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Bargaining According to the Baron-Ferejohn Model, Taking into Account Need

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Nils Springhorn*

Abstract: Baron and Ferejohn in *Bargaining in Legislature* identified theoretical solutions to specific bargaining situations. Their model is widely accepted, widely used and has a great impact. However, the model does not consider that players may have a need for the resource that is to be distributed which is higher than the value for approval determined by Baron and Ferejohn. If such a case is given, in extreme cases - if meeting the need is necessary for survival - it can end fatally if a player adopts the supposedly rational solution of the BF model. In this article, strategies and solutions are developed for the 3 player/2 sessions case of the BF model to avoid dying due to a neglect of need. In addition, it is shown that even when death is not imminent, individual need or, more generally, individual lower approval thresholds have a massive impact on the bargaining process.

Keywords: Baron Ferejohn-Model, Multilateral Bargaining, Legislative Bargaining, Need

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Introduction

The Baron-Ferejohn Model is first presented by Baron and Ferejohn in *Bargaining in Legislature* (1989). It is widely accepted, widely used and has a great impact. Agranov and Tergiman (2014) refer to the model as "the most popular formal model used to study multilateral bargaining" (75). The goal is to negotiate the distribution of a resource. The general setting is - roughly summarised - the following: successive rounds of bargaining follow until a majority is reached in favour of a proposal for the distribution of a resource. The proposal is made by a group member that is randomly selected for each round. Baron and Ferejohn show that in many cases there are unique strategies for (risk neutral) proposers and respondents, which lead to a proposal being made in the first round that achieves a majority. Crucial for these results is that Baron and Ferejohn assume that a respondent agrees to a distribution proposal if it provides at least as much for her or him as she or he expects for the following round(s). Baron and Ferejohn begin their investigation with the simplest case, that three players negotiate a maximum of two sessions. This case is also the basis for the research in this article. They show that under these conditions, the bargaining ends in the first round when the proposer offers 1/3 of the resource to a (randomly chosen) respondent and keeps 2/3 for himself. This advantage of the proposer is also shown in other versions of the BF model and is called proposer-power. The details are presented in the chapter "The Baron Ferejohn-Model".

What is meant by a need in the sense used here is discussed in the chapter "Need". In addition to some basic remarks and a brief historical and scientific classification, this chapter takes a first step into formalization: A need is an individual quantity whose (absolute) value indicates how much an individual needs of the good available for distribution in order to survive[1]. Examples can be the (individual) number of calories an individual needs to consume (in a given period of time) or a corresponding financial endowment that allows the purchase of a sufficiently large quantity.

The relevance of need in the BF model arises because - as will be shown in this article – strategies and solutions taking into account (individual) need turn out significantly different than when - as in the BF model - need is neglected. An essential and easily comprehensible reason for this is the following: If the (individual) need of a player is above the (general) value for approval determined by Baron and Ferejohn, she cannot approve without acting in a highly irrational manner. This becomes especially clear if the need for survival is assumed and a player dies if his need is not met. Neglecting needs would therefore only be legitimate if it could be assumed that participating players, at least in general, have a need for the good to be distributed that is less than the value determined by Baron and Ferejohn. However, this is a very strong assumption, which is generally not fulfilled, especially in the area of scarce goods.

To address this, the main section of this article, entitled "Bargaining in the Baron Ferejohn Model under Consideration of Need," develops and discusses strategies that consider need and the associated bargaining solutions.

Following the obligatory first chapter of the main part "Definitions, Notations and Desiderata", an approach is presented under the title "A Naive Approach", which is convincing at first sight, but which is described as naive in anticipation of the further results: (Individual) lower limits of approval defined by (individual) needs are introduced into the BF model. A respondent agrees if he is offered the maximum of the value determined analogously to the procedure of Baron and Ferjohn and his need.

This seems to disadvantage players with large needs because their approval may be more expensive than the approval of players with small needs and the former therefore do not receive

[1] That it is the value an individual needs to *survive* is a simplification made to focus on essential aspects.

an offer from the proposer. Accordingly, players with small needs seem to benefit. This is referred to as the "plausible assumption" and is also referred to as low-need power. By the way, anyone who has the impression that this model is more unfair than the BF model is mistaken[2], because this modelling shows only generally existing power imbalances, which are ignored in the BF model.

In the following chapter "Counter Intuitive Results of the Naive Approach" examples are given which show that the plausible assumption does not seem to be correct: players with small need turn out to be the most expensive players, players with large need turn out to be the cheapest. The latter therefore receive an offer which they agree to, bringing the negotiation to an end and leaving players with smaller needs empty-handed and not even having their needs met. That this surprising result can be easily explained from the right perspective is a first point that makes the heart of a theorist beat faster: because players with low need assume to be preferred, they have a high expectation value. These high expectation values make their approval expensive and possibly more expensive than the approval of players with high need.

However, it is not the plausible assumption that is wrong, but the naive approach, which is why it is called this in advance. For if players have different large needs, they have different scope for bargaining, in the sense that a player with a small need can accept a smaller offer than a player with a large need. A player with a small need can, in a sense, undercut a player with a large need by being willing to accept an offer that is smaller than the offer that a player with a large need can accept (as long as it is at least as large as its own need). This is referred to as the "cheaper rule". In this way, a player may not realize his expected value, but at least covers his need. Here, the player prevents a competing player from receiving an offer, which is the only rational decision because it also stops the bargaining from ending abruptly because said other player accepts said offer. The details of these considerations can be found in the chapter "An improved approach".

Thus, the complexity increases significantly compared to the BF model. Whereas in the BF model a respondent must consider only one parameter - the expected value, which is the same for all players - here a respondent must not only determine his own lower limit for approval from his own expected value and his own need according to the naive approach, but also these values for a competing respondent, in order to avoid being undercut. However, this is not the end of the matter, because similar considerations show that an upper limit can be derived from the resource, the expected value, and the need of the proposer, with respect to what the proposer can offer a respondent at most, because a proposer - analogous to the considerations from the perspective of a respondent - does not make any offers, with which it is associated that less than the maximum of its own expected value and its own need remains for himself from the resource minus the offer to the most favourable respondent. In order to determine whether a bargaining solution can be reached at all and, if so, what it looks like, the needs and expected values of all players as well as the size of the resource have to be taken into account[3]. A simple – admittedly very poor - purely combinatorial estimation of the complexity on the basis of these seven variables shows more than 3 million case distinctions[4].

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[2] The impression of such injustice could arise because, following Matthew's principle "To those who have will be given.", it could be said: "To those who need little will be given (and not to those who need it more urgently)".

[3] More precisely, it turns out that - case-specifically - the expected value of one player does not have to be taken into account.

[4] If you permute the seven variables (without repetition) you get 7! = 5,040 cases. Each permutation results in 3^6 case distinctions depending on the expression of three possible relations between the sizes (greater than, equal to, less than), so that there are purely combinatorial 5,040*3^6 = 3,674,160 case distinctions.

That this complexity can be reduced to only 50 case distinctions for the first session and only 12 more for the second makes the heart of a theorist beat faster a second time and is an essential result of this article. And even this comparatively small number of case distinctions still gives an overcomplex impression of the solutions. For on the one hand, the solution of a concrete bargaining situation can be read off from the scheme developed for this purpose in only three steps, and on the other hand many of these cases could be summarized (which is refrained from in order to give priority to a presentation that is as systematic and comprehensible as possible over a presentation that is as concise as possible, and because possible criteria according to which the cases could be summarized are partly opposing each other). In this respect, it must be emphasized that it is thanks to the scheme developed for this purpose that the results are nevertheless so comparatively clear and simple. This scheme as well as the derivation of the case differentiations and the case specific solutions can be found in the appendix ("Case Differentiations and Solutions - Derivation"), the results are summarized in the chapter "Case Differentiations and Solutions - Overview". This is followed by the chapter "Numerical Examples and Discussion". Central results are:

a) There is not, as in the BF model, the *one* proposition in which everything cumulates. The complexity does not allow that. But - this is also an important result - the complexity does not increase (contrary to what combinatorial estimates suggest) compared to the BF model to an extent that it would no longer be manageable or comprehensible.

b) The plausible assumption or the low-need-power (according to which a player with a small need benefits from the consideration of need) is confirmed, albeit in a much more differentiated form than initially assumed. The central statement in this context is that if a bargaining solution is reached, then the player with the smallest need - if this is unambiguous - is involved and his need is (at least) covered. However, this may be associated with the player receiving much less than he would receive according to the BF model.

c) Closely related to this, proposer power is relativized (according to which the proposer receives significantly more than the respondent). Proposer power can decrease significantly compared to the BF model, but it can also increase significantly. Particularly interesting in this context are indications that players who do not have need information may assume conditions that lead to a weaker proposer-power than the one predicted by the BF model when taking need into account. It may be possible to derive explanations for empirical findings that the BF model overestimates proposer power.

The detailed description of the 3 player/2 sessions case under consideration of need is followed in the chapter "An Outlook on Generalizations" by a short outlook on the treatment of bargaining with more than 3 players and more than 2 sessions under consideration of need. The chapter "Summary and Conclusion" gives an overview of the main results and concludes the article.

The Baron Ferejohn-Model

The basic setting of the (closed rule) Baron Ferejohn model is the following: Given is a normalized resource of size 1 to be distributed. Risk-neutral players are assumed. Among the players, a proposer is chosen at random with equal probability to propose a distribution. This distribution proposal is formulated in the form of shares of the resource for the individual players. It is assumed that the proposer agrees with the distribution proposal. The other players vote on the distribution proposal (on the distribution proposal as a whole, not just on the allocation that the proposal provides for the voting respondent). If a majority (including the proposer's presumed approval) approves the proposal, it is adopted. Each player then receives the share of the resource that the distribution proposal provides for him, and the bargaining ends. If no majority is reached, the bargaining goes to the next session. In the next session, the same procedure is followed as before, a proposer is again chosen randomly among all players (and not only among the players who were not chosen as proposer before) with equal probability. If the number of sessions is finite and there is no majority in the last session, all players are left empty-handed. A respondent agrees if he is offered at least his expected value for the next session.

In the case that 3 players are involved and the bargaining ends after 2 sessions (as assumed in this article), Baron and Ferejohn show that already in the first session the following bargaining solution achieves a majority: The proposer offers exactly one respondent one third of the resource and keeps the rest of the resource in the amount of two thirds for himself. The respondent agrees as a risk-neutral player because the share of the resource offered to him corresponds to his expected value[5]. This advantage of the proposer is called proposer-power.

The basic-setting of the BF model for the 3-player/2-sessions-case is adopted in this paper with the following modification: Because the (absolute) size of the resource determines whether the needs of the players needed for a majority can be met, and therefore different sized resources can lead to different outcomes, it is simpler not to assume the resource to be normalized and to negotiate not for shares but for absolute values. It is assumed that not only the size of the resource but also the individual needs are known to all players.

Need

The question of what the need of an individual is cannot by discussed without reference to the question what the endowment of the individual should be sufficient for. The answers differ widely. The possibly most influential approach to classify needs is Maslow's theory of human motivation (1943). The theory holds that there is a hierarchy of needs with physiological needs at the bottom, outranked by safety, love/belonging, and esteem needs, and with self-actualization at the top. The lower the need is in the hierarchy, the stronger it is. Absolute concepts of poverty like the basic needs approach (see Streeten 1981) rely on expert knowledge regarding the minimum cost diet that secures physical survival of an individual (Seidl 1988). Studying the living conditions of the working class in York, Rowntree (1901) for instance defined “families whose total earnings are insufficient to obtain the minimum necessaries for the maintenance of merely physical efficiency” (86) as poor. Living standards in a society can grow or shrink. Restricting needs purely to physical survival would ignore the fact that the satisfaction of both physiological and psychological needs contributes to mental health and thus the well-being of people (Deci and Ryan 2000; Ryan and Deci 2000). Sociological relative deprivation theories (Runciman 1966; Townsend 1974) carry the subjectivity of needs to the extreme. According to Runciman (1966) a person is in need if she does not have something, somebody else has it, she wants to have it, and she thinks that obtaining it is realistic. Both the purely absolute and the relative view of need and poverty were harshly criticized by Sen (1983). He proposed the concept of absolute neediness instead. A person is absolutely needy if she does not have the capability, say, in terms of income, to partake in the commonly accepted activities of the community. This is the so-called capabilities approach (Sen 2009; Nussbaum 2000, 2011). But there are more. For example, Braybrooke (1987) argues for the goal of a normal course of life, Daniels (1981) for a normal range of opportunities, Schuppert (2013) for agency and Sher (2014) for ‘leverage’, meaning the capacity to acquire additional goods. A general discussion of the goals at which need fulfilment could aim is given by Miller (1999, chapter 10).

[5] Since all players in the last session go away empty-handed if no majority is reached, Baron and Ferejohn assume that every respondent in the last session agrees to a distribution proposal that provides for an allocation of 0 for him and all other respondents. This results in an expected value of 1/3 for all players in the first session. Springhorn shows in " Capitulate for Nothing? Does Baron and Ferejohn's Bargaining Model Fail Because No One Would Give Everything for Nothing?" that this assumption is not plausible, but also that the expected values and other results of Baron and Ferejohn hold even under plausible assumptions.

So, it is easy to see that the question of what need is not easy to answer. It should be noted that it is generally assumed that need is something individual. Collective variables such as a general poverty line, which - in a certain sense - could be used to understand what need is, are simplifications (which have their justification in certain contexts), but the need for a good of one individual is generally not equal to the need of another individual. Accordingly, individual needs are assumed here as well (although the case where everyone has the same need is considered as one case among many). The statements in this article aim to take into account a concept of need that is as comprehensive as possible. Therefore, it is only assumed that need is quantifiable. In addition, it shall be assumed that need sets clearly defined limits, which a player does not fall below in his decision making. This is done in order to exclude trade-offs, so that a player could agree to distribution proposals even if the allocation intended for him is "somewhat" below his need. Faced with the alternative of going empty otherwise, such trade-offs can play a large role. For example, put simply, a little social participation is better than none at all. To illustrate this point, a player is assumed to die if it falls below an approval floor imposed by need, and thus it has no option to approve to a proposal that does not award it at least an allocation equal to its need. That such lower limits exist is undisputed and is discussed in the basic needs approach. However, the remarks are not intended and should not be understood as recommending or suggesting that it is appropriate to negotiate for essential goods according to the models discussed here. The distribution of goods necessary for survival must undoubtedly be carried out according to other standards. The fact that in this article the term of need is treated in such a way that an individual dies if her need is not met is a purely theoretical aggravation. The statements apply even if survival does not depend on it, but in such a case fewer clear limits are to be assumed, which would make the explanations more difficult.

Basic Definitions, Notations and Desiderata

The set of players is denoted by P.

The resource to be distributed is denoted by res; it is assumed that res > 0[6].

An arbitrary player is referred to by p, different arbitrary players are distinguished by an index: p_x, p_y, ..., certain (fixed) players are denoted by i, j, k, ...

An arbitrary session is referred to by s, different arbitrary sessions are distinguished by an index: s_x, s_y, ...

The (necessary for survival) need of a player is denoted by n(p).

$E(p, s_x, s_y)$ denotes the expected value of a player p in session s_x for session s_y. Analogous to the BF model, it is assumed that all players go away empty handed if the number of sessions to be played is finite and no majority is obtained in the last session. Therefore, the expected value in a last session is set to 0.

The proposer in session s is denoted by p(s).

An arbitrary respondent in session s is denoted by r(s), if different respondents are to be distinguished in the same session, this is indicated by an indexing: $r_x(s)$, $r_y(s)$, ... The set of respondents in session s is denoted by $R(s) \coloneqq \{p \in P: p \neq p(s)\}$.

$O(p_x, s | p(s) = p_y)$ denotes the offer to player p_x in session s, given that player p_y is chosen as proposer. If the chosen proposer results from the context or if the offer (or a statement related to it) is independent of the choice of the proposer, the condition is omitted for simplicity.

[6] res < 0 can make sense if the distribution is not of profits but of burden. res = 0 cannot be interpreted meaningfully, as it is equivalent to having nothing to distribute (which makes negotiation superfluous).

The offers of the proposer p(s) to each individual player in session s are summarized in a sequence with #P entries - the distribution proposal: DP(p(s), s) ≔ (O(i, s), O(j, s), ..., O(#P,s)).

$MV(p_x, s|p(s) = p_y)$ denotes the minimum approval value of player p_x in session s, given that player p_y is chosen as proposer. If the chosen proposer results from the context or if the MV (or a statement related to it) is independent of the choice of proposer, the condition is omitted for simplicity. The minimum approval value can be role-dependent (proposer or respondent) and strategy-dependent (and is indexed according to the chosen strategy if this is not clear from the context). For a respondent, the minimum approval value indicates how much must be offered to the respondent in order for him to agree (since the proposer anticipates this, she will not offer more). For the proposer this designation is not quite appropriate, because according to the (adopted) construction of Baron and Ferejohn the approval of the proposer is assumed for his proposal, so strictly speaking the proposer does not approve (or reject). However, the proposer is only required to make distributional proposals that provide for at least his minimum approval value. In order not to have to introduce a further variable (for example - and perhaps more obvious - a maximum offer value, which indicates how much the proposer can distribute at most, if he wants to keep his minimum approval value or a minmal residual value = 1 - maximum offer value), this notation/definition is also used for the proposer, despite the not quite appropriate designation.

If $MV(p_x, s)$ </=/> $MV(p_y, s)$, it is said that the approval of p_x in session s is cheaper (more favorable)/equally cheap or equally expensive/more expensive than that of p_y, or shorter, that p_x in session s is cheaper/equally cheap or equally expensive/more expensive than p_y.

The cheapest respondent in session s, given that p_y is chosen as proposer, is denoted by $r_c(s|p(s)= p_y)$. If the chosen proposer results from the context or if the chepest respondent (or a statement related to it) is independent of the choice of proposer, the condition is omitted for simplicity. Since the cheapest respondent does not have to be unambiguous, it is strictly speaking a set $r_c(s)$ ≔ $\{p_x \in R(s): \nexists p_z \in R(s): MV(p_z,s) < MV(p_x,s)\}$. Since, when speaking of the cheapest respondent in the following, all players from this set are to be treated equivalently, it is spoken of the cheapest respondent in the singular for the sake of simplicity.

It is said that the condition for a majority is given if and only if an r(s) exists such that $MV(p(s), s) + MV(r(s), s) \leq res$, so the resource is at least as large as the sum of the minimal approval values of the proposer and at least one respondent.

Desideratum[7] 1 (D1):

> Given p_y is the proposer in session s, then the following holds:
>
> (a) the proposer p_y only makes distributional proposals $DP(p_y, s)$ in session s for which it holds that she herself obtains at least her minimum approval value for session s: $O(p_y, s|p(s) = p_y) \geq MV(p_y,s)$,
> (b) a respondent r(s) agrees to a distribution proposal $DP(p_y, s)$ in session s if and only if he is offered at least his minimal approval value for session s: $O(r(s), s|p(s) = p_y) \geq MV(r(s), s)$.

Desideratum 1 is also valid in the original BF model, but in the BF model the minimal approval value for all players (independent of the role) is given by the expected value (which is the same for all players) and the minimal approval value is formulated relatively (as a share of the

[7] Desiderata are, so to speak, the axioms of the theory. Since in general certain requirements are placed on an axiom system, such as that the axioms are independent (do not imply each other), but such considerations are irrelevant here, the weaker term of desiderata is used.

resource), so that the question of the condition for a majority (in the sense mentioned above) does not arise. Taking need into account, however, D1 results in the following

Lemma 1 (L1):

> Given p_y is the proposer in session s, then the following holds:
>
> if the condition for a majority in session s does not hold (i.e., if it holds that no respondent r(s) exists such that $MV(p_y, s) + MV(r(s), s) \leq res$), then there can be no majority in session s for a distributional proposal.

If the condition for a majority is not met, the bargaining moves to the next session or, if there is no further session, the bargaining ends without a solution and all players go away empty-handed. In contrast to the BF-model, this is a principal constraint, which is given by the size of the resource and does not depend on the decisions of the players. For the sake of simplicity, I assume in such a case that the proposer does not make an offer to all players, including himself, represented in the form of a distribution proposal (0, 0, ..., 0) (this avoids costly discussions of distribution proposals for which no majority can be reached for reasons of principle).

Desideratum 2 (D2):

> For an arbitrary player p and an arbitrary session s holds: $MV(p, s) \geq n(p)$.

This is the first and crucial difference to the BF model. This introduces the described (individual) lower limit for approval given by the (individual) need.

Bargaining in the Baron Ferejohn-Model under Consideration of Need

A Naive Approach

Respondents' Strategy in the Naive Approach

The strategy of a respondent in session s, r(s), is essentially determined by its minimal approval value. Starting from the BF model, where this value is determined by the expected value of the player, taking into account need, the expected value is bounded downward by the need of the respondent.

$$MV_n(r(s), s) \coloneqq \max\{E(r(s), s, s+1), n(r(s))\}$$

Here, the subscript n is used to denote the MV in the naive approach.

A respondent r(s) agrees to the distribution proposal of a proposer p(s) if he is offered at least his expected value, unless his expected value is smaller than his need - in such a case he must be offered at least his need.

Proposer's Strategy in the Naive Approach

Also, the strategy of the proposer in session s, r(s), is essentially determined by its minimal approval value, which in the case of the naive approach is equal to the minimal approval value of a respondent.

$$MV_n(p(s), s) \coloneqq \max\{E(p(s), s, s+1), n(p(s))\}$$

A proposer p(s) only makes proposals with which he has at least his expected value left from the resource (after subtracting the offers to a respondent), unless his expected value is smaller than his need - in such a case he only makes proposals with which he has at least his need left.

If the condition for a majority in the current session is not given, Lemma 1 applies and the proposer proposes 0 for all players.

If the condition for a majority in the current session is given, the proposer offers the MV to the respondent with the smallest MV and keeps the rest of the resource for himself. If the respondent with the smallest MV is not unique, the proposer chooses one of the respondents with the smallest MV randomly and with equal probability, analogous to the BF model.

Counter-intuitive Results of the Naive Approach

The following example, where i has the smallest need, k has the largest need, and j's need lies in between, supports the plausible assumption (according to which the player with the smallest need benefits from the consideration of need)[8]:

<table>
<tr><th colspan="5"></th><th colspan="15">Session 2</th><th colspan="15">Session 1</th></tr>
<tr><th>Example</th><th>Ressource</th><th>n(i)</th><th>n(j)</th><th>n(k)</th><th>E(i,2,3)</th><th>E(j,2,3)</th><th>E(k,2,3)</th><th>MV(i,2)</th><th>MV(j,2)</th><th>MV(k,2)</th><th>O(i,2|p(2)=i)</th><th>O(j,2|p(2)=i)</th><th>O(k,2|p(2)=i)</th><th>O(i,2|p(2)=j)</th><th>O(j,2|p(2)=j)</th><th>O(k,2|p(2)=j)</th><th>O(i,2|p(2)=k)</th><th>O(j,2|p(2)=k)</th><th>O(k,2|p(2)=k)</th><th>E(i,1,2)</th><th>E(j,1,2)</th><th>E(k,1,2)</th><th>MV_n(i,1)</th><th>MV_n(j,1)</th><th>MV_n(k,1)</th><th>O(i,1|p(1)=i)</th><th>O(j,1|p(1)=i)</th><th>O(k,1|p(1)=i)</th><th>O(i,1|p(1)=j)</th><th>O(j,1|p(1)=j)</th><th>O(k,1|p(1)=j)</th><th>O(i,1|p(1)=k)</th><th>O(j,1|p(1)=k)</th><th>O(k,1|p(1)=k)</th></tr>
<tr><td>1</td><td>100</td><td>5</td><td>20</td><td>30</td><td>0,0</td><td>0,0</td><td>0,0</td><td>5,0</td><td>20,0</td><td>30,0</td><td>80,0</td><td>20,0</td><td>0,0</td><td>5,0</td><td>95,0</td><td>0,0</td><td>5,0</td><td>0,0</td><td>95,0</td><td>30,0</td><td>38,3</td><td>31,7</td><td>30,0</td><td>38,3</td><td>31,7</td><td>68,3</td><td>0,0</td><td>31,7</td><td>30,0</td><td>70,0</td><td>0,0</td><td>30,0</td><td>0,0</td><td>70,0</td></tr>
</table>

Table 1

The player with the smallest need, player i, has the smallest MV in the first session and - regardless of whether j or k is chosen as proposer - is preferred to the other respondent because he is cheaper (i's MV is smaller than those of j and k). If j is chosen as proposer, j makes i an offer equal to i's MV, i agrees, the bargaining ends, and k goes away empty-handed. If k is chosen as proposer, he also makes i an offer equal to i's MV, i agrees, the bargaining ends, and j goes away empty-handed.

However, the next example shows that this cannot be generalized:

<table>
<tr><th colspan="5"></th><th colspan="15">Session 2</th><th colspan="15">Session 1</th></tr>
<tr><th>Example</th><th>Ressource</th><th>n(i)</th><th>n(j)</th><th>n(k)</th><th>E(i,2,3)</th><th>E(j,2,3)</th><th>E(k,2,3)</th><th>MV(i,2)</th><th>MV(j,2)</th><th>MV(k,2)</th><th>O(i,2|p(2)=i)</th><th>O(j,2|p(2)=i)</th><th>O(k,2|p(2)=i)</th><th>O(i,2|p(2)=j)</th><th>O(j,2|p(2)=j)</th><th>O(k,2|p(2)=j)</th><th>O(i,2|p(2)=k)</th><th>O(j,2|p(2)=k)</th><th>O(k,2|p(2)=k)</th><th>E(i,1,2)</th><th>E(j,1,2)</th><th>E(k,1,2)</th><th>MV_n(i,1)</th><th>MV_n(j,1)</th><th>MV_n(k,1)</th><th>O(i,1|p(1)=i)</th><th>O(j,1|p(1)=i)</th><th>O(k,1|p(1)=i)</th><th>O(i,1|p(1)=j)</th><th>O(j,1|p(1)=j)</th><th>O(k,1|p(1)=j)</th><th>O(i,1|p(1)=k)</th><th>O(j,1|p(1)=k)</th><th>O(k,1|p(1)=k)</th></tr>
<tr><td>2</td><td>100</td><td>10</td><td>20</td><td>30</td><td>0,0</td><td>0,0</td><td>0,0</td><td>10,0</td><td>20,0</td><td>30,0</td><td>80,0</td><td>20,0</td><td>0,0</td><td>10,0</td><td>90,0</td><td>0,0</td><td>10,0</td><td>0,0</td><td>90,0</td><td>33,3</td><td>36,7</td><td>30,0</td><td>33,3</td><td>36,7</td><td>30,0</td><td>70,0</td><td>0,0</td><td>30,0</td><td>0,0</td><td>70,0</td><td>30,0</td><td>33,3</td><td>0,0</td><td>66,7</td></tr>
</table>

Table 2

Unchanged, i has the smallest need, k the largest, and j's need is in between. Despite the fact that i has the smallest need, in this example i is more expensive than k - the player with the greatest need (i's MV is higher than k's MV). Accordingly, k is favoured by j and receives an offer equal to its MV and agrees, ending the bargaining and leaving i empty-handed.

This can be taken to an extreme, so to speak: In the next example, i - again as the player with the smallest need - is not only more expensive than k, whose need is also the largest in this example, but also more expensive than player j, whose need - also as in the previous examples - lies between that of i and j. Despite having the smallest need, i is the most expensive of all the players (its MV is larger than that of j and k):

<table>
<tr><th colspan="5"></th><th colspan="15">Session 2</th><th colspan="15">Session 1</th></tr>
<tr><th>Example</th><th>Ressource</th><th>n(i)</th><th>n(j)</th><th>n(k)</th><th>E(i,2,3)</th><th>E(j,2,3)</th><th>E(k,2,3)</th><th>MV(i,2)</th><th>MV(j,2)</th><th>MV(k,2)</th><th>O(i,2|p(2)=i)</th><th>O(j,2|p(2)=i)</th><th>O(k,2|p(2)=i)</th><th>O(i,2|p(2)=j)</th><th>O(j,2|p(2)=j)</th><th>O(k,2|p(2)=j)</th><th>O(i,2|p(2)=k)</th><th>O(j,2|p(2)=k)</th><th>O(k,2|p(2)=k)</th><th>E(i,1,2)</th><th>E(j,1,2)</th><th>E(k,1,2)</th><th>MV_n(i,1)</th><th>MV_n(j,1)</th><th>MV_n(k,1)</th><th>O(i,1|p(1)=i)</th><th>O(j,1|p(1)=i)</th><th>O(k,1|p(1)=i)</th><th>O(i,1|p(1)=j)</th><th>O(j,1|p(1)=j)</th><th>O(k,1|p(1)=j)</th><th>O(i,1|p(1)=k)</th><th>O(j,1|p(1)=k)</th><th>O(k,1|p(1)=k)</th></tr>
<tr><td>3</td><td>100</td><td>15</td><td>20</td><td>30</td><td>0,0</td><td>0,0</td><td>0,0</td><td>15,0</td><td>20,0</td><td>30,0</td><td>80,0</td><td>20,0</td><td>0,0</td><td>15,0</td><td>85,0</td><td>0,0</td><td>15,0</td><td>0,0</td><td>85,0</td><td>36,7</td><td>35,0</td><td>28,3</td><td>36,7</td><td>35,0</td><td>30,0</td><td>70,0</td><td>0,0</td><td>30,0</td><td>0,0</td><td>70,0</td><td>30,0</td><td>0,0</td><td>35,0</td><td>65,0</td></tr>
</table>

Table 3

Thus, the plausible assumption (in its naive version) is refuted.

[8] Of particular interest are always the values for the first session. These are derived by backward induction from the values of the second session (and the variables that are constant over all rounds).

However, the presentation of the examples also shows why this is the case: The expected value of i increases with i's increasing need (ceteris paribus - as always to be assumed). As long as i is the cheapest player, he benefits because he has to be offered more for his approval. However, it also becomes more expensive and at a certain point so expensive that other players - despite a larger need - are cheaper than i. A small need can correlate with a large expected value - which ironically can be understood in a certain sense as a confirmation of the plausible assumption: *players with a small need expect especially much and possibly too much.*

This effect is amplified because, conversely, the expected value of the other players decreases as i's need grows, because they, as proposer, have to offer i more as his need grows and if he is the cheapest player despite the growing need, in order to win his approval. It is worth noting that, according to this interaction, the effect observed in the examples occurs not only when i's need grows, but also when the need of the others or one other player decreases, as in the following examples, where j's need decreases:

					Session 2															Session 1														
Example	Ressource	n(i)	n(j)	n(k)	E(i,2,3)	E(j,2,3)	E(k,2,3)	MV(i,2)	MV(j,2)	MV(k,2)	O(i,2|p(2)=i)	O(j,2|p(2)=i)	O(k,2|p(2)=i)	O(i,2|p(2)=j)	O(j,2|p(2)=j)	O(k,2|p(2)=j)	O(i,2|p(2)=k)	O(j,2|p(2)=k)	O(k,2|p(2)=k)	E(i,1,2)	E(j,1,2)	E(k,1,2)	MV_n(i,1)	MV_n(j,1)	MV_n(k,1)	O(i,1|p(1)=i)	O(j,1|p(1)=i)	O(k,1|p(1)=i)	O(i,1|p(1)=j)	O(j,1|p(1)=j)	O(k,1|p(1)=j)	O(i,1|p(1)=k)	O(j,1|p(1)=k)	O(k,1|p(1)=k)
4	100	5	20	30	0,0	0,0	0,0	5,0	20,0	30,0	80,0	20,0	0,0	5,0	95,0	0,0	5,0	0,0	95,0	30,0	38,3	31,7	30,0	38,3	31,7	68,3	0,0	31,7	30,0	70,0	0,0	30,0	0,0	70,0
5	100	5	13	30	0,0	0,0	0,0	5,0	13,0	30,0	87,0	13,0	0,0	5,0	95,0	0,0	5,0	0,0	95,0	32,3	36,0	31,7	32,3	36,0	31,7	68,3	0,0	31,7	0,0	68,3	31,7	32,3	0,0	67,7
6	100	5	6	30	0,0	0,0	0,0	5,0	6,0	30,0	94,0	6,0	0,0	5,0	95,0	0,0	5,0	0,0	95,0	34,7	33,7	31,7	34,7	33,7	31,7	68,3	0,0	31,7	0,0	68,3	31,7	0,0	33,7	66,3

Table 4

One observes essentially the same effects as in the examples where i's need grows. In example 4, i is the cheapest player, in example 5 it is more expensive than k, and in example 6 it is the most expensive player.

Once these relationships are clear, it is far from surprising that the player with the smallest need can be the most expensive player. However, it is difficult to assess the interrelationships correctly because, as will be examined in detail below, they are determined by a direct as well as elusive indirect interplay of the influencing variables n(i), n(j), n(k) and res.

The Improved Approach

The statement that the player with the smallest need can be the most expensive player is, however, only valid under the assumption that the strategies formulated in the naive approach are appropriate. That this is not the case is indicated in the examples, because i could prevent himself from going away empty-handed. i could *undercut* the respective competing respondent in the sense that he accepts offers that are below the minimum approval value of the competing respondent (which would make him cheaper than the competing respondent), but above his own need. Thus, he would have to refrain - in certain cases - from realizing his expected value, but he would prevent that a majority for a distribution proposal is reached without him being considered, the negotiation ends and he goes away empty-handed. Faced with the choice of insisting on an offer equal to his own expected value and thus going away empty-handed, or accepting a smaller offer and thus at least covering his own needs, a rational decision-maker should not find the decision difficult: She undercuts her competitors. Anything else would be considered highly irrational, regardless of theoretically determined solutions (by the BF model) or definitions (of a risk-neutral player whose basis for decision is, by definition, its expected value). To take this into account, the strategy of a respondent in the improved approach is extended by the following desideratum (the cheaper rule) compared to the strategy of a respondent in the naive approach:

Desideratum 3 (D3 „Cheaper-Rule“):

If it is given that for a respondent $r_x(s)$ and the competing respondent $r_y(s)$ holds: $n(r_x(s)) < n(r_y(s)) \leq E(r_x(s),s,s+1)$, then holds:

the respondent $r_x(s)$ agrees to a distribution proposal if and only if she is offered at least $n(r_y(s))$-ε.

By applying the cheaper-rule, the respondent $r_x(s)$ *avoids* that she goes away empty-handed and her need is not met, because the competing respondent $r_y(s)$ is cheaper, therefore an offer is made to him, he agrees to the offer and the bargaining ends, although $r_x(s)$ due to a small need *could* accept offers that are below those that the competing respondent $r_y(s)$ can accept ($r_x(s)$ *undercuts* $r_y(s)$ (by ε)).

$r_x(s)$ achieves an optimal result if it undercuts $r_y(s)$ by the smallest possible amount ε. In order to avoid complex calculations of limit values, it shall be assumed for the sake of simplicity that all calculations are in a discrete space of real numbers, whose elements have a finite number of decimal places and a sufficient number of decimal places to solve all occurring equations. For example, if $(n(r_x(s)) + n(r_y(s)))/2$ with $n(r_x(s)) = 1$ and $n(r_y(s)) = 2$ is to be calculated ((1+2)/2 = 1.5), a space is to be assumed which contains numbers with exactly one decimal place, so that the smallest possible ε is 0.1. If $r_x(s)$ chooses a smallest possible ε under the conditions formulated in the cheaper-rule, which results in $(n(r_x(s)) + n(r_y(s)))/2$ (the "middle" between the needs of $r_x(s)$ and $r_y(s)$), $r_x(s)$'s need is met.

$r_x(s)$ does not undercut the expected value of $r_y(s)$, but its need. If $r_x(s)$ would underbid the expected value of $r_y(s)$, $r_y(s)$ would anticipate this and in turn underbid $r_x(s)$. This regression only ends when $r_x(s)$ underbids the need of $r_y(s)$, because then $r_y(s)$ cannot follow suit.

It is more favorable for $r_x(s)$ to undercut the need of $r_y(s)$ than to bind its approval to an offer equal to the need of $r_y(s)$. If both respondents would agree when offered $n(r_y(s))$, one shall assume, analogously to the BF model, that the proposer in such a situation chooses one of the respondents at random and with equal probability. Thus the statement is valid, because the expected value of $r_x(s)$ for the current session - before the proposer has chosen a respondent - $E(r_y(s),s,s)$ (this is not to be confused with the expected value for the next session $E(r_y(s),s,s+1)$), is $n(r_y(s))/2$. Since $n(r_y(s))-\varepsilon \geq n(r_y(s))/2$ (qua construction of ε), $r_x(s)$ obtains a better result by binding its approval to an offer equal to $n(r_y(s))$-ε than by binding it to an offer equal to $n(r_y(s))$.

Note in this context the following fact: With the consideration of need in the BF model, for a respondent *not only the necessity to need more for approval than in the BF model* may arise, *the necessity to accept less than in the BF model* may also arise. While the former may be obvious, the latter is quite surprising.

Respondents' Strategy in the Improved Approach

Considering the cheaper rule changes the minimum approval value of a respondent $r_x(s)$ compared to the naive approach. Taking the Cheaper Rule into account, this results as follows:

$$MV(r_x(s), s) := \max\{\min\{E(r_x(s), s, s+1), n(r_y(s))-\varepsilon\}, n(r_x(s))\},$$

where $r_y(s)$ is the competing respondent.

A respondent $r_x(s)$ agrees with the distribution proposal of a proposer p(s) if he is offered at least his expected value, unless the need of the competing respondent $r_y(s)$ is smaller than his expected value - in such a case he agrees if he is offered at least $n(r_y(s))$-ε, unless $n(r_y(s))$-ε is smaller than his need - in such a case he must be offered at least his need for agreement.

Proposer's Strategy in the Improved Approach

Unlike a respondent, the proposer cannot get into the situation where he has to prevent another player from receiving an offer and he himself does not. Therefore, the proposer's strategy in the improved approach does not change compared to his strategy in the naive approach (but the amount of the offers does, since the respondents' MVs generally change with the changed strategy).

Case Differentiations and Solutions – Overview

In contrast to the BF model, the solution depends not only on the individual expected value (which is the same for all players in the BF model), but also on the choice of the proposer, the needs of all players, the size of the resource and - which is difficult to recognize (a very descriptive explanation is given in the chapter "Numerical Examples and Discussion") - *the relationships between the needs and the resource*. In this respect, the *conditions* for the MVs that are possible in principle (and not the MVs that are possible in principle) should be considered in the following. The following list is complete, it includes *all* case distinctions that lead to different bargaining solutions and the associated distribution proposals (repetitions have not been removed; as explained in the appendix, with the aim of making the presentation as systematic as possible from the beginning to the end of the game). Their derivation can be found in the appendix. Numerical examples, discussion and main results can be found in the following chapters.

1) If for any two players p_x and p_y the following holds: $n(p_x) + n(p_y) > res$, then

1.1) p(1) = i	DP(i,1) = (0, 0, 0)
1.1.1) p(2) = i:	DP(i,2) = (0, 0, 0)
1.1.2) p(2) = j:	DP(j,2) = (0, 0, 0)
1.1.3) p(2) = k:	DP(k,2) = (0, 0, 0)
1.2) p(1) = j	DP(j,1) = (0, 0, 0)
1.2.1) p(2) = i:	DP(i,2) = (0, 0, 0)
1.2.2) p(2) = j:	DP(j,2) = (0, 0, 0)
1.2.3) p(2) = k:	DP(k,2) = (0, 0, 0)
1.3) p(1) = k	DP(k,1) = (0, 0, 0)
1.3.1) p(2) = i:	DP(i,2) = (0, 0, 0)
1.3.2) p(2) = j:	DP(j,2) = (0, 0, 0)
1.3.3) p(2) = k:	DP(k,2) = (0, 0, 0)

2.1) If all players have equal needs: n(i) = n(j) = n(k)
and n(i) + n(j) = n(i) + n(k) = n(j) + n(k) ≤ res, then

2.1.1) p(1) = i	with 50% probability	DP(i,1) = (res-n(j), n(j), 0) and
	with 50% probability	DP(i,1) = (res-n(k), 0, n(k))
2.1.2) p(1) = j	with 50% probability	DP(j,1) = (n(i), res-n(i), 0) and
	with 50% probability	DP(j,1) = (0, res-n(k), 0, n(k))
2.1.3) p(1) = k	with 50% probability	DP(k,1) = (n(i), 0, res-n(i)) and
	with 50% probability	DP(k,1) = (0, n(j), res-n(j))

2.2.1) If there are two players with equal need and one with smaller need: n(i) < n(j) = n(k)
and n(i) + n(j) = n(i) + n(k) ≤ res, then

E(i,1,2) = 1/3*res + 2/3*n(i) – 1/3*n(j)
E(j,1,2) = 1/3*res – 1/3*n(i) + 1/6*n(j)
E(k,1,2) = 1/3*res – 1/3*n(i) + 1/6*n(k)

2.2.1.1) p(1) = i with 50% probability DP(i,1) = (res-n(j), n(j), 0) and
with 50% probability DP(i,1) = (res-n(k), 0, n(k))

2.2.1.2) p(1) = j

2.2.1.2.1) res ≤ n(i) + n(k) DP(j,1) = (n(i), res-n(i), 0)
2.2.1.2.2) n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k) DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.1.2.3) res ≥ -2*n(i) + n(j) + 3*n(k) DP(j,1) = (n(i), res-n(i), 0)

2.2.1.3) p(1) = k

2.2.1.3.1) res ≤ n(i) + n(k) DP(k,1) = (n(i), 0, res-n(i))
2.2.1.3.2) n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k) DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))
2.2.1.3.3) res ≥ -2*n(i) + n(j) + 3*n(k) DP(k,1) = (n(j)-ε, 0, res-n(j)-ε)

2.2.2.1) If there are two players with equal need and one with greater need: n(i) = n(j) < n(k) and n(i) + n(j) ≤ res and n(i) + n(k) = n(j) + n(k) > res, then

E(i,1,2) = 1/3*res
E(j,1,2) = 1/3*res
E(k,1,2) = 0

2.2.2.1.1) p(1) = i

2.2.2.1.1.1) res ≤ 3*n(j) DP(i,1) = (res-n(j), n(j), 0)
2.2.2.1.1.2) 3*n(j) < res < 3*n(k) DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)
2.2.2.1.1.3) res ≥ 3*n(k) DP(i,1) = (res-n(k)+ε, n(k)-ε, 0)

2.2.2.1.2) p(1) = j

2.2.2.1.2.1) res ≤ 3*n(j) DP(j,1) = (n(i), res-n(i), 0)
2.2.2.1.2.2) 3*n(j) < res < 3*n(k) DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.2.1.2.3) res ≥ 3*n(k) DP(j,1) = (n(k)-ε, res-n(k)+ε, 0)

2.2.2.1.3) p(1) = k D(k,1) = (0, 0, 0)

2.2.2.1.3.1) p(2) = i: DP(i,2) = (res-n(j), n(j), 0)
2.2.2.1.3.2) p(2) = j: DP(j,2) = (n(i), res-n(i), 0)
2.2.2.1.3.3) p(2) = k: DP(k,2) = (0, 0, 0)

2.2.2.2) If there are two players with equal need and one with greater need: n(i) = n(j) < n(k) and n(i) + n(j) ≤ res and n(i) + n(k) = n(j) + n(k) ≤ res, then

E(i,1,2) = 1/3*res + 1/6*n(i)
E(j,1,2) = 1/3*res + 1/6*n(j)
E(k,1,2) = 1/3*res – 1/3*n(i)

2.2.2.2.1) p(1) = i

2.2.2.2.1.1) res ≤ 5/2*n(j) DP(i,1) = (res-n(j), n(j), 0)
2.2.2.2.1.2) 5/2*n(j) < res < -1/2*n(j) + 3*n(k) DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)
2.2.2.2.1.3) res ≥ -1/2*n(j) + 3*n(k) DP(i,1) = (res-n(k)+ε, n(k)-ε, 0)

2.2.2.2.2) p(1) = j

2.2.2.2.2.1) res ≤ 5/2*n(j) DP(j,1) = (n(i), res-n(i), 0)
2.2.2.2.2.2) 5/2*n(j) < res < -1/2*n(j) + 3*n(k) DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.2.2.2.3) res ≥ -1/2*n(j) + 3*n(k) DP(j,1) = (n(k)-ε, res-n(k)+ε, 0)

2.2.2.2.3) p(1) = k with 50% probability DP(k,1) = (n(i), 0, res-n(i)) and with 50% probability $DP(k,1) = (0, n(j), res-n(j))$

2.2.3.1) If all players have different needs: $n(i) < n(j) < n(k)$
and $n(i) + n(j) \le res$
and $n(i) + n(k) > res$ (and thus also $n(j) + n(k) > res$), then

$E(i,1,2) = 1/3*res + 1/3*n(i) - 1/3*n(j)$
$E(j,1,2) = 1/3*res - 1/3*n(i) + 1/3*n(j)$
$E(k,1,2) = 0$

2.2.3.1.1) p(1) = i

2.2.3.1.1) $res \le n(i) + 2*n(j)$	$DP(i,1) = (res-n(j), n(j), 0)$
2.2.3.1.2) $n(i) + 2*n(j) < res < n(i) - n(j) + 3*n(k)$	$DP(i,1) = (E(i,1,2), res-E(i,1,2), 0)$
2.2.3.1.3) $res \ge n(i) - n(j) + 3*n(k)$	$DP(i,1) = (n(k)-\varepsilon, res-n(k)-\varepsilon, 0)$

2.2.3.1.2) p(1) = j

2.2.3.1.2.1) $res \le -n(i) + 4*n(j)$	$DP(j,1) = (n(j), res-n(j), 0)$
2.2.3.1.2.2) $-n(i) + 4*n(j) < res < -n(i) + n(j) + 3*n(k)$	$DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)$
2.2.3.1.2.3) $res \ge -n(i) + n(j) + 3*n(k)$	$DP(j,1) = (n(k)-\varepsilon, res-n(k)-\varepsilon, 0)$

2.2.3.1.3) p(1) = k $DP(k,1) = (0, 0, 0)$

2.2.3.1.3.1) p(2) = i:	$DP(i,2) = (res-n(j), n(j), 0)$
2.2.3.1.3.2) p(2) = j:	$DP(j,2) = (n(i), res-n(i), 0)$
2.2.3.1.3.3) p(2) = k:	$DP(k,2) = (0, 0, 0)$

2.2.3.2) If all players have different needs: $n(i) < n(j) < n(k)$
and $n(i) + n(j) \le res$
and $n(i) + n(k) \le res$
and $n(j) + n(k) > res$, then

$E(i,1,2) = 1/3*res + 2/3*n(i) - 1/3*n(j)$
$E(j,1,2) = 1/3*res - 1/3*n(i) + 1/3*n(j)$
$E(k,1,2) = 1/3*res - 1/3*n(i)$

2.2.3.2.1) p(1) = i

2.2.3.2.1.1) $res \le n(i) + 2*n(j)$	$DP(i,1) = (res-n(j), n(j), 0)$
2.2.3.2.1.2) $n(i) + 2*n(j) < res < n(i) - n(j) + 3*n(k)$	$DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)$
2.2.3.2.1.3) $res \ge n(i) - n(j) + 3*n(k)$	$DP(i,1) = (res-n(k)+\varepsilon, n(k)-\varepsilon, 0)$

2.2.3.2.2) p(1) = j

2.2.3.2.2.1) $res < -2*n(i) + n(j) + 3*n(k)$	$DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)$
2.2.3.2.2.3) $res \ge -2*n(i) + n(j) + 3*n(k)$	$DP(j,1) = (n(k)-\varepsilon, res-n(k)+\varepsilon, 0)$

2.2.3.2.3) p(1) = k

2.2.3.2.3.1) $res < n(i) - 1/2*n(j) + 3/2*n(k)$	$DP(k,1) = (0, 0, 0)$
2.2.3.2.3.1.1) p(2) = i:	$DP(i,2) = (res-n(j), n(j), 0)$
2.2.3.2.3.1.2) p(2) = j:	$DP(j,2) = (n(i), res-n(i), 0)$
2.2.3.2.3.1.3) p(2) = k:	$DP(k,2) = (n(i), 0, res-n(i))$
2.2.3.2.3.2) $n(i) - 1/2*n(j) + 3/2*n(k) \le res < -2*n(i) + 4*n(j)$	$DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))$
2.2.3.2.3.3.1) $res \ge -2*n(i) + 4*n(j)$ und $res \le n(i) + 3*n(k)$	$DP(k,1) = (0, 0, 0)$
2.2.3.2.3.3.1.1) p(2) = i:	$DP(i,2) = (res-n(j), n(j), 0)$
2.2.3.2.3.3.1.2) p(2) = j:	$DP(j,2) = (n(i), res-n(i), 0)$

2.2.3.2.3.3.1.3) p(2) = k: DP(k,2) = (n(i), 0, res-n(i))

2.2.3.2.3.3.2) res ≥ -2*n(i) + 4*n(j) und res ≥ n(i) + 3*n(k) DP(k,1) = (n(j)-ε, 0, res-n(j)+ε)

2.2.3.3) If all players have different needs: n(i) < n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) ≤ res
and n(j) + n(k) ≤ res, then

E(i,1,2) = 1/3*res + 2/3*n(i) – 1/3*n(j)
E(j,1,2) = 1/3*res – 1/3*n(i) + 1/3*n(j)
E(k,1,2) = 1/3*res – 1/3*n(i)

2.2.3.3.1) p(1) = i

2.2.3.3.1.1) res ≤ n(i) + 2*n(j) DP(i,1) = (res-n(j), n(j), 0)
2.2.3.3.1.2) n(i) + 2*n(j) < res < n(i) – n(j) + 3*n(k) DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)
2.2.3.3.1.3) res ≥ n(i) – n(j) + 3*n(k) DP(i,1) = (res-n(k)+ε, n(k)-ε, 0)

2.2.3.3.2) p(1) = j

2.2.3.3.2.1) res < -2*n(i) + n(j) + 3*n(k) DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.3.3.2.2) res ≥ -2*n(i) + n(j) + 3*n(k) DP(j,1) = (n(k)-ε, res-n(k)+ε, 0)

2.2.3.3.3) p(1) = k

2.2.3.3.3.1) res < -2*n(i) + 4*n(j) DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))
2.2.3.3.3.2) res ≥ -2*n(i) + 4*n(j) DP(k,1) = (n(j)-ε, 0, res-n(j)+ε)

Numerical Examples and Discussion
The following examples all fall under case 2.2.3.3), i.e.: All players have different needs: n(i) < n(j) < n(k) and for any two players p_x and p_y each, $n(p_x) + n(p_y) \leq$ res. The second session results as in the examples above.

					Session 2															Session 1, p(1) = j								
Example	Ressource	n(i)	n(j)	n(k)	E(i,2,3)	E(j,2,3)	E(j,2,3)	MV(i,2)	MV(j,2)	MV(k,2)	O(i,2|p(2)=i)	O(j,2|p(2)=i)	O(k,2|p(2)=i)	O(i,2|p(2)=j)	O(j,2|p(2)=j)	O(k,2|p(2)=j)	O(i,2|p(2)=k)	O(j,2|p(2)=k)	O(k,2|p(2)=k)	E(i,1,2)	Threshold -2*n(i) + n(j) + 3*n(k)	res < -2*n(i) + n(j) + 3*n(k)	res >= -2*n(i) + n(j) + 3*n(k)	MV(i,1|p(1)=j)	O(i,1|p(1)=j)	O(j,1|p(1)=j)	O(k,1|p(1)=j)
7	100	2	15	30	0,0	0,0	0,0	2,0	15,0	30,0	85,0	15,0	0,0	2,0	98,0	0,0	2,0	0,0	98,0	29,7	101,0	x		29,7	29,7	70,3	0
8	100	3	15	30	0,0	0,0	0,0	3,0	15,0	30,0	85,0	15,0	0,0	3,0	97,0	0,0	3,0	0,0	97,0	30,3	99,0		x	30 - ε	30 - ε	70 + ε	0
9	100	2	14	30	0,0	0,0	0,0	2,0	14,0	30,0	86,0	14,0	0,0	2,0	98,0	0,0	2,0	0,0	98,0	30,0	100,0		x	30 - ε	30 - ε	70 + ε	0
10	100	2	15	29	0,0	0,0	0,0	2,0	15,0	29,0	85,0	15,0	0,0	2,0	98,0	0,0	2,0	0,0	98,0	29,7	98,0		x	29 - ε	29 - ε	71 + ε	0
11	101	2	15	30	0,0	0,0	0,0	2,0	15,0	30,0	86,0	15,0	0,0	2,0	99,0	0,0	2,0	0,0	99,0	30,0	101,0		x	30 - ε	30 - ε	71 + ε	0

Table 5

For the first session, examples 7 to 11 consider the case where j is chosen as proposer (case 2.2.3.3.2), cf. the Overview and maybe the Derivation). In this case, i is the cheapest respondent, which is why its expected value and MV are given. In this (special) case, it is impossible for i's MV to be equal to its need, which is why there are two case distinctions (see the Appendix for derivation). For better understanding, a kind of case-differentiating threshold -2*n(i) + n(j) + 3*n(k) is given. Case- differentiating is this threshold in the sense that if res is smaller than this threshold, MV(i,1) = E(i,1,2) holds and if res is greater than or equal to this threshold, MV(i,1) = n(k)-ε holds.

In Example 7, the condition for MV(i,1) = E(i,1,2): res < -2*n(i) + n(j) + 3*n(k) is satisfied. j offers i the expected value (i accepts because this is its MV under these conditions) and keeps the rest of the resource for herself.

In Example 8, the condition for MV(i,1) = n(k)-ε: res ≥ -2*n(i) + n(j) + 3*n(k) is satisfied. j offers i n(k)-ε (i accepts because this is its MV under these conditions) and keeps the rest of the resource for itself. Here, the cheaper rule applies because i's expected value is greater than k's need (and i would go away empty handed according to the naive approach).

Examples 9 to 11 are intended to provide a better understanding, in particular, of why things turn out to be so complex and why so many case distinctions are necessary. The results do not differ, or do not differ significantly, from those in Example 8. However, while Example 8 differs from Example 7 (ceteris paribus) in that the need of i is larger, Example 9 differs from Example 7 in that the need of j is smaller, Example 10 differs from Example 7 in that the need of k is smaller, and Example 11 differs from Example 7 in that the resource is larger. All these differences - as can be seen from the overview, the derivation and the numerical examples - influence the MV-determining conditions either by shifting the threshold or by changing the resource.

If you think about such changes further, the (initial) conditions change fundamentally and thus which case is given. For example, if the resource were to decrease to 16, not even the need of the two players with the smallest need (i and j) could be met and no solution would be found (we would be in case 1). If the need of i would increase to 30, one would be in case 2.2.1) (two players with equal need and one with smaller need). Would k's need drop to 2, one would be in case 2.2.2.2) (two players with equal need and one with greater need) and so on. In each case, this is associated with different expected values and different MV-determining thresholds. Which threshold are valid (whether they exist at all and how large the expectation values are) results from a complex interplay of all the needs and the resource, and it is ultimately surprising that this complexity is not reflected in significantly more case distinctions.

In the following examples 12 and 13, the case is considered that in the first session i is chosen as proposer (case 2.2.3.3.1) - the details can be found in the Overview and in the Appendix, respectively).

					Session 2															Session 1, p(1) = i										
Example	Ressource	n(i)	n(j)	n(k)	E(i,2,3)	E(j,2,3)	E(j,2,3)	MV(i,2)	MV(j,2)	MV(k,2)	O(i,2|p(2)=i)	O(j,2|p(2)=i)	O(k,2|p(2)=i)	O(i,2|p(2)=j)	O(j,2|p(2)=j)	O(k,2|p(2)=j)	O(i,2|p(2)=k)	O(j,2|p(2)=k)	O(k,2|p(2)=k)	E(j,1,2)	Threshold n(i) + 2*n(j)	Threshold n(i) – n(j) + 3*n(k)	res <= n(i) + 2*n(j)	n(i) + 2*n(j) < res < n(i) – n(j) + 3*n(k)	res >= n(i) – n(j) + 3*n(k)	MV(j,1|p(1)=i)	O(i,1|p(1)=i)	O(j,1|p(1)=i)	O(k,1|p(1)=i)
12	100	1	2	3	0,0	0,0	0,0	1,0	2,0	3,0	98,0	2,0	0,0	1,0	99,0	0,0	1,0	0,0	99,0	33,7	5,0	8,0			x	3 - ε	97 + ε	3 - ε	0
13	100	1	98	99	0,0	0,0	0,0	1,0	98,0	99,0	2,0	98,0	0,0	1,0	99,0	0,0	1,0	0,0	99,0	65,7	197,0	200,0	x			98	2	98	0

Table 6

In examples 15 and 16 the case is considered that in the first session k is chosen as proposer (case 2.2.3.3.3)) - also for this the details can be found in the Overview and in the Appendix, respectively).

					Session 2															Session 1, p(1) = k								
Example	Ressource	n(i)	n(j)	n(k)	E(i,2,3)	E(j,2,3)	E(j,2,3)	MV(i,2)	MV(j,2)	MV(k,2)	O(i,2|p(2)=i)	O(j,2|p(2)=i)	O(k,2|p(2)=i)	O(i,2|p(2)=j)	O(j,2|p(2)=j)	O(k,2|p(2)=j)	O(i,2|p(2)=k)	O(j,2|p(2)=k)	O(k,2|p(2)=k)	E(i,1,2)	Threshold -2*n(i) + 4*n(j)	res < -2*n(i) + 4*n(j)	res >= -2*n(i) + 4*n(j)	MV(i,1|p(1)=k)	O(i,1|p(1)=k)	O(j,1|p(1)=k)	O(k,1|p(1)=k)
14	100	47	49	50	0,0	0,0	0,0	47,0	49,0	50,0	51,0	49,0	0,0	47,0	53,0	0,0	47,0	0,0	53,0	48,3	102,0	x		48,3	48,3	0,0	51,67
15	100	1	2	3	0,0	0,0	0,0	1,0	2,0	3,0	98,0	2,0	0,0	1,0	99,0	0,0	1,0	0,0	99,0	33,3	6,0		x	2 - ε	2 - ε	0,0	98 + ε

Table 7

The examples are for discussion of the plausible assumption and low-need power. i is the player with the smallest need in each case, followed by j and then k.

In example 12, the low-need power is confirmed. i, as the proposer, benefits from the fact that need is taken into account. He receives almost 100 percent of the resource (in the BF model he receives 2/3), but only because the needs of j and k, even if they are larger than i's need, are also small. j - the cheapest respondent - cannot come close to realizing hers expected value because she has to undercut the competing respondent k. Likewise, in Example 14, i, as a respondent, receives almost 50 percent as a respondent with certainty (versus 1/3 of the resource with 50'percent probability in the BF model). Here, he benefits from the fact that his high expectation value is below the need of the competing respondent j and is (just) small enough that the resource covers his MV and the MV of proposer k.

In contrast, examples 13 and 15 show that there is not always low-need power. In example 13, I, as the proposer, has to offer respondent j almost 100 percent. He does so because he still retains more than his need (1) and also more than his expected value ((2+1+1)/3 ≈ 1.3). In Example 15, i, as a respondent, is in the unfortunate situation of having to underbid the competing respondent j, who also has a very small need, making the offer to i very small. Nevertheless, he agrees, since his need is covered and he would otherwise be left empty-handed.

Example 15 is also interesting in comparison to example 12, since the same values for the needs and the resource are assumed for both examples and they differ only with respect to the proposer choice. Here, the choice of the proposer determines - under otherwise identical conditions - an offer to i in the amount of almost 100 percent and an offer in the amount of little more than 0 percent. This illustrates particularly clearly how fragile the solutions are with respect to allegedly small changes.

Furthermore, the examples are chosen to show the variability of proposer power depending on the given conditions. The fact that the offer to i, both when he is chosen as proposer and when he is respondent, can be both larger and smaller than in the BF model implies that proposer power can also be both larger and smaller than in the BF model. In examples 12 and 15, the proposer power is (significantly) larger than in the BF model (the proposer receives close to 100 percent). In examples 13 and 14, proposer power is (significantly) smaller than in the BF model (in example 13, the proposer gets almost nothing, in example 14, only about 50 percent).

It should also be noted that proposer power and low-need power are not correlated. In example 12, both low-need power and proposer power are large, in example 13 both low-need power and proposer power are small, in example 14 low-need power is large and proposer power is small, and in example 15 low-need power is small and proposer power is large.

An Outlook on Generalizations

Generalizations regarding more than 3 players and more than 2 sessions can essentially follow the procedure described here for the 3 players/2 sessions case. It can be assumed that although further case distinctions are added, neither the strategies of the respondents and the proposer change fundamentally, nor the solution structures.

For a generalization regarding n>3 players it is helpful that not all n-1 respondents have to be considered, but at most h ≔ ceil((n - 1)/2) + 1, i.e. the (in case of an odd number of respondents rounded up) half of the respondents needed for a majority plus one respondent. The half of the respondents to be considered are the cheapest respondents, the additional respondent is the next most expensive respondent, which is to be undercut if necessary and possible. As in the 3 players/2 sessions case, the corresponding respondents can be identified on the basis of their needs. Note that the respondent to be undercut (as well as the set of cheapest respondents) does not have to be unique. The 3 players/2 sessions case is a special case in which h is equal to the number of all respondents. Even if not all respondents have to be considered, the scope of the investigations increases significantly, because in general all combinations of the three principally possible expressions of the MV of h respondents (need, expected value and need of the competing respondent minus ε) and the two principally possible expressions of the MV of the proposer (need and expected value) have to be investigated to see if the resource is large enough to cover them together. Besides a significant increase of case distinctions, there should be no hurdles in determining the solutions.

Probably more difficult is the situation with respect to the extension to more than 2 sessions. Baron and Ferejohn make their extension to more than 2 sessions under the assumption of "stationary equillibria". This essentially means that the expected values of the player do not change from session to session - except for the last session. This assumption allows them to extend the results for the 2 sessions case comparatively easily to cases with more than 2 sessions.[9]. Taking into account need stationary equillibria are rather the exception[10], which is why a corresponding extension is much more challenging. A crucial and open question in this context is the importance of the cheaper rule in the case of more than 2 sessions. It is possible that even in the case of an infinite number of sessions the bargaining is terminated (early) because respondents want to prevent certain bargaining processes and undercut competing respondents. This would facilitate the determination of solutions in the case of a finite number of sessions and possibly enable the solution in the case of an infinite number of sessions.

Summary and Conclusion

The BF model does not take into account that players may have a need for the resource that - from the point of view of a respondent - is above the value determined by Baron and Ferejohn for approval, or - from the point of view of the proposer - above the value that remains for the proposer from the resource after subtracting the offer to a respondent. If a player's need is greater than the corresponding value, he cannot agree as a respondent and cannot make a corresponding distribution proposal as a proposer. This becomes especially clear if the need is a survival need and a player dies if his need is not met. The neglect of needs would therefore only be justified if it could be assumed that the values determined by Baron and Ferejohn cover the needs of the players at least in general. However, this is a very strong assumption for which there is no evidence, and which cannot be assumed, especially in the area of scarce goods. Irrespective of this, it has been shown in this article that the strategies and solutions that emerge

[9] The elegance of the BF model for more than 2 sessions is precisely due to the fact that Baron and Ferejohn, with the definition of stationary equillibria, create exactly the conditions under which simple and elegant solutions are possible - which is not to diminish their achievement, because even the identification of conditions for simple solutions is in general not trivial.

[10] One can even observe cases of alternating MVs: Under certain conditions, the MV of a player increases from session to session because its expected value increases from session to session, but then falls again because the high expected value makes him unattractive for coalitions, until he becomes attractive again due to a sufficiently small expected value, which results in an increasing expected value and so on.

when taking need into account are fundamentally very different from those in the BF model. The main reason for this is that the generally existing individual differences in the size of needs create power imbalances that are decisive for bargaining. Players with small needs can accept smaller offers as respondents than players with large needs (and thus undercut competing respondents with large needs) and make larger offers as proposers than players with large needs.

In order to counter the weaknesses of the BF model, an approach was first presented that seems convincing at first glance, but which is described as naive in anticipation of the following results. According to this approach, a respondent agrees to a distribution proposal if he is offered at least the maximum of the value determined analogously to the procedure of Baron and Ferjohn and it's need. As a risk-neutral player he wants at least his expected value to be realized, as a player with a certain need he wants at least this need to be covered. The same applies to a proposer. However, this approach does not take into account that the respondent with the smaller need can have such a large expected value that she is more expensive than the competing respondent and can therefore go away empty-handed, although she should not go away empty-handed. If she wants to avoid this, she may have to accept offers that are lower than those that the other respondent can accept - she must undercut the other respondent in this sense. If she does so, she can - provided that the resource is sufficiently large that a distribution is made - avoid going empty and achieve that her need is covered. Compared to the alternative of going short and not covering one's own need, this is the only rational strategy, even if it means that a risk-neutral player, who by definition should orientate himself to his expected value, has to move away from such a lower limit of approval. The improved approach builds on such considerations.

Thus, the complexity increases significantly compared to the BF model, not only with respect to the strategies of the respondents and the proposer, but also and especially - under additional consideration of the size of the resource - with respect to the solutions that are possible in principle and the case-specific solutions. A simple statement about the solution(s), comparable to the one in the BF-model, is not possible due to the complexity; rather 50 case distinctions[11] have to be made, for each of which different solutions or thresholds for certain solutions result[12]. In view of the complexity which becomes apparent at first (also and in particular on the basis of combinatorial estimations), it is almost surprising that only this still manageable number of case distinctions arises, the solutions of which, moreover, generally follow a tripartite division:

i) There are the more or less trivial cases, in which it follows directly from the initial conditions and eventually the choice of the proposer, that there cannot be a solution in the first round and possibly also not in the second round (this includes in particular case 1)).
ii) Similarly trivial (because similar to the BF model) are the cases in which it follows directly from the initial conditions and eventually the choice of the proposer that a respondent is chosen randomly and with equal probability and that he receives his need (unlike in the BF model) (this includes in particular case 2)).
iii) The remaining cases (the majority of cases) essentially follow a further tripartite division: Either the respondent with the smallest need is offered its need or its expected value or the need of the competing respondent minus a small amount ε. However, the conditions for this are very different, depending on which players can form a successful coalition in the second session (determined by the size of the resource

[11] Strictly speaking, there are 50 case distinctions with respect to the first session and 12 more, but trivial, ones with respect to the second session.
[12] The count includes some repetitions that have not been removed in order to give preference to a simple, systematic structure rather than to the shortest possible presentation, and because the criteria according to which the cases could be grouped together are not clear and are sometimes in dispute.

and the sum of their needs) and which player is chosen as proposer. In a few cases, the initial conditions under the choice of a particular proposer are such that the case cannot occur that the (cheapest) respondent accepts an offer equal to his need[13]. Finally, only the case 2.2.3.2.3) falls outside this framework. Here it is shown that under the initial conditions, the choice of proposer and the conditions for a certain MV of the (cheapest) respondent, the situation can occur that the MV of the (cheapest) respondent and the MV of the proposer are not covered in total by the resource. For example, if the conditions are given under which the respondent agrees only if he is offered at least his expected value, the situation may arise that the resource is not large enough to cover the MV of the proposer as well, which is why there is no distribution. It is surprising that such a situation does not occur more often, or, to put it differently (and positively), that the conditions for a certain MV of the (cheapest) respondent are generally (but not always) only given if the resource is sufficiently large to cover next to it the MV of the proposer that is associated with these conditions (so that a distribution occurs in which both receive their MV). To show this, however, proves to be quite complex and makes up a comparatively large part of the appendix.

From this point of view, the complexity is relativized. This is also true from the point of view that it is generally possible to read off the solution in a maximum of three steps (initial condition, choice of proposer, condition for one of a maximum of three possible MVs of the (cheapest) respondent) (this may be followed by a fourth but trivial (because simple and always the same) step with regard to the second round). It should be emphasized in this respect that it is thanks to the systematics developed for this purpose that these results can be read comparatively easily. Without this systematics and the associated extensive investigations in the appendix, this would not be possible.

The extent to which the solutions deviate from those of the BF model under consideration of need can be seen in the overview of the solutions and is clearly shown in the numerical examples given. Particularly noteworthy are the results regarding the plausible assumption and the low-need power (i.e., the question of whether players with low need benefit from taking need into account compared to the BF model) and regarding the proposer power (i.e., the prediction of the BF model that the proposer receives significantly more from the resource than the respondent selected by him).

With regard to the plausible assumption and the low-need power, the picture is different. As can be seen from the overview of the solutions, the player with the smallest need - if this is unique - is always involved in the distribution - if this occurs - and his need is met. If the player with the smallest need is respondent (if he is not chosen as proposer), the competing respondent has the same need and if the condition for a majority is fulfilled, he is in a similar situation as in the BF model and receives his need with a 50% probability, which can at most be considered as an advantage or disadvantage with regard to the following aspects. If the player with the smallest need is the respondent, if the competing respondent has a larger need and if the condition for a majority is fulfilled, the player with the smallest need receives at least his need, but the offer to him can, as can be seen from the overview of the solutions and the numerical examples, be significantly smaller than his expected value and also significantly smaller than in the BF model (the offer can be close to 0 percent, despite a high expected value and while in the BF model it is always 1/3 of the resource for a (randomly) selected respondent). In short, the advantage that one's own need is covered with certainty (at least) in most cases, is opposed

[13] In these cases, there is a bipartition (and not a tripartition): Either the respondent with the smallest need is offered its expected value or the need of the competing respondent minus ε.

by the disadvantage of a possibly very small offer. It is hardly possible to weigh this against each other, especially since it must be taken into account that, as can also be seen from the overview of the solutions and the numerical examples, the offer can also be significantly larger than in the BF model (it can be close to 100 percent), but also - unlike in the BF model - the case can occur that the resource is not distributed because the condition for a majority is not fulfilled - which means that everyone goes away empty-handed. Whether one recognizes a low-need power in this respect probably depends on one's own risk profile.

Closely related to this is the fact that the proposer power is relativized[14]. As can be seen from the overview of the solutions and the numerical examples, it exists in the sense and analogously to the low-need power, that the need of the proposer - if the condition for a majority is fulfilled - is covered. Unlike the player with the smallest need, the proposer with the consideration of need does not have an advantage in the sense that it is considered with certainty - if the condition for a majority is fulfilled - since this is also the case in the BF model. However, he may receive much less than he would under the BF model (his allocation may be 0 percent if there is no distribution and close to 0 percent if there is a distribution) or much more (his allocation may be close to 100 percent). Compared to the BF model, in which the proposer always receives 2/3 of the resource, the proposer power can be significantly smaller as well as (significantly) larger. It should be emphasized in this respect that the overview of the solutions and the numerical examples give an indication of a possible explanation of empirical findings, according to which the proposer power is smaller than predicted by the BF model[15]: If the needs of all players are approximately equal and if they are of such a size that the sum of the MV's of two players is approximately equal to the resource (but not greater, so that the resource covers the needs), proposer power weakens compared to the BF model (see in particular case 2) and example 14). It could be that participants in studies with reference to the BF model without need information assume a similar situation: In the absence of information, they assume that all players have similar needs (or, more generally, approval floors). And since shares of the resource to be distributed between two players are negotiated, the situation is similar to the situation where the sum of the needs (the approval floors) of two players equals the resource. Investigating this further could be a profitable endeavor.

This work provides a complete solution to the 3-players/2-sessions case of the BF model considering individual need. The strategies determined for this case and the ways developed for determining solutions are a good starting point for determining solutions for cases with more than 3 players and more than 2 sessions as well. Furthermore, the work provides a starting point to explain deviations between theoretical prediction and empirical findings in a central point of the BF model.

References

Agranov, Marina, and Chloe Tergiman. 2014. “Communication in Multilateral Bargaining.” *Journal of Public Economics* 118 (October): 75–85.

Baranski, Andrzej, and Rebecca Morton. 2020. “The Determinants of Multilateral Bargaining: A Comprehensive Analysis of Baron and Ferejohn Majoritarian Bargaining Experiments.” Division of Social Science Working Paper Series, Working Paper 0037, New York University Abu Dhabi, Abu Dhabi.

[14] This follows directly from the previous paragraph if the player with the smallest need is chosen as proposer, but also indirectly if the player with the smallest need is not chosen as proposer, because then the proposer is faced with a player whose MV can be between almost 0 and almost 100 per cent of the resource, leaving him with between almost 100 and almost 0 per cent after subtracting the offer from the resource.

[15] Baranski and Morton (2020) summarize in "The Determinants of Multilateral Bargaining: A Comprehensive Analysis of Baron and Ferejohn Majoritarian Bargaining Experiments" all relevant studies (chapter 2) and show in a meta-analysis (chapter 4.1) that proposer power is overestimated by the BF model.

Baron, David P., and John A. Ferejohn. 1989. “Bargaining in Legislatures.” *The American Political Science Review* 83 (4): 1181–1206.
Braybrooke, David. 1987. *Meeting Needs*. Princeton: Princeton University Press.
Daniels, Norman. 1979. “Wide Reflective Equilibrium and Theory Acceptance in Ethics.” *The Journal of Philosophy* 76 (5): 256–82.
Deci, Edward L., and Richard M. Ryan. 2000. “The ‘What’ and ‘Why’ of Goal Pursuits: Human Needs and the Self-Determination of Behavior.” *Psychological Inquiry* 11 (4): 227–68.
Maslow, A. H. 1943. “A Theory of Human Motivation.” *Psychological Review* 50 (4): 370–96.
Miller, David. 1999. *Principles of Social Justice*. Cambridge, Massachusetts; London, England: Harvard University Press.
Nussbaum, Martha C. 2000. *Women and Human Development: The Capabilities Approach*. Cambridge: Cambridge University Press.
———. 2011. *Creating Capabilities: The Human Development Approach*. Cambridge, Mass: Belknap Press of Harvard University Press.
Rowntree, Benjamin S. 1901. *Poverty, a Study of Town Life*. London: Macmillan and Co.
Runciman, Walter G. 1966. *Relative Deprivation and Social Justice: A Study of Attitudes to Social Inequality in Twentieth-Century England*. London: Rontledge and Kegan Paul.
Ryan, Richard M., and Edward L. Deci. 2000. “Self-Determination Theory and the Facilitation of Intrinsic Motivation, Social Development, and Well-Being.” *American Psychologist* 55 (1): 68–78.
Schuppert, Fabian. 2013. “Distinguishing Basic Needs and Fundamental Interests.” *Critical Review of International Social and Political Philosophy* 16 (1): 24–44.
Seidl, Christian. 1988. “Poverty Measurement: A Survey.” In *Welfare and Efficiency in Public Economics*, edited by Dieter Bös, Manfred Rose, and Christian Seidl, 71–147. Berlin, Heidelberg: Springer.
Sen, Amartya. 1983. “Poor, Relatively Speaking.” *Oxford Economic Papers* 35 (2): 153–69.
———. 2009. *The Idea of Justice*. Harvard University Press.
Sher, George. 2014. *Equality for Inegalitarians*. Cambridge: Cambridge University Press.
Springhorn, Nils. 2021. “Capitulate for Nothing? Does Baron and Ferejohn’s Bargaining Model Fail Because No One Would Give Everything for Nothing?” FOR2104 Working Paper, Helmut Schmidt University, Hamburg.
Streeten, Paul, Shahid Burki, M. Ul-Haq, Norman Hicks, and Frances Stewart. 1981. *First Things First: Meeting Basic Human Needs in Developing Countries*. New York: Oxford University Press.
Townsend, Peter. 1974. “Poverty as Relative Deprivation: Resources and Style of Living.” In *Poverty, Inequality and Class Structure*, edited by Dorothy Wedderburn, 15–41. Cambridge: Cambridge University Press.

Appendix

Case Differentiations and Solutions - Derivation

For a rough orientation, the following procedure for determining the cases with different expected values and solutions (or limits of solutions) can be described as a disjoint decomposition of "all possible case-defining conditions". There is no example - known to the author - of this procedure. One should not be deterred by this description, because starting from initially simple case distinctions, the procedure quickly becomes understandable.

In a first step, the case that in principle no bargaining solution is possible, because the resource does not even cover the needs of any two players, is separated from the other cases. In the other cases, it is assumed that there are at least two players whose combined needs are less than or equal to the resource:

1) For any two players p_x and p_y, $n(p_x) + n(p_y) > res$.
2) For at least two players p_x and p_y holds: $n(p_x) + n(p_y) \leq res$.

In case 1) there can be no majority for a proposer in either the first or the second session. Regardless of who is chosen as proposer, 0 is proposed for all players in the first as well as in the second session (see the remarks on Lemma 1)). With the aim of a systematic representation from the beginning to the end of the game, the following distinctions are made:

1.1) p(1) = i: DP(i,1) = (0, 0, 0)

1.1.1) p(2) = i: DP(i,2) = (0, 0, 0)
1.1.2) p(2) = j: DP(j,2) = (0, 0, 0)
1.1.3) p(2) = k: DP(k,2) = (0, 0, 0)

1.2) p(1) = j: DP(j,1) = (0, 0, 0)

1.2.1) p(2) = i: DP(i,2) = (0, 0, 0)
1.2.2) p(2) = j: DP(j,2) = (0, 0, 0)
1.2.3) p(2) = k: DP(k,2) = (0, 0, 0)

1.3) p(1) = k: DP(k,1) = (0, 0, 0)

1.3.1) p(2) = i: DP(i,2) = (0, 0, 0)
1.3.2) p(2) = j: DP(j,2) = (0, 0, 0)
1.3.3) p(2) = k: DP(k,2) = (0, 0, 0)

In case 2), further differentiation must be made. The cases can be given that

2.1) all players have equal needs, or
2.2) not all players have equal needs.

In case 2.1) there is no need ;-) to differentiate between players. The expected value of each player in session 1 for session 2 depends on whether the resource is large enough to satisfy the needs of any two players (because they all have the same). If this is not the case, case 1) is given. Therefore, in case 2.1) only the case has to be considered that

2.1) all three players have the same need: $n(i) = n(j) = n(k)$
and $n(i) + n(j) = n(i) + n(k) = n(j) + n(k) \leq res$[16].

The expected value of each player is obtained as follows:

$$E(p,1,2) = (res-n(p) + 0.5*n(p) + 0.5*n(p))/3 = 1/3*res$$[17].

According to the strategy of a respondent, the following applies (quite fundamentally) to a respondent:

$MV(r_x(1),1) = E(r_x(1),1,2)$, if $n(r_x(1)) < E(r_x(1),1,2) < n(r_y(1))$,
$MV(r_x(1),1) = n(r_y(1))-\varepsilon$, if $n(r_x(1)) < n(r_y(1)) \leq E(r_x(1),1,2)$,
$MV(r_x(1),1) = n(r_x(1))$, else,

[16] This case is similar to the case studied by Baron and Ferejohn (the closest among all the cases listed here). If the resource is equal to 1 and the needs are equal to the expected values, it is a correspondence.
[17] Because - since the respondents are equally cheap - the proposer randomly selects one of the respondents and makes him an offer.

where r_y is the competing respondent. The cases $MV(r_x(1),1) = E(r_x(1),1,2)$ and $MV(r_x(1),1) = n(r_y(1))-\varepsilon$ can be excluded under the conditions for 2.1), because the conditions $n(r_x(1)) < E(r_x(1),1,2) < n(r_y(1))$ and $n(r_x(1)) < n(r_y(1)) \leq E(r_x(1),1,2)$ cannot be given under 2.1)[18].

As an interim result, it can be stated the following: If the proposer makes a distribution proposal, he offers - because both respondents are equally expensive - a randomly selected respondent his need. The respondent accepts because it is his MV and the negotiation ends.

However, the proposer makes such an offer only if the condition for a majority is given. According to the strategy of a proposer, its MV is given as follows (quite fundamentally):

$MV(p(1),1) = E(p(1),1,2)$, if $n(p(1)) < E(p(1),1,2)$,
$MV(p(1),1) = n(p(1))$, else.

The conditions for 2.1) guarantee that $MV(p(1),1) = n(p(1))$ is covered by the resource in addition to $MV(r_x(1),1) = n(r_x(1))$. Less obviously, $MV(p(1),1) = E(p(1),1,2)$ can be covered by the resource alongside $MV(r_x(1),1) = n(r_x(1))$. The case $MV(p(1),1) = E(p(1),1,2)$ is given when $n(p(1)) < E(p(1),1,2)$. This can be transformed in 2.1) to $n(p(1)) < 1/3*res$. Thus, if $n(p(1)) < 1/3*res$, $MV(p(1),1) = E(p(1),1,2)$. Since the needs of all players are equal, $n(r(1)) < 1/3*res$ is also true and thus $res - n(r(1)) > 1/3*res = MV(p(1),1)$, which means that the proposer has enough left to cover his MV after subtracting the offers. Thus, it cannot be the case that the MVs of proposer and respondent cannot be covered together.

Therefore, the solution stated in the interim result is realized. Again, with the aim of a systematic representation from the beginning to the end of the game, a distinction is made:

2.1.1) $p(1) = i$: $DP(i,1) = (res-n(j), n(j), 0)$ with 50% probability and
$DP(i,1) = (res-n(k), 0, n(k))$ with 50% probability.

2.1.2) $p(1) = j$: $DP(j,1) = (n(i), res-n(i), 0)$ with 50% probability and
$DP(j,1) = (0, res-n(k), 0, n(k))$ with 50% probability.

2.1.3) $p(1) = k$: $DP(k,1) = (n(i), 0, res-n(i))$ with 50%'s probability and
$DP(k,1) = (0, n(j), res-n(j))$ with 50% probability.

The procedure for 2.1) can be summarized and generalized as follows:

i) Once a differentiation is found that allows the expected values to be calculated, it is determined whether it makes a difference which player is chosen as the proposer (this is not the case in 2.1)).

ii) For each of these cases, the cheapest respondent is determined, its case-specific possible MVs and their case-specific conditions (in 2.1) all players are equally cheap, there is only one possible MV for the respondent and thus no further differentiating conditions).

iii) One obtains an interim result in the form that when the proposer makes a distribution proposal, it offers the cheapest respondent its MV, the amount of which depends on the previously determined conditions.

iv) It remains to determine whether the resource is large enough for such a proposal to be made, in the sense that the condition for a majority is satisfied. For this purpose, we do not calculate how large the proposer's MV is under the conditions for a given MV of the respondent. Instead, the question is answered whether the case can occur

[18] The cheaper rule forces each respondent to be cheaper than the competitor, his need forbids him to accept an amount smaller than his need.

that MV(p(1),1) + MV(r_c(1),1) < res. If the question is answered in the negative, the interim result is used to derive a proposed distribution, which is accepted because, by construction, the respondent receives his MV and the propo-ser receives at least his MV. If the question is answered in the affirmative, further case distinctions follow.

In individual cases, there are more elegant and shorter procedures. However, this procedure has the advantage that it can be applied systematically to all further case differentiations.

In case 2.2) (not all players have equal needs), further differentiation is necessary. First, among the cases where not all players have equal needs, the case is examined where two players have equal needs and the third has a smaller need than the others. Analogous to case 2.1), it is assumed that the resource covers the need of i and k and thus also j and k together, since otherwise one would be in case 1):

2.2.1) There are two players with equal need and one with smaller need: n(i) < n(j) = n(k) *and* n(i) + n(j) = n(i) + n(k) ≤ res.

The expected values are obtained as follows:

E(i,1,2) = (res-n(j) + n(i) + n(i))/3 = 1/3*res + 2/3*n(i) - 1/3*n(j)
E(j,1,2) = (0.5*n(j) + res-n(i) + 0)/3 = 1/3*res - 1/3*n(i) + 1/6*n(j) =
E(k,1,2) = (0.5*n(k) + 0 + res-n(i))/3 = 1/3*res - 1/3*n(i) + 1/6*n(k)

Unlike in case 2.1), one has to distinguish whether one of the players with the smallest need (i) or one of the other players is chosen as proposer:

2.2.1.1) p(1) = i

Since n(j) = n(k), i - if i makes a distribution proposal - randomly chooses one of the respondents j or k to make it an offer. Analogous to 2.1) the cases MV(j,1) = E(j,1,2) and MV(j,1) = n(k)-ε or MV(k,1) = E(k,1,2) and MV(k,1) = n(j)-ε can be excluded, because the conditions for this in 2.2.1) cannot be fulfilled. Thus, if i makes an offer, then with equal probability to j or k in the amount of their need.

The important and very laborious question is: *Can the case occur that i cannot make such an offer because the resource is not large enough?*

Given MV(i,1) = n(i) this is not the case, since the conditions of 2.2.1) guarantee that n(i) + n(j) = n(i) + n(k) ≤ res. Given MV(i,1) = E(i,1,2) this would be the case if holds:

res < n(j) + E(i,1,2)
⇔ res < n(j) + 1/3*res + 2/3*n(i) – 1/3*n(j)
⇔ 3*res < 3*n(j) + res + 2*n(i) – n(j)
⇔ 2*res < 2*n(i) + 2*n(j)
⇔ res < n(i) + n(j)

However, res < n(i) + n(j) is impossible under the condition of 2.2.1), which states that n(i) + n(j) ≤ res.

Thus, the solution from the interim result is realized:

2.2.1.1) DP(i,1) = (res-n(j), n(j), 0) with 50% probability and
DP(i,1) = (res-n(k), 0, n(k)) with 50% probability.

2.2.1.2) p(1) = j

If j makes a distribution proposal, j offers i its MV since i is cheaper than k (because i's need is smaller than k's, i can undercut k if necessary).

None of the MV's of i that are possible in principle (MV(i,1) = n(i), MV(i,1) = E(i,1,2), and MV(i,1) = n(k)-ε) can be excluded under the conditions for 2.2.1) because:

$$MV(i,1) = E(i,1,2), \text{ if } \quad n(i) < E(i,1,2) < n(k).$$

$$\begin{aligned} & n(i) < E(i,1,2) < n(k) \\ \Leftrightarrow\quad & n(i) < 1/3*res + 2/3*n(i) - 1/3*n(j) < n(k) \\ \Leftrightarrow\quad & 3*n(i) < res + 2*n(i) - n(j) < 3*n(k) \\ \Leftrightarrow\quad & n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k) \end{aligned}$$

(n(i) + n(j) < res may be given in 2.2.1) (if this is not satisfied, then under the conditions of 2.2.1) n(i) + n(j) = res). res < -2*n(i) + n(j) + 3*n(k) can be given, since n(i) < n(k), thus -2*n(i) + 3*n(k) > n(i) and thus -2*n(i) + n(j) + 3*n(k) > n(i) + n(j). Thus, the case may be given that n(i) + n(j) ≤ res < -2*n(i) + n(j) + 3*n(k) and thus MV(i,1) = E(i,1,2). For the sake of comprehensiveness, such a detailed presentation will not be given in the following; the reader is kindly requested to use this model for the following presentations).

This results in:

MV(i,1) = n(k)-ε, if res ≥ -2*n(i) + n(j) + 3*n(k)[19] and

MV(i,1) = n(i), else (i.e., if res ≤ n(i) + n(j)) (since res < n(i) + n(j) is impossible under the conditions for 2.2.1), this can be simplified to res = n(i) + n(j)).

As an interim result, if j proposes a distribution, it will result according to the following case distinctions:

2.2.1.2.1) If res ≤ n(i) + n(k), then DP(j,1) = (n(i), res-n(i), 0).
2.2.1.2.2) If n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k), then DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.1.2.3) If res ≥ -2*n(i) + n(j) + 3*n(k), then DP(j,1) = (n(i), res-n(i), 0)

Again, it remains to be determined whether in any of these cases MV(j,1) + MV(i,1) < res can be given (so that proposer j cannot make an appropriate distribution proposal).

For that the proposer side must be considered. Also for the proposer j none of the principally possible MV's of j (MV(j,1) = n(i) and MV(j,1) = E(j,1,2)) can be excluded under the conditions for 2.2.1), since:

$$MV(j,1) = E(j,1,2), \quad \text{if} \quad n(j) < E(j,1,2)$$

$$\begin{aligned} & n(j) < E(j,1,2) \\ \Leftrightarrow\quad & n(j) < 1/3*res - 1/3*n(i) + 1/6*n(k) \\ \Leftrightarrow\quad & 6*n(j) < 2*res - 2*n(i) + n(k) \end{aligned}$$

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[19] MV(i,1) = n(k)-ε, if n(i) < n(k) ≤ E(i,1,2)

$$\begin{aligned} & n(i) < n(k) \le E(i,1,2) \\ \Leftrightarrow\quad & n(k) \le E(i,1,2) \text{ [since } n(i) < n(k) \text{ is assumed]} \\ \Leftrightarrow\quad & n(k) \le 1/3*res + 2/3*n(i) - 1/3*n(j) \\ \Leftrightarrow\quad & 3*n(k) \le res + 2*n(i) - n(j) \\ \Leftrightarrow\quad & res \ge -2*n(i) + n(j) + 3*n(k) \end{aligned}$$

$\Leftrightarrow$ 2*res > 2*n(i) + 6*n(j) – n(k)
$\Leftrightarrow$ res > n(i) + 3*n(j) – 1/2*n(k)

MV(j,1) = n(j), else (i.e. if res ≤ n(i) + 3*n(j) – 1/2*n(k))

Therefore, for all 6 combinations

MV(i,1) = n(i) and MV(j,1) = n(j),
MV(i,1) = n(i) and MV(j,1) = E(j,1,2),
MV(i,1) = n(i) and MV(j,1) = E(j,1,2),
MV(i,1) = E(i,1,2) and MV(j,1) = E(j,1,2),
MV(i,1) = n(k)-ε and MV(j,1) = n(j),
MV(i,1) = n(k)-ε and MV(j,1) = n(j),

it is to show that the case MV(i,1) + MV(j,1) > res cannot occur.

MV(i,1) = n(i) und MV(j,1) = n(j):

res < n(i) + n(j)

res < n(i) + n(j) impossible under the condition for 2.2.1): n(i) + n(j) ≤ res

MV(i,1) = n(i) and MV(j,1) = E(j,1,2):

res < n(i) + E(j,1,2)
$\Leftrightarrow$ res < n(i) + 1/3*res - 1/3*n(i) + 1/6*n(j)
$\Leftrightarrow$ 6*res < 6*n(i) + 2*res - 2*n(i) + n(j)
$\Leftrightarrow$ 4*res < 4*n(i) + n(j)
$\Leftrightarrow$ res < n(i) + 1/4*n(j)

res < n(i) + 1/4*n(j) impossible under the condition for 2.2.1): n(i) + n(j) ≤ res

MV(i,1) = n(i) and MV(j,1) = E(j,1,2):

res < E(i,1,2) + n(j)
$\Leftrightarrow$ res < 1/3*res + 2/3*n(i) – 1/3*n(j) + n(j)
$\Leftrightarrow$ 3*res < res + 2*n(i) – n(j) + 3*n(j)
$\Leftrightarrow$ 2*res < 2*n(i) + 2*n(j)
$\Leftrightarrow$ res < n(i) + n(j)

res < n(i) + n(j) impossible under the condition for 2.2.1): n(i) + n(j) ≤ res

MV(i,1) = E(i,1,2) and MV(j,1) = E(j,1,2):

res < E(i,1,2) + E(j,1,2)
$\Leftrightarrow$ res < 1/3*res + 2/3*n(i) – 1/3*n(j) + 1/3*res – 1/3*n(i) + 1/6*n(j)
$\Leftrightarrow$ 6*res < 2*res + 4*n(i) – 2*n(j) + 2*res – 2*n(i) + n(j)
$\Leftrightarrow$ 2*res < 2*n(i) – n(j)
$\Leftrightarrow$ res < n(i) – 1/2*n(j)

res < n(i) – 1/2*n(j) impossible under the condition for 2.2.1): n(i) + n(j) ≤ res

MV(i,1) = n(k)-ε and MV(j,1) = n(j):

res < n(k)-ε + n(j)

res < n(j) + n(k) – ε impossible under the condition for MV(i,1) = n(k)-ε: res ≥ -2*n(i) + n(j) + 3*n(k)

$MV(i,1) = n(k)-\varepsilon$ and $MV(j,1) = n(j)$:

$$
\begin{aligned}
& res < n(k)-\varepsilon + E(j,1,2) \\
\Leftrightarrow\quad & res < n(k)-\varepsilon + 1/3*res - 1/3*n(i) + 1/6*n(j) \\
\Leftrightarrow\quad & 6*res < 6*n(k) - 6*\varepsilon + 2*res - 2*n(i) + n(j) \\
\Leftrightarrow\quad & 4*res < -2*n(i) + n(j) + 6*n(k) - 6*\varepsilon \\
\Leftrightarrow\quad & res < -1/2*n(i) + 1/4*n(j) + 3/2*n(k) - 3/2*\varepsilon
\end{aligned}
$$

$res < -1/2*n(i) + 1/4*n(j) + 3/2*n(k) - 3/2*\varepsilon = -1/2*n(i) + 7/4*n(j) - 3/2*\varepsilon$ impossible under the condition for $MV(j,1) = E(j,1,2)$: $res > n(i) + 3*n(j) - 1/2*n(k) = n(i) + 10/4*n(j)$

Therefore, the case $MV(j,1) + MV(i,1) < res$ cannot occur and the solutions recorded in the interim result are realized.

2.2.1.3) $p(1) = k$

This case results analogous to case 2.2.1.2):

2.2.1.3.1) If $res \leq n(i) + n(k)$, then $DP(k,1) = (n(i), 0, res-n(i))$
2.2.1.3.2) If $n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k)$, then $DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))$
2.2.1.3.3) If $res \geq -2*n(i) + n(j) + 3*n(k)$, then $DP(k,1) = (n(j)-\varepsilon, 0, res-n(j)-\varepsilon)$

Starting from case 2.2) (not all players have the same need), the following two cases in addition to case 2.2.1) (two players have the same need and the third has a smaller need) has to be distinguished (which results in a disjoint decomposition of case 2.2)):

2.2.2) Two players have equal needs and the third has a greater need.
2.2.3) All players have needs of different sizes.

These cases (unlike the cases before) are to be further differentiated with respect to the size of the resource. In the previous case differentiations, the conditions were such that the resource either covers the needs of any pair of players, or of no pair of players (which puts us in case 1)). In cases 2.2.2) and 2.2.3), however, depending on the size of the resource, the case can occur that the resource only covers the needs of two specific players (namely the one with the smallest needs), but not the needs of the player with the largest need and another player, thus ruling out the possibility that the player with the largest need can participate in a coalition that reaches a majority. Accordingly, it must be distinguished:

2.2.2.1) There are two players with equal need and one with greater need: $n(i) = n(j) < n(k)$
and $n(i) + n(j) \leq res$
and $n(i) + n(k) = n(j) + n(k) > res$.

2.2.2.2) There are two players with equal need and one with greater need: $n(i) = n(j) < n(k)$
and $n(i) + n(j) \leq res$
and $n(i) + n(k) = n(j) + n(k) \leq res$.

2.2.3.1) All players have different needs: $n(i) < n(j) < n(k)$
and $n(i) + n(j) \leq res$
and $n(i) + n(k) > res$ (and hence also $n(j) + n(k) > res$).

2.2.3.2) All players have different needs: $n(i) < n(j) < n(k)$
and $n(i) + n(j) \leq res$
and $n(i) + n(k) \leq res$

and n(j) + n(k) > res.

2.2.3.3) All players have different needs: n(i) < n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) ≤ res
and n(j) + n(k) ≤ res.

The study of the cases 2.2.2.1), 2.2.2.2), 2.2.3.1) and 2.2.3.3) do not differ essentially from the investigation of the case 2.2.1.2) and are treated very briefly. In case 2.2.3.2) the case occurs that MV(j,1) + MV(i,1) < res cannot be excluded, which is why it is treated in more detail.

2.2.2.1) There are two players with equal need and one with greater need: n(i) = n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) = n(j) + n(k) > res.

E(i,1,2) = (res-n(j) + n(i) + 0)/3 = 1/3*res =
E(j,1,2) = (n(j) + res-n(i) + 0)/3 = 1/3*res
E(k,1,2) = (0 + 0 + 0)/3 = 0

2.2.2.1.1) p(1) = i:

r_c(1|p(1)=i) = {j}

MV(j,1) = E(j,1,2), if n(j) < E(j,1,2) < n(k)

n(j) < E(j,1,2) < n(k)
⇔ n(j) < 1/3*res < n(k)
⇔ 3*n(j) < res < 3*n(k)
⇔ res > 3*n(j) and res < 3*n(k)
⇔ 3*n(j) < res < 3*n(k)

MV(j,1) = n(k)-ε, if n(j) < n(k) ≤ E(j,1,2)

n(j) < n(k) ≤ E(j,1,2)
⇔ n(j) < n(k) and n(k) ≤ 1/3*res
⇔ n(k) ≤ 1/3*res
⇔ 3*n(k) ≤ res
⇔ res ≥ 3*n(k)

MV(j,1) = n(j), else (i.e., if res ≤ 3*n(j))

MV(i,1) = E(i,1,2), if n(i) < E(i,1,2)

n(i) < E(i,1,2)
⇔ n(i) < 1/3*res
⇔ 3*n(i) < res
⇔ res > 3*n(i)

MV(i,1) = n(i), else (i.e., if res ≤ 3*n(i)).

Can the case occur that MV(i,1) + MV(j,1) > res?

MV(j,1) = n(j) and MV(i,1) = n(i):

res < n(i) + n(j)

res < n(i) + n(j) impossible under the condition for 2.2.2.1): n(i) + n(j) ≤ res

$MV(j,1) = n(j)$ and $MV(i,1) = E(i,1,2)$:

$res < n(j) + E(i,1,2)$
$\Leftrightarrow \quad res < n(j) + 1/3*res$
$\Leftrightarrow \quad 3*res < 3*n(j) + res$
$\Leftrightarrow \quad 2*res < 3*n(j)$
$\Leftrightarrow \quad res < 3/2*n(j)$

$res < 3/2*n(j)$ impossible under the condition for 2.2.2.1): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = n(i)$:

$res < E(j,1,2) + n(i)$
$\Leftrightarrow \quad res < 1/3*res + n(i)$
$\Leftrightarrow \quad 3*res < res + 3*n(i)$
$\Leftrightarrow \quad 2*res < 3*n(i)$
$\Leftrightarrow \quad res < 3/2*n(i)$

$res < 3/2*n(i)$ impossible under the condition for 2.2.2.1): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = E(i,1,2)$:

$res < E(j,1,2) + E(i,1,2)$
$\Leftrightarrow \quad res < 1/3*res + 1/3*res$

$res < 1/3*res + 1/3*res$ impossible

$MV(j,1) = n(k)-\varepsilon$ und $MV(i,1) = n(i)$:

$res < n(k)-\varepsilon + n(i)$

$res < n(i) + n(k) - \varepsilon$ impossible under the condition for $MAV(j,1) = n(k)-\varepsilon$: $res \geq 3*n(k)$

$MV(j,1) = n(k)-\varepsilon$ and $MV(i,1) = E(i,1,2)$:

$res < n(k)-\varepsilon + E(i,1,2)$
$\Leftrightarrow \quad res < n(k)-\varepsilon + 1/3*res$
$\Leftrightarrow \quad 3*res < 3*n(k) - 3*\varepsilon + res$
$\Leftrightarrow \quad 2*res < 3*n(k) - 3*\varepsilon$
$\Leftrightarrow \quad res < 3/2*n(k) - 3/2*\varepsilon$

$res < 3/2*n(k) - 3/2*\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq 3*n(k)$

Therefore, the case $MV(i,1) + MV(j,1) < res$ cannot occur.

2.2.2.1.1) Distribution proposals

2.2.2.1.1.1) If $res \leq 3*n(j)$, then $DP(i,1) = (res-n(j), n(j), 0)$
2.2.2.1.1.2) If $3*n(j) < res < 3*n(k)$, then $DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)$
2.2.2.1.1.3) If $res \geq 3*n(k)$, then $DP(i,1) = (res-n(k)+\varepsilon, n(k)-\varepsilon, 0)$

2.2.2.1.2) $p(1) = j$:

Analogous to 2.2.2.1.1)

2.2.2.1.2) Distribution proposals

2.2.2.1.2.1) If $res \le 3*n(j)$, then $DP(j,1) = (n(i), res-n(i), 0)$
2.2.2.1.2.2) If $3*n(j) < res < 3*n(k)$, then $DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)$
2.2.2.1.2.3) If $res \ge 3*n(k)$, then $DP(j,1) = (n(k)-\varepsilon, res-n(k)+\varepsilon, 0)$

2.2.2.1.3) $p(1) = k$:

k has no possibility to be part of a coalition that can reach a majority under the condition of 2.2.2.1): $n(i) + n(k) = n(j) + n(k) > res$.

2.2.2.1.3) Distribution proposals

2.2.2.1.3) $DP(k,1) = (0, 0, 0)$

2.2.2.1.3.1) $p(2) = i$: $DP(i,2) = (res-n(j), n(j), 0)$
2.2.2.1.3.2) $p(2) = j$: $DP(j,2) = (n(i), res-n(i), 0)$
2.2.2.1.3.3) $p(2) = k$: $DP(k,2) = (0, 0, 0)$

2.2.2.2) There are two players with equal need and one with greater need: $n(i) = n(j) < n(k)$ and $n(i) + n(j) \le res$ and $n(i) + n(k) = n(j) + n(k) \le res$.

$E(i,1,2) = (res-n(j) + n(i) + 0.5*n(i))/3 = 1/3*res + 1/6*n(i) =$
$E(j,1,2) = (n(j) + res-n(i) + 0.5*n(j))/3 = 1/3*res + 1/6*n(j)$
$E(k,1,2) = (0 + 0 + res-n(i))/3 = 1/3*res – 1/3*n(i)$

2.2.2.2.1) $p(1) = i$

$r\_c(1|p(1)=i) = \{j\}$

$MV(j,1) = E(j,1,2)$, if $n(j) < E(j,1,2) < n(k)$

$n(j) < E(j,1,2) < n(k)$
$\Leftrightarrow n(j) < 1/3*res + 1/6*n(j) < n(k)$
$\Leftrightarrow 6*n(j) < 2*res + n(j) < 6*n(k)$
$\Leftrightarrow 2*res > 5*n(j)$ and $2*res < -n(j) + 6*n(k)$
$\Leftrightarrow res > 5/2*n(j)$ and $res < -1/2*n(j) + 3*n(k)$
$\Leftrightarrow 5/2*n(j) < res < -1/2*n(j) + 3*n(k)$

$MV(j,1) = n(k)-\varepsilon$, if $n(j) < n(k) \le E(j,1,2)$

$n(j) < n(k) \le E(j,1,2)$
$\Leftrightarrow n(j) < n(k) \le 1/3*res + 1/6*n(j)$
$\Leftrightarrow n(k) \le 1/3*res + 1/6*n(j)$
$\Leftrightarrow 6*n(k) \le 2*res + n(j)$
$\Leftrightarrow 2*res \ge -2*n(j) + 6*n(k)$
$\Leftrightarrow res \ge -n(j) + 3*n(k)$

$MV(j,1) = n(j)$, else (i.e. if $res \le 5/2*n(j)$)

$MV(i,1) = E(i,1,2)$, if $n(i) < E(i,1,2)$

$n(i) < E(i,1,2)$
$\Leftrightarrow n(i) < 1/3*res + 1/6*n(j)$
$\Leftrightarrow 6*n(i) < 2*res + n(j)$
$\Leftrightarrow 2*res > 6*n(i) – n(j)$
$\Leftrightarrow res > 3*n(i) – 1/2*n(j)$

$MV(i,1) = n(i)$, else (i.e. if $res \leq 3*n(i) - 1/2*n(j)$)

Can the case occur that $MV(i,1) + MV(j,1) > res$?

$MV(j,1) = n(j)$ und $MV(i,1) = n(i)$:

$$res < n(i) + n(j)$$

$res < n(i) + n(j)$ impossible under the condition for 2.2.2.2): $n(i) + n(j) \leq res$

$MV(j,1) = n(j)$ and $MV(i,1) = E(i,1,2)$:

$$\begin{aligned} & res < n(j) + E(i,1,2) \\ \Leftrightarrow\ & res < n(j) + 1/3*res + 1/6*n(i) \\ \Leftrightarrow\ & 6*res < 6*n(j) + 2*res + n(i) \\ \Leftrightarrow\ & 4*res < n(i) + 6*n(j) \\ \Leftrightarrow\ & res < 1/4*n(i) + 3/2*n(j) \end{aligned}$$

$res < 1/4*n(i) + 3/2*n(j)$ impossible under the condition for 2.2.2.2): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = n(i)$:

$$\begin{aligned} & res < E(j,1,2) + n(i) \\ \Leftrightarrow\ & res < 1/3*res + 1/6*n(j) + n(i) \\ \Leftrightarrow\ & 6*res < 2*res + n(j) + 6*n(i) \\ \Leftrightarrow\ & 4*res < n(j) + 6*n(i) \\ \Leftrightarrow\ & res < 1/4*n(j) + 3/2*n(i) \end{aligned}$$

$res < 1/4*n(j) + 3/2*n(i)$ impossible under the condition for 2.2.2.2): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = E(i,1,2)$:

$$\begin{aligned} & res < E(j,1,2) + E(i,1,2) \\ \Leftrightarrow\ & res < 1/3*res + 1/6*n(j) + 1/3*res + 1/6*n(i) \\ \Leftrightarrow\ & 6*res < 2*res + n(j) + 2*res + n(i) \\ \Leftrightarrow\ & 2*res < n(j) + n(i) \\ \Leftrightarrow\ & res < 1/2*n(i) + 1/2*n(j) \end{aligned}$$

$res < 1/2*n(i) + 1/2*n(j)$ impossible under the condition for 2.2.2.2): $n(i) + n(j) \leq res$

$MV(j,1) = n(k)-\varepsilon$ and $MV(i,1) = n(i)$:

$$res < n(k)-\varepsilon + n(i)$$

$res < n(i) + n(k)-\varepsilon$ impossible under the condition for 2.2.2.2): $n(i) + n(k) \leq res$

$MV(j,1) = n(k)-\varepsilon$ and $MV(i,1) = E(i,1,2)$:

$$\begin{aligned} & res < n(k)-\varepsilon + E(i,1,2) \\ \Leftrightarrow\ & res < n(k)-\varepsilon + 1/3*res + 1/6*n(i) \\ \Leftrightarrow\ & 6*res < 6*n(k) - 6*\varepsilon + 2*res + n(i) \\ \Leftrightarrow\ & 4*res < n(i) + 6*n(k) - 6*\varepsilon \\ \Leftrightarrow\ & res < 1/4*n(i) + 3/2*n(k) - 3/2*\varepsilon \end{aligned}$$

$res < 1/4*n(j) + 3/2*n(k) - 3/2*\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq -n(j) + 3*n(k)$.

Therefore, the case MV(i,1) + MV(j,1) < res cannot occur.

2.2.2.1) Distribution proposals

2.2.2.1.1) If res ≤ 5/2*n(j), then DP(i,1) = (res-n(j), n(j), 0)
2.2.2.1.2) If 5/2*n(j) < res < -1/2*n(j) + 3*n(k),
then DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)
2.2.2.1.3) If res ≥ -1/2*n(j) + 3*n(k), then DP(i,1) = (res-n(k)+ε, n(k)-ε, 0)

2.2.2.2.2) p(1) = j

Analogous to 2.2.2.2.1)

2.2.2.2.2) Distribution proposals

2.2.2.2.2.1) If res ≤ 5/2*n(j), then DP(j,1) = (n(i), res-n(i), 0)
2.2.2.2.2.2) If 5/2*n(j) < res < -1/2*n(j) + 3*n(k),
then DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.2.2.2.3) If res ≥ -1/2*n(j) + 3*n(k), then DP(j,1) = (n(k)-ε, res-n(k)+ε, 0)

2.2.2.2.3) p(1) = k

r_c(1|p(1)=k) = {i,j}

MV(i,1) = E(i,1,2) if n(i) < E(i,1,2) < n(j)

n(i) < n(j) impossible under 2.2.2.2)

MV(i,1) = n(j)-ε, if n(i) < n(j) ≤ E(i,1,2)

n(i) < n(j) impossible under 2.2.2.2)

MV(i,1) = n(i), else (i.e. always under 2.2.2.2.3)

MV(k,1) = E(k,1,2), if n(k) < E(k,1,2)

n(k) < E(k,1,2)
⇔ n(k) < 1/3*res - 1/3*n(i)
⇔ 3*n(k) < res - n(i)
⇔ res > n(i) + 3*n(k)

MV(k,1) = n(k), else (i.e. if res ≤ n(i) + 3*n(k)).

Can the case occur that MV(i,1) + MV(k,1) > res?

MV(i,1) = n(i) and MV(k,1) = n(k):

res < n(i) + n(k)

res < n(i) + n(k) impossible under the condition for 2.2.2.2): n(i) + n(k) ≤ res

MV(i,1) = n(i) and MV(k,1) = E(k,1,2)

res < n(i) + E(j,1,2)
⇔ res < n(i) + 1/3*res - 1/3*n(i)
⇔ 3*res < 3*n(i) + res - n(i)
⇔ 2*res < 2*n(i)
⇔ res < n(i)
res < n(i) impossible under the condition for 2.2.2.2): n(i) + n(j) ≤ res.

Therefore, the case MV(j,1) + MV(k,1) < res cannot occur.

2.2.2.2.3) Distribution proposals

DP(k,1) = (n(i), 0, res-n(i)) with 50% probability and
DP(k,1) = (0, n(j), res-n(j)) with 50% probability.

2.2.3.1) All players have different needs: n(i) < n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) > res (and hence also n(j) + n(k) > res).

E(i,1,2) = (res-n(j) + n(i) + 0)/3 = 1/3*res + 1/3*n(i) – 1/3*n(j)
E(j,1,2) = (n(j) + res-n(i) + 0)/3 = 1/3*res – 1/3*n(i) + 1/3*n(j)
E(k,1,2) = (0 + 0 + 0)/3 = 0

2.2.3.1.1) p(1) = i

r_c(1|p(1)=i) = {j}

MV(j,1) = E(j,1,2), if n(j) < E(j,1,2) < n(k)

n(j) < E(j,1,2) < n(k)
⇔ n(j) < 1/3*res – 1/3*n(i) + 1/3*n(j) < n(k)
⇔ 3*n(j) < res – n(i) + n(j) < 3*n(k)
⇔ res > n(i) + 2*n(j) und res < n(i) – n(j) + 3*n(k)
⇔ n(i) + 2*n(j) < res < n(i) – n(j) + 3*n(k)

MV(j,1) = n(k)-ε, if n(j) < n(k) ≤ E(j,1,2)

n(j) < n(k) ≤ E(j,1,2)
⇔ n(j) < n(k) ≤ 1/3*res – 1/3*n(i) + 1/3*n(j)
⇔ n(k) ≤ 1/3*res – 1/3*n(i) + 1/3*n(j)
⇔ 3*n(k) ≤ res – n(i) + n(j)
⇔ res ≥ n(i) – n(j) + 3*n(k)

MV(j,1) = n(j), else (i.e. res ≤ n(i) + 2*n(j))

MV(i,1) = E(i,1,2), if n(i) < E(i,1,2)

n(i) < E(i,1,2)
⇔ n(i) < 1/3*res + 1/3*n(i) – 1/3*n(j)
⇔ 3*n(i) < res + n(i) – n(j)
⇔ res > 2*n(i) + n(j)

MV(i,1) = n(i), else (i.e. if res ≤ 2*n(i) + n(j))

Can the case occur that MV(i,1) + MV(j,1) > res?

MV(j,1) = n(j) and MV(i,1) = n(i):

res < n(j) + n(i)

res < n(i) + n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(j,1) = n(j) and MV(i,1) = E(i,1,2):

res < n(j) + E(i,1,2)

$\Leftrightarrow$ res < n(j) + 1/3*res + 1/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ 3*res < 3*n(j) + res + n(i) – n(j)
$\Leftrightarrow$ 2*res < n(i) + 2*n(j)
$\Leftrightarrow$ res < 1/2*n(i) + n(j)

res < 1/2*n(i) + n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(j,1) = E(j,1,2) and MV(i,1) = n(i):

res < E(j,1,2) + n(i)
$\Leftrightarrow$ res < 1/3*res – 1/3*n(i) + 1/3*n(j) + n(i)
$\Leftrightarrow$ 3*res < res – n(i) + n(j) + 3*n(i)
$\Leftrightarrow$ 2*res < 2*n(i) + n(j)
$\Leftrightarrow$ res < n(i) + 1/2*n(j)

res < n(i) + 1/2*n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(j,1) = E(j,1,2) and MV(i,1) = E(i,1,2):

res < E(j,1,2) + E(i,1,2)
$\Leftrightarrow$ res < 1/3*res – 1/3*n(i) + 1/3*n(j) + 1/3*res + 1/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ res < 0

res < 0 impossible according to the basic assumption

MV(j,1) = n(k)-ε and MV(i,1) = n(i):

res < n(k)-ε + n(i)

res < n(i) + n(k) – ε impossible under the condition for MV(j,1) = n(k)-ε: res ≥ n(i) – n(j) + 3*n(k)

MV(j,1) = n(k)-ε and MV(i,1) = E(i,1,2):

res < n(k)-ε + E(i,1,2)
$\Leftrightarrow$ res < n(k)-ε + 1/3*res + 1/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ 3*res < 3*n(k) – 3*ε + res + n(i) – n(j)
$\Leftrightarrow$ 2*res < n(i) – n(j) + 3*n(k) – 3*ε
$\Leftrightarrow$ res < 1/2*n(i) – 1/2*n(j) + 3/2*n(k) – 3/2*ε

res < 1/2*n(i) – 1/2*n(j) + 3/2*n(k) – 3/2*ε impossible under the condition for MV(j,1) = n(k)-ε: res ≥ n(i) – n(j) + 3*n(k)

Therefore, the case of MV(i,1) + MV(j,1) < res cannot occur.

2.2.3.1.1) Distribution proposals

2.2.3.1.1) If res ≤ n(i) + 2*n(j), then DP(i,1) = (res-n(j), n(j), 0)
2.2.3.1.2) If n(i) + 2*n(j) < res < n(i) – n(j) + 3*n(k), then DP(i,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.3.1.3) If res ≥ n(i) – n(j) + 3*n(k), then DP(i,1) = (n(k)-ε, res-n(k)-ε, 0)

2.2.3.1.2) p(1) = j

r_c(1|p(1)=j) = {i}

MV(i,1) = E(i,1,2), if n(i) < E(i,1,2) < n(k)

n(i) < E(i,1,2) < n(k)

$\Leftrightarrow$ n(j) < 1/3*res + 1/3*n(i) – 1/3*n(j) < n(k)
$\Leftrightarrow$ 3*n(j) < res + n(i) – n(j) < 3*n(k)
$\Leftrightarrow$ -n(i) + 4*n(j) < res < -n(i) + n(j) + 3*n(k)

MV(i,1) = n(k)-ε, if n(i) < n(k) ≤ E(i,1,2)

n(i) < n(k) ≤ E(i,1,2)
$\Leftrightarrow$ n(j) < n(k) und n(k) ≤ 1/3*res + 1/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ n(k) ≤ 1/3*res + 1/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ 3*n(k) ≤ res + n(i) – n(j)
$\Leftrightarrow$ res ≥ -n(i) + n(j) + 3*n(k)

MV(i,1) = n(i), else (i.e if res ≤ -n(i) + 4*n(j))

MV(j,1) = E(j,1,2), if n(j) < E(j,1,2)

n(i) < E(j,1,2)
$\Leftrightarrow$ n(i) < 1/3*res – 1/3*n(i) + 1/3*n(j)
$\Leftrightarrow$ 3*n(i) < res – n(i) + n(j)
$\Leftrightarrow$ res > 4*n(i) – n(j)

MV(j,1) = n(j), else (i.e. if res ≤ 4*n(i) – n(j))

Can the case that MV(i,1) + MV(j,1) > res occur?

MV(i,1) = n(i) and MV(j,1) = n(j):

res < n(i) + n(j)

res < n(i) + n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(i,1) = n(i) and MV(j,1) = E(j,1,2):

res < n(i) + E(j,1,2)
$\Leftrightarrow$ res < n(i) + 1/3*res – 1/3*n(i) + 1/3*n(j)
$\Leftrightarrow$ 3*res < 3*n(i) + res – n(i) + n(j)
$\Leftrightarrow$ 2*res < 2*n(i) + n(j)
$\Leftrightarrow$ res < n(i) + 1/2*n(j)

res < n(i) + 1/2*n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(i,1) = E(i,1,2) and MV(j,1) = n(j):

res < E(i,1,2) + n(j)
$\Leftrightarrow$ res < 1/3*res + 1/3*n(i) – 1/3*n(j) + n(j)
$\Leftrightarrow$ 3*res < res + n(i) – n(j) + 3*n(j)
$\Leftrightarrow$ 2*res < n(i) + 2*n(j)
$\Leftrightarrow$ res < 1/2*n(i) + n(j)

res < 1/2*n(i) + n(j) impossible under the condition for 2.2.3.1): n(i) + n(j) ≤ res

MV(i,1) = E(i,1,2) and MV(j,1) = E(j,1,2):

res < E(i,1,2) + E(j,1,2)
$\Leftrightarrow$ res < 1/3*res + 1/3*n(i) – 1/3*n(j) + 1/3*res – 1/3*n(i) + 1/3*n(j)
$\Leftrightarrow$ res < 0

res < 0 impossible according to base assumption.

MV(i,1) = n(k)-ε and MV(j,1) = n(j):

$\text{res} < n(k)-\varepsilon + n(j)$

res < n(j) + n(k) – ε impossible under the condition for MV(i,1) = n(k)-ε: res ≥ -n(i) + n(j) + 3*n(k)

MV(i,1) = n(k)-ε and MV(j,1) = E(j,1,2):

$$
\begin{aligned}
& \text{res} < n(k)-\varepsilon + E(j,1,2) \\
\Leftrightarrow\quad & \text{res} < n(k)-\varepsilon + 1/3*\text{res} - 1/3*n(i) + 1/3*n(j) \\
\Leftrightarrow\quad & 3*\text{res} < 3*n(k) - 3*\varepsilon + \text{res} - n(i) + n(j) \\
\Leftrightarrow\quad & 2*\text{res} < -n(i) + n(j) + 3*n(k) - 3*\varepsilon \\
\Leftrightarrow\quad & \text{res} < -1/2*n(i) + 1/2*n(j) + 3/2*n(k) - 3/2*\varepsilon
\end{aligned}
$$

res < -1/2*n(i) + 1/2*n(j) + 3/2*n(k) – 3/2*ε impossible under the condition for MAV(i,1) = n(k)-ε: res ≥ -n(i) + n(j) + 3*n(k)

Therefore, the case MV(i,1) + MV(j,1) < res cannot occur.

2.2.3.1.2) Distribution proposals

2.2.3.1.2.1) If res ≤ -n(i) + 4*n(j), then DP(j,1) = (n(j), res-n(j), 0)
2.2.3.1.2.2) If -n(i) + 4*n(j) < res < -n(i) + n(j) + 3*n(k) then DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.3.1.2.3) If res ≥ -n(i) + n(j) + 3*n(k), then DP(j,1) = (n(k)-ε, res-n(k)-ε, 0)

2.2.3.1.3) p(1) = k

k has no possibility to be part of a coalition that can reach a majority under the condition of 2.2.3.1): n(i) + n(k) > res (and therefore also n(j) + n(k) > res).

2.2.3.1.3) Distribution proposals

DP(k,1) = (0, 0, 0)

2.2.3.1.3.1) p(2) = i: DP(i,2) = (res-n(j), n(j), 0)
2.2.3.1.3.2) p(2) = j: DP(j,2) = (n(i), res-n(i), 0)
2.2.3.1.3.3) p(2) = k: DP(k,2) = (0, 0, 0)

2.2.3.2) All players have different needs: n(i) < n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) ≤ res
and n(j) + n(k) > res.

E(i,1,2) = (res-n(j) + n(i) + n(i))/3 = 1/3*res + 2/3*n(i) – 1/3*n(j)
E(j,1,2) = (n(j) + res-n(i) + 0)/3 = 1/3*res – 1/3*n(i) + 1/3*n(j)
E(k,1,2) = (0 + 0 + res-n(i))/3 = 1/3*res – 1/3*n(i)

2.2.3.2.1) p(1) = i

r_c(1|p(1)=i) = {j}

MV(j,1) = E(j,1,2), if n(j) < E(j,1,2) < n(k)

n(j) < E(j,1,2) < n(k)

$\Leftrightarrow \quad n(j) < 1/3*res - 1/3*n(i) + 1/3*n(j) < n(k)$
$\Leftrightarrow \quad 3*n(j) < res - n(i) + n(j) < 3*n(k)$
$\Leftrightarrow \quad n(i) + 2*n(j) < res < n(i) - n(j) + 3*n(k)$

$MV(j,1) = n(k)-\varepsilon$, if $n(j) < n(k) \leq E(j,1,2)$

$n(j) < n(k) \leq E(j,1,2)$
$\Leftrightarrow \quad n(j) < n(k) \leq 1/3*res - 1/3*n(i) + 1/3*n(j)$
$\Leftrightarrow \quad n(k) \leq 1/3*res - 1/3*n(i) + 1/3*n(j)$
$\Leftrightarrow \quad 3*n(k) \leq res - n(i) + n(j)$
$\Leftrightarrow \quad res \geq n(i) - n(j) + 3*n(k)$

$MV(j,1) = n(j)$, else (i.e.. $res \leq n(i) + 2*n(j)$)

$MV(i,1) = E(i,1,2)$, if $n(i) < E(i,1,2)$

$n(i) < E(i,1,2)$
$\Leftrightarrow \quad n(i) < 1/3*res + 2/3*n(i) - 1/3*n(j)$
$\Leftrightarrow \quad 3*n(i) < res + 2*n(i) - n(j)$
$\Leftrightarrow \quad res > n(i) + n(j)$

$MV(i,1) = n(i)$, else (i.e if $res = n(i) + n(j)$, see 2.2.3.2): $n(i) + n(j) \leq res$)

Can the case occur that $MV(i,1) + MV(j,1) > res$?

$MV(j,1) = n(j)$ and $MV(i,1) = n(i)$:

$res < n(j) + n(i)$

$res < n(i) + n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(j,1) = n(j)$ and $MV(i,1) = E(i,1,2)$:

$res < n(j) + E(i,1,2)$
$\Leftrightarrow \quad res < n(j) + 1/3*res + 2/3*n(i) - 1/3*n(j)$
$\Leftrightarrow \quad 3*res < 3*n(j) + res + 2*n(i) - n(j)$
$\Leftrightarrow \quad 2*res < 2*n(i) + 2*n(j)$
$\Leftrightarrow \quad res < n(i) + n(j)$

$res < n(i) + n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = n(i)$:

$res < E(j,1,2) + n(i)$
$\Leftrightarrow \quad res < 1/3*res - 1/3*n(i) + 1/3*n(j) + n(i)$
$\Leftrightarrow \quad 3*res < res - n(i) + n(j) + 3*n(i)$
$\Leftrightarrow \quad 2*res < 2*n(i) + n(j)$
$\Leftrightarrow \quad res < n(i) + 1/2*n(j)$

$res < n(i) + 1/2*n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(j,1) = E(j,1,2)$ and $MV(i,1) = E(i,1,2)$:

$res < E(j,1,2) + E(i,1,2)$
$\Leftrightarrow \quad res < 1/3*res - 1/3*n(i) + 1/3*n(j) + 1/3*res + 2/3*n(i) - 1/3*n(j)$
$\Leftrightarrow \quad 3*res < res - n(i) + n(j) + res + 2*n(i) - n(j)$
$\Leftrightarrow \quad res < n(i)$

res < n(i) impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(j,1) = n(k)-\varepsilon$ and $MV(i,1) = n(i)$:

$$res < n(k)-\varepsilon + n(i)$$

$res < n(k)-\varepsilon + n(i)$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq n(i) - n(j) + 3*n(k)$

$MV(j,1) = n(k)-\varepsilon$ and $MV(i,1) = E(i,1,2)$:

$$
\begin{aligned}
& res < n(k)-\varepsilon + E(i,1,2) \\
\Leftrightarrow\quad & res < n(k)-\varepsilon + 1/3*res + 2/3*n(i) - 1/3*n(j) \\
\Leftrightarrow\quad & 3*res < 3*n(k) - 3*\varepsilon + res + 2*n(i) - n(j) \\
\Leftrightarrow\quad & 2*res < 2*n(i) - n(j) + 3*n(k) - 3*\varepsilon \\
\Leftrightarrow\quad & res < n(i) - 1/2*n(j) + 3/2*n(k) - 3*\varepsilon
\end{aligned}
$$

$res < n(i) - 1/2*n(j) + 3/2*n(k) - 3*\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq n(i) - n(j) + 3*n(k)$

Therefore, the case $MV(i,1) + MV(j,1) < res$ cannot occur.

2.2.3.2.1) Distribution proposals

2.2.3.2.1.1) If $res \leq n(i) + 2*n(j)$, then $DP(i,1) = (res-n(j), n(j), 0)$
2.2.3.2.1.2) If $n(i) + 2*n(j) < res < n(i) - n(j) + 3*n(k)$,
then $DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)$
2.2.3.2.1.3) If $res \geq n(i) - n(j) + 3*n(k)$, then $DP(i,1) = (res-n(k)+\varepsilon, n(k)-\varepsilon, 0)$

2.2.3.2.2) p(1) = j

$r\_c(1|p(1)=j) = \{i\}$

$MV(i,1) = E(i,1,2)$, if $n(i) < E(i,1,2) < n(k)$

$$
\begin{aligned}
& n(i) < E(i,1,2) < n(k) \\
\Leftrightarrow\quad & n(i) < 1/3*res + 2/3*n(i) - 1/3*n(j) < n(k) \\
\Leftrightarrow\quad & 3*n(i) < res + 2*n(i) - n(j) < 3*n(k) \\
\Leftrightarrow\quad & n(i) + n(j) < res < -2*n(i) + n(j) + 3*n(k) \\
\Leftrightarrow\quad & res < -2*n(i) + n(j) + 3*n(k)
\end{aligned}
$$

(since under the conditions for 2.2.3.2): $n(i) < n(j) < n(k)$ and $n(i) + n(k) \leq res$ holds: $n(i) + n(j) < res$)

$MV(i,1) = n(k)-\varepsilon$, if $n(i) < n(k) \leq E(i,1,2)$

$$
\begin{aligned}
& n(i) < n(k) \leq E(i,1,2) \\
\Leftrightarrow\quad & n(j) < n(k) \text{ and } n(k) \leq 1/3*res + 2/3*n(i) - 1/3*n(j) \\
\Leftrightarrow\quad & n(k) \leq 1/3*res + 2/3*n(i) - 1/3*n(j) \\
\Leftrightarrow\quad & 3*n(k) \leq res + 2*n(i) - n(j) \\
\Leftrightarrow\quad & res \geq -2*n(i) + n(j) + 3*n(k)
\end{aligned}
$$

$MV(i,1) = n(i)$, impossible (see $MV(i,1) = E(i,1,2)$)

$MV(j,1) = E(j,1,2)$, if $n(j) < E(j,1,2)$

$$
\begin{aligned}
& n(i) < E(j,1,2) \\
\Leftrightarrow\quad & n(i) < 1/3*res - 1/3*n(i) + 1/3*n(j) \\
\Leftrightarrow\quad & 3*n(i) < res - n(i) + n(j)
\end{aligned}
$$

$\Leftrightarrow \quad res > 4*n(i) – n(j)$

$MV(j,1) = n(j)$, else (i.e. if $res \leq 4*n(i) – n(j)$)

Can the case occur that $MV(i,1) + MV(j,1) > res$?

$MV(i,1) = E(i,1,2)$ and $MV(j,1) = n(j)$:

$$
\begin{aligned}
& res < E(i,1,2) + n(j) \\
\Leftrightarrow \quad & res < 1/3*res + 2/3*n(i) – 1/3*n(j) + n(j) \\
\Leftrightarrow \quad & 3*res < res + 2*n(i) – n(j) + 3*n(j) \\
\Leftrightarrow \quad & 2*res < 2*n(i) + 2*n(j) \\
\Leftrightarrow \quad & res < n(i) + n(j)
\end{aligned}
$$

$res < n(i) + n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(i,1) = E(i,1,2)$ and $MV(j,1) = E(j,1,2)$:

$$
\begin{aligned}
& res < E(i,1,2) + E(j,1,2) \\
\Leftrightarrow \quad & res < 1/3*res + 2/3*n(i) – 1/3*n(j) + 1/3*res – 1/3*n(i) + 1/3*n(j) \\
\Leftrightarrow \quad & 3*res < res + 2*n(i) – n(j) + res – n(i) + n(j) \\
\Leftrightarrow \quad & res < n(i)
\end{aligned}
$$

$res < n(i)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(i,1) = n(k)-\varepsilon$ and $MV(j,1) = n(j)$:

$$res < n(k)-\varepsilon + n(j)$$

$res < n(j) + n(k)-\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq -2*n(i) + n(j) + 3*n(k)$

$MV(i,1) = n(k)-\varepsilon$ and $MV(j,1) = E(j,1,2)$:

$$
\begin{aligned}
& res < n(k)-\varepsilon + E(j,1,2) \\
\Leftrightarrow \quad & res < n(k)-\varepsilon + 1/3*res – 1/3*n(i) + 1/3*n(j) \\
\Leftrightarrow \quad & 3*res < 3*n(k) – 3*\varepsilon + res – n(i) + n(j) \\
\Leftrightarrow \quad & 2*res < -n(i) + n(j) + 3*n(k) – 3*\varepsilon \\
\Leftrightarrow \quad & res < -1/2*n(i) + 1/2*n(j) + 3/2*n(k) – 3/2*\varepsilon
\end{aligned}
$$

$res < -1/2*n(i) + 1/2*n(j) + 3/2*n(k) – 3/2*\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq -2*n(i) + n(j) + 3*n(k)$

Therefore, the case of $MV(i,1) + MV(j,1) < res$ cannot occur.

2.2.3.2.2) Distribution proposals

2.2.3.2.2.1) If $res < -2*n(i) + n(j) + 3*n(k)$, then $DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)$
2.2.3.2.2.2) If $res \geq -2*n(i) + n(j) + 3*n(k)$, then $DP(j,1) = (n(k)-\varepsilon, res-n(k)+\varepsilon, 0)$

2.2.3.2.3) p(1) = k

$r\_c(1|p(1)=k) = \{i\}$

$MV(i,1) = E(i,1,2)$, if $n(i) < E(i,1,2) < n(j)$

$$
\begin{aligned}
& n(i) < E(i,1,2) < n(j) \\
\Leftrightarrow \quad & n(i) < 1/3*res + 2/3*n(i) – 1/3*n(j) < n(j) \\
\Leftrightarrow \quad & 3*n(i) < res + 2*n(i) – n(j) < 3*n(j)
\end{aligned}
$$

$\Leftrightarrow$ n(i) + n(j) < res < -2*n(i) + 4*n(j)
$\Leftrightarrow$ res < -2*n(i) + 4*n(j)

(since under the conditions for 2.2.3.2): n(i) < n(j) < n(k) and n(i) + n(k) $\leq$ res holds n(i) + n(j) < res)

MV(i,1) = n(j)-ε, if n(i) < n(j) $\leq$ E(i,1,2)

n(i) < n(j) $\leq$ E(i,1,2)
$\Leftrightarrow$ n(j) $\leq$ 1/3*res + 2/3*n(i) – 1/3*n(j)
$\Leftrightarrow$ 3*n(j) $\leq$ res + 2*n(i) – n(j)
$\Leftrightarrow$ res $\geq$ -2*n(i) + 4*n(j)

MV(i,1) = n(i), impossible (see MV(i,1) = E(i,1,2))

MV(k,1) = E(k,1,2), if n(k) < E(k,1,2)

n(k) < E(k,1,2)
$\Leftrightarrow$ n(k) < 1/3*res – 1/3*n(i)
$\Leftrightarrow$ 3*n(k) < res – n(i)
$\Leftrightarrow$ res > n(i) + 3*n(k)

MV(k,1) = n(k), else (i.e. if res $\leq$ n(i) + 3*n(k))

Can the case occur that MV(i,1) + MV(k,1) > res?

MV(i,1) = E(i,1,2) und MV(k,1) = n(k):

res < E(i,1,2) + n(k)
$\Leftrightarrow$ res < 1/3*res + 2/3*n(i) – 1/3*n(j) + n(k)
$\Leftrightarrow$ 3*res < res + 2*n(i) – n(j) + 3*n(k)
$\Leftrightarrow$ 2*res < 2*n(i) – n(j) + 3*n(k)
$\Leftrightarrow$ res < n(i) – 1/2*n(j) + 3/2*n(k)

res < n(i) – 1/2*n(j) + 3/2*n(k) possible

Proof: Let res = 53, n(i) = 9, n(j) = 30, and n(k) = 40. Then the case-defining conditions are satisfied:

n(i) = 9 < n(j) = 30 < n(k) = 40
n(i) + n(j) = 9 + 30 = 39 $\leq$ res = 53
n(i) + n(k) = 9 + 40 $\leq$ res = 53
n(j) + n(k) = 30 + 40 > res = 53
MV(i,1) = E(i,1,2) $\Leftrightarrow$ n(i) + n(j) = 9 + 30 = 39 < res = 53 < -2*n(i) + 4*n(j) = -2*9 + 4*30 = 102
MV(k,1) = n(k) $\Leftrightarrow$ res = 53 $\leq$ n(i) + 3*n(k) = 9 + 3*40 = 111

At the same time res = 53 < n(i) – 1/2*n(j) + 3/2*n(k) = 9 – 1/2*30 + 3/2*40 = 66 bzw. MV(i,1) + MV(k,1) = E(i,1,2) + n(k) = 1/3*res + 2/3*n(i) – 1/3*n(j) + n(k) = 1/3*53 + 2/3*9 – 1/3*30 + 40 = 1/3*53 + 36 = 53,66 > res = 53. QED

There is no reason for i to agree if he is not offered at least his expectation value and there is no reason for k to make an offer that does not leave him with at least his need. Therefore, the following case distinction has to be made:

If MV(i,1) = E(i,1,2) and res < n(i) – 1/2*n(j) + 3/2*n(k),
then DP(k,1) = (0, 0, 0) and

if $MV(i,1) = E(i,1,2)$ and $res \geq n(i) - 1/2*n(j) + 3/2*n(k)$,
then $DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))$.

$MV(i,1) = E(i,1,2)$ and $MV(k,1) = E(k,1,2)$:

$$\begin{aligned} & res < E(i,1,2) + E(k,1,2) \\ \Leftrightarrow\quad & res < 1/3*res + 2/3*n(i) - 1/3*n(j) + 1/3*res - 1/3*n(i) \\ \Leftrightarrow\quad & 3*res < res + 2*n(i) - n(j) + res - n(i) \\ \Leftrightarrow\quad & res < n(i) - n(j) \end{aligned}$$

$res < n(i) - n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \leq res$

$MV(i,1) = n(j)-\varepsilon$ and $MV(k,1) = n(k)$:

$$res < n(j)-\varepsilon + n(k)$$

$res < n(j) + n(k) - \varepsilon$ possible

Proof: Let $res = 11$, $n(i) = 1$, $n(j) = 3$ and $n(k) = 9$. Then the case-defining conditions are satisfied:

$$\begin{aligned} & n(i) = 1 < n(j) = 3 < n(k) = 9 \\ & n(i) + n(j) = 4 \leq res = 11 \\ & n(i) + n(k) = 10 \leq res = 11 \\ & n(j) + n(k) = 12 > res = 11 \\ & MV(i,1) = n(j)-\varepsilon \Leftrightarrow -2*n(i) + 4*n(j) = -2*1 + 4*3 = 10 \leq res = 11 \\ & MV(k,1) = n(k) \Leftrightarrow res = 11 \leq n(i) + 3*n(k) = 1 + 3*9 = 28 \end{aligned}$$

At the same time $res = 11 < n(j) + n(k) - \varepsilon = 3 + 9 - \varepsilon = 12 - \varepsilon$ or $MV(i,1) + MV(k,1) = n(j)-\varepsilon + n(k) = 3-\varepsilon + 9 = 12 - \varepsilon > res = 11$ for appropriately chosen ε. QED

However, under the case-defining condition $n(j) + n(k) > res$, it is impossible for j and k to form a coalition that could achieve a majority. k cannot make j an offer that j could accept. Therefore, it is not necessary for i to underbid j (this is basically done only for the purpose of preventing the proposer from forming a coalition with the competing respondent and leaving the respondent empty-handed, even though he could accept smaller offers). Therefore, k makes no offer under the conditions for $MV(i,1) = n(j)-\varepsilon$ and $MV(k,1) = n(k)$. As can be seen from the following case, the limiting condition is not the condition for $MV(i,1) = n(j)-\varepsilon$, but the condition for $MV(k,1) = n(k)$.

$MV(i,1) = n(j)-\varepsilon$ und $MV(k,1) = E(k,1,2)$:

$$\begin{aligned} & res < n(j)-\varepsilon + E(k,1,2) \\ \Leftrightarrow\quad & res < n(j)-\varepsilon + 1/3*res - 1/3*n(i) \\ \Leftrightarrow\quad & 3*res < 3*n(j) - 3*\varepsilon + res - n(i) \\ \Leftrightarrow\quad & 2*res < -n(i) + 3*n(j) - 3*\varepsilon \\ \Leftrightarrow\quad & res < -1/2*n(i) + 3/2*n(j) - 3/2*\varepsilon \end{aligned}$$

$res < -1/2*n(i) + 3/2*n(j) - 3/2*\varepsilon$ impossible under the condition for $MV(j,1) = n(k)-\varepsilon$: $res \geq -2*n(i) + 4*n(j)$

2.2.3.2.3) Distribution proposals

2.2.3.2.3.1) If $res < n(i) - 1/2*n(j) + 3/2*n(k)$, then $DP(k,1) = (0, 0, 0)$

2.2.3.2.3.1.1) p(2) = i: DP(i,2) = (res-n(j), n(j), 0)
2.2.3.2.3.1.2) p(2) = j: DP(j,2) = (n(i), res-n(i), 0)
2.2.3.2.3.1.3) p(2) = k: DP(k,2) = (n(i), 0, res-n(i))

2.2.3.2.3.2) If n(i) - 1/2*n(j) + 3/2*n(k) ≤ res < -2*n(i) + 4*n(j),
then DP(k,1) = (E(i,1,2), 0, res-E(i,1,2))

2.2.3.2.3.3.1) If res ≥ -2*n(i) + 4*n(j) and res ≤ n(i) + 3*n(k)
then DP(k,1) = (0, 0, 0)

2.2.3.2.3.3.1.1) p(2) = i: DP(i,2) = (res-n(j), n(j), 0)
2.2.3.2.3.3.1.2) p(2) = j: DP(j,2) = (n(i), res-n(i), 0)
2.2.3.2.3.3.1.3) p(2) = k: DP(k,2) = (n(i), 0, res-n(i))

2.2.3.2.3.3.2) If res ≥ -2*n(i) + 4*n(j) and res ≥ n(i) + 3*n(k)
then DP(k,1) = (n(j)-ε, 0, res-n(j)+ε

2.2.3.3) All players have different needs: n(i) < n(j) < n(k)
and n(i) + n(j) ≤ res
and n(i) + n(k) ≤ res
and n(j) + n(k) ≤ res.

Compared to 2.2.3.2, n(j) + n(k) ≤ res. Since i and j prefer each other over k, and k unchanged prefers i over j, this has no effect on the expected values and the solutions under the conditions that i or j are chosen as proposer. Differences arise for the solutions when k is chosen as proposer.

E(i,1,2) = (res-n(j) + n(i) + n(i))/3 = 1/3*res + 2/3*n(i) – 1/3*n(j)
E(j,1,2) = (n(j) + res-n(i) + 0)/3 = 1/3*res – 1/3*n(i) + 1/3*n(j)
E(k,1,2) = (0 + 0 + res-n(i))/3 = 1/3*res – 1/3*n(i)

2.2.3.3.1) p(1) = i

Analog to 2.2.3.2.1)

2.2.3.3.1) Distribution proposals

2.2.3.3.1.1) If res ≤ n(i) + 2*n(j), then DP(i,1) = (res-n(j), n(j), 0)
2.2.3.3.1.2) If n(i) + 2*n(j) < res < n(i) – n(j) + 3*n(k),
then DP(i,1) = (res-E(j,1,2), E(j,1,2), 0)
2.2.3.3.1.3) If res ≥ n(i) – n(j) + 3*n(k), then DP(i,1) = (res-n(k)+ε, n(k)-ε, 0)

2.2.3.3.2) p(1) = j

Analogous to 2.2.3.2.2)

2.2.3.3.2) Distribution proposals

2.2.3.3.2.1) If res < -2*n(i) + n(j) + 3*n(k), then DP(j,1) = (E(i,1,2), res-E(i,1,2), 0)
2.2.3.3.2.2) If res ≥ -2*n(i) + n(j) + 3*n(k), then DP(j,1) = (n(k)-ε, res-n(k)+ε, 0)

2.2.3.3.3) p(1) = k

r_c(1|p(1)=k) = {i}

MV(i,1) = E(i,1,2), if n(i) < E(i,1,2) < n(j)

$n(i) < E(i,1,2) < n(j)$
$\Leftrightarrow \quad n(i) < 1/3*res + 2/3*n(i) – 1/3*n(j) < n(j)$
$\Leftrightarrow \quad 3*n(i) < res + 2*n(i) – n(j) < 3*n(j)$
$\Leftrightarrow \quad n(i) + n(j) < res < -2*n(i) + 4*n(j)$
$\Leftrightarrow \quad res < -2*n(i) + 4*n(j)$

(since under the conditions for 2.2.3.3): $n(i) < n(j) < n(k)$ und $n(i) + n(k) \le res$ holds: $n(i) + n(j) < res$)

MV(i,1) = n(j)-ε, if $n(i) < n(j) \le E(i,1,2)$

$n(i) < n(j) \le E(i,1,2)$
$\Leftrightarrow \quad n(j) \le 1/3*res + 2/3*n(i) – 1/3*n(j)$
$\Leftrightarrow \quad 3*n(j) \le res + 2*n(i) – n(j)$
$\Leftrightarrow \quad res \ge -2*n(i) + 4*n(j)$

MV(i,1) = n(i), impossible (see MV(i,1) = E(i,1,2))

MV(k,1) = E(k,1,2), if $n(k) < E(k,1,2)$

$n(k) < E(k,1,2)$
$\Leftrightarrow \quad n(k) < 1/3*res – 1/3*n(i)$
$\Leftrightarrow \quad 3*n(k) < res – n(i)$
$\Leftrightarrow \quad res > n(i) + 3*n(k)$

MV(k,1) = n(k), else (i.e. if $res \le n(i) + 3*n(k)$)

Can the case occur that MV(i,1) + MV(k,1) > res?

MV(i,1) = E(i,1,2) and MV(k,1) = n(k):

$res < E(i,1,2) + n(k)$
$\Leftrightarrow \quad res < 1/3*res + 2/3*n(i) – 1/3*n(j) + n(k)$
$\Leftrightarrow \quad 3*res < res + 2*n(i) – n(j) + 3*n(k)$
$\Leftrightarrow \quad 2*res < 2*n(i) – n(j) + 3*n(k)$
$\Leftrightarrow \quad res < n(i) – 1/2*n(j) + 3/2*n(k)$

$res < n(i) – 1/2*n(j) + 3/2*n(k)$ impossible under the condition for 2.2.3.3): $n(j) + n(k) \le res$

MV(i,1) = E(i,1,2) and MV(k,1) = E(k,1,2):

$res < E(i,1,2) + E(k,1,2)$
$\Leftrightarrow \quad res < 1/3*res + 2/3*n(i) – 1/3*n(j) + 1/3*res – 1/3*n(i)$
$\Leftrightarrow \quad 3*res < res + 2*n(i) – n(j) + res – n(i)$
$\Leftrightarrow \quad res < n(i) – n(j)$

$res < n(i) – n(j)$ impossible under the condition for 2.2.3.2): $n(i) + n(j) \le res$

MV(i,1) = n(j)-ε and MV(k,1) = n(k):

$res < n(j)-ε + n(k)$

$res < n(j) + n(k) – ε$ impossible under the condition for 2.2.3.3): $n(j) + n(k) \le res$

MV(i,1) = n(j)-ε and MV(k,1) = E(k,1,2):

$res < n(j)-ε + E(k,1,2)$

⇔ $res < n(j) - \varepsilon + 1/3 * res - 1/3 * n(i)$
⇔ $3 * res < 3 * n(j) - 3 * \varepsilon + res - n(i)$
⇔ $2 * res < -n(i) + 3 * n(j) - 3 * \varepsilon$
⇔ $res < -1/2 * n(i) + 3/2 * n(j) - 3/2 * \varepsilon$

$res < -1/2 * n(i) + 3/2 * n(j) - 3/2 * \varepsilon$ impossible under the condition for $MV(j,1) = n(k) - \varepsilon$: $res \geq -2 * n(i) + 4 * n(j)$

Therefore, the case $MV(i,1) + MV(j,1) < res$ cannot occur.

2.2.3.3.3) Distribution proposals

2.2.3.3.3.1) If $res < -2 * n(i) + 4 * n(j)$, then $DP(k,1) = (E(i,1,2), 0, res - E(i,1,2))$
2.2.3.3.3.2) If $res \geq -2 * n(i) + 4 * n(j)$, then $DP(k,1) = (n(j) - \varepsilon, 0, res - n(j) + \varepsilon)$

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Traub, Stefan, Schwaninger, Manuel, Paetzel, Fabian and Neuhofer, Sabine: Evidence on Need-sensitive Giving Behavior: An Experimental Approach to the Acknowledgment of Needs. Working Paper Nr. 2021-01. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2021-01.pdf

Bauer, Alexander Max, Meyer, Frauke, Romann, Jan, Siebel, Mark and Traub, Stefan: Need, Equity, and Accountability: Evidence on Third-Party Distributive Decisions from an Online Experiment. Working Paper Nr. 2020-01. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2020-01.pdf

Bauer, Alexander Max: Sated but Thirsty. Towards a Multidimensional Measure of Need-Based Justice. Working Paper Nr. 2018-03. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2018-03.pdf

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Bauer, Alexander Max: Monotonie und Monotoniesensitivität als Desiderata für Maße der Bedarfs-gerechtigkeit. Working Paper Nr. 2018-01. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2018-01.pdf

Schramme, Thomas: Mill and Miller: Some thoughts on the methodology of political theory. Working Paper Nr. 2017-25. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2017-25.pdf

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Chugunova, Marina, Nicklisch, Andreas and Schnapp, Kai-Uwe: On the effects of transparency and reciprocity on labor supply in the redistribution systems. Working Paper Nr. 2017-19. http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2017-19.pdf

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Chugunova, Marina, Nicklisch, Andreas and Schnapp, Kai-Uwe Redistribution and Production with the Subsistence Income Constraint: a Real-Effort Experiment. Working Paper Nr. 2017-18.
http://bedarfsgerechtigkeit.hsu-hh.de/dropbox/wp/2017-18.pdf

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