are a mixture of styling and mathematics, as well as aspects "belonging" more to either of the two knowledge areas, while at the same time identifying the praxis (task and technique) and logos (technology and theory) aspects.

Examples from collaborative teaching

When it comes to the setting of the makeup lesson, the location was the styling teaching rooms with the consequence that the learning resources connected to styling were there, with mirrors, hair and makeup materials, et cetera. The mathematics teacher added learning resources to the teaching as well, as will be shown below. The curriculum in the sense of teaching content derived from both styling and mathematics, with knowing aspects concerning carrying out makeup through highlighting and contouring, and handling triangles from a mathematical perspective. The institutional norm, that there is value in drawing on vocational knowing in the teaching of mathematics, is very much present. The overarching assignment of the lesson was a combination of these knowledge areas: to carry out facial makeup through the use of triangles. This overarching assignment consisted of task types, for example to know which parts of the face to highlight, and which parts to make darker through contouring. In order to make the modes and resources clear, the video episode is transcribed multimodally, with columns addressing various modes (see Excerpt 1).

In the first example, the styling teacher (D) discusses how the students should proceed to place triangles on the face (see Figure 1b). Normally, a stylist does not draw triangles on a person's face with clear lines, but during this lesson students would do this to emphasize the moment of seeing triangles in the face while doing makeup. Seeing the triangles facilitates, among other things, the work of making a face more symmetrical through make-up (which is a theoretical aspect, not highlighted by the teacher during this event). Before this event, D has asked some students where they can find relevant triangles on the face:

Time	**Speech**	**Body movements and resources**
20:07	D: You are really good; you know exactly where on the face the triangles should be.	Dips his makeup brush on the back of the hand, stands slightly forward leaning
20:08		Carries out make-up on a student under the eyebrows with light strokes
	[...]	
20:16	D: Here, here we should have light, right? And then we take all that	D puts a lot of makeup on the student's jawbone and tries to show a clear triangle

Excerpt 1. The styling teacher (D=Divo) discusses positions of triangles with a student (S).

The focus in Excerpt 2 is on the actual *technique* of doing makeup with the support of triangles. D articulates this in words ("…you know exactly where on the face the triangles should be") in coordination with body movements and resources. The word "triangle" communicated through speech is transformed by D into body movements when he shows triangles as part of carrying out the technique of makeup. This transformation is part of making the technique clear to the students. This event received the following position in the framework (Figure 3):

	Styling				Mathematics
Task					
Technique					
Technology					
Theory					

Figure 3. The interactions in Excerpt 1 interpreted as a technique with the focus equally on styling and mathematics.

The placement in the middle column in Figure 3 is due to the fact that D largely directs the students' attention to the triangles of the face (part of mathematical knowing), at the same time as he does this from a styling perspective.

The next example is that the mathematics teacher (M) and a student are looking for triangles on a face chart, as a basis for the makeup to be done (Excerpt 2). As mentioned previously, *theory*, in praxeology by Chevallard (2006), is about overall knowing and ideas which form the basis as to why *technologies* can explain certain *techniques*. The example in Excerpt 2 was interpreted to display knowing reflecting different areas of the model in the framework.

Time	**Speech**	**Body movements and resources**
23:01	M: Exactly mm… and then you can imagine that you have a triangle kind of like this… or?	Is squatting beside S. S looks at M, who makes a triangle in the forehead with her fingers
23:04	M: Or you want it so that it…	Shows with her fingers in the opposite direction.
	[...]	
23:24	M: Hey,look at you! You can really find many triangles.	Looks at the student.
	[...]	

23:32	M: Here we can make rather small triangles. Like this, right?	Shows with her thumb and index finger how thin the triangle may be.
23:34	M: Which has a small base.	Shows with both index fingers in the air, how small it may be.
23:26	M: But with higher sides, kind of.	Draws upwards with the index fingers. S nods.
23:29	M: So… there, right? Right under...	Points at the cheek of the face of the student's paper. S nods.

Excerpt 2: The mathematics teacher (M) and a student (S) are looking for triangles on a face chart, as a basis for the makeup to be done.

Excerpt 2 shows that M emphasizes that triangles can look very different from each other. The aspects of the interaction where they looked for triangles of the face, led to the middle box in the row of technology, since they were justifying how and why to use triangles when doing makeup. At the same time, they discussed the properties of the different triangles found, and this was inferred to concern theoretical aspects of mathematics:

	Styling				Mathematics
Task					
Technique					
Technology					
Theory					

Figure 4. The interactions in Excerpt 2 interpreted as a technology with an equal focus on styling and mathematics, drawing on mathematical theory.

M also uses various terms that are relevant to descriptions of triangles, such as the word "base." This belongs to the overall mathematical knowing, which justifies the placement in the box in the lower right corner of Figure 4.

Discussion

In this paper, we have illuminated how it is possible to carry out a detailed analysis of data from collaborative teaching, with attention to aspects concerning for example transformations between modes/resources, and practical and theoretical aspects of knowing. With the attention to how theoretical and practical aspects of knowing can be understood as part of practices, we have contributed with a framework which challenges a dichotomous understanding of mathematics in relation to vocational knowing. The framework of this paper can be viewed as a didactic model possible to adopt for teachers and researchers with a specific interest, such as collaborative teaching in mathematics and for example vocational subjects.

From a design theoretical perspective, this paper illuminates the setting of the teaching in terms of the institutional norms of the school, where it was promoted that mathematics should be integrated with vocational teaching for one lesson a week. Another setting aspect concerns resources where the paper describes how the styling classroom had many authentic artefacts which helped to strengthen aspects of knowing in styling, and also in mathematics. This is similar to the findings by Frejd and Muhrman (2020), although the framework of this paper extends the multimodal focus of the analysis, which also deepens the understanding of the meaning making that was afforded through the teaching. One example in the form of an artefact was the face chart which is normally used in styling practices, but was now also used with a focus on mathematical aspects such as symmetry and triangles. This focus, in turn, helped the styling content to be articulated more clearly. Through our attention to how knowing aspects were transformed between modes, we also identified the extent to which the knowing aspects reflected mathematics and styling, and also whether they were mainly about praxis or logos.

This paper aimed to take both mathematics and vocational knowing seriously, and to not take for granted that mathematics in vocational activities is not simply materialised as school mathematics. The framework of this paper adds ways of capturing the character of mathematical and vocational knowing in an educational setting, and how these knowledge areas can relate to each other in many different ways, and through a variety of modes. We argue that such a framework is helpful in making mathematics relevant for students attending vocational education programs, not least since it was shown in the project that a deliberate epistemic focus created opportunities for teaching and learning in both mathematics *and* styling.

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References

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique.* Lanham, MD: Rowman & Littlefield.

Boistrup, L. B., Bellander, E., & Blaesild, M. (2018). Mathematics in pre-vocational education: A model for interfaces between two different teaching contents. In A. Rogers, B. Street, K. Yasukawa, & K. Jackson (Eds.), *Numeracy as social practice: Global and local perspectives* (pp. 76–98). London: Routledge.

Boistrup, L. B., & Hällback, M. (in press). Designing and researching vocational mathematics education. In L. B. Boistrup & S. Selander (Eds.), *Designs in research, teaching and learning: A framework for future education.* London: Routledge.

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories in mathematics education* (pp. 67–83). London: Springer.

Castela, C. (2015). When praxeologies move from an institution to another one: The transpositive effects. In *Mathematics, science and technology education for empowerment and equity, 23rd Annual meeting of the SAARMSTE* (pp. 6–19).

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the 4th conference of the European society for research in mathematics education* (pp. 21–30). Sant Feliu de Guíxols: CERME 4.

Engeström, Y. (2001). Expansive Learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), 133–156.

FitzSimons, G. E. (2015). Learning mathematics in and out of school: A workplace education perspective. In U. Gellert, J. Giménez Rodríguez, C. Hahn, & S. Kafoussi (Eds.), *Educational paths to mathematics. A C.I.E.A.E.M. sourcebook* (pp. 99-115). Cham: Springer.

Frejd, P. & Muhrman, K. (2020). Is the mathematics classroom a suitable learning space for making workplace mathematics visible? – An analysis of a subject integrated team-teaching approach applied in different learning spaces. *Journal of Vocational Education & Training*. Advance online publication. https://doi.org/10.1080/13636820.2020.1760337

Ladage, C., Achiam, M., & Marandino, M. (2019). Research on ATD outside mathematics. In M. Bosch, Y. Chevallard, F. J. García, & J. Monaghan (Eds.), *Working with the Anthropological theory of the didactic in mathematics education: A comprehensive casebook* (pp. 12–30). London: Routledge.

Quéré, P. V. (2017). French engineers' training and their mathematical needs in the workplace: Interlinking tools and reasoning. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Mathematical Society for Research in Mathematics Education* (pp. 2233–2240).

Rosvall, P.-A., Hjelmér, C., & Lappalainen, S. (2017). Staying in the comfort zones: Low expectations in vocational education and training mathematics teaching in Sweden and Finland. *European Educational Research Journal, 16*(4), 425–439.

Selander, S. (2008). Designs for learning: A theoretical perspective. *Designs for Learning, 1*(1), 4–22.

Straehler-Pohl, H., & Gellert, U. (2013). Towards a Bernsteinian language of description for mathematics classroom discourse. *British Journal of Sociology of Education, 34*(3), 313–332.

Anarchism as a methodological foundation in mathematics education: A portrait of resistance

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A large swath of research in mathematics education claims to serve an equity agenda. However, too often this research is conducted atheoretically, failing to disrupt the worldviews that produce injustice and oppression, for both the researcher and the reader. Responding to this commonplace divergence of intent and impact, we propose a methodological approach that anchors research and praxis in a sociopolitical foundation of anarchism. We seek cohesion of theory and practice by consciously demanding that values common to the equity agenda—cooperation, mutual aid, and freedom from hierarchy—provide explicit grounding for method and methodology.

> *Whenever we call something mathematics education research, we either reify existing lines along which something is included or excluded from the foam of mathematics education research, or we perturb them. We can blow additional air into bubbles that exist, we can reach in with our fingers and pop them, or we can blow—and hope—that a new bubble will emerge. The beauty of it all is that we cannot be sure what will happen. We can, however, be sure that things can change. Mathematics education research has not, and does not currently, have a fixed definition. Mathematics education research does not have a fixed, and proper, object of study. And it should not. (Dubbs, 2021, p. 165)*

Mathematics education is not a monolith with a singular identity. Instead, mathematics education is an amorphous collection of bubbles—conversations are raised or dropped; motivations expressed, rescinded, or revised; new conversants join a conversation while others leave, or perhaps just daydream for a time… Mathematics education is froth and foam, and we may at any moment shatter or breath new life into its constituent transience (Dubbs, 2021). At the same time, mathematics education research is commonly atheoretical, a term we use here to reference the commonplace disconnect (or lack of conscious interrogation) between researcher worldview and researcher methods (Stinson, 2020; Walter & Andersen, 2013). Framed in Dubbs' metaphor, researchers do not always know which bubbles they are growing, and which they are dissipating.

The purpose of this manuscript is thus twofold: To breathe new life into a bubble still not often explicitly explored in mathematics education (Bowers & Lawler, 2020), and to do so in

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a way that might give readers lost in the translucence an opportunity to notice and explore aspects of their own worldview that might previously have operated below the level of consciousness. Thus, in this text we explore the methodology—the combined worldview and method—of anarchist mathematics education. In so doing, we aim to contribute to the reflective awareness of ourselves and others regarding the bubble(s) that comprise anarchic methodology, as well as offer a counterpoint from which those of different perspectives might consciously notice aspects of their own worldview and its interrelationship or conflict with their methods (Wheatley, 2005).

Anarchism and the researcher: Conceptualizing anarchist methodology

> In the end, considerations of ontology, epistemology, ethics, values, subjective and ideological grounds, and so on—that is, the researcher's worldview—should precede not follow theoretical and methodological considerations. Explicitly and critically interrogating one's worldview should be the starting point of any research project. (Stinson, 2020, p. 13)

> We also noted that students and scholars (often of European descent), on hearing us present our work, would ask how our methodology differed from theirs. In response, we asked them to articulate exactly which aspects of standard quantitative methodologies they wanted us to contrast Indigenous methodology with [...]. What intrigued us about such questioning was not that our audience wanted to know how an Indigenous quantitative methodology differed from other methodologies, but that they wanted us to provide a coherent picture of our methodologies when they could not provide a coherent picture of theirs. (Walter & Andersen, 2013, p. 43)

Anarchists and (mathematics) education researchers generally share convergent interests regarding social (in)equality—that is to say, both groups share a concern over noticing inequality and making efforts to pursue social justice. "Anarchists are principally and generally motivated by the presence of social inequality and domination to take action" (Williams, 2012, p. 10), and educational researchers have now spent decades producing such a volume of research noticing/analyzing inequality that printing the aggregate work (even of only the subset written exclusively by white cis-hetero men) might well blot out the sun. However, when looking beneath the surface, differences quickly materialize. Of particular note for our purposes, anarchism has clear theoretical (and, dare I say, methodological) underpinning, while substantial work in mathematics education is functionally atheoretical (Stinson, 2020) in the sense defined previously (i.e., commonplace disconnect, or lack of conscious interrogation, between researcher worldview and researcher methods). This atheoretical quality is, from our perspective, deeply concerning—even extremely well-intentioned equity work can actively operate against the goals of social justice when the researcher's worldview isn't consciously and critically interrogated, as for example in the case of work that fetishizes gap-gazing (Gutiérrez, 2008). Positivist framing and its analogues have been discarded but not replaced (Walter & Andersen, 2013), leaving static—white noise (pun intended). Thus, we seek in this section to utilize the strengths of anarchism to outline a broad but firmly grounded methodological approach for (mathematics) education research, one which we offer to those still lost in the static.

To frame this chiaroscuro sketch of anarchism as methodology, we will employ the framework of Walter and Andersen (2013), visually represented in Figure 1. Thus we begin with four key aspects of the researcher's worldview, which we will explore in this order: axiology (philosophy of ethical and aesthetic value), ontology (philosophy of being), epistemology (philosophy of knowing), and social position. These four are deeply interrelated and can't be meaningfully separated (in spite of our pragmatic choice for constructing this section), and we will surface a few of these connections as we go. Our decision to start with axiology rather than ontology might seem surprising, as the latter is suggested by the lingering positivist-shaped void many of us have experienced enculturation towards, but in truth the precedence of ethic is entailed in the selection of anarchism for our methodological framing (Bowers & Lawler, 2020)—anarchism is a methodology and lifestyle (Vellanki & Fendler, 2018) built upon a choice of ethical value. In short, anarchism is minimally built on the values of cooperation, mutual aid, and freedom from unjustified, coercive hierarchy (e.g., ablist cis-hetero patriarchal white-supremacist Capitalism).

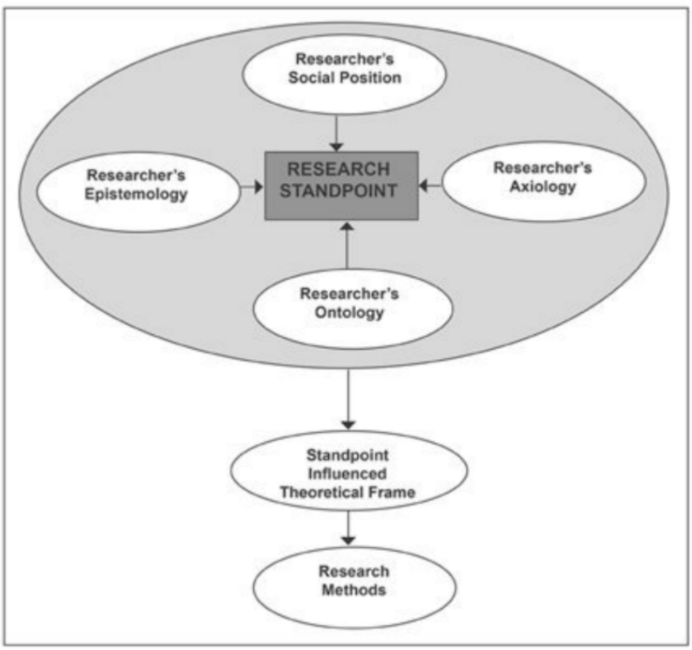

Figure 1: Conceptualization of Methodology (Walter & Andersen, 2013, p. 45)

Axiology

Axiology, the study of ethical and aesthetic value, deals with the intrinsic and extrinsic principles that shape our perception and practice. Axiology palpably shapes every aspect of research: our sense of how we do *good* while doing research, our sense of what questions are *interesting* or *worthwhile*. Adding complexity, these values exist not just in the researcher, but also separately and rarely identically in the products and practices of research itself (Walter & Andersen, 2013).

As an articulation of ethics, anarchism is a mode of human organization with social self-determination, rooted in the experiencing of daily life. Anarchism, specifically social or communal anarchisms, holds a conceptual connection between freedoms of the individual and social equality, emphasizing cooperation, mutual aid, and rejection of hierarchy. "I am not myself free or human until or unless I recognize the freedom and humanity of all my fellow men" (Bakunin, in Suissa, 2010, p. 44). Anarchism values humanizing relationships that minimize if not eliminate coercive structures and interactions, taking seriously the hope for an equal and free society.

Ontology

Ontology, the philosophy of being, deals with how we perceive and operationalize a notion of reality. Far from concrete or immutable, our sense of what is *real* and how we respond to that *reality* can be fluid and even contradictory. Ontology establishes invisible boundaries around what is considered possible or meaningful, made visible in the ongoing clash between marginalized ontologies (e.g., indigenous, queered) and governing societal understandings (Walter & Andersen, 2013).

While the anarchic principles on which we build this methodological frame do not presuppose a particular ontology (or epistemology), they do foreground particular ontological (and epistemological) directions, substantively narrowing the array of ontologies (and epistemologies) we might consider reasonable. Phrased in the most concise possible terms, the ontological stance of an anarchist researcher is one of profound humility: that humans and nonhumans are complex, and that we are complex in ways that resist meaningful simplification (Smedslund, 2009; Weaver & Snaza, 2017). Efforts to simplify the human are, by definition, dehumanizing, and would thus conflict with our anarchic axiological foundation.

To efficiently convey a sense of the magnitude of this complexity within the confines of this manuscript, here we will surface four ontological characteristics of the human (note that the complexity amplifies when one extends beyond sole consideration of the human) and observe how they constrain one of the oft-touted goals of educational research—namely, the goal of surfacing absolute or general principles related to given measures based on the noticing of empirical regularity (Goddard & Wierzbicka, 1994; Smedslund, 2004; Wierzbicka, 1996). These principles have been associated with the ongoing disconnect between educational research and educational practice; they are principles that practitioners must take for granted, while research commonly tries to evade or methodologically exclude them (Smedslund, 2009; Weaver & Snaza, 2017).

Principle 1: Openness. The openness of the human means that, in principle, every single psychological/behavioral measure, and hence every composite measure, is open to an indefinite number of possible influences, depending on how the situation is varied and how the task is understood.

Principle 2: Irreversibility. People are intentional (e.g., trying to pursue good outcomes or avoid bad outcomes), and do not completely unlearn or forget. Thus, observed regularities are conditional upon stable perception of outcomes, rendering absolute or general principles impossible. A valid general principle would entail a limit to intentionality, since it could not be modified by changing outcomes.

Principle 3: Shared Meaning Systems. People are forever part and parcel to innumerable overlapping shared meaning systems—the cultures of family, friends, workplaces, countries, regions, religions, ethnicities, and so forth. Regularities surfaced in research commonly reproduce what we already know (explicitly or tacitly) by virtue of their contingency on shared meaning systems.

Principle 4: Uniqueness. Chance plays a prodigious role in all aspects of our lived experiences, both inward and outward (e.g., Bandura, 1982). Serendipity and misfortune shape people in surreptitious ways, imposing a rigid barrier betwixt the ways the human is and the possibility of developing absolute or general principles.

Epistemology

Epistemology, the philosophy of knowledge and knowing, is central to the work of researchers ostensibly (per our positivist-shaped void) tasked with knowledge production. Whereas traditional Western philosophy constructed epistemology as outside of or prior to culture, the true span of epistemic consideration is wider: considerations of the (oft unwritten) rules of what counts as knowledge, who can be considered knowledgeable, and what knowledges are valorized or marginalized are key to epistemic consideration (Walter & Andersen, 2013).

Feyerabend (2010) argued for and outlines an anarchistic theory of knowledge (this was the subtitle of the first edition). Like us, Feyerabend's motivations in writing that text were built upon an axiological foundation:

> Anger at the wanton destruction of cultural achievements from which we all could have learned, at the conceited assurance with which some intellectuals interfere with the lives of people, and contempt for the treacly phrases they use to embellish their misdeeds. (p. 265)

The overarching thesis statement of Feyerabend's text is simply stated at the outset:

> Science is an essentially anarchic enterprise: theoretical anarchism is more humanitarian and more likely to encourage progress than its law-and-order alternatives… history generally, and the history of revolution in particular, is always richer in content, more varied, more many-sided, more lively and subtle than even the best historian and the best methodologist can imagine. (p. 1)

In essence, just as we describe the anarchist researcher as ontologically humble, Feyerabend describes an anarchist researcher as epistemologically humble. Indeed, we note that ontological humility almost demands epistemological humility, as we illustrated above in relating ontological principles to barriers to one of the commonly touted epistemic goals of hegemonic science.

To the anarchist researcher, anything goes; or rather, you are always more free than you realize you are. An anarchist researcher might use virtually any epistemic method to make sense of or shape the world, even methods that seem contrary to their perspective, as when anti-rationalist Feyerabend regularly made rationalist arguments to discomfit his rationalist opponents and friends (e.g., Lakatos). Viewing science (in our broad conceptualization) as a pedagogic project, one wherein we are forever learning with and from others, and imagining hegemonic science as institutionalized schooling, we find this reflection rooted in Freire and Rancière to be particularly meaningful:

> The unschooled world is only feared by those who have been thoroughly schooled. The emancipated world has no enemies among the truant. None among children and none among artists. None among those who would take equality as a point of departure. (Bingham & Biesta, 2010, p. 157).

Social Position

Our social position comprises much of who we are socially, economically, culturally, and racially. Social position is not just about the individual or individual choices—class, culture, race, gender, sexuality, (dis)ability/neurodivergence, and so forth deeply shape worldview, not least because so much is taken for granted. Social position is thus a verb rather than a noun. As researchers and as people, we do, live, and embody a social position (Walter & Andersen, 2013).

Obviously, we can make no particular observations about the specific social positioning of you, dear reader. However, the anarchist researcher does adopt a particular relationship with social positioning. In short, the anarchist researcher takes social positioning seriously, and may use it as a guide for noticing blind spots or disproportionate emotional/physical labor demands. Additionally, anarchist positioning is worthy of note in-of-itself, for it necessarily runs deep (per our axiological foundation), contrary to the disparate professional positionings that others may tend to write off as "just part of the job".

Anarchists take social positioning seriously (though people who don't have been known to attempt to co-opt the title). Per our axiological foundation, anarchists oppose unjustified, coercive hierarchies, including those of race, gender/sexuality, class, dis/ability, religion, nationality, and so forth. None of these hierarchies are individual monoliths—they silently embrace and dance a dance of violence, holding each other so close that their boundaries blur and disappear. Thus, anarchists are also deeply invested in intersectionality, variously in terms of: noticing and responding to the complex ways various intersections of identity shape lived experiences, accounting for constructed invisibility and cultural obfuscation of the multiply marginalized, and working to build solidarity across those of disparate backgrounds suffering at the hands of the same deathly waltz.

Furthermore, there are additional aspects of social positioning and hierarchy that are relevant to (mathematics) education researchers in particular. An anarchist researcher recognizes that they bear unique experience, knowledges, or tools that another might not have immediate access to, but will nonetheless express extreme skepticism of the many coercive and unjustified aspects of the hierarchy which places researchers epistemically above others as knowers and learners. Relatedly, an anarchist researcher is deeply skeptical of the epistemic and social hierarchy that perceives established scholars as superior to emerging scholars, as well as the epistemic and social hierarchy that perceives “teachers” as superior to “students.” An anarchist researcher further opposes hierarchies of discipline, such as the *Romance of Mathematics* (Lakoff & Núñez, 2000), the still commonplace mythology that contemporary disciplinary mathematics is epistemically superior to other disciplines or disciplinary perspectives.

As one final note that distinguishes this methodological approach from those typically constructed as existing at the center rather than the margins (work constructed on the margins, such as indigenous methodologies, is more likely to intentionally share this characteristic), anarchism as methodology is not limited to professional situations—it is a lifestyle. To borrow a metaphor from educational philosopher Lynn Fendler, imagine yourself as a chef, passionately dedicated to your craft. Can you dissociate the qualities of your ingredients from the qualities of the food they are used to create? By the same token, in our work as researchers, is it really possible to dissociate the qualities of the researcher from the qualities of the research they produce? To create the most delicious dish, we use ingredients that carry the qualities we wish to be present in that dish. To create the most ethical research, we must use ingredients that carry the axiological qualities we wish to be present in the research (Vellanki & Fendler, 2018).

Anarchism and research: Tracing the possibility space of anarchist method

> In [critically engaging with one’s worldview], the frantic search that novice (and even seasoned) researchers experience in selecting theoretical frameworks and methodological approaches more times than not becomes self-evident and trivial. (Stinson, 2020, p. 13)

Having sketched the outlines of the worldview(s) associated with anarchism above, in this section we use that foundation to infer elements of the possibility space of anarchist research method. This space is vast, even as it excludes swathes of normative research methods (for example, psychometrics in its normative context—that is, the context of the worldviews and purposes that commonly underlay it—fall in steep conflict with an anarchist worldview). Thus, our goal is to be illustrative rather than exhaustive. In particular, we draw attention to three categories of method that have proved meaningful in our own work: collaborative action and design research, discourse-shaping and radicalizing research, and work that stands as iconoclasm of the oppressor within.

Collaborative action and design research

Despite ill-informed representations perpetuated by the media and other sources, the most likely places you might find anarchists in your community are at your local community garden, baby pantry, workers union, or co-op (we, the authors, have participated in each of these). These are loci of cooperation and mutual aid, places where people have noticed their community has a need that they can help to fill. As researchers with specialized knowledge(s) that can be leveraged for the benefit of our communities, one notable analogue to these aforementioned spaces in research is collaborative action and design research. Action and design research represent

> an orientation to knowledge creation that arises in a context of practice and requires researchers to work with practitioners [and other stakeholders] […] its purpose is not primarily or solely to understand social arrangements, but also to effect desired change as a path to generating knowledge and empowering stakeholders. (Huang, 2010, p. 93)

Note that whereas more normative research might prioritize *knowledge* as a means towards *change and empowerment*, Huang instead describes pursuing desired *change* as a means to *knowledge and empowerment*. Along with relating to our axiological foundation, this also ties into the ontological and epistemological humility we referenced previously. In short, the goal of collaborative action and design research is to support communities, as for example through the collaborative development of "new theories, artifacts, and practices that can be generalized to other schools" (Barab, 2014, p. 151) or areas of praxis.

Discourse-shaping

Every publication, presentation, seminar, lesson, and conversation is necessarily a political act. Interaction either reifies or perturbs boundaries and beliefs, forever modifying the ideological and material translucence we occupy (Dubbs, 2021). Whenever we converse, the question is never "should I be political or not," for we are necessarily political in manners either hidebound or progressive. Instead, the question is, "in what ways should I be political?" or "in what ways should I shape this discourse?" Conscious of this, the anarchist researcher seeks to shape discourse in ways that advance our ethical goals of cooperation, mutual aid, and freedom from unjustified, coercive hierarchy. We might, for example, publish critical works in typically acritical spaces (Bowers & Küchle, 2020) in an effort to normalize critical perspectives therein, thus paving the way for further change or revolution. Discourse shaping is always a component of the work of any researcher (consciously or not), but for the anarchist researcher it can also serve as a goal in-of-itself, whether that means mobilizing/radicalizing potential future researcher-activists or simply offering a moment of critical introspection to an audience not often engaged in such.

Iconoclasm of the oppressor within

White supremecy, cis-heterosexual male supremecy, abled/neurotypical supremecy, capital supremecy… each of these (and other) oppressive paradigms/hierarchies surround us and can act through us.

> The true focus of revolutionary change is never merely the oppressive situations which we seek to escape, but that piece of the oppressor which is planted deep within each of us, and which knows only the oppressors' tactics, the oppressors' relationships (Lorde, 2007, p. 118)

The anarchist researcher has an ethical responsibility to always consider the ways oppressive systems may act through us, below the level of consciousness, notably (but not solely) along continua wherein our identity and/or positioning might place us among oppressors. For example, a white researcher might be aware that whiteness is acting through a system in which they participate, such as mathematics doctoral coursework. It is common for disproportionate burden to be placed on BI-POC (Black, Indigenous, and People of Color) to surface white supremacy (with analogues applicable to each other system of oppression), so the white researcher might dedicate careful time, energy, and attention to analysing and thinking through how whiteness is acting in those spaces, then share what they surface with other researchers subject to similar positional blindness (e.g., Bowers, 2019). With this sort of persistent, critical reflection, we seek to free ourselves from such hierarchies of domination—by sharing this work with others, we engage in yet another type of cooperation and mutual aid. While we emphasize the ways positional blindnesses and disproportionate systemic expectations might make this work more important along the lines of our oppressive identities/positionings, we do wish to mention that such supremacies are commonly internalized along the lines of our marginalized identities as well, as when internalized neurotypical supremacy or cis-heterosexual male supremacy rears its ugly head in the work, thought, or action of this neurodivergent (autistic, ADHD), genderqueer author.

Conclusion

Anarchism is present in a significant portion of modern equity and justice research in mathematics education (Bowers & Lawler, 2020). Explicit identification of and attention to anarchist methodology provides researcher and reader the opportunity to more explicitly identify the manner in which knowledge production operates in harmony (or conflict) with stated aims of a just an equitable society: cooperation; mutual aid; and freedom from unjustified, coercive hierarchy. Phrased differently, building methodology from the foundation of anarchism allows for a cohesion of theory and praxis that would be beneficial in any work, but which carries particular weight when one's goals are emancipatory or anti-oppressive.

Critical theory is necessarily an inadequate force of change when not accompanied by critical praxis. The additional step of critical praxis has presented a hurdle to many, a fact at once shocking and wholly unsurprising—unsurprising, because critical and social justice work can't be built on a foundation of oppression such as that symbolically and materially reified in the norms and methods of much mathematics education research, but shocking, because a new reality lies just out of sight over the horizon. We hope this glimpse of one such reality, one such critical praxis developed through a cohesion of theory and practice, might offer a glimpse over such a horizon. While throwing off the reigns of oppression might seem at times an insurmountable challenge, we look forward, in solidarity, to basking under the warmth of new suns.

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References

Bandura, A. (1982). The psychology of chance encounters and life paths. *American Psychologist, 37*, 747–755.

Barab, S. (2014). Design-based research: A methodological toolkit for engineering change. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 151–169). Cambridge University Press.

Bingham, C. & Biesta, G. J. J. (2010). *Jacques Rancière: Education, truth, emancipation.* Continuum International Publishing Group.

Bowers, D. M. (2019). Critical analysis of whiteness in a mathematics education proseminar syllabus. In J. Subramanian (Ed.), *Proceedings of the 10th international Mathematics Education and Society Conference* (pp. 284–293). Hyderabad, India.

Bowers, D. M. & Küchle, V. A. B. (2020). Mathematical proof and genre theory. *The Mathematical Intelligencer, 42*(2), 48–55.

Bowers, D. M. & Lawler, B. R. (2020). Reconciling tensions in equity discourse through an anti-hierarchical (anarchist) theory of action. *Proceedings of the 42nd annual meeting of the North American Chapter of the International Group for Psychology in Mathematics Education* (pp. 518-524). Mazatlán, Mexico.

Dubbs, C. H. (2021). *Mathematics education atlas: Mapping the field of mathematics education research.* Crave Press.

Feyerabend, P. (2010). *Against method* (4th ed.). Verso.

Goddard, C., & Wierzbicka, A. (Eds.). (1994). *Semantics and lexical universals.* John Benjamins.

Gutiérrez, R. (2008). A "gap-gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education, 39*(4), 357–364.

Huang, H. B. (2010). What is good action research?: Why the resurgent interest? *Action Research, 8*(1), 93–109

Lakoff, G. & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* Basic Books.

Lorde, A. (2007). *Sister outsider: Essays and speeches.* Crossing Press.

Smedslund, J. (2004). *Dialogues about a new psychology.* Taos Institute.

Smedslund, J. (2009). The mismatch between current research methods and the nature of psychological phenomena: What researchers must learn from practitioners. *Theory & Psychology, 19*(6), 778–794.

Stinson, D. (2020). Scholars before researchers: Philosophical considerations in the preparation of mathematics education researchers. *Philosophy of Mathematics Education Journal, 36.*

Suissa, J. (2010). *Anarchism and education: A philosophical perspective.* PM Press.

Vellanki, V. & Fendler, L. (2018). *Methodology as lifestyle choice* [Video]. YouTube. https://www.youtube.com/watch?v=Rfy_y1E8EoY&

Walter, M. & Andersen, C. (2013). *Indigenous statistics: A quantitative research methodology.* Taylor & Francis Group.

Weaver, J. A., & Snaza, N. (2017). Against methodocentrism in educational research. *Educational Philosophy and Theory, 49*(11), 1055–1065.

Wheatley, M. J. (2005). *Finding our way: Leadership for an uncertain time.* Berrett-Koehler Publishers.

Wierzbicka, A. (1996). *Semantics: Primes and universals.* Oxford University Press.

Williams, D. (2012). From top to bottom, a thoroughly stratified world: An anarchist view of methodology and domination. *Race, Gender, & Class, 19*(3), 9–34.

Mapping conocimiento and desconocimiento in collaborative mathematics teacher professional development

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Using Case Study, this paper demonstrates the ways a group of teachers in Chile experienced collaborative knowing as they participated in Lesson Study for professional development. I use Anzaldúa's conception of Conocimiento, reimagined for math teacher learning by Gutiérrez, to analyse data and understand the ways this team of teachers co-created knowledge together. Data includes transcribed audio/video recordings of planning meetings, lessons, and post-lesson reflections as well as participant interviews and focus groups. This study contributes to the discussion on mathematics teacher professional development including the preparation of teachers for advocating for their students within systems of oppression.

Lesson Study in Chile

Lesson Study incorporates teachers working together to plan a lesson, enact and learn from that lesson and then participate in collective reflection on student learning observed in that lesson (Fernandez & Yoshida, 2004). It is a system of teacher learning that was developed in a grassroots way for over one-hundred years in Japan and prioritizes teachers' perspectives in their learning. Lesson Study can be done alone as its key feature is reflection on one's practice through attention on student learning. Collaboration with other teachers, coaches and administrators provides additional lenses for that reflection (Fernandez et al., 2003). Juxtaposed to more popular models where millions of dollars are spent on professional development, where many studies have shown that professional development for teachers is fragmented and superficial, largely ignoring what research has demonstrated about teacher learning (Ball & Cohen, 1999; Borko, 2004; Putnam & Borko, 1997). Gellert et al. (2013) found that with elementary math teachers in Chile nationwide professional development efforts had the opposite effect of what was intended. Due to Chile's neoliberal past, living in the shadow of the Pinochet dictatorship that produced the high-risk teacher evaluations coupled with job insecurity, researchers have found teachers will teach 'safe' lessons that refrain from activities that might demonstrate errors or misconceptions (Araya & Dartnell, 2008). It is likely these same precautions are demonstrated in Lesson Study.

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Methodology

This research sought to answer the following questions: *What cycles of Conocimiento do teachers of Mathematics experience while engaged in Lesson Study in Chile? and What do we as math teacher educators and math teacher education researchers learn about teacher learning by focusing on uncertainties as they arise when the group is trying to construct new knowledge?* Coming from a social constructivist perspective and using Anzaldua's framework of Nepantla/ Conocimiento and Gutierrez's reimagining of Nepantla for mathematics teacher learning in the Path of Conocimiento, I used Case Study with an ethnographic, qualitative approach to explore teacher learning through several cycles if Lesson Study as they were enacted over the course of one year. Data included analytic memos, audio and video transcribed recordings of collaborative lesson planning, lesson implementation, and collaborative lesson reflections. Data also includes teacher personal reflections, artifacts from the lessons and lesson planning. Lastly, teacher interviews and member checking sessions were also recorded and analyzed. Data was analyzed based on themes from the Path of Conocimiento as well as using inductive analysis. Nepantla was used to represent tensions or liminal spaces observed in the data. This is later represented by a 20-pointed star. When participants rejected interpretations in ways that shut down the exploration, they were coded as Desconocimiento. If comments or actions built on interpretations, clarified, or posited additional interpretations which respected both speakers, they were coded as Conocimiento, knowing. This longitudinal, situated Case Study design employed holistic single-case design outlined by Yin, (2018).

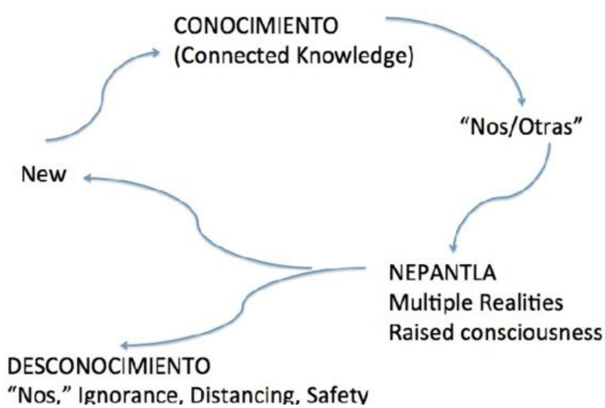

Figure 1: The Path of Conocimiento from Gutiérrez (2012)

Teacher	Grade/Subject	Experience (yrs.)	Yrs. at School	Gender
Valentina	Instructional Coach	30	30	F
Emilio	5th to 8th Grade Math	11	1	M
Violeta	2nd Grade	7	1	F
Martina	3rd Grade	6	0.5	F
Paloma	Classroom Aid	13	13	F
Elena	1st Grade	2	1	F

Table 1: Teacher/ Administrative Participants.

Context

This paper looks at a specific instance of conocimiento that emerged in the second of three cycles of Lesson Study carried out in one school. The objective of the lesson was to build understanding of the concept of tens and ones among a class of 1st grade students. As a secondary goal, the team sought to explore student collaboration through teacher-lead learning centers. The head teacher for the class, Elena, designed four centers which were introduced at the beginning of the class. Students were then sent to one of the four centers and then rotated as the teacher instructed. Video cameras were positioned around the room to capture the work in the classroom as a whole as well as at each of the centers. These videos then were edited to follow three focal students as they moved from center to center, so that the teachers could collaboratively reflect on one small set of students as the progressed through the centers. In the reflection meeting, teachers were given reflection sheets to record thoughts independently before they watched the edited video of the focal students.

Data analysis

After the lesson, students were dismissed to the school patio where they enjoyed a recess with the other classes. As we walked out of the classroom Martina and Violeta said that they felt the lesson had “gone well” and enjoyed “working together in the same room” but did not mention the lesson or their observations made in the lesson. Later, in Emilio’s home, he began to process the events at his center. He explained that his student did not know her numbers yet, “She confused the ten and the one.” He explained that generally the students performed poorly in his school and he cited poverty factors for why the school scores were so low on the state exams.

> And why do you see this? Why does this happen? It’s because the parents have low income. I’ve checked those statistics and the parents sometimes have an eighth grade [education]… (Interview, June 28) [A]

Emilio’s referencing poverty as an explanation of student performance was the beginning of the representations from the teaching group indicating that there were problems in student performance. The second indication was found in the common language in the reflection, predominantly speaking about student engagement in general but positive terms. Specifically, teacher referred to the students as “happy” and “motived” or “happy and “excited to participate” (see Table 2.) Only three “suggestions” were given in all reflections. Two where on Emilio’s reflection page, meaning that most did not offer suggestions.

Observations | Suggestions

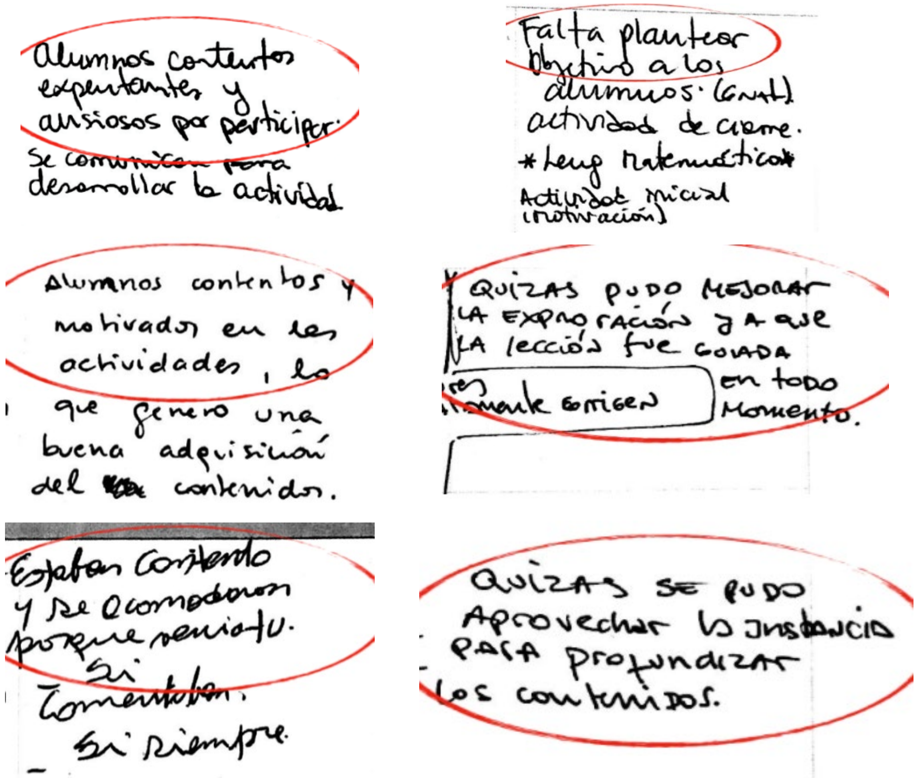

Table 2: Sample Comments from Teacher Reflection.

Despite predominantly positive comments from the independent reflections, there was a perceivable difference in mood among the teachers in the reflection compared to the planning meeting and the lesson. Where our planning time at the beginning of the week was energetic and full of ideas, the reflection was more reserved. One way to see this was in the coding: as the teachers were eager to share, they began to speak over one another, making transcribing difficult. These moments were coded as cross-speak. There were 3 cross-speak moments in this reflection.

Planning Meeting 1	Reflection 1	Planning Meeting 2	Reflection 2	Impromptu Reflection (3)	Reflection 3	Totals
24	0	9	3	12	2	50

Table 3: Cross-speech across lesson study meetings. Note that the first and second reflections had 2 and 4 participants, respectively.

We watched Emilio's center, as the focus student counted with tiles, "17, 18, 19, 30." To remediate, Emilio tried to get the student to count her tens and then to count on from the tens. He repeated the activity with different sets of tiles. When she answered correctly, she did so with an inflection at the end, indicating that she was not sure of her answer.

As the video played, it became evident that the teacher of the class, Elena, was uncomfortable. She shrunk down in her seat and kept her hands folded in front of her, partially obscuring her face, see Figures 2. (Note that the figures include a timestamp, this is only to communicate the passage of time.) As other teachers began mirroring her stance (see the third image in Figure 2), I was alerted to a tension happening in the group. As the video played, the mentor, Valentina quietly asked Elena (not the group) if the student knew the material. Elena responded, "Yes, she was just very nervous because of all the people and the cameras...".[B]

Figure 2: Teacher Tension

After watching one center, I paused the video and asked if the student had learned the objective (23:45.81). The teachers all said "yes".[C] These were the three moments leading up to this conversation that demonstrated the teachers were experiencing some conflict with what was observed in the lesson. A: Emilio explained that the student was mixing her tens and ones and then cited poverty. B: When asked whether the student knew the material, Elena said yes but that the situation in the room had made her nervous. C: When asked if the student had learned, everyone said yes, and then said nothing. These pre-indicators are representing by triangles in Figure 3.

2 (24:01.09) Valentina: This is a concept we teach through many grades. We have been working on it since kindergarten.

Comments A, B, C and 2 demonstrated that the teachers might have been expecting a fault to land, that I or others were looking for a person or factors that were to take the blame. This progression is represented in a straight path, opposed to the turn represented in conocimiento. In the straight path, we see desconocimiento, we see missed opportunities. The desconocimiento presents a refusal to see students learning as a place to hone practice, but as simply as a factor in grading teachers in summative ways. With new perspective/ interpretations, with challenges, we see the opportunity to begin making new knowledge collectively. A conocimiento move from 1 to 2 could have been *yes, and*: "Yes I noticed that she was using the numbers interchangeable, but I think it relates to her confusion of ones and tens." or "yes, but I wonder if it is a language thing... not a number thing?", Instead comment 2 re-centers the conversation and reiterates that this is something learned over time and that there is nothing to discuss concerning what was observed.

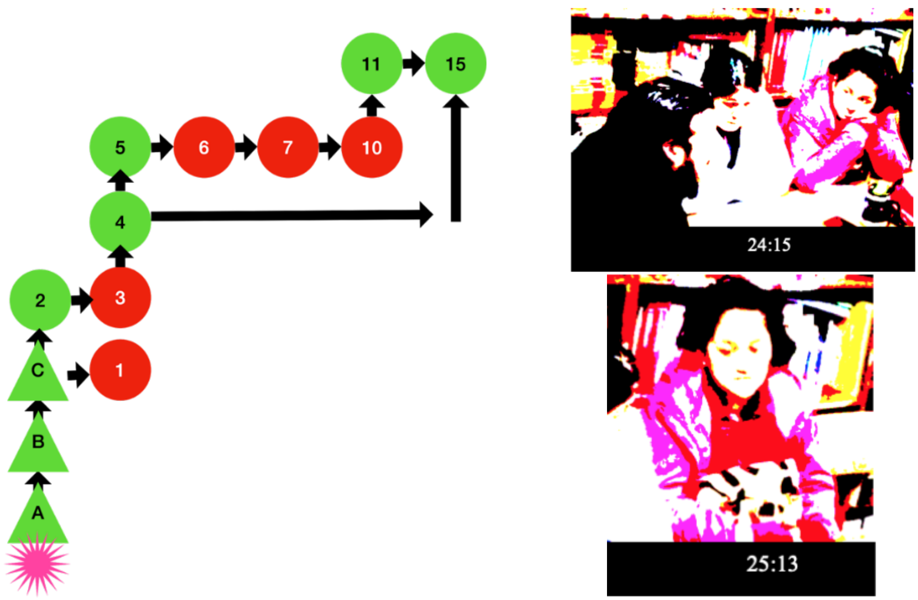

Figure 3: Building Conocimiento, A conversation diagram and a shift in body-language, observing Emilio (3) and agreeing (4).

3 (24:09:25) Emilio: I thought they understood that a ten was ten ones.... *maybe it was me. I think when I counted backward, going down and down and down, it confused her.* Also, all the people and the cameras could have made her nervous.

4 (25:07.31) Elena: Yes, because in fact, in class the kids do well. *But they struggle with complexity.* We committed a lot of time to it because it was hard for them to understand. They could understand that in tens there were 10 units representing 10 disks. Then for one group of tens they had a hard time because they had already said there were 18… for example, there are 18 already, but we have only one ten, 'How many were ones?' *[the children would respond] 'One!' What? No, a group of 10!*

Emilio presented himself as possibly making an error, an attention to discuss. Elena uses this moment to discuss an observation in the class, the confusion with the ten and the one. In this, she introduces an obstacle to understanding tens and ones well-documented in research (Fuson et al., 1997; Guerrero et al., 2020; Kevin Miller et al., 2000).

5 (25:53.11) Emilio: ...talking with another colleague... She did an activity where they gathered 10 balls of dough. *And those 10 balls of playdough, she combined… then perhaps the children will not be confused with a ones and the tens….* that they are not the same, because ten is bigger. Maybe that's what happened to the children with this activity. Because they confused one with a ten, when they are bigger... perhaps does not make sense, because it is obviously the ten largest. But for them as they were confused there.

6 (26:33.80) Elena: But first they were shown the loose quantity. There are 10 chips. Then I joined them and said, 'Look at these tens and there are ten. And this one, each of these pieces is called unit.' So, I show the units and they gather ten units…

Here Elena answers Emilio's suggestion with examples of what she did in class, and what she did at her center. Here we see Elena challenging Emilio's suggestion.

7 (27:06.61) Author: But maybe it was the way it was written? because it looks very similar… *Because normally we write 18 like this….* They might be guessing what we want.

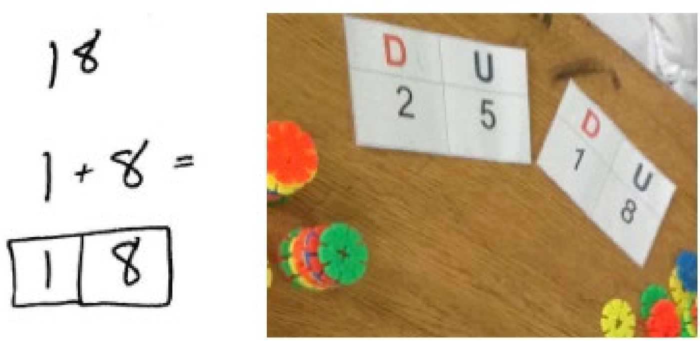

Figure 4: Artifacts from the reflection, My depiction in my notes and screen capture of cards at the observed center.

8 (27:19.83) Emilio:	Maybe they did not understand the instructions.
9 (27:25.34) Valentina:	Maybe it is the other way around…
10 (28:14.05) Author:	Judging by her facial expressions and how she was mixing 20 and 30… *[cross speak] …* I wonder if she was not hearing the difference between the sounds.

This marks our first cross-speech. We had entered the problem-solving stage; we had been building and considering each other's ideas/interpretations since Emilio presented that maybe he had confused her. Here we see increase excitement and engagement. We then received important information from the classroom aid, who had worked with the student across grades.

11 (28:43.93) Elena:	In fact, with her, you have to enunciate your words
12 (28:45.73) Paloma:	She *watches the mouth a lot.* [cross speak] I had her in prekindergarten. If she did not see your mouth, she did not understand.
13 (29:06.90) Emilio:	Was she evaluated?
14 (29:12.69) Paloma:	In kindergarten, she had hearing aids... These tests are done periodically; the mom controls that. [Cross talk] The mom says that it got better and then fell.
15 (29:24.46) Violeta:	… she must have gone many years without being diagnosed then *she learned to listen to people by the movement of their mouth.* Then it takes a few years with hearing aids... A year and a half? It is just processing the movement of the mouth with the sound coming out. *I also think the concept is difficult.*

Here we see the combination of observation culminating in identifying what will be specifically difficult for this student. In the post-reflection interview, it was revealed that one obstacle to a productive collaborative reflection was that criticizing felt like betrayal.

> It is my personal desire to support and not to betray my colleague, accusing her, accusing her that her work is not well done… It was Elena's class, her student. Elena said the student knew, but she was nervous… Then I also think that she was justifying herself… to not look bad, because that was her student. (Emilio, post-reflection interview)

Emilio's ability to have empathy for his colleagues while choosing to engage in difficult conversations helped to provide the brave space the group needed to learn from their practice. He could have simply agreed that the student was nervous in the reflection meeting, and the learning would have stopped there. Alternatively, he could have simply stated what he had told me, "The student doesn't know how to count." It would provide a missed opportunity to learn and to engage in professional development as the silence would likely have continued. It is clear for him, the tension included his interpretation, the care of his colleagues and their collaborative relationships and the interpretations being given from the group. Emilio prioritized the solidarity of the group to navigate the tension within the perspectives because he felt that an environment of solidarity is key to developing a productive learning space.

As a part of the member checking process, in zoom meetings with the teachers involved in this lesson and reflection, I showed the teachers the clip of this phenomena and asked them to remember what they were thinking. In the follow-up interview, Emilio stated, “I am a normal teacher and we all make mistakes. So, *I give the foot* or give the opportunity… I speak honestly and I think it drives others to see that we are all equal and they can tell their real experience without putting on a show.” By modeling with self-criticism, he made space to engage with the differing perspectives in way that was candid and productive.

Discussion

Using this mapping tool, I was able to document how the group was able to turn into the tension for making knowledge together. This was represented by right turns, away from the straight path with the group’s current conocimiento or desconocimiento. This tool then mapped the multiple ways the perspectives were combining to build rich, context-oriented complexity within the group. This tool made the combining of ideas as the group co-created knowledge explicit, visually representing teacher making sense together by expanding… verses a more linear path that often happened as teachers did not offer or incorporate each other’s lenses.

Findings

By entering in a discussion, interpreting and making sense together, we developed a more complex interpretation of what was happening with the focal student in the lesson. In part, she may have been nervous. I and the other teachers had created a disruption and we were doing something very new. Yes, there is a well-documented struggle for children to understand the differences of tens and ones. Additionally, maybe the way we represented the tens and ones (with boxes) contributed to the confusion as to what were tens and what were ones and yes, in this center, we saw a student who had a history with hearing problems. This revelation about the hearing aid sparked an ongoing conversation at the school on student files so teachers could get a general report on each student’s progress and understand abilities and student backgrounds that might impact teaching. Having overcome the pervasive high-risk nature of the neo-liberal education system in Chile, the teachers were able to address student thinking rich with context, combining student thinking, abilities, with research. The lessons learned in watching this focal student relied on open communication that was not initially happening. When they embraced the tension they paved the way for a more nuanced understanding among the group. That nuance resided in each person’s privately held perspective of what was happening with that student, waiting to be held against the other perspectives. In this way we saw the collaborative making of new knowledge between the teachers, the research, through close observation of the student, and background knowledge of the classroom aid.

References

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32).

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3–15. https://doi.org/10.3102/0013189X033008003

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US–Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education*, *19*(2), 171–185. https://doi.org/10.1016/S0742-051X(02)00102-6

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Erlbaum.

Fuson, K. C., Smith, S. T., & Cicero, A. M. L. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*(6), 738–766.

Gellert, U., Espinoza, L., & Barbé, J. (2013). Being a mathematics teacher in times of reform. *ZDM Mathematics Education*, *45*(4), 535–545. https://doi.org/10.1007/s11858-013-0499-1

Guerrero, D., Hwang, J., Boutin, B., Roeper, T., & Park, J. (2020). Is thirty-two three tens and two ones? The embedded structure of cardinal numbers. *Cognition*, *203*, Article 104331. https://doi.org/10.1016/j.cognition.2020.104331

Gutiérrez, R. (2012). Embracing Nepantla: Rethinking "knowledge" and its use in mathematics teaching. *Journal of Research in Mathematics Education*, *1*(1), 29–56. https://doi.org/10.4471/redimat.2012.02

Miller, K., Major, S. M., Shu, H., & Zhang, H. (2000). Ordinal knowledge: Number names and number concepts in Chinese and English. *Canadian Journal of Experimental Psychology*, *54*(2), 129–139.

Putnam, R. T., & Borko, H. (1997). Teacher learning: Implications of new views of cognition. In B. J. Biddle, T. L. Good, & I. F. Goodson (Eds.), *International handbook of teachers and teaching* (Vol. 3, pp. 1223–1296). Springer. https://doi.org/10.1007/978-94-011-4942-6_30

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, *29*(1), 4–15. https://doi.org/10.2307/1176586

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage.

About the useful uselessness and unimportant importance of mathematics

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In this presentation we explore the symbolic power of mathematics and mathematics education (ME), focusing on the trait of the socio-political perception of usefulness-importance of mathematics. We contribute excerpts from discussion groups with students from 2nd and 3rd year of secondary education, from a public school of Madrid. Then, we analyse and discuss those excerpts with the aim of understanding how the students live through school mathematics and its aforementioned "usefulness-importance".

Introduction

This chapter is an excerpt (and adaptation to the theme of this MES conference) of a chapter from a recently finished doctoral thesis (Bruno, 2020). Which in turns is heavily influenced by the young tradition of the socio-political perspectives in mathematics education (ME), as explained by Valero & Pais (2015), along many other authors. As we explain and elaborate throughout the mentioned Thesis, from a socio-political perspective in ME, the phenomena of the mathematics class or of the school cease (to a greater or lesser extent, as Valero & Pais, 2015, explain) to be the exclusive center of research attention. And they are understood as a fundamental, but not exclusive, node of an interweaving of social practices of different kinds: technologies of power and government, devices for evaluation and accreditation / social stratification, global agendas, dominant cultural discourses and evaluations, creation of subjectivities through schooling, "regimes of truth" (in Foucauldian language), and so on. All of them indeed closely related to mathematics, their schooling, and research in ME.

A relevant issue, both in the doctoral research and in the theoretical approach there adopted, is what authors like Skovsmose (1994) and Sáenz and García (2015) call the *symbolic power* of mathematics. We characterize this *symbolic power* (based on the developments of such authors) with the following fundamental features:

- Mathematics is considered an objective knowledge, aseptic of any ideological-political interest or struggles, neutral in values.
- Mathematics have, politically / symbolically, an almost unappealable truth value ("ideology of certainty", according to Skovsmose & Borba, 1997). "The numbers speak for themselves / they don't lie."
- Mathematics is very useful / important ("for everything", "they are everywhere", "in science and technology…but hey, also in the arts and… and in everyday life"). And

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therefore they are/should be a significant component of any educational curriculum in any country/community that claims to be up to date on any timeline of progress and innovation.

– Furthermore, considering the global aspirations of democratization, equity, social justice, "global citizenship" promoted from high levels of international organizations (i.e., the UN 2030 agenda; UN, 2015), teaching-schooling in Mathematics is promoted as a universal right / duty, as a key element to achieve such aspirations. Valero (2017) genealogically analyzes this ideal of "Mathematics for all" and how it has been developing in recent decades.

However, despite this broad socio-cultural legitimation, for a large part of the "ordinary people", mathematics is impenetrable or incomprehensible, if one does not have an important level of initiation and practice with them (i.e., university degree or profession that applies them in depth). Therefore, Mathematics has the remarkable political-cultural position of being a symbolic system that is both accepted and legitimized, but largely misunderstood and even feared-rejected by a large part of the population (Spanish, world). In our experience, both as mathematics teachers at different educational levels and as researchers in ME, this fear-rejection could be related to the systemic and generalized phenomenon of school failure in mathematics.

Now, considering the enormous embedding that mathematics has in decision-making and power spaces (in states, in international agencies, in economic and financial schemes, in electoral systems, in health systems, in modern ICT, even in the understanding/measurement of space-time…) the political consequences of this status of symbolic power of Mathematics are far-reaching. As expressed by Sáenz and García (2015, p. 26):

> By presenting itself so legitimized and at the same time so impenetrable for the uninitiated, it goes so far as to make the dominated assume the legitimacy of their submission […]. Clouded in the shadow of this unattainable knowledge, anyone who ignores the language and mathematical methods by which social processes are usually expressed remains unarmed, without an answer, sometimes he must accept his bad luck, quietly waiting until the wisdom of who decided for him bears fruit.

As a part of a doctoral thesis project, field work has been developed in different Secondary Education schools in Madrid, Spain. The students involved are from 2nd and 3rd year of secondary education ("ESO", by its acronym in Spanish).

In the field experiences mentioned above, we seek to investigate the different aspects of the "symbolic power" of mathematics. In this presentation we will communicate some results referring to the socio-political and cultural *importance-usefulness* trait assigned to mathematics and its education.

For that purpose, we studied the perceptions and experiences of the participating students about mathematics and school math, considering different points of view: difficulty, liking or rejection, being a "person of sciences" or being a "person of letters", self-concept, etc. A significant part of the inquiries was around the question of the importance-usefulness of mathematics (in society, in daily life, in school, etc.).

Methodology of research

In her doctoral thesis, Suavita (2017) proposes an operational construction around the notion of *social imaginary*, that is, both notions and theoretical framework, as a methodology to study these socio-political, cultural and historical phenomena. His thesis is entitled "Imaginaries in mathematics teachers in training". Among others, Suavita collects the developments of the Ibero-American Research Network on Imaginaries and Representations (RIIR), which in turn bases its theoretical frameworks on the research of Castoriadis, Durand, Maffesoli, Baeza, Carretero, Silva and others.

In this chapter we follow her methodological approach in her study of social imaginaries. One of the main techniques she uses for obtaining data and evidence is the *focus group* or *discussion group*, and we thus will apply it for addressing our research concerns.

We organized the participant students in up to 7 focus groups, and in those we proposed to the students the discussion around topics that implied the trait of the use-importance of mathematics, like: why is mathematics taught in school? What are they used for in different areas of life? Why are they (or are they not) important-useful?

Mathematics are useful-useless, but not really important-unimportant

We will now transcribe excerpts from some of the discussion groups relevant to *use-importance* trait, and its perception by the participant students. We will follow each with analysis and commentaries considering the theoretical perspectives posited above.

The "R" abbreviation means "Researcher", the person who was conducting the discussion groups. The other abbreviations refer to the names of the participant students.

Group 1:

R: Let's see, one more question: if you're not good at math, can you be a sciences person?
(The students discuss, argue a bit before answering)
R: And if you are good at math, can you be a letters person?
A3: I'm bad at math, and I'm going for sciences.
A1 (to A3): And are you going for sciences? Well, not me, because I believe that everything is united, you know [...] That if you don't know mathematics, then, as you progress, you will not know physics and chemistry either, which is what is happening to me.
L (to A3): For example, Physics and Chemistry are not mathematics[sic], but there are formulas... and they are mathematics after all.
R: And if you were bad at math, it wouldn't be convenient for you to do science, or would it be convenient...?
A3: Let's see, for the career I want to do, yes. Because that also depends on what you want to do. I, for example, want to be a midwife.
A1: And what use is mathematics for you to get a child? One-two-three!
A2: Three minus one, two, minus one, one...
A3: If I want to go for that profession, then I must go for sciences. Then it [mathematics] will be useful to me. Let's see, math is useless to me, but it is necessary in sciences, so it's a no brainer.

We first observe that A3 classmates themselves question the decision to "follow a sciences degree", because science has/is a lot of math; or, without math there is no science, or without math you cannot do sciences. And they affirm it empirically, because at least in the current schooling-accreditation system, this is considered true. Whether or not math is used in one or another discipline, study, trade or criminal activity, to do "science" and anything that has been so labeled, you have to go through many and difficult math... and their assessments.

Of course, the issue of the usefulness of mathematics for an obstetrician is a sweet plate of non-sense, but we will return to it later.

Group 2:

C: Math is a bummer, but it is very important.

Q: That depends on what you study ...

R: And if you did not study engineering or "sciences", would they be useful to you? (Controversy, mixed voices).

B1: In certain things, yes. But like, above all, if you have a subject that has to do with mathematics. For example, if you go to "Sciences" ... also in "Letters" I think you must do mathematics. Or maybe it helps me to know the area of a certain something if I study Plastic Arts. But there are a lot of things that have nothing to do with what we are going to do.

S: But, for example, if I want to study psychology, mathematics will not help me to study psychology. For example, "basic math", as we said before, in real life does work for me, but ...

B1: In everything we have lived through, we haven't used a square root at all. Rule of three I have used, very few, but square root for nothing, knowing the apothem of whatever, either... I do not walk down the street calculating apothems.

R: And, for example, if any of you were a journalist ... Do you think math could help you to be a good journalist?

B: All knowledge helps you to come to more knowledge. But I think it could also do well without math. That you can do better, yes. But if you do not like it, why are you going to put in the effort? [sic!]

R: Do you think I could use math in a particular situation as a journalist?

E: For example, maybe you have to do a survey, and for that you do need math.

B2: You must calculate the schedule, what time do you have to be to do certain something...

B1: But, for example, with the survey, you can hire a worker to do it. But, to calculate the time, you need "basic" math, you don't need to know a rule ...

S: But, the basic thing, what we use in life, is addition, subtraction, multiplication, and division, and studying the hours. And the percentages, that is it.

L: Yes, things that are of no use to you. For example, equations of the second degree, me, not at all. I mean, and I am terrible at these.

E: Sure, imagine that you are a lifeguard, you are not going to calculate the distance you must jump to catch [someone drowning] ... it is already dead in the water.

The first answer of the students is an automatic “it’s very important (despite its ugliness)”, i.e., a conformity with the socio-political trait we mentioned at the beginning of this presentation. But shortly afterwards, the contradictions, questionings and even ironies emerge again, as in the first group. It seems that math is not so valuable in everyday life, but also in many professional activities; is useful mostly for those things labelled as “science”, whatever that means. Only basic and elementary school math has proven to be useful to them.

Group 3:

R: Why do you think math is compulsorily taught in ESO?
G: It is something we have to know... at least a minimum.
A: For our future, I don’t know, so that we can...
S: There is always the same excuse. What are you studying it for? To approve the test, but...
G: Let’s see, until the ESO, it’s fine. Then, more math, well ... whoever wants to aspire, well.
[...]
R: How do you see it?
S: Well, I say, math is going to be used and such, but to a certain extent. There are other things that you look at or whatever, but because you have to approve it. But that’s the excuse you’ve always been given. You study it, whatever, because you have to pass. Because another thing’s, what are you going to use them for?
G: I don’t know, it’s what they made us do. Study to pass, and the same with everything.
[...]
J: Man, people have to perfect themselves, and humanity also, if we stayed at a certain point, we could never move forward, we could not help other people.
G: But it’s considered good [sic] to study just to pass. And not that they teach you to learn. Because, if you study it, you will forget it; If you learn it, it will stay with you. Someday you will forget, but ...

It is again difficult for students to justify the supposed importance-usefulness of school mathematics, at least from what they are studying in the ESO courses (2nd-3rd year at the time of the discussion groups). In general, the immediate or automatic response is “because we have to know it-it’s very important-it’s good for life”. But then the doubts and questions appear...

¡No, wait! Math is important, you have to study it “to pass” (the test, the courses), to “be approved” ... to go on with more studies. To progress.

And beyond that, it’s not so clear for the students.

Group 4:

R: What do you think mathematics is taught you for, and what do you think mathematics is used for? Please be sincere.
D: Let’s see, mathematics is taught, really speaking, to teach us (sic!) ... to know how to unwind...
(J. laughs at the non-sense)
D: J. you’re going to take one...

J: (ironic)… Mathematics is taught and used to teach…

D: Yes, they are taught because if you don't know mathematics, you will hardly know anything else...

J (sarcastic) But also, they are used to teach ...

D (annoyed) He won't let me finish the sentence... do you want to shut up? I didn't say that...

(Brief argument)

D: Anyway, we said... mathematics seems to me to be necessary, as J. said. It is "necessary" that they teach it. But sometimes I also thought that it is a lot to give #@*%*...

M: And that there are times when they are not understood ... Why do I have to learn this formula, or five hundred thousand formulas [...], if then maybe a mathematics teacher comes to give classes or is going to dedicate to some math, and he is going to go to the notebook and look at the formulas ... Why do you have to learn those formulas? Sometimes it doesn't make sense.

D: Because if they don't understand it then they won't know how to explain it ...

R: I mean, but you, think about it, why are Mathematics taught to you ...

(Discussion)

D: Let's see, let's be clear. Mathematics give a lot of #@*%*. And even more so when we are... consider that mathematics can make you repeat a whole course where you are doing well, and because you don't pass math, you repeat the course! I mean, that's really annoying!

R: If you don't pass math and language, you repeat the course. That is fashionable, it has become fashionable.

All: Yes, yes.

"They are important-useful because they teach us and they are useful-important." In all the discussion groups of the research experiences emerged some variant of what is shown in these excerpts: in a first automatic response, someone affirmed that mathematics "is very important-useful"; but shortly after the discussion began, the supposed importance-usefulness of math was either (not) justified with non-sensical expressions, or it was quickly questioned by the group. That is, there are contents of the school mathematics that are too abstract (difficult, without context?). And it is not at all obvious how they would be used in "daily life" (or in most careers, professions, vices, felonies); they may be used in specialized applications of certain professions, in "sciences" or for "engineers", not in "everyday life". Arguments of "common sense", but arguments after all...

All in all, in the discussion groups emerged a marked skepticism about the importance-usefulness of school mathematics, despite the first norm-conforming automatic responses.

However, another *sense-event* comes to light, perhaps more clearly in the last excerpt: the compulsory nature of school math, and its strong character of selection/exclusion through the assessment device. There are many expressions like "you study them because you have to pass", and D.'s elegant rhetoric in the last excerpt. Math is useful, but not really, and yet, they are tough and compulsory and can jeopardize the future perspective of students. What?

Let's go back now to the student who wanted to study obstetrics: school mathematics would hardly be important-useful in her intended profession, at the time of assisting in childbirth (we can't imagine how an obstetrician would need to calculate the sum of an infinite series at that given moment; perhaps we are wrong ...). But if she doesn't perform "well" on school math (that is, high enough grades), she won't be able to earn enough political credits to be a midwife. It is not difficult to extend this idea to the paradox of useless-usefulness of school mathematics: "you have to study them to approve them, because they will be useful to you, not in themselves (rather the opposite, they are applied little to nothing in most circumstances of "daily life" and many trades); but because if you do not pass them or do not have good enough grades, your personal possibilities in the future will be limited". Assessment, the sine-qua-non device of this stratification machinery, is implicit in all these disquisitions and *non-sense*…

Non-sense in a plate: math is useful and useless, essential but not really important…

Prudentia

In his own personal experience as students (teen and adults), we never had major problems with math and even enjoyed studying them... However, about tastes and colors... mathematics is a very particular discipline, and even when taught wonderfully, the students may or may not like it ... no, yes?

Well… we are also a bit fed up with seeing over and over the same disconnected, non-sensical, useless topics with our students; repetitive assessments, the suffering in our own flesh of the failures of our students and having to repeat again and again ... What we did not personally experienced in person of this selection/exclusion system, we are living it now through our present students. (And yet, somehow, we need the failure of the students... because in part we gain our bread out of it, and this chapter is and the participation in this conference is partially financed with that income...).

Furthermore, the students do not have such a naïve perspective, nor do they lack a critical view of the arbitrariness of ME, as a selection/exclusion mechanism; and of the doubtful importance of the "contents" of school mathematics, in daily life. We do not necessarily agree with the perceptions and evaluations of the students (rather, the opposite in many cases…). It cannot be denied that in their own way, the kids question some features of the symbolic power of mathematics and ME, as explained in the first part of this chapter.

So yes... there is something paradoxical, non-sensical, in the students' perception regarding the importance-utility of mathematics. Somehow, like in a Lewis Carroll novel or an El Chavo del Ocho episode, they consider mathematics useful and useless, important and unimportant, at the same time. What can the students, their teachers, we researchers, do about it?

Well, first of all acknowledge that, most likely, we also have a lot of paradoxes and internal, repressed non-senses about the sociopolitical place of mathematics, ME and the school as political technologies. So, we adults can, at least in principle, assume that this intrinsic tension is present.

As Cancino (2011) explains, in the individual psyche-social imaginary tension, "society imposes socialization on the psychē through its institutions." In contrast, "the psychē imposes an essential requirement on the social institution: the social institution must provide it with meaning" (Cancino, 2011, p. 73; Castoriadis, 2002, p. 268).

So, we also propose to interpret this non-sense as a way of experiencing the symbolic power of mathematics. More precisely, as a way of conforming their (our) own subjectivity in face of the symbolic power of mathematics, and especially of ME and school mathematics as technologies of selection/exclusion and/or accreditation/stratification in contemporary capitalist societies.

Valero and Pais, in an article from 2015, summarize:

> What if school mathematics is not important in society due to the exceptional and intrinsic characteristics of the academic field that gives this school subject its name, but rather due to the place it occupies within a particular social configuration of power? [...] A political approach thus assumes that the teaching and learning of mathematics are not neutral practices, but that they insert people —be it children, youth, teachers, adults— in socially valued mathematical rationalities and forms of knowing. Such insertion is part of larger processes of selection of people that schooling operates in society. It results in differential positioning of inclusion or exclusion of learners in relation to access to socially privileged resources such as further education, labor market, cultural goods, etc. (Valero & Pais, 2015, p. 178).

As Gilles Deleuze elaborates in *La logique du Sens* (1969), *non-sense* and paradox are intrinsic to *sense*. Either in the proposition or in the (emerging) event and singularity, non-sense allows for the operation of *donation of sense*. By exploring (exploiting without mercy) these internal tensions of *non-sense* we attempt to reveal some chin in the armor of the symbolic power of mathematics (education).

References

Borba, M., & Skovsmose, O. (1997). The ideology of certainty in mathematics education. *For the Learning of Mathematics, 17*, 17–23.

Bruno, G. (2020). *Educación matemática y justicia social: Una experiencia de aprendizaje y servicio solidario* [Doctoral dissertation, Universidad Autónoma de Madrid].

Cancino, L. (2011). Aportes de la noción de imaginario social para el estudio de los movimientos sociales. *Polis, Revista de la Universidad Bolivariana, 10*(28), 69–83.

Castoriadis, C. (2002). *Figuras de lo pensable*. Fondo de Cultura Económica.

Deleuze, G. (1969). *Logique du sens*. Les éditions de Minuits.

Sáenz, C. & García, X. (2015). *Matemáticas: Placer, poder, a veces dolor. Una mirada crítica sobre la matemática y su enseñanza*. UAM ediciones.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Kluwer.

Suavita, M. (2017). *Imaginarios del profesorado en formación sobre las matemáticas: Hacia una cultura matemática para la justicia social* [Doctoral dissertation, Universidad Autónoma de Madrid].

UN General Assembly (2015). Transforming our world: The 2030 Agenda for Sustainable Development. https://www.refworld.org/docid/57b6e3e44.html

Valero, P. (2017). Mathematics for all, economic growth, and the making of the citizen-worker. In T. S. Popkewitz, J. Diaz, & C. Kirchgasler (Eds.), *A political sociology of educational knowledge: Studies of exclusions and difference* (pp. 117–132). Routledge.

Valero, P., & Pais, A. (2015). Examining political perspectives in mathematics education. In C. Bergsten & B. Sriraman (Eds.), *Refractions of mathematics education* (pp. 173–196). Information Age.

White intellectual alibies in use: A critical analysis of preservice teachers' rhetoric

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The demographic disconnect in the U.S. between the majority white, female teacher workforce and the diverse students they serve perpetuates white supremacy in various ways. These relationships can be especially problematic in mathematics settings, where race issues are often disguised behind discourses of neutrality, intelligence, and meritocracy. To further understand how white supremacy is enacted in educational spaces, we applied Leonardo's theory of "white intellectual alibis" to critically analyze interview data involving a pair of white-identifying preservice teachers engaging with novel hypothetical scenarios. Findings show that participants utilized various alibis that reinforced racist narratives and silenced possible antiracist conversations. Implications for teacher education are discussed.

Introduction

One way that white supremacy is perpetuated in U.S. classrooms is through disproportions between white teachers (79%) and students of color (50%) (Taie & Goldring, 2020; Yoon, 2012). Often, white teachers fall back on "hidden expressions of disgust for the Other" (Matias & Zembylas, 2014) and rely on the privilege of whiteness afforded them in existing systems. Within schools, mathematics spaces can be especially problematic as white privilege is further exacerbated by narratives of meritocracy, knowledge neutrality, normative intelligence discourses about who is capable and who is not, and white masculinity (Bullock, 2017; Martin, 2009; Warburton, 2015).

For the white teachers who wish to take part in mending this broken system, and would like to understand their role in perpetuating its preservation, the view from their classrooms can seem daunting. Perhaps white teachers are aware of centuries of inequality in the U.S. (Kendi, 2016; Ewing, 2018), which result in disparate, racialized, educational outcomes that significantly disadvantage students of color (e.g., academic outcomes, graduation rates, and college admissions). They might wonder if their daily interactions with students will have any impact on students' lives inside and outside of the classroom, or affect lasting change in helping reform inherently racist systems.

Although it is tempting to argue that white teachers are not the *problem*, but rather the *symptom* of these racist structures, we choose to focus on them because we contend that the

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actions of individuals can either rescript existing narratives or challenge them (Perry, 2011). Teachers who enter their professions with antiracist identities and the skills necessary to enact antiracist education are better equipped to be part of positive systemic change (e.g., Lewis, 2018; Tilson, Sandretto & Pratt, 2017). Thus, we seek to address the practices of white teachers as leverage points for potential change.

Teacher education programs are potential sites to address these issues. However, it has been argued that teacher education has itself been a site of whiteness's remaking (Daniels & Varghese, 2020, p. 57; Jupp, Leckie, Cabrera & Utt, 2019). Even within teacher education programs that are dedicated to social-justice issues, there continues to be disconnects between antiracist education and teacher practices (e.g., Agarwal, et al., 2010; White, Crespo, & Civil, 2016). This is often due to how white preservice teachers (PSTs) work to avoid and deflect interrogations into their own positionalities, consequences of whiteness, and how racism and white privilege are enacted (e.g., Buchanan, 2016; Lewis, 2018; Matias & Zembylas, 2014). A recent review of 39 peer-reviewed journal articles (all focused on the intersection of preservice teacher education and race) found that "teacher educators continue to face the problems of race talk evasion, colorblind racism, and even retaliation" that were evident in early white teacher identity studies (Hambacher & Ginn, 2020, p. 339).

Examining how PSTs affective responses and white subjectivities are enacted during "uncomfortable conversations devoted to naming the consequences of racism" has potential to illuminate various leverage-points and potential pitfalls for teachers and teacher educators to design and implement antiracist education (Buchanan, 2016; Lewis, 2018; Matias & Zembylas, 2014; Sharma & Lazar, 2014). In this study, we look carefully at the discourse of two preservice elementary teachers, Abby and Sarah, as a window into practices that evade race and racism. Data come from a pair interview; during which, the pair were given several scenarios and were asked to imagine that they were teachers in a fictitious classroom and would need to provide next pedagogical steps that would support students pertaining to content or issues of equity within a mathematics or science setting.

This paper focuses on one of these discussions in which participants were faced with hypothetical scenarios where a student shared a concern regarding how they perceived that "white kids" were being unfairly called on more during math class. Their discussion offers clues into ways which white teachers might unknowingly perpetuate racist status quos, and how they can often see defensive and race-evasive behaviors in another teacher's reactions (e.g., denial, avoidance, unfair blaming), though not in their own, similar, reactions.

Theoretical framework: White intellectual alibis

In this paper, we focus on how the participants used *white intellectual alibis* (Leonardo & Zembylas, 2013) to "prove" their innocence when faced with the possibility of their own racism. The alibi is a spatial metaphor: "The criminal is ruled out as a suspect once he furnishes a fail-proof accounting for his whereabouts [...] he cannot be in two places at once" (p. 152). White intellectual alibis create a racist binary; whites positioning themselves as good nonracists, and "other" whites (or their former selves) as bad racists.

White intellectual alibis leave no room for the possibility that one could espouse antiracism yet maintain discriminatory practices (Leonardo & Zembylas, 2013; Perry, 2011). This suggests an extant nonracist, rather than keeping focus on possible antiracist narratives. Through avoidance and silences, racism proceeds unchecked and internalized messages about white superiority are perpetuated, rather than investigated and challenged (Souto-Manning, 2013). In this way, *acting* antiracist takes precedence over *advocating* for antiracism projects.

There are many possible alibis, such as: "my best friend is black"; "the n-word is just a word"; or claiming "I don't see race" (Nishi, Matias, and Montoya, 2015, p. 462). The findings presented here show how participants Abby and Sarah created white intellectual alibis for themselves through various discursive strategies. Here we pay special attention to three of these alibis—*I'm not like bad racist "others"; I'm a strategic problem-solving teacher;* and *Well, life isn't fair.* Through the lens of white intellectual alibis, we analyze how discourse can work to "prove racist innocence" and avoid "difficult" conversations; we suggest the growing list of white intellectual alibis in teacher education might be expanded to include our findings.

This framework, as well as the white intellectual alibies described in this paper, are informed by a rich tradition of CRT and whiteness theory. For example, Bonilla-Silva (2018) provides a useful guide to recognize rhetorical moves that signify "color blind racism": In this data corpus we see how, as Bonilla-Silva outlines, meritocratic-thinking functions as race evasion (seen in *the strategic problem-solving teacher* alibi). Additionally, in avoiding race talk by discussing the how life may not be fair, or focusing on the behaviour of others, participants were discussing "anything but race" in order to "dismiss the fact that race affects an aspect of [their] life" (Bonilla-Silva, 2018, p. 86). In focusing this analysis on white intellectual alibies, we do not intend to overstate the power of this theoretical lens. We use it to show one of the ways whiteness functions in action – as "evidence" for white innocence.

Participants and methodology

The qualitative data for this study come from a pair interview with two preservice elementary teachers (Abby and Sarah) enrolled in a university teacher training program. Both participants and the interviewer (Author 1) identified as white women. The interviewer approached the participants (and approaches this work) in a spirit of complicity–understanding that her work as a white teacher has been problematic at times, and she must confront and challenge her own white intellectual alibis.

The interview protocol was created as part of a larger project focused on improving math and science content and methods courses in a university teaching licensure program. As part of this project, a team of researchers gathered qualitative audio data of PSTs discussing various classroom case-study scenarios, which we call *hypothetical teaching scenarios* (HTSs). Using hypothetical scenarios in teacher education settings to approximate real classroom interactions is common practice (Shaughness & Boerst, 2018). However, what is less common is gathering and analyzing qualitative data of teachers *collaboratively* engaging with HTSs, in small groups or dyadic interviews, for example. The HTSs in this project were designed to elicit rich discussion about issues that might arise in a mathematics or science classroom and covered a wide range of content and pedagogy (e.g., using representations in mathematics; evaluating arguments in science; addressing sexism, racism, or intelligence discourses in STEM settings; see Gutiérrez et al., 2019).

This paper focuses on two HTSs (Figures 1 & 2). Next, we describe our protocols.

Responding to a Student Grievance

Imagine you are teaching a 5th grade math class. One of your students asks to talk with you privately and you agree to meet with them. They start the conversation as following:

Student: "I don't know how to say this, but… it seems like you only call on the same three white students to show their work at the board when we're doing math. And I have my hand up too! But you never call on me… What's up with that?"

Figure 1: "Responding to a Student Grievance," a hypothetical teaching task.

After reading this scenario, small groups of PSTs were prompted to imagine how they might respond to the student, then write and discuss their responses. From analyzing these responses, the research team created a coding scheme that highlighted common ways PSTs used language to respond (e.g., apologies, explanations, or "solutions"; Gutiérrez et al., 2020). The research team proposed taking this HTS into an interview setting where the interviewer would ask follow-up questions and provide individualized prompts. We chose to utilize a pair interview format, where participants could interact back-and forth with a friend as well as the interviewer. We hypothesized this format would create an environment for participants to discuss difficult or uncomfortable topics, such as race, in ways that they might not otherwise. In this setting, at times, the interviewer prompted participants to discuss responses with one another and took an active observer role (Morgan, Ataie, Carder, & Hoffman, 2013). At other times, the interviewer jumped into the conversation, sharing relevant experiences and prior racial assumptions or beliefs.

For this pair interview study, we also sought to understand how PSTs viewed another teacher's response; to see if they might notice whiteness discourse outside of themselves. Thus, we added an HTS, *Student Grievance Conversation* (Figure 2), that included an imaginary conversation between the hypothetical teacher and student. The hypothetical teacher's responses in this HTS were crafted to closely follow common themes from the small-group discussion analysis (Gutiérrez et al., 2020). During the interview, Abby and Sarah were presented with paper versions of these scenarios, one at a time. For *Responding to a Student Grievance* they were asked to write how they would respond to the student, then share their responses with one another and discuss. They were also asked follow-up questions and prompted several times throughout. Next they were asked to read the *Student Grievance Conversation* and were given similar protocol prompts. However, in this case they were asked to discuss their thoughts about the teacher's response, rather than their own responses.

Student Grievance Conversation

Imagine the same student and a teacher (other than yourself) had the following conversation after the student shares the same concern:

Student: "I don't know how to say this, but... it seems like you only call on the same three white students to show their work at the board when we're doing math. And I have my hand up too! But you never call on me... What's up with that?"
Teacher: "I'm so sorry you feel this way. I never meant for this to happen. I will try and call on students more fairly in the future."
Student: "Thanks. That's cool. I don't think it's just that I feel this way, though. I notice the same thing happening with my friends or at the store. I just feel invisible sometimes. I just want white people to admit that they treat me this way."
Teacher: "Well, again, I'm sorry you feel this way. Not every white person treats you this way. I care for all my students. I can't control your friends or people at the store. But I can control what I do in my classroom."
Student: "OK. Thanks." [walks out of the room, but seems disappointed and has head down.]

Figure 2: "Student Grievance Conversation," a follow-up HTS.

The interview was video and audio recorded, and transcribed. We applied Critical Discourse Analysis methods to analyze, understand, and explain the data in order to "speak to and, perhaps, intervene in institutional, social, or political issues, problems, and controversies in the world" (Gee & Handford, 2013, p. 9).

Findings

Here, we highlight three of the white intellectual alibis consistently employed by Abby and Sarah. Much of the discourse seen here is discussed in whiteness literature to explain, for example, ways in which avoidance, deflection, image management, or cultures of caring work to shift conversations away from antiracist ones (e.g., Lewis, 2018; Orozco, 2019). We present these data under the lens of white intellectual alibis in order to further highlight ways that rhetorical moves are used as spatial dividers between "innocent" nonracists and "guilty" racists.

The strategic, problem-solving teacher alibi

Both Abby and Sarah wanted to "solve the problem" as evidenced in Abby's statement, "I would feel horrible if a kid felt that way, and I would want to fix it." Solving problems and fixing things is generally considered an important part of a teacher's job; however, using pedagogies and classroom practices as comprehensive solutions to racialized situations is, we submit, a type of white intellectual alibi: *I'm a strategic, problem-solving teacher.* The use of this alibi both exonerates white teachers from being racist, and from participating in further discussions of race. Notice how this alibi functions by considering Sarah's initial response to the first scenario:

> I would never call on the same three white kids [...] I wouldn't teach like that [...] [I would] pull sticks [...] making sure every child can answer the question.

Although Sarah used the term "white" (a rare occurrence in the data corpus), there was no prior or further discussion about race or racism. She proceeded as though she had solved the problem by pulling sticks in order to randomize student participation (her solution for equity.) This illustrates how an alibi works to "solve the problem," and shut down possible conversations about race. Sarah could not be in two places at once: with strong pedagogies and classroom management skills, she precluded herself from committing a racist crime and interrogating potential racism or biases.

This alibi also worked to pacify the student and "prove to him that they were not doing it on purpose" (Sarah). Instead of engaging in critical self-reflection, Abby and Sarah addressed the student in order to "fix the problem," proposing several solutions. For example, at one point Sarah offered the following solution: "I will make sure to wait a little bit longer for people to put their hands up as well." And another time, Abby sought to involve the student in the solution: "Like, do you have a solution [...] so how can I fix this for you? What I'm doing might not be working for you. What do you have in mind?" In focusing on pragmatic solutions for the issue at hand, and in working to "fix" things by getting the student's input, Abby distanced herself from the guilty racist charge. There was no further discussion about racism afterward; these conversations about classroom pedagogies were used to absolve Abby and Sarah and convince themselves (and others) that there was no possibility that a racist crime had been committed.

Well, life isn't fair alibi

Another alibi can be seen in the following excerpt that occurred after Abby scanned the second HTS and audibly sighed:

> I can see [...] it talks about the white people, and stuff, and the sad thing is, you can't control every aspect of their lives [...] I can only do so much [...] Like, I can't go everywhere and make sure everybody treats them fairly.

This statement seemed to put-an-end-to, or take place of, possible discussions of racism. As with other alibis, on its face, this is a true statement: Abby cannot "control every aspect" of a student's life. However, the work is not done through the statement alone, but through the insinuation that this fact is not compatible with problematizing racist selves and practices.

We call this alibi *Well, life isn't fair*. One of the most notable incidents of this alibi was near the end of the interview when Sarah explicitly addressed the possibility of her own racism by saying:

> I feel like I'm coming off racist [...] I'm not [...] Like it's always a battle [...] but you're not going to win every battle [...] like there are going to be really hard things to deal with in life [...] and there's some really crappy people in the world, but we need to, like, be above those people.

Sarah's contention that she was "not" racist was followed immediately with the *Well, life isn't fair* alibi (e.g., "Like it's always a battle [...] but you're not going to win every battle"). The way that the nonracist comment was paired so closely with this alibi is evidence that Sarah saw the two as mutually exclusive. Her racism couldn't coexist with the fact that life was, in general, unjust and "hard." For Sarah, the two ideas could not occupy the same space at the same time.

I'm not like bad racist "others" alibi

The last part of the excerpt above, "there's some really crappy people in the world, but we need to, like, be above those people," is an example of a common theme in Abby and Sarah's speech: racism does exist, but only in others ("crappy people".) This alibi, *I'm not like bad racist "others,"* was especially apparent when Abby and Sarah, positioned as observers of another teacher, responded to the second HTS. They spoke as though the teacher's response was racially motivated, whereas, they projected their own motivations as unintentional—creating a nonracist alibi while distancing themselves from the racist "other" white in the scenario (Leonardo & Zembylas, 2013).

Especially revealing of this was when Sarah critiqued the teacher's apology:

> The *but* takes away the sincerity of it. It's like, 'blah, blah, blah, this, this, and this. *But* I can't really help you.' I hate the comment, 'I'm sorry you feel this way.' To me, that's like, it's almost like, 'Oh, yeah. I'm sorry you feel this way!' [sarcastic tone] That's not validating their feelings at all [...] it's shutting it down and trying to make it a smaller problem.

Here, Abby's critique of the teacher's apology ("I'm so sorry you feel this way") was very similar to Abby's apology from the first scenario ("I'm so sorry if, like, that made you feel [...] I'm so sorry if that made you think that I never call on you."). Despite the obvious similarities in the apologies, Sarah never mentioned this irony. In fact, she "hates" the teacher's apology. Looking at this phenomenon through a white intellectual alibi lens, we can see that it functions similar to a legal battle where the defendant's character is either defamed or commended. Again, we see how the spatial binary works: a defendant either has "good" behavior and, thus, is not capable of committing the crime of racism, or has "bad" behavior, and thus, is likely to be racist. Further, in pointing-out the bad behavior of others, the defendant separates themselves even more from the scene of the crime.

Discussion and future research

White intellectual alibis

This analysis shows how, as Abby and Sarah navigated a racialized discussion, they spent a significant amount of time creating white intellectual alibis that carefully managed their images as nonracist. Although both teachers seemed genuinely interested in addressing and rectifying the problem broached by the hypothetical student, they were rarely direct in addressing racism as a possibility. Abby and Sarah seemed, primarily, concerned with proving their innocence to themselves and the hypothetical student through the use of alibis

These findings validate the claim that "turning to whiteness in education means that the subjects who are least individually prepared and collectively underdeveloped for race dialogue occupy a central place at the table" (Leonardo & Zembylas, 2013, p. 155). This is a call to white educators like me (Author 1) to interrogate the alibis we use that allow us to excuse ourselves from racist practices. So doing, I hope to model for PSTs a process of critical self-reflection, attempting to expose *my* white intellectual alibis and engage in race-visible pedagogy (Jupp et al., 2019).

Hypothetical teaching scenarios

The novel use of HTSs in a pair interview has several implications for teacher education and critical education research. Our findings reveal that, although the participants were engaged in discourse, they carefully avoided discussing racism and possible antiracist pedagogy. This implies that teacher educators who wish to use HTSs as antiracist pedagogies will have to carefully consider design that nudges participants to explore issues of racism more directly. Otherwise, group discussions might be spaces where whiteness continues to have a primary role, instead of discussions where whiteness is questioned and PSTs are able to see their roles in problematic racist practices. In the case of Sarah and Abby, one possible tactic might be to circle back to their response to the first scenario and highlight the similarities in apologies. This might create a generative tension that can carry them through a crucial dialogue and confronting their potential racial biases.

Limitations and extensions

This HTS was couched in a mathematics-centric protocol; however, narratives about race in mathematics settings were backgrounded in this particular scenario. We were curious to see if participants would be primed (by the previous HTSs) or notice that the student referred to math in his complaint; however, none of the participants focused on this point. In future studies, we hope to find a way to include race issues while simultaneously highlighting the mathematics setting, in order to see how PSTs discuss these issues in tandem.

Through much feedback and discussion, we imagine there are many variations of this HTS that might extend our understanding of how whiteness functions in teacher education settings. For example, as suggested by an MES reviewer, the HTS might be structured to "reveal the kind of anti-racist response" that we might encourage in PST learners. As per his suggestion, perhaps the teacher would be directed to "ignore the dominating white male students in order to allow other voices to emerge." In this setting we could investigate how PSTs respond to the idea of "using discrimination to overcome discrimination?"

Finally, this work might be extended to other settings where whiteness is at work. It seems natural, for example, to use HTSs in professional trainings for university faculty and staff in STEM degree settings where, typically, student diversity is low, unexamined biases are high, and individual faculty members are engaged in practices that can either reinscribe or challenge this status quo (Killpack & Melón, 2016).

References

Agarwal, R., Epstein, S., Oppenheim, R., Oyler, C., & Sonu, D. (2010). From ideal to practice and back again. *Journal of Teacher Education, 61*(3), 237–247.

Bonilla-Silva, E. (2018). *Racism without racists: Color-blind racism and the persistence of racial inequality in America.* Rowman & Littlefield.

Buchanan, L. (2016). Elementary preservice teachers' navigation of racism and whiteness through inquiry with historical documentary film. *The Journal of Social Studies Research, 40*(2), 137–154.

Bullock, E. (2017). Only STEM can save us? *Educational Studies, 53*(6), 628–641.

Daniels, J., & Varghese, M. (2020). Troubling practice: Exploring the relationship between whiteness and practice-based teacher education in considering a raciolinguicized teacher subjectivity. *Educational Researcher, 49*(1), 56–63.

Ewing, E. L. (2018). *Ghosts in the schoolyard.* University of Chicago Press.

Gee, J. P., & Handford, M. (Eds.). (2013). *The Routledge handbook of discourse analysis.* Routledge.

Gutiérrez, J. F., Barth-Cohen, L.A., Francom, R., Greenberg, K., MacArthur, K., & Dobie, T. (2019). An emerging methodology for the study of preservice teachers' learning about equity in STEM education. In S. Otten, A. Candela, Z. de Araujo, C. Haines, & C. Munter (Eds.), *"... against a new horizon." Proceedings of the 41st annual meeting of the North-American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA).* University of Missouri.

Gutiérrez, J. F., Dobie, T., Greenberg, K., Francom, R., & Barth-Cohen, L. (2020). Examining preservice teachers' written responses to a hypothetical classroom scenario involving race. In M. Gresalfi & I.S. Horn (Eds.), *The Interdisciplinarity of the Learning Sciences: Proceedings of the 14th International Conference of the Learning Sciences (ICLS 2020)* (Vol. 4, pp. 2325–2328). Vanderbilt University.

Hambacher, E., & Ginn, K. (2021). Race-visible teacher education: A review of the literature from 2002 to 2018. *Journal of Teacher Education*, *72*(3), 329–341.

Jupp, J. C., Leckie, A., Cabrera, N. L., & Utt, J. (2019). Race-evasive White teacher identity studies 1990-2015. *Teachers College Record*, *121*(2), 1–58.

Kendi, I. X. (2016). *Stamped from the beginning.* Hachette UK.

Killpack, T. L, & Melón, L. C. (2016). Toward Inclusive STEM Classrooms. *CBE Life Sciences Education*, *15*(3), es3. https://doi.org/10.1187/cbe.16-01-0020

Leonardo, Z., & Zembylas, M. (2013). Whiteness as technology of affect. *Equity & Excellence in Education, 46*(1), 150–165.

Lewis, T. (2018). "But I'm not a racist!" Phenomenology, racism, and the body schema in white, pre-service teacher education. *Race Ethnicity & Education, 21*(1), 118–131.

Martin, D. B. (2009). Researching race in mathematics education. *Teachers College Record, 111*(2), 295–338.

Matias, C. E., & Zembylas, M. (2014). 'When saying you care is not really caring': emotions of disgust, whiteness ideology, and teacher education, *Critical Studies in Education, 55*(3), 319–337.

Morgan, D., Ataie, J., Carder, P., & Hoffman, K. (2013). Introducing dyadic interviews as a method for collecting qualitative data. *Qualitative Health Research, 23*(9), 1276–1284.

Nishi, N., Matias, C., & Montoya, R. (2015). Exposing the white avatar. *Social Identities, 21*(5), 459–473.

Orozco, R. (2019). The method of avoidance. *Whiteness and Education, 4*(2), 128–145.

Perry, I. (2011). *More beautiful and more terrible.* New York University Press.

Sharma, S., & Lazar, A. (2014). Pedagogies of discomfort. *Advances in Research on Teaching, 21*, 3–29.

Shaughnessy, M., & Boerst, T. A. (2018). Uncovering the skills that preservice teachers bring to teacher education. *Journal of Teacher Education, 69*(1), 40–55.

Souto-Manning, M. (2014). Critical narrative analysis. *International Journal of Qualitative Studies in Education, 27*(2), 159–180.

Taie, S., & Goldring, R. (2020). *Characteristics of Public and Private Elementary and Secondary School Teachers in the United States*. Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Tilson, J., Sandretto, S., & Pratt, K. (2017). Using preservice teachers' beliefs, theories and video-recorded teaching to prompt a cycle of praxis. *Teaching and Teacher Education, 67*, 454–463.

Warburton, T. (2015). *Solving for irrational zeros: Whiteness in mathematics teacher education* [Doctoral dissertation, University of Utah]. https://collections.lib.utah.edu

White, D., Crespo, S., & Civil, M. (2016). *Cases for mathematics teacher educators: Facilitating conversations about inequities in mathematics classrooms*. AMTE.

Yoon, I. (2012). The paradoxical nature of whiteness-at-work in the daily life of schools and teacher communities. *Race Ethnicity & Education, 15*(5), 587–613.

Mathematics in action: an approach in civil engineering and ecology courses

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This paper presents part of the results of a doctoral research linked to the teaching and learning of mathematics in higher education. This study aims to reflect on the ways in which mathematics can be perceived in real life contexts – this conception is known as Mathematics in Action. The data were produced from activities developed based on the perspective of Problem-Based Learning, using a qualitative approach. They were developed in Civil Engineering and Ecology courses at two public higher education institutions in the state of São Paulo, Brazil. The results show evidence that learning contexts that present mathematical formulas based on reality can favor reflections on actions or decisions based on mathematics. They also contribute to the production of other studies involving the field of mathematics education in higher education.

Introduction

In recent years, concerns about the challenges and perspectives of higher education have been the subject of international level discussions. The aspects involving the teaching and learning processes, as well as the understanding and use of knowledge, in different courses, are linked to future actions of individuals in society, whether personal or professional. As a result, concerns also emerge regarding the ways in which the teaching and learning processes occur.

These concerns also extend to the field of mathematics education. After all, the notions linked to mathematical knowledge move in several directions. Reflecting on them can stimulate the development of critical perspectives. In addition, the aspects that revolve around the teaching of mathematics can be explored through an environment that encourages investigation processes.

The main objective of this paper is to propose discussions about the perceptions of mathematics being put into action in real life contexts, which is understood in the literature as the concept called mathematics in action, proposed by Skovsmose (2006, 2014a, 2014b, 2020). The inclinations of the respective work encompass perspectives based on Problem-Based Learning (PBL). The intention was to establish greater approximations between theoretical studies involving mathematical knowledge and its practical applications projected in real life contexts.

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In the academic world in general, there are already studies related to mathematics and learning based on problems and projects in the context of universities. Among them, it is possible to mention works such as: Vithal, Christiansen and Skovsmose (1995); Christensen (2008); Hernandez, Valero and Ravn (2015); Gouvêa (2016); Souza (2016); Valero and Ravn (2017). However, there are no empirical studies related to the concept of mathematics in action in Higher Education from PBL perspectives. This is a fundamental characteristic that demonstrates the originality of this research. Making investigations possible in this sense is something relevant to students, the academic community and our society. Thus, the purpose of this study is to reflect on some learning contexts that are based on reality and the use of mathematical concepts or formulas, in two bachelor degrees: Civil Engineering and Ecology.

Initially, this paper presents the methodological perspective and some procedures used in the referred research. In addition to describing the elaboration and development of the proposed activities, which were outlined based on a real situation, inspired by the PBL. Following, the theoretical assumptions related to the development of mathematics in action, which can be perceived in reality through five performance aspects are highlighted.

To conclude, are presented discussions about elements that were present in the construction of one of the three themes elaborated from the analysis and interpretation of the data, which was entitled “Learning contexts with formulas or mathematical concepts can tell us many things”. In this stage, we highlighted three aspects of mathematics in action: technological imagination, hypothetical reasoning and realization, in addition to relevant considerations about the learning principles related to PBL.

The study proposes reflections on the different ways of perceiving the presence of mathematics in real life. Actions or decisions based on mathematics can be analyzed and questioned in universities through practices that lead to reflection, in addition to collaborating with the personal and professional training of students. Contributions such as these can favor studies in the field of mathematical education and concerns related to the challenges and perspectives of higher education.

The study design

The research molds were built from a qualitative approach, which seeks to interpret in a careful way the dynamics of the observed study.

The data production stage took place at two different times, in two public higher education institutions in the state of São Paulo, Brazil, during the second semester of 2018, through weekly meetings. In the first institution, named in the research as institution A, a group of first year students of Civil Engineering participated in the investigation, during extracurricular hours. In the case of the second institution, called institution B, the participants were second year students of the Ecology course and the whole process took place during some classes in the course of Differential and Integral Calculus II.

The study and analysis of a real problem mobilized the dynamics of meetings. There were four meetings at both institutions. At institution A, activities were carried out by the researcher herself. In the case of institution B, in addition to the students and the researcher, there was the participation of the class teacher and a research assistant, who was part of the group of studies and research in mathematics education to which the author of this paper is linked. In this same institution, the students were divided into small groups and the discussions held took place through three meetings. Furthermore, the fourth meeting was destined for the final presentation. At this stage, the students could approach themes addressed throughout the process.

All data produced were recorded through audio recordings, and the researcher wrote comments in field notes, respecting the research's ethical principles.

Regarding the data presentation, there were concerns about demonstrating the potential of the work based on the discussions generated by a particular problem. The transcription of the data resulted in the first drafts, which were read and rewritten a few times. This process culminated in the final production of two texts, presented in two distinct chapters, one focused on data produced at institution A and the other, focused on institution B.

The data analysis process was inspired by the theoretical assumptions of Creswell (2007). For the author, the constitution of the analysis occurs through some steps that range from the organization and preparation of the data to the final interpretation of the results obtained.

After carrying out the analysis some categories emerged, which led, finally, to the stage of elaboration and description of the themes. According to Creswell (2007), these themes represent the main results in qualitative studies. Furthermore, they must be based on the theoretical contributions of the study and on specific evidence related to the research issue. In this thesis, three central themes emerged, which are directly or indirectly linked to the five aspects of mathematics in action. The three themes are:

1. Learning contexts with formulas or mathematical concepts can tell us many things
2. Elaborating legitimations and justifications through mathematics
3. Reflections on responsibility, ethics and valuation

In this paper, the discussions refer to the first theme, which discusses the use of certain concepts or mathematical formulas in different situations. The contexts analyzed by the participants mobilized different reflections about possible applications of a mathematical model in reality.

The problem that was used as the guiding thread of the investigation process is presented below.

From the organization to the understanding of the problem used

The material organization that led some of the face-to-face meetings as part of the master's research developed by the researcher (Souza, 2016). Its structuring followed theoretical assumptions related to the PBL's use. At that time, four problems were elaborated, which could be worked on in the exact, humanities and natural science areas, for example. These problems were structured by the researcher based on real or potentially real contexts and, among them, there was a suggestion of possibilities to assist in conducting the activities.

Soon, during the doctorate, there was a restructuring of one of the problems presented in the dissertation, based on new theoretical contributions. The intention was to use the chosen problem as a trigger for the discussions that would be held during the face-to-face meetings, seeking to ascertain whether the choice would open up possibilities to foster different reflections, in addition to the field of mathematical education.

"Environmental impacts caused by chemical pollutants" was the title of the problem. It consisted of two approaches: one involved a description of a fire that occurred in fuel tanks, in 2015, in the Alemoa neighborhood, in the city of Santos, state of São Paulo, Brazil, causing contamination of the region and several environmental impacts: social, political and economic. The other approach called "Support study" was elaborated from some adaptations of an activity found in a Calculation book. It addressed a fictitious situation, similar to the real case that occurred in Santos, and sought to contribute to discussions related to different mathematical knowledge worked in university.

The subsequent section includes theoretical references to the themes involved in the present study.

Understanding the concept called mathematics in action

The concept called mathematics in action was developed by Ole Skovsmose and is part of the concerns associated with the field of critical mathematics education. We understand that mathematics is associated with a variety of situations and practices, like social, economic and political aspects. Thus, it is possible to propose discussions about the operationalization of mathematics in different contexts.

When talking about mathematics in action we are referring to

> All those practices that include mathematics as a constituting part. It could be: technological innovation; forms of production; automation; management and decision making; financial transactions; risk estimation; cost-benefit analysis, etc. (Skovsmose, 2006, p. 323).

For this author, practices like these contain actions based on mathematics and, therefore, there is a need to promote reflections around them. In this way, this concept can be understood from five aspects: *technological imagination*, *hypothetical reasoning*, *legitimation or justification*, *realization* and *dissolution of responsibility*.

The technological imagination refers to the possibilities of exploring the construction and development of hypothetical situations in the form of technological alternatives, based on imagination. Every type of enterprise can be based on imagination, that is, on an imaginary scenario. Hypothetical situations can be raised and mathematics can provide materials for the construction of such situations.

Elaborate the hypothetical reasoning it concerns the analysis and evaluation of the possible consequences of an imaginary scenario. In other words, this aspect contributes to investigations about a given situation before an undertaking is executed or a decision is made. Mathematics can help in the approaches related to hypothetical reasoning because the hypotheses and investigations about the probable results of something not yet realized can be made through the use of mathematical models.

The construction of justifications and legitimations math based can be used for certain actions or decisions be taken. The aspect of realization takes action when the mathematics becomes part of reality in everyday life, but many effects may be hidden in these models and the categories and discourses produced from them are varied and their consequences can sometimes be pleasant and beneficial, sometimes they can present risks and disadvantages.

Finally, the dissolution of responsibility occurs when actions supported in mathematics include an exemption of liability. Normally, any action refers to an agent subject, but, in mathematics in action, that subject's performance does not seem to exist and the relevance of responsibility seems to be in charge of the mathematical model used. When addressing the question of responsibility, it is necessary to analyze whether the methods and tools used are reliable; whether the calculations performed are reasonable; whether (and which) aspects were ignored in the formulation of the model; etc.

Next, we presented some analyzes regarding the understanding of three aspects of mathematics in action constructed from the constitution of scenarios that encompass concepts or mathematical formulas.

What learning contexts involving reality and mathematical formulas can tell us

This section presents some previous results of the doctoral research. In different moments of data production, the use of some concepts or mathematical formulas in different situations mobilized many reflections. These contexts contributed to some aspects of mathematics in action that emerged during the discussions.

The construction of the theme "What learning contexts with mathematical formulas based on reality can tell us" contemplates three aspects of mathematics in action: technological imagination, hypothetical reasoning and realization. Thus, to contemplate them, another perspective is presented to approach the different mathematical content in universities.

A teacher putting different mathematical relationships on the board. A textbook or handout material. Students copying examples or solving exercises. Usually, this landscape prevails in many classes that contemplate mathematics in higher education.

The contexts presented below involve a blackboard, study materials, examples or exercises. However, everything was contemplated through the students' protagonism. The perceptions that emerged during the meetings were possible through the material delivered to the students about the case of the accident in Santos-SP. To start the discussions, the following a student's speech from institution A:

Rogério: [...] we could work with some mathematical models thinking about the influence that [the fire in Alemoa] would have on the traffic of that region. They could be linked to a project that was done or that we want to do. You can see how much this will influence the fauna, you will have to move the animals that are around and you have to have all this planning part of how much this would change in the surroundings. For example, statistical data ... I think it would be interesting to include them in the analysis to quantify how much would change and to take the respective measures to mitigate the impacts, as much as possible.

According to Rogério's speech, we understand that mathematics is associated with the aspect of *technological imagination*. The examples he describes reveal the development of hypothetical situations and show that there are possibilities to think about different alternatives based on the analysis and understanding of mathematical models and the use of statistics. The notes made by the student concern the forms of organization, know-how and procedures related to the execution of projects and undertakings, as well as decision-making.

Another issue that emerged in the discussions was present in the "Support study", and that referred to the self-purification process. At institution A, the participants worked effectively with the material delivered at the beginning of the activities. When elaborating and developing it with the students, there were no specific explanations as to the meaning of the term self-purification. Interpretations related to that word were developed over the meetings and were made by the students themselves, based on the analysis of a bay's self-cleaning ability. The following dialogue was based on the following information and took place between researcher Débora and students Luara and Renata, both from institution A.

> After a careful analysis of the situation, environmental scientists have guaranteed that the bay has a capacity for self-cleaning at a rate of 20% per year.

Débora: Here I would like to ask just one question: "Ability to self-purify at a rate of 20%. What is your understanding of this?"

Luara: I don't know what self-cleaning is, but it must be related to something that is over.

Renata: I think it must be the rate that contamination takes to decrease, for example, in one year, it will return to 20% of what it was before.

The situation presented contributed to the engineering students elaborated interpretations of what would be the representation of a scenario elaborated according to this information. And the raising of these hypotheses was associated with the following mathematical model covered in the support study:

> Based on this hypothesis, the specialists then established a model for the concentration of Oily Agent over time:

$$f(1) = 10$$
$$f(x+1) = 0{,}8f(x)$$

Soon, the students stated:

Luara: So, it will always decrease by 20% and 20%; it will remove more and more oil and there will always be a little bit leftover.

Renata: It means that the next [value] will always be 80% of the previous one.

It can be seen that although they did not have specific knowledge about self-purification, this was being built from the set of data analyzed. There was an opening for the *technological imagination* emerged. The students explored hypothetical situations based on imagination. The organization of the mathematical information constructed in this imaginary scenario allowed them to analyze and evaluate the consequences of that scenario in reality. These perceptions are in line with Skovsmose (2014b, p. 96) when he says that "one can reflect on the nature of technological imagination supported by mathematics in view of specific issues [...]. This imagination can generate new guidelines [...] It can enable actions that otherwise would not be possible". With this, if the relations given were placed to investigate the decontamination processes in that bay, it would be necessary to put into action the *hypothetical reasoning* aspect because through it the hypothesis survey would be analyzed using the models considered.

Luara and Renata stated that self-purification was associated with the rates of decrease in the concentration of pollutants. A similar placement was also made by the ecology students, Katia and Carlos, from institution B. They highlighting other important elements:

Katia: In this process, depending on the location, if it is in the sea, in the lake, it can have a difference in oxidation. As in the seas, the current is faster and larger, it occurs faster than in the river.

Carlos: And self-purification also depends both on the speed of the watercourse and on the morphology of the riverbed or the quantity of the substance that is emptied, we can associate it with the transparency of the river.

This proved to be relevant as the analysis of the participants evolved, highlighting that as proposed by Skovsmose (2008), the analysis of an imaginary scenario is not only related to mathematics, after all, it involves other fields of knowledge. Katia and Carlos pointed out to evaluate the self-purification process, it would also be necessary to understand the location's oxidation, the speed of the watercourse, among other factors related to environmental issues. This placement of students reinforces that hypothetical reasoning should not be supported by mathematics alone. It is in line with Skovsmose (2008) when stating that in situations like these, the details of the investigation are only represented within a specific mathematical construction within a given alternative and, thus, there are limitations regarding real reasoning because the reasoning itself is founded on mathematics.

When advancing in the reading and analysis of the support study, the engineering students deepened the discussions. They hypothesized about the limit formula of a function: $\lim_{x \to \infty} f(x) = 0$. For example, Luara, in his first perception, deduced:

Luara: I think there is a limit to how much the place can get rid of the pollutant. [...] It seems that it will never reach zero. We know that this is decreasing ...

As the readings progressed, in association with the tables and graphs of the support study, Luara and Renata elaborated more interpretations about this formula:

Luara: It makes no sense! That's because, in the given expression, x tends to infinity! So, it will only return to the natural, to the infinite ... only that we will never reach the infinite because it is infinite.

Renata: It may be a very small amount, but it will still be there.

The hypotheses raised by the students demonstrate reflections on the real impacts of applying a mathematical formula in reality, which is related to the aspect of *realization*. For them, the limit formula interpretation in the given context could be somewhat wrong. For them, the bay, would not be totally decontaminated, because even over the years there would still be some pollutant in the place, from the spill of the oily agent. The students' statements reveal that certain effects may be hidden in a situation like this. In addition, the speeches produced from such an interpretation, could present risks and disadvantages.

Subsequently, in the context of the material used, the expression of the limit of a function was used as an argument by the defense lawyer of the company responsible for the accident. According to him, the formula supported the thesis that the cleaning of the bay would occur naturally over the years, which, therefore, would influence the value of the fine for the environmental impacts caused. Even before reaching the part of the material where this conclusion would be made, student Rogério emphasized:

> Rogério: As I understand it, the damage that was caused in that environment, over time, would gradually be cured. Over time, the more it would improve, until it reached the point where he would be completely cured, so the lawyer approached this for a time equal to infinite. So based on that he gave a natural response, as if not as if there was no real damage to nature because over time she can fix things herself [...]

The student imagined a scenario constituted by the understanding of several elements that emerged during the meetings: the limit formula, the tables and graphs of the support study, the deepening of research on self-purification and, such analyzes, would influence in reality, leading us to perceive the aspect of *realization* as well.

In these analyzes it was possible to verify that the knowledge used was applied in a particular situation and new possibilities of learning, of studies, emerged from the materials and the discussions provided. With that, different actions and decisions could be taken in face of the conclusions obtained. For example, the entire context involving these self-cleaning processes would contribute to the assessment of the analyzed aquatic environment and, consequently, this would allow different actions to be planned and executed, such as the release of fishing, the ways of compensating the damages caused, among other factors.

This entire process of discussion and reflection was supported by theoretical foundations by Skovsmose. The construction of a set of hypothetical situations is a powerful act. It is understood that opening possibilities for technological imagination, hypothetical reasoning and realization to appear in higher education mathematics classes is something powerful. Thus, this opening can transform learning contexts supported by the predominance of conventional classes of the explanation-exercises-correction type.

Concluding remarks

In this paper we presented some results of a PhD research in progress. In it, it is possible to contemplate ways of approaching some aspects of mathematics in action in higher education, inspired by a work perspective based on the PBL.

We tried to highlight the path of the investigative process, describing from the organization to the stages of development of the activities carried out. There was also a concern to explain the understanding of the five aspects of mathematics in action, even though only three of them were actually worked on in the presentation of the results.

Develop this theme in the thesis revealed some research potentialities. Inspired by Skovsmose it was possible to perceive, through learning around a problem, how the mathematics in action's aspects could reveal themselves, even in different university contexts.

Learning contexts that present mathematical formulas based on reality can promote different reflections. This theme emerged from moments when participants made interpretations regarding the use of formulas, equations, mathematical indexes or even concepts related to this field of knowledge. In different circumstances they sought to understand the ways in which mathematics operated in certain situations, whether implicitly or explicitly. In certain cases, the participants seemed to perceive the context described in each case addressed. They raised different hypotheses, sought to identify and assess the possible consequences of a project, undertaking or deciding, and also seemed to understand the real application of mathematical models in their fields of activity. It is understood that these reflections were mobilized by the work with the PBL. This teaching methodology contributed to the students' concerns being expressed. The problem adopted in the investigation was used to trigger the discussions. Several of them related to technological imagination, hypothetical reasoning, and achievement. However, the notes made by the participants did not only concern mathematics. They were associated with a real event and was structured to promote reflections on social, environmental and economic aspects, for example. Furthermore, the students worked in groups, discussed real issues and problems, there were perceptions about different fields of knowledge, they deepened research involving their fields of interest and presented criticisms regarding mathematics and their future area of expertise in this case, as engineers or ecologists, etc. Finally, the elaboration of this theme aimed to highlight these perceptions, in addition to proposing discussions that aim to contribute to the field of mathematics education in universities.

This was a way of presenting some reflections in this article. By linking the mathematical field to different learning situations, it is possible to perceive the ways in which mathematics can present itself in reality. Furthermore, this allows students to reflect on the knowledge learned and analyze how the presence of mathematical models in reality can actually happen. Being aware of such perceptions is something that cannot be lacking in the field of mathematical education and this can contribute to the adoption of different critical thoughts and attitudes in society.

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References

Christensen, O. R. (2008). Closing the gap between formalism and application: PBL and mathematical skills in engineering. *Teaching Mathematic and its Applications, 27*(3), 131–139.

Creswell, J. W. (2007). Projeto de pesquisa: métodos qualitativo, quantitativo e misto. Artmed.

Hernandez, C.; Ravn, O. & Valero, P. (2015). The Aalborg University PO-PBL Model from a socio-cultural learning perspective. *Journal Problem-Based Learning in Higher Education, 3*(2), 16–36.

Gouvêa, G. de (2016). *Reflexões acerca do uso da aprendizagem baseada em problemas no ensino de conceitos matemáticos* [Master's thesis]. Programa de Pós-Graduação em Ensino de Ciências e Matemática, Instituto Federal de Educação, Ciência e Tecnologia de São Paulo, São Paulo, Brazil.

Skovsmose, O. (2006). Reflections as a challenge. *ZDM Mathematics Education*, *38*(4), 323–332.

Skovsmose, O. (2008). *Desafios da reflexão em educação matemática crítica.* Papirus.

Skovsmose, O. (2014a). *Um convite à educação matemática crítica.* Papirus.

Skovsmose, O. (2014b). *Critique as uncertainty.* Information Age.

Skovsmose, O. (2020). Banality of mathematical expertise. *ZDM Mathematics Education, 52*(6). https://doi.org/10.1007/s11858-020-01168-4

Souza, D. V. de (2016). *O ensino de noções de cálculo diferencial e integral por meio da aprendizagem baseada em problemas* [Master's thesis]. Programa de Pós-Graduação em Ensino de Ciências e Matemática, Instituto Federal de Educação, Ciência e Tecnologia de São Paulo, São Paulo, Brazil.

Valero, P. & Ravn, O. (2017). Recontextualizaciones y ensamblajes ABP y matemáticas universitárias. *Didactiae, 1*(1), 4–25.

Valero, P.; Christensen, O. R. & Skovsmose, O. (2009). The multi-layered transitions of knowledge production and university education in science and mathematics. In O. Skovsmose, P. Valero, & O. R. Christensen (Eds.), *University sciences and mathematics education in transition* (pp. 1–13). Springer.

Vithal, R.; Christiansen, I. M. & Skovsmose, O. (1995). Project work in university mathematics education: A Danish experience. *Educational Studies in Mathematics, 29*, 199–223.