DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 85-053
June 1985

GAUGE THEORIES, SPONTANEOUS SYMMETRY BREAKING AND ELECTROWEAK INTERACTIONS

by

R.D. Peccei

Deutsches Elektronen-Synchrotron DESY, Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX ,
send them to the following address (if possible by air mail) :

DESY
Bibliothek
Notkestrasse 85
2 Hamburg 52
Germany

DESY 85-053
June 1985

ISSN 0418-9833

GAUGE THEORIES, SPONTANEOUS SYMMETRY BREAKING AND ELECTROWEAK INTERACTIONS

R.D. Peccei
DESY, Hamburg, Federal Republic of Germany

Lectures given at the Advanced Course in Theoretical Physics (Quarks and Leptons), Univ. of Stellenbosch, Rep. of South Africa, Jan. 21 to Feb. 1, 1985. To be published by Springer Verlag in the Lecture Notes in Physics Series.

Prologue

The aim of these lectures is to present the physics of the spontaneously broken gauge theories of the weak and electromagnetic interactions at a level suitable for non specialists. Thus, in general, I shall try to emphasize more the conceptual rather than the technical side of the subject - although, inevitably, some technical material will have to be included. For those readers interested in a more detailed exposition of electroweak interactions, there exist both recent conference reviews /1/ as well as summer school lectures /2/ /3/, which cover this topic in more depth.

The first part of these lectures is devoted to developing the concepts necessary for understanding how the electroweak gauge theories are built. I discuss, in particular, how global symmetries are realized in nature and how one can make a globally symmetric theory locally symmetric, by introducing gauge fields. The consequence of spontaneous symmetry breakdown for the spectrum of excitations and its role in mass generation are also emphasized here. All these ideas are illustrated in the context of simple models.

Having developed all the necessary tools, in the second part of these lectures I construct the SU(2) x U(1) model for electroweak interactions of Glashow, Salam and Weinberg /4/. After describing the structure of interactions of the model, I discuss some aspects of the phenomenology of neutral current experiments. Both purely leptonic as well as deep inelastic experiments are considered. A brief discussion of parity violation effects in atoms is also included. As a final topic in this section, some properties of the W and Z bosons, discovered recently at the CERN collider, are examined.

The last part of these lectures is devoted to the open problems of the Glashow Salam Weinberg theory. These problems are centered in the symmetry breaking sector of the model, in which the symmetry breakdown is triggered by the vacuum expectation value of an elementary scalar field. Some of the theoretical ideas proposed to replace this elementary Higgs mechanism by something more dynamical are discussed, along with the difficulties that they encounter. Both the Technicolor scheme of dynamical symmetry breakdown, as well as the idea that quarks and leptons themselves may have some structure, are briefly touched upon.

I. Symmetries in Field Theory: their Realization and their Dynamics

The natural language for elementary particle physics is that of quantum field theory. To each fundamental excitation one assigns a corresponding quantum field. Symmetries of nature are incorporated by constructing Lagrangian densities, made up of these quantum fields, which are explicitly invariant under the given symmetry in question *.

I shall consider specifically only continuous symmetries, which reflect particular transformation properties of the quantum fields of the theory under some group of transformations. Let me denote a generic quantum field by $\chi_\alpha(x)$, where x is its space-time location and α is an (internal) index which runs over the possible components of χ. If a is one of the operations of the symmetry group G of transformations, and if the quantum fields χ_α are members of an (irreducible) multiplet, then under this operation one has:

$$\chi_\alpha(x) \underset{a}{\longrightarrow} \chi'_\alpha(x) = \mathcal{R}_{\alpha\beta}(a)\,\chi_\beta(x) \tag{I.1}$$

That is, under the group transformation the new components of χ are linear combinations of the old components.

The matrices $\mathcal{R}(a)$, characterized by a, constitute a representation matrix of the group G. That is, if one performs the sequence of transformations

$$\chi_\alpha(x) \underset{a}{\longrightarrow} \chi'_\alpha(x) \underset{a'}{\longrightarrow} \chi''_\alpha(x) \tag{I.2}$$

which is equivalent to

$$\chi_\alpha(x) \underset{a''}{\longrightarrow} \chi''_\alpha(x) \tag{I.3}$$

then one has

$$\mathcal{R}_{\alpha\beta}(a')\,\mathcal{R}_{\beta\gamma}(a) = \mathcal{R}_{\alpha\gamma}(a'') \tag{I.4}$$

In the Hilbert space of the quantum operators $\chi_\alpha(x)$ the transformation (I.1) is induced by a Unitary operator $U(a)$. One has

$$U^{-1}(a)\,\chi_\alpha(x)\,U(a) = \chi'_\alpha(x) = \mathcal{R}_{\alpha\beta}(a)\,\chi_\beta(x) \tag{I.5}$$

The composition property (I.4) has its counterpart in terms of the unitary operators U. One easily sees that

* More precisely, it is only necessary that the action be invariant.

$$U(a)\,U(a') = U(a'') \tag{I.6}$$

Since continuous symmetry transformations are being considered, it suffices to study infinitesimal transformations. Finite transformations can always be built up from these infinitesimal transformations by repeated (infinite) compounding. A given group G is characterized by the number of parameters associated with these infinitesimal transformations and by the algebra obeyed by its generators. The generators G_i just describe the infinitesimal form of the unitary operators $U(\delta a)$. That is, one writes for infinitesimal transformations

$$U(\delta a) = 1 + i\,\delta a_i\,G_i \tag{I.7}$$

The parameters δa_i can be taken to be real, without loss of generality. Thus the generators G_i are Hermitian operators. The composition property (I.6) implies a group algebra (Lie algebra) for the generators:

$$[G_i, G_j] = i\,c_{ijk}\,G_k \tag{I.8}$$

The coefficients c_{ijk}, which can be chosen to be totally antisymmetric in i, j and k, are called the structure constants of the group.

For infinitesimal transformations, the representation matrices $\mathcal{R}(\delta a)$ will also be close to the identity. One may write

$$\mathcal{R}_{\alpha\beta}(\delta a) = \delta_{\alpha\beta} + i\,\delta a_i\,(g_i)_{\alpha\beta} \tag{I.9}$$

It is not hard to show that the matrices g_i furnish a representation for the generators G_i and thus obey the same algebra as (I.8). This can be demonstrated as follows. For an infinitesimal transformation, using (I.5) one has:

$$[1 - i\,\delta a_i G_i]\,\chi_\alpha(x)\,[1 + i\,\delta a_i G_i] = \chi_\alpha(x) + i\,\delta a_i\,(g_i)_{\alpha\beta}\,\chi_\beta(x)$$

whence it follows that

$$[G_i, \chi_\alpha(x)] = -(g_i)_{\alpha\beta}\,\chi_\beta(x) \tag{I.10}$$

This equation embodies succinctly how the quantum fields χ_α transform under the group, and will be of much use later. Using (I.10) in the Jacobi identity

$$[G_i,[G_j,\chi_\alpha]] + [\chi_\alpha,[G_i,G_j]] + [G_j,[\chi_\alpha,G_i]] = 0$$

one readily checks that it is necessary that the matrices g_i obey the algebra

$$[g_i,g_j] = i\, c_{ijk} g_k \tag{I.11}$$

which is the desired result.

Let me imagine a theory, built out of the quantum fields χ_α, which is invariant under the transformations of the group G. What are the implications of this statement? The invariance of the theory means that the quantum action does not change if the theory is expressed either in terms of the χ_α or χ'_α fields. Since the action is just the space time integral of the Lagrangian density $\mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha)$, the statement of invariance of the theory under G is simply that

$$W = \int d^4x\, \mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha) = \int d^4x\, \mathcal{L}(\chi'_\alpha, \partial_\mu \chi'_\alpha) \tag{I.12}$$

The stationarity of the action under G implies the existence of conserved currents J_i^μ. This is easily seen by looking at the case in which the fields χ'_α are infinitesimally different from χ_α. One has then

$$0 = \delta W = \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \chi_\alpha} \delta\chi_\alpha + \frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha} \delta \partial_\mu \chi_\alpha \right\}$$

$$= \int d^4x \left\{ \left[\frac{\partial \mathcal{L}}{\partial \chi_\alpha} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha} \right) \right] \delta\chi_\alpha + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha} \delta\chi_\alpha \right] \right\}$$

The first term above vanishes by virtue of the Euler-Lagrange equations of motion. The second term can be rewritten using

$$\delta\chi_\alpha = \chi'_\alpha - \chi_\alpha = i\, \delta a_i\, (g_i)_{\alpha\beta} \chi_\beta$$

as

$$0 = \delta W = - \int d^4x\, \delta a_i\, \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha} \frac{1}{i} (g_i)_{\alpha\beta} \chi_\beta \right]$$

Since the parameters δa_i are independent, it follows that the currents

$$J_i^\mu(x) = \frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha(x)} \frac{1}{i} (g_i)_{\alpha\beta} \chi_\beta(x) \tag{I.13}$$

are conserved

$$\partial_\mu J_i^\mu(x) = 0 \tag{I.14}$$

The generators G_i of the transformation can be identified as the space integrals of the time components of these currents

$$G_i = \int d^3x\, J_i^0(x) = \int d^3x \left[\frac{\partial \mathcal{L}}{\partial \partial_0 \chi_\alpha} \frac{1}{i} (g_i)_{\alpha\beta} \chi_\beta \right] \tag{I.15}$$

Because the currents are conserved, the generators G_i are time independent and thus they are constants of the motion. If H is the Hamiltonian of the theory, then

$$[H, G_i] = 0 \tag{I.16}$$

This last statement may be a more familiar way to express the invariance of the theory under the transformations of the group G.

It is easy to check that the G_i constructed in (I-15) are indeed the generators. The quantity $\frac{\partial \mathcal{L}}{\partial \partial_0 \chi_\alpha}$ is just the canonical momentum, conjugate to the field χ_α:

$$\pi_\alpha(x) = \frac{\partial \mathcal{L}}{\partial \partial_0 \chi_\alpha(x)} \tag{I.17}$$

Consider then the commutator of G_i with a field $\chi_\gamma(y)$

$$[G_i, \chi_\gamma(y)] = \int d^3x \left[\pi_\alpha(x) \frac{1}{i} (g_i)_{\alpha\beta} \chi_\beta(x), \chi_\gamma(y) \right]$$

Because G_i is time independent, it is possible to set $x^0 = y^0$ in the above and thus evaluate the commutator using the canonical equal time commutation relations *:

* For fermionic fields these have to be replaced by anti-commutation relations, but the result still obtains

$$[\pi_\alpha(x), \chi_\beta(y)]_{x^0=y^0} = \frac{1}{i}\,\delta^3(\vec{x}-\vec{y})\,\delta_{\alpha\beta} \tag{I.18}$$

$$[\pi_\alpha(x), \pi_\beta(y)]_{x^0=y^0} = [\chi_\alpha(x), \chi_\beta(y)]_{x^0=y^0} = 0$$

The result is just Eq. (I.10). In the same way, one can show that the G_i's obey the Lie algebra (I.8), which establishes the identification.

Up to now in my discussion of symmetries I focussed on the transformations of the quantum fields $\chi_\alpha(x)$. Eq. (I.5), for instance, informs me that under a transformation of G these fields mix in a well defined way. It is natural to suppose that the single particle states associated with the fields $\chi_\alpha(x)$ will transform in an analogous way. Let me denote these states by $|p;\alpha\rangle$, where p^μ is the 4-momentum of the state and $p^2 = -m_\alpha^2$, since these states are supposed to describe particles of a given mass. These states can be constructed, by the well known LSZ procedure /5/, by applying χ_α on the vacuum state of the theory. Corresponding to Eq. (I.5) one expects that under a group rotation one has

$$U^{-1}(a)\,|p;\alpha\rangle = R_{\alpha\beta}(a)\,|p;\beta\rangle \tag{I.19}$$

This equation can be used to deduce that all the states of the multiplet $|p;\alpha\rangle$ have the same mass. Let $|p;\alpha\rangle_{rest}$ denote the state corresponding to 4-momentum $p^\mu = (m_\alpha, \vec{0})$. Then, by definition, the action of the Hamiltonian on this state is just

$$H\,|p;\alpha\rangle_{rest} = m_\alpha\,|p;\alpha\rangle_{rest} \tag{I.20}$$

However, if the theory is invariant under G, the Hamiltonian commutes with all the generators (c.f. Eq. (I.16)) and hence also with $U^{-1}(a)$:

$$[H, U^{-1}(a)] = 0$$

Consider applying this commutator on the rest states $|p;\alpha\rangle_{rest}$:

$$0 = [H, U^{-1}(a)]\,|p;\alpha\rangle_{rest} = (H U^{-1}(a) - U^{-1}(a) H)\,|p;\alpha\rangle_{rest}$$
$$= R_{\alpha\beta}(a)\,(m_\beta - m_\alpha)\,|p;\beta\rangle_{rest}$$

Since $R_{\alpha\beta}(a)$ is arbitrary it follows that $m_\beta = m_\alpha$

I have shown that if G is a symmetry of the theory, and if Eq. (I.19) applies, then particles associated with fields that transform irreducibly under G have the same mass. This is known as a Wigner-Weyl realization of the symmetry. There are many examples in nature of symmetries which are realized in a Wigner Weyl way. Perhaps one of the best known is the (approximate) SU(2) symmetry of strong interactions, which leads to the (near) equality in the masses of the charged and neutral pions and of the neutron and proton.

There is, however, another way in which symmetries can be realized in nature, called the Nambu-Goldstone realization. In this case G is still an invariance of the theory, so that Eq. (I.16) applies. However, Eq. (I.19) ceases to be valid because the ground state - vacuum state - is not invariant under G. Eq. (I.19) follows directly from the transformation property (I.5), provided that the vacuum state is G invariant. That is

$$U(a)\,|0\rangle = |0\rangle \quad \text{or} \quad G_i\,|0\rangle = 0 \tag{I.21}$$

The one particle state $|p;\alpha\rangle$ is given by the LSZ formula *

$$|p;\alpha\rangle = \lim_{x^0\to\pm\infty} \int d^3x\, e^{ipx}\,\frac{1}{i}\overleftrightarrow{\partial_0}\,\chi_\alpha(x)\,|0\rangle \tag{I.22}$$

If (I.21) holds, then the application of $U(a)^{-1}$ on $|p;\alpha\rangle$ is easily seen to give (I.19):

$$U^{-1}(a)\,|p;\alpha\rangle = \lim_{x^0\to\pm\infty} \int d^3x\, e^{ipx}\,\frac{1}{i}\overleftrightarrow{\partial_0}\,U^{-1}(a)\,\chi_\alpha(x)\,|0\rangle$$

* This is written for the case of a Bose field. Similar arguments hold for a Fermion field

$$= \lim_{x^0 \to \pm\infty} \int d^3x \; e^{ipx} \; \frac{1}{i} \overleftrightarrow{\partial_0} \; U^{-1}(a) \, \chi_\alpha(x) \, U(a) \, |0\rangle$$

$$= \lim_{x^0 \to \pm\infty} \int d^3x \; e^{ipx} \; \frac{1}{i} \overleftrightarrow{\partial_0} \; R_{\alpha\beta}(a) \, \chi_\beta(x) \, |0\rangle$$

$$= R_{\alpha\beta}(a) \, |p;\beta\rangle$$

However, if the vacuum is not left invariant by the transformations of G, the second line above no longer follows.

If

$$U(a)|0\rangle \neq |0\rangle \tag{I.23}$$

the vacuum state is not unique. It is degenerate. Under these circumstances it no longer follows that the symmetry implies multiplets of particles of the same mass. Rather, what happens is that there appear in the theory massless excitations, the so called <u>Goldstone bosons</u>. To see how this ensues, let me consider again the fields χ_α and take the vacuum expectation value of Eq. (I.10):

$$\langle 0 | [G_i, \chi_\alpha(x)] | 0\rangle = -(g_i)_{\alpha\beta} \langle 0 | \chi_\beta(x) | 0\rangle \tag{I.24}$$

If one is in the Wigner-Weyl case, Eq. (I.21) applies and it follows that the vacuum expectation value of the fields χ_β vanishes

$$\langle 0 | \chi_\beta(x) | 0\rangle = 0 \qquad \text{Wigner-Weyl} \tag{I.25}$$

If χ_β carries spin or parity, this is an expected result which follows independently of the internal symmetry group, just by demanding that the vacuum state be a Lorentz scalar and not violate parity. For a scalar field, however, the Lorentz properties do not force Eq. (I.25) to hold.

Imagine, therefore, that one is dealing with scalar fields. If the symmetry is realized in a Nambu-Goldstone way, the generators G_i no longer annihilate the vacuum and there is at least one field for which

$$\langle 0 | \chi_\beta(x) | 0\rangle \neq 0 \qquad \text{Nambu-Goldstone} \tag{I.26}$$

Recalling the definition of the generators, Eq. (I.15), one may rewrite Eq. (I.24) as

$$-(g_i)_{\alpha\beta} \langle 0 | \chi_\beta(x) | 0\rangle = \int d^3y \; \langle 0 | J_i^0(y) \chi_\alpha(x) - \chi_\alpha(x) J_i^0(y) | 0\rangle$$

It proves convenient to insert a complete set of states $|n\rangle$ in the LHS of this equation and use translational invariance:

$$J_i^0(y) = e^{-iPy} \; J_i^0(0) \; e^{iPy} \tag{I.27}$$

where P^μ is the generator of space time translations. It follows that

$$\begin{aligned}
\text{LHS} &= \sum_n \int d^3y \left\{ \langle 0 | J_i^0(y) | n\rangle \langle n | \chi_\alpha(x) | 0\rangle - \langle 0 | \chi_\alpha(x) | n\rangle \langle n | J_i^0(y) | 0\rangle \right\} \\
&= \sum_n \int d^3y \left\{ e^{iP_n y} \langle 0 | J_i^0(0) | n\rangle \langle n | \chi_\alpha(x) | 0\rangle - e^{-iP_n y} \langle 0 | \chi_\alpha(x) | n\rangle \langle n | J_i^0(0) | 0\rangle \right\} \\
&= \sum_n (2\pi)^3 \delta^3(\vec{p}_n) \left\{ e^{-iP_n^0 y^0} \langle 0 | J_i^0(0) | n\rangle \langle n | \chi_\alpha(x) | 0\rangle \right. \\
&\qquad \left. - \, e^{+iP_n^0 y^0} \langle 0 | \chi_\alpha(x) | n\rangle \langle n | J_i^0(0) | 0\rangle \right\}
\end{aligned} \tag{I.28}$$

By assumption this expression <u>does not</u> vanish. Furthermore since the RHS does not depend on y^0 the LHS must also be independent of y^0. This can only happen if in the theory there exist one particle states $|n\rangle$ which have zero mass, and only these states contribute in the sum. These zero mass states are the Goldstone bosons /6/.

It is not difficult to convince oneself that for each generator G_i that does not annihilate the vacuum there exists a Goldstone boson. After all the action of the "broken" generators G_i on the vacuum must give some state and these are the Goldstone bosons. Let me write the Goldstone boson states as $|p;j\rangle$, where $p^2 = 0$. Then it follows that the matrix element of the currents associated with the broken generators between the vacuum and these states are non vanishing:

$$\langle 0 | J_i^\mu(0) | p;j\rangle = i f_j \, \delta_{ij} \, p^\mu \tag{I.29}$$

where the f_j are some non vanishing constants. Using the fact that for a one particle state

$$\sum_m \equiv \int \frac{d^3 p_m}{(2\pi)^3\, 2p_m^0} \qquad \text{(I.30)}$$

and Eq. (I.29), the constants f_i are identified essentially in terms of the non vanishing expectation value of χ :

$$i(g_i)_{\alpha\beta}\langle 0|\chi_\beta(0)|0\rangle = \frac{1}{i}\left\{ f_i\langle p_i|\chi_\alpha(0)|0\rangle + f_i^*\langle 0|\chi_\alpha(0)|p_i\rangle\right\} \qquad \text{(I.31)}$$

where $p^\mu \rightarrow 0$ is understood.

The Nambu-Goldstone realization of a symmetry, because it is so much less familiar, ought to be exemplified in a simple context. For these purposes consider the following Lagrangian density describing the interaction of a complex scalar field ϕ, with itself

$$\mathcal{L} = -\partial_\mu\phi^\dagger(x)\,\partial^\mu\phi(x) - \lambda\left(\phi^\dagger(x)\phi(x) - f\right)^2 \qquad \text{(I.32)}$$

Clearly this theory is invariant under a U(1) phase transformation

$$\begin{aligned} \phi(x) &\rightarrow \phi'(x) = e^{i\alpha}\,\phi(x) \\ \phi^\dagger(x) &\rightarrow \phi^{\dagger\prime}(x) = e^{-i\alpha}\,\phi^\dagger(x) \end{aligned} \qquad \text{(I.33)}$$

The conserved current J^μ associated with this symmetry can be constructed from the general formula (I-13) and is simply

$$J^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\,\frac{1}{i}(1)\,\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\,\frac{1}{i}(-1)\,\phi^\dagger = i\left[(\partial^\mu\phi^\dagger)\phi - (\partial^\mu\phi)\phi^\dagger\right] \qquad \text{(I.34)}$$

The generator

$$G = \int d^3x\, J^0(x) = i\int d^3x\left[(\partial^0\phi^\dagger(x))\phi(x) - (\partial^0\phi(x))\phi^\dagger(x)\right] \qquad \text{(I.35)}$$

has the commutation relations

$$\begin{aligned} [G, \phi(x)] &= -\phi(x) \\ [G, \phi^\dagger(x)] &= +\phi^\dagger(x) \end{aligned} \qquad \text{(I.36)}$$

In a classical sense one may think of the second term in the Lagrangian (I.34) as a potential

$$V(\phi^\dagger, \phi) = \lambda\left(\phi^\dagger\phi - f\right)^2 \qquad \text{(I.37)}$$

Clearly, for the positivity of the theory one must demand that $\lambda > 0$. The physics is considerably different depending on the sign of the parameter f. If $f < 0$ the potential has a unique minimum at $\phi = 0$ and the theory is realized in a Wigner-Weyl way, leading to a degenerate multiplet of massive states. If $f > 0$, on the other hand, the potential has an infinity of minima given by the condition that $\phi^\dagger\phi = f$. The theory is realized in a Nambu-Goldstone way and there is both a massive and a massless state in the theory. The latter state is the Goldstone boson, expected from general principles.

For $f < 0$, because the minimum of the potential is at $\phi = 0$, it is sensible to expand the potential about this value and consider the quadratic terms as mass terms.

$$V(\phi^\dagger, \phi) = \lambda f^2 - 2\lambda f\,\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

This identifies

$$m^2_\phi = m^2_{\phi^\dagger} = -2\lambda f > 0 \qquad \text{(I.38)}$$

One has a degenerate multiplet of two (charge-conjugate) particles, which are interacting via the $\lambda(\phi^\dagger\phi)^2$ term.

If $f > 0$, on the other hand, an expansion about $\phi = 0$ makes no sense. The potential has a local maximum there and is unstable. The only sensible place to expand the potential is about its minimum point which occurs at $\phi = \sqrt{f}\,e^{i\theta}$, with θ arbitrary. In fact if $f > 0$ in no way can the quadratic term in $V(\phi^\dagger, \phi)$ be interpreted as a mass squared term, since it is negative. Quantum mechanically the non zero value of ϕ at the potential minimum implies that ϕ has a non vanishing vacuum expectation value

$$\langle 0|\phi(x)|0\rangle = \sqrt{f}\,e^{i\theta} \qquad \text{(I.39)}$$

The phase θ is in fact irrelevant and can be rotated away. It is a reflection of the non uniqueness of the vacuum state of the theory. Since under a U(1) transformation

$$U^{-1}(\alpha)\ \phi(x)\, U(\alpha) = e^{i\alpha}\,\phi(x) \tag{I.40}$$

it is clear that the expectation of ϕ between the states $U(-\theta)|0\rangle$ is purely real:

$$\langle 0|\, U^{-1}(-\theta)\ \phi(x)\ U(-\theta)|0\rangle = \sqrt{f} \tag{I.41}$$

Obviously $U(-\theta)|0\rangle$ is just as good a vacuum state as $|0\rangle$.

Without loss of generality one can therefore write ϕ as

$$\phi(x) = \sqrt{f} + \chi(x) \tag{I.42}$$

and look for quadratic terms in an expansion of V about $\chi = 0$. One finds

$$V = \lambda f\,(\chi+\chi^{+})^{2} + 2\lambda\sqrt{f}\ \chi^{+}\chi\,(\chi+\chi^{+}) + \lambda\,(\chi^{+}\chi)^{2} \tag{I.43}$$

If one introduces two real fields $\chi_{\pm}$, related by a canonical transformation to χ and χ^{+}:

$$\chi_{+} = \frac{1}{\sqrt{2}}\,(\chi+\chi^{+}) \quad ; \quad \chi_{-} = \frac{i}{\sqrt{2}}\,(\chi^{+}-\chi) \tag{I.44}$$

it is clear from (I.43) that χ_{+} has a mass

$$m^{2}_{+} = 4\lambda f > 0 \tag{I.45}$$

but χ_{-} is massless. Even though the Lagrangian is U(1) symmetric, this symmetry is not reflected in the spectrum of the theory. There is, however, a Goldstone excitation.

The identification of χ_{-} as the Goldstone excitation follows also directly from the commutation relations it has with the generator G. Using (I.36) one has

$$[\,G, \chi_{-}\,] = i\,(\sqrt{2f} + \chi_{+}) \tag{I.46}$$

Whence, taking vacuum expectation values, one obtains

$$\langle 0|\,[G, \chi_{-}]\,|0\rangle = i\sqrt{2f} \tag{I.47}$$

Clearly this equation singles out χ_{-} as the Goldstone boson field. Neglecting non-linearities one expects therefore

$$\langle 0|\,\chi_{-}(0)\,|p\rangle = 1 \tag{I.48}$$

with $|p\rangle$ being the Goldstone boson state. Eq. (I.47) then gives, in this same approximation

$$\langle 0|\, J^{\mu}(0)\,|p\rangle = i\,(\sqrt{2f}\,)\,p^{\mu} \tag{I.49}$$

which identifies the "decay constant" f_i of Eq. (I.29) as $\sqrt{2f}$. This same result also follows directly by rewriting the current J^{μ} of Eq. (I.34) in terms of χ_{+} and χ_{-}. One finds

$$J^{\mu}(x) = \sqrt{2f}\ \partial^{\mu}\chi_{-}(x) + \text{non linear terms} \tag{I.50}$$

which implies (I.49).

To summarize, there are two ways in which symmetries ($[H,U] = 0$) in nature can be realized. If the vacuum state is unique ($U|0\rangle = |0\rangle$) then we have a Wigner-Weyl realization with degenerate particle multiplets. If, on the other hand, the vacuum state is not unique ($U|0\rangle \neq |0\rangle$) we have a Nambu-Goldstone realization with a number of massless excitations, one for each of the generators of the group which does not annihilate the vacuum. In this latter case, one often refers to the phenomena as a spontaneous symmetry breakdown because, although the symmetry exists, it is not reflected in the spectrum of states.

In all the preceding discussion I have implicitly only talked about <u>global</u> symmetry transformations. That is, the parameters of the transformations were assumed to be independent of space-time. These global transformations, as exemplified by Eq. (I.1), transform fields at different space time points in the same way. One may well ask what happens if the group parameters are space time dependent. In this case, in general, the fields at x would be rotated by a different amount than those at another space-time point x'. Transformations where this happens are called <u>local</u>, to distinguish them from the global transformations of Eq. (I.1). Under a local transformation one has

$$\chi_\alpha(x) \xrightarrow[a(x)]{} \chi'_\alpha(x) = R_{\alpha\beta}(a(x))\,\chi_\beta(x) \tag{I.51}$$

It is quite clear that even though one might have constructed an action

$$W = \int d^4x\ \mathcal{L}\,(\chi_\alpha(x), \partial_\mu \chi_\alpha(x))$$

which is invariant under a certain set of global transformations, this action will fail to be invariant under local transformations. The Lagrangian density through its kinetic terms depends on $\partial_\mu \chi_\alpha$ and these quantities transform differently under local transformations than under global ones. One has

$$\begin{aligned}\partial_\mu \chi_\alpha(x) \xrightarrow[a(x)]{} \partial_\mu \chi'_\alpha(x) &= \partial_\mu \left(R_{\alpha\beta}(a(x))\,\chi_\beta(x)\right) \\ &= R_{\alpha\beta}(a(x))\,\partial_\mu \chi_\beta(x) + (\partial_\mu R_{\alpha\beta}(a(x)))\,\chi_\beta(x)\end{aligned} \tag{I.52}$$

The presence of the second term above destroys the local invariance of $\mathcal{L}$. If one wants the action really to be locally invariant, one must add to $\mathcal{L}$ additional fields (gauge fields) which will serve to cancel these extra terms. Thus one sees that Lagrangian densities that are locally invariant must necessarily contain more degrees of freedom.

I want to illustrate this point with a simple, but very helpful, example. Consider a free Dirac field, whose Lagrangian density is

$$\mathcal{L} = -\bar{\psi}(x)\,(\gamma^\mu \tfrac{1}{i}\partial_\mu + m)\,\psi(x) \tag{I.53}$$

Clearly $\mathcal{L}$ is invariant under the U(1) transformation

$$\psi(x) \to \psi'(x) = e^{i\alpha}\,\psi(x) \tag{I.54}$$

and the associated conserved current is easily seen to be

$$J^\mu(x) = \bar{\psi}(x)\,\gamma^\mu\,\psi(x) \tag{I.55}$$

If $\alpha = \alpha(x)$, however, the Lagrangian ceases to be invariant since

$$\partial_\mu \psi(x) \to \partial_\mu \psi'(x) = e^{i\alpha(x)}\,\partial_\mu \psi(x) + i\,(\partial_\mu \alpha(x))\,e^{i\alpha(x)}\,\psi(x) \tag{I.56}$$

It is quite simple to enlarge the Lagrangian (I.53) so that it is invariant also under local U(1) transformations. For that purpose all one needs to do is to introduce a field $A_\mu(x)$ and postulate that under local U(1) transformations:

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\,\psi(x) \tag{I.57}$$

it transforms as

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \frac{1}{e}\,\partial_\mu \alpha(x) \tag{I.58}$$

where e is a parameter, which will eventually play the role of a coupling constant. Clearly the combination

$$D_\mu \psi(x) \equiv (\partial_\mu - ie\,A_\mu(x))\,\psi(x) \tag{I.59}$$

transforms under local transformations without any inhomogeneous terms. That is

$$D_\mu \psi(x) \longrightarrow D'_\mu \psi'(x) = e^{i\alpha(x)}\,D_\mu \psi(x) \tag{I.60}$$

So $D_\mu \psi(x)$ transforms under <u>local</u> transformations precisely as $\partial_\mu \psi(x)$ transforms under <u>global</u> transformations. Hence the Lagrangian density

$$\begin{aligned}\mathcal{L} &= -\bar{\psi}(x)\,(\gamma^\mu \tfrac{1}{i} D_\mu + m)\,\psi(x) \\ &= -\bar{\psi}(x)\,(\gamma^\mu \tfrac{1}{i}\partial_\mu + m)\,\psi(x) + e\,\bar{\psi}(x)\,\gamma^\mu\,\psi(x)\,A_\mu(x)\end{aligned} \tag{I.61}$$

is obviously invariant under local U(1) transformations.

The Lagrangian (I.61), identifying the gauge field A_μ with the electromagnetic potential, describes the interaction of a Dirac particle with electromagnetism. What is missing in Eq. (I.61) is the term

$$\mathcal{L}_{Kin} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{I.62}$$

which describes the kinetic energy of the photon field A_μ. The field strength

$$F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) \tag{I.63}$$

because it involves a curl is invariant under the transformation (I.58). Hence the total Lagrangian

$$\mathcal{L} = -\bar{\psi}(x)(\gamma^\mu \tfrac{1}{i} D_\mu + m)\psi(x) - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x) \tag{I.64}$$

is completely locally U(1) invariant.

Two comments are in order. First, I note again that the requirement of local invariance has necessitated an enlargement of the original globally invariant theory. The compensating gauge field introduced thereby has fixed interactions with the original Dirac field. This is a marvelous result: demanding local invariance fixes the interactions. Second, one should note that the local invariance actually forbids a mass for the A_μ field. A mass term

$$\mathcal{L}_{mass} = -\frac{1}{2} m_A^2 A^\mu(x) A_\mu(x)$$

actually breaks the local U(1) transformation (I.58). Furthermore, the form of the kinetic energy terms for the gauge field - expressed in terms of $F_{\mu\nu}$ - is also fixed by the demand of local U(1) symmetry.

The above simple example demonstrates how easy it is to make a theory which is globally invariant under a symmetry also locally invariant. Of course, in the example, the symmetry was a simple Abelian U(1) symmetry. The same procedure, however, also applies for non Abelian groups - groups whose structure constants are non vanishing. I want to describe here the steps one must follow to construct from a globally invariant Lagrangian, under some group G, a locally invariant Lagrangian.

Consider a Lagrangian density $\mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha)$ composed of fields χ_α and their derivatives $\partial_\mu \chi_\alpha$. Suppose that the fields χ_α transform irreducibly under a group of continuous transformations G and that $\mathcal{L}$ is globally G invariant. Under the global G transformations therefore one has:

$$\begin{aligned} \chi_\alpha(x) &\xrightarrow[a]{} \chi'_\alpha(x) = R_{\alpha\beta}(a)\,\chi_\beta(x) \\ \partial_\mu \chi_\alpha(x) &\xrightarrow[a]{} \partial_\mu \chi'_\alpha(x) = R_{\alpha\beta}(a)\,\partial_\mu \chi_\beta(x) \\ \mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha) &\xrightarrow[a]{} \mathcal{L}(\chi'_\alpha, \partial_\mu \chi'_\alpha) = \mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha) \end{aligned} \tag{I.65}$$

Clearly, if one could construct a covariant derivative for the fields χ_α, $D_\mu \chi_\alpha$, which under local transformations transforms precisely as $\partial_\mu \chi_\alpha$ does under global transformations, then the job will be done. To get a locally invariant Lagrangian one needs just to replace throughout $\partial_\mu \chi_\alpha \leftrightarrow D_\mu \chi_\alpha$. In addition, of course, one must provide appropriate locally invariant field strengths for the gauge fields needed to construct the covariant derivatives $D_\mu \chi_\alpha$.

The covariant derivatives under local transformations are required to obey

$$D_\mu \chi_\alpha(x) \xrightarrow[a(x)]{} D'_\mu \chi'_\alpha(x) = R_{\alpha\beta}(a(x))\, D_\mu \chi_\beta(x) \tag{I.66}$$

In analogy to what was done in the U(1) example let us introduce a gauge field $A_i^\mu(x)$ for each of the parameters of the group G. Since the gauge fields are supposed to compensate for the local variations, it is clear that one wants to have a gauge field for each variation. If the fields χ_α transform under the group G as

$$[G_i, \chi_\alpha(x)] = -(g_i)_{\alpha\beta}\, \chi_\beta(x)$$

then the U(1) example suggests writing for the covariant derivative:

$$D_\mu \chi_\alpha(x) = [\partial_\mu \delta_{\alpha\beta} - i g\, (g_i)_{\alpha\beta} A_{\mu i}(x)]\, \chi_\beta(x) \tag{I.67}$$

where g is some coupling constant.

For Eq. (I.66) to be satisfied the $A_i^\mu(x)$ fields must respond appropriately under the local transformations. Let me compute the transformed covariant derivative $D'_\mu \chi'_\alpha$:

$$D'_\mu \chi'_\alpha(x) = \partial_\mu \chi'_\alpha(x) - ig(g_i)_{\alpha\beta} A'_{\mu i}(x)\, \chi'_\beta(x)$$

$$= \partial_\mu \left[\mathcal{R}_{\alpha\beta}(a(x))\, \chi_\beta(x) \right] - ig(g_i)_{\alpha\beta} A'_{\mu i}(x)\, \mathcal{R}_{\beta\gamma}(a(x))\, \chi_\gamma(x)$$

$$= \mathcal{R}_{\alpha\beta}(a(x)) (\partial_\mu \chi_\beta(x)) + (\partial_\mu \mathcal{R}_{\alpha\beta}(a(x)))\, \chi_\beta(x)$$
$$- ig(g_i)_{\alpha\beta} A'_{\mu i}(x)\, \mathcal{R}_{\beta\gamma}(a(x))\, \chi_\gamma(x)$$

Clearly this will equal the desired result only if

$$(g_i)_{\alpha\beta} A'_{\mu i}(x) = \left[\mathcal{R}_{\alpha\delta}(a(x)) (g_i)_{\delta\gamma} \mathcal{R}^{-1}_{\gamma\beta}(a(x)) \right] A_{\mu i}(x)$$
$$+ \frac{1}{ig} (\partial_\mu \mathcal{R}_{\alpha\delta}(a(x)))\, \mathcal{R}^{-1}_{\delta\beta}(a(x)) \qquad \text{(I.68)}$$

It is easy to check that this formula agrees with Eq. (I.58) in the Abelian case. There $\mathcal{R} = e^{i\alpha}$, $g_i = 1$, and the coupling constant is e. In principle, however, Eq. (I.68) appears very troublesome. The transformation property for the gauge fields A'^{μ}_i seems to depend on how the fields χ_α transform under the group. If true this would be disastrous, for if one had two different fields in the theory transforming according to different irreducible representations of G, then one would have to introduce two kinds of compensating gauge fields! Fortunately, the dependence of the transformation (I.68) on the way χ_α transforms is illusory.

To demonstrate this important point I will consider infinitesimal transformations, where the matrices $\mathcal{R}$ have the decomposition (I.9):

$$\mathcal{R}_{\alpha\beta}(\delta a(x)) = \delta_{\alpha\beta} + i\, \delta a_i(x) (g_i)_{\alpha\beta}$$

Let me then rewrite Eq. (I.68) for these transformations, using an obvious matrix notation:

$$g_i A'_{\mu i} = [1 + i\delta a_j g_j]\, g_i\, [1 - i\delta a_k g_k]\, A_{\mu i}$$
$$+ \frac{1}{ig} [\partial_\mu (1 + i\delta a_j g_j)] [1 - i\delta a_k g_k]$$
$$= g_i A_{\mu i} + i\, \delta a_j [g_j, g_i] A_{\mu i} + \frac{1}{g} (\partial_\mu a_i) g_i + O((\delta a)^2)$$

However, the matrices g_i obey the algebra of the group (cf Eq. (I.11))

$$[g_j, g_i] = i c_{jik} g_k = -i c_{ijk} g_k$$

Hence one sees that in fact the transformation of A'^{μ}_i is independent of the matrices g_i. One finds explicitly

$$A'_{\mu k}(x) = A_{\mu k}(x) + \delta a_j(x)\, c_{ijk} A_{\mu i}(x) + \frac{1}{g} \partial_\mu (\delta a_k(x)) \qquad \text{(I.69)}$$

For global transformations, where δa_k is independent of x, the transformation (I.69) can be written in the standard form one would expect for a quantum field, namely:

$$A'_{\mu k}(x) = A_{\mu k}(x) + i\, \delta a_j (\tilde{g}_j)_{ki} A_{\mu i}(x) \qquad \text{(I.70)}$$

Here the matrices $\tilde{g}_j$ can be expressed in terms of the structure constants of the group:

$$(\tilde{g}_j)_{ki} = -i c_{ijk} = -i c_{jki} \qquad \text{(I.71)}$$

and play the role of the generators of the infinitesimal transformation. That this identification is correct can be checked, by using the Jacobi identity. It is not hard to see in this way that the $\tilde{g}_i$ matrices also obey the group Lie algebra.

The above discussion shows that the gauge fields $A^{\mu}_i(x)$ introduced in the covariant derivative (I.67) transform in a specific way under the group G of transformations. If the group has n parameters then the gauge fields transform under the nxn dimensional representation of the group, whose generators can be expressed in terms of the structure constants by Eq. (I.71). This representation is known as the adjoint and obviously has nothing necessarily to do with how the fields χ_α transform. The gauge fields needed to make the Lagrangian locally invariant under G depend purely on G and not on what fields enter in $\mathcal{L}$. Indeed, since the representation matrices for the A^μ_i are connected to the structure constants of G, one begins to see how fundamental a rôle the gauge fields play.

Having made $\mathcal{L}$ locally invariant under G transformations by introducing the gauge fields A^μ_i, through the covariant derivatives, it remains to endow these fields with kinetic energy terms. Clearly one needs the analog of $F^{\mu\nu}$ for this non Abelian example. The naive choice

$$F^{\mu\nu}_i = \partial^\mu A^\nu_i - \partial^\nu A^\mu_i$$

is easily seen not to do, since under local G transformations it does not transform homogeneously. It is not difficult to show, however, that

$$F^{\mu\nu}_i = \partial^\mu A^\nu_i - \partial^\nu A^\mu_i + g\, c_{ijk} A^\mu_j A^\nu_k \tag{I.72}$$

transforms, under local transformations, according to the adjoint representation with no additional terms.

To check this contention, it is convenient to define matrices $F^{\mu\nu}(x)$ and $A^\mu(x)$ obtained by contracting $F^{\mu\nu}_i(x)$ and $A^\mu_i(x)$ with the generator matrices g_i *:

$$F^{\mu\nu}(x) \equiv g_i F^{\mu\nu}_i(x) \quad ; \quad A^\mu(x) \equiv g_i A^\mu_i(x) \tag{I.73}$$

It is easy to check that (I.72) implies that

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu] \tag{I.74}$$

Consider what happens to this object under a local transformation. Making use of Eq. (I.68) one has:

$$F^{\mu\nu} \xrightarrow[a(x)]{} F'^{\mu\nu} = \partial^\mu A'^\nu - \partial^\nu A'^\mu - ig\,[A'^\mu, A'^\nu]$$

(cont.)

* In principle any generator matrices g_i will do

$$\begin{aligned}
F'^{\mu\nu} &= \partial^\mu\left[\mathcal{R}A^\nu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\nu\mathcal{R})\mathcal{R}^{-1}\right] - \partial^\nu\left[\mathcal{R}A^\mu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\mu\mathcal{R})\mathcal{R}^{-1}\right] \\
&\quad - ig\left[\mathcal{R}A^\mu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\mu\mathcal{R})\mathcal{R}^{-1}\right]\left[\mathcal{R}A^\nu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\nu\mathcal{R})\mathcal{R}^{-1}\right] \\
&\quad + ig\left[\mathcal{R}A^\nu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\nu\mathcal{R})\mathcal{R}^{-1}\right]\left[\mathcal{R}A^\mu\mathcal{R}^{-1} + \frac{1}{ig}(\partial^\mu\mathcal{R})\mathcal{R}^{-1}\right] \\
&= \mathcal{R}F^{\mu\nu}\mathcal{R}^{-1} + \frac{1}{ig}\left[(\partial^\nu\mathcal{R})(\partial^\mu\mathcal{R}^{-1}) - (\partial^\mu\mathcal{R})(\partial^\nu\mathcal{R}^{-1})\right] \\
&\quad - \frac{1}{ig}\left[(\partial^\mu\mathcal{R})\mathcal{R}^{-1}(\partial^\nu\mathcal{R})\mathcal{R}^{-1} - (\partial^\nu\mathcal{R})\mathcal{R}^{-1}(\partial^\mu\mathcal{R})\mathcal{R}^{-1}\right] \\
&\quad + \left[(\partial^\mu\mathcal{R})A^\nu\mathcal{R}^{-1} + \mathcal{R}A^\nu(\partial^\mu\mathcal{R}^{-1}) - (\partial^\nu\mathcal{R})A^\mu\mathcal{R}^{-1} - \mathcal{R}A^\mu(\partial^\nu\mathcal{R}^{-1})\right] \\
&\quad - \left[(\partial^\mu\mathcal{R})A^\nu\mathcal{R}^{-1} + \mathcal{R}A^\mu\mathcal{R}^{-1}(\partial^\nu\mathcal{R})\mathcal{R}^{-1} - \mathcal{R}A^\nu\mathcal{R}^{-1}(\partial^\mu\mathcal{R})\mathcal{R}^{-1} - (\partial^\nu\mathcal{R})A^\mu\mathcal{R}^{-1}\right]
\end{aligned}$$

All the terms above except for the first can be seen to cancel using that

$$\mathcal{R}^{-1}(\partial^\nu\mathcal{R})\mathcal{R}^{-1} = -(\partial^\nu\mathcal{R}^{-1}) \tag{I.75}$$

So indeed:

$$F^{\mu\nu}(x) \xrightarrow[a(x)]{} F'^{\mu\nu}(x) = \mathcal{R}(a(x))\, F^{\mu\nu}(x)\, \mathcal{R}^{-1}(a(x)) \tag{I.76}$$

This transformation is precisely that expected for an object that transforms according to the adjoint representation (compare Eq. (I.68)). It follows that for infinitesimal transformations then

$$F^{\mu\nu}_k(x) \xrightarrow[\delta a(x)]{} F'^{\mu\nu}_k(x) = F^{\mu\nu}_k(x) + \delta a_j(x)\, c_{ijk} F^{\mu\nu}_i(x) \tag{I.77}$$

Clearly therefore the term $F_i^{\mu\nu} F_{i\mu\nu}$ is a group invariant, due to the antisymmetry of the structure constants.

To summarize: the Lagrangian density $\mathcal{L}(\chi_\alpha, \partial_\mu \chi_\alpha)$ assumed to be invariant under some global transformation G can be made locally invariant by introducing appropriate gauge fields. The locally invariant Lagrangian, including the contribution of the gauge fields, reads

$$\mathcal{L}_{local} = \mathcal{L}(\chi_\alpha, D_\mu \chi_\alpha) - \frac{1}{4} F_i^{\mu\nu} F_{i\mu\nu} \tag{I.78}$$

and is totally fixed, knowing $\mathcal{L}_{global}$. A number of points should be commented upon:

(1) The pure gauge field Lagrangian in (I.78) is already a nonlinear field theory, because $F_i^{\mu\nu}$ contains terms quadratic in the gauge fields A^μ_i. For the Abelian case, where the structure functions vanish these nonlinear terms are absent.

(2) Because the A_i^μ transforms nontrivially under G, as far as global transformations go, the symmetry currents of the theory now also get contributions from the gauge fields. That is

$$J_i^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \chi_\alpha} \frac{1}{i} (g_i)_{\alpha\beta} \chi_\beta + \frac{\partial \mathcal{L}}{\partial \partial_\mu A_j^\nu} \frac{1}{i} (\tilde{g}_i)_{jk} A_{\nu k} \tag{I.79}$$

(3) No mass terms for the gauge fields are allowed if one wants to preserve the local symmetry.

For the case of global symmetries the symmetry could be realized either in a Wigner-Weyl way or in a Nambu-Goldstone way, depending on whether the vacuum state of the theory was invariant under the symmetry or not. It is obviously very interesting to investigate what happens in each case, when the global symmetry is made local. In the Wigner-Weyl case, nothing much happens. Besides the various degenerate multiplets of the global symmetry, one now has also a degenerate zero mass multiplet of gauge field excitations. In the Nambu-Goldstone case, however, some remarkable things happen. When the global symmetry is gauged, the Goldstone bosons associated with the broken generators disappear and the corresponding gauge fields acquire a mass. This is the celebrated Higgs mechanism /7/.

To explore this phenomena it is useful to return to the simple example of the Abelian model of Eq. (I.32) and try to make the global symmetry local. Recall the Lagrangian was

$$\mathcal{L} = -\partial_\mu \phi^\dagger \partial^\mu \phi - \lambda (\phi^\dagger \phi - f)^2$$

where f < 0 corresponded to the Wigner Weyl case and f > 0 corresponded to the Nambu-Goldstone case. To make this Lagrangian locally invariant one just needs to replace the divergence $\partial_\mu \phi$ by the covariant divergence *:

$$D_\mu \phi = (\partial_\mu - i g A_\mu) \phi \tag{I.80}$$

and introduce a kinetic energy term for the gauge field. The Lagrangian

$$\mathcal{L} = -(D_\mu \phi)^\dagger (D^\mu \phi) - \lambda (\phi^\dagger \phi - f)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{I.81}$$

is obviously invariant under the local transformations:

$$\begin{aligned} \phi(x) &\to \phi'(x) = e^{i\alpha(x)} \phi(x) \\ A_\mu(x) &\to A'_\mu(x) = A_\mu(x) + \frac{1}{g} \partial_\mu \alpha(x) \end{aligned} \tag{I.82}$$

If f < 0, so that the global symmetry is Wigner-Weyl realized, the above Lagrangian is suitable for computation. It describes the interaction of a degenerate multiplet of scalar fields ϕ and $\phi^\dagger$ with themselves and with the massless gauge field A_μ. Because the fields ϕ are scalar, and the kinetic energy is therefore quadratic in the divergence of ϕ, the interactions with the gauge fields contain both a linear term

$$\mathcal{L}^{(1)}_{int} = g A_\mu [i (\partial^\mu \phi^\dagger) \phi - i (\partial^\mu \phi) \phi^\dagger] = g A_\mu J^\mu \tag{I.83}$$

as well as a "sea-gull" term

$$\mathcal{L}^{(2)}_{int} = -g^2 A^\mu A_\mu \phi^\dagger \phi \tag{I.84}$$

This latter term's presence is dictated by gauge invariance.

If, on the other hand, the parameter f > 0 one must reparametrize the theory in terms

---

* I use the coupling constant g here to emphasize that the field A^μ need not have anything to do with the photon field.

of fields which have vanishing expectation value. This reparametrization (cf Eq. (I.42)) is such that one is computing quantum oscillations around the minimum of the potential $V(\phi^\dagger,\phi)$. Thus one has

$$\phi^\dagger\phi = f + \text{terms involving quantum fields}$$

This necessary shift implies that the seagull term (I.84) gives rise to a mass for the A^μ field

$$\mathcal{L}_{mass} = -g^2 f\, A^\mu A_\mu = -\tfrac{1}{2} m_A^2\, A^\mu A_\mu \tag{I.85}$$

Now if the gauge field acquires a mass, it follows that it cannot be purely transverse (like the photon) but must also have a longitudinal component. This extra degree of freedom must come from somewhere. It is not difficult to show that it arises from the disappearance of the Goldstone excitation that would ordinarily result from the spontaneous breakdown of the U(1) symmetry.

To check this assertion, in the case of $f>0$, it is convenient to reparametrize the field ϕ not as in Eq. (I.42) but to choose an exponential parametrization *:

$$\phi(x) = \frac{1}{\sqrt{2}}\left[\sqrt{2f} + \rho(x)\right] e^{i\xi(x)/\sqrt{2f}} \tag{I.86}$$

Here $\rho(x)$ and $\xi(x)$ are real fields and the Goldstone excitations are connected with the "phase" field $\xi(x)$. Indeed, the potential

$$V(\phi^\dagger,\phi) = \lambda\,(\phi^\dagger\phi - f)^2$$

is clearly independent of ξ so that, in the absence of the gauge interactions, one would identify it with the zero mass Goldstone excitation . Using Eq. (I.86) the potential becomes

$$V = \lambda\left(\tfrac{1}{2}\rho^2 + \sqrt{2f}\,\rho\right)^2$$

so that the field ρ has a mass

$$m_\rho^2 = 4\lambda f \tag{I.87}$$

* The physics of the theory is independent of what parametrization one chooses. Different choices are akin to different gauge choices

which was the value obtained previously for the massive scalar field in the theory.

It is easy to check that the field $\xi(x)$ enters in the covariant derivative $D_\mu\phi$ in a rather trivial way. One has

$$D_\mu\phi = (\partial_\mu - igA_\mu)\phi = (\partial_\mu - igA_\mu)\frac{1}{\sqrt{2}}\left[\sqrt{2f}+\rho\right]e^{i\xi/\sqrt{2f}}$$

$$= \frac{e^{i\xi/\sqrt{2f}}}{\sqrt{2}}\left[\partial_\mu\rho - i(\rho+\sqrt{2f})\,g\left\{A_\mu + \frac{1}{g\sqrt{2f}}\partial_\mu\xi\right\}\right] \tag{I.88}$$

The factor in front in Eq. (I.88) obviously will not appear in the Lagrangian (I.81) since one has $(D^\mu\phi)^\dagger(D_\mu\phi)$. Furthermore, the quantity in the curly bracket in Eq. (I.88) is just a gauge transformed field (cf. Eq. (I.82)) with $\alpha = \xi/\sqrt{2f}$. By fixing the gauge so that everything is expressed in terms of a new gauge field

$$B_\mu = A_\mu + \frac{1}{g\sqrt{2f}}\partial_\mu\xi \tag{I.89}$$

one sees that the Goldstone field ξ disappears entirely from the theory.

If the U(1) global symmetry is spontaneously broken $(f>0)$ the Lagrangian (I.81) can be rewritten entirely in terms of a massive vector field B_μ and a massive real field ρ. The resulting Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\rho\,\partial^\mu\rho - \tfrac{1}{2}m_\rho^2\rho^2 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}m_A^2 B^\mu B_\mu$$
$$- g^2\left(\sqrt{2f}\,\rho + \tfrac{1}{2}\rho^2\right)B^\mu B_\mu - \lambda\left(\tfrac{1}{4}\rho^4 + \sqrt{2f}\,\rho^3\right) \tag{I.90}$$

where

$$m_\rho^2 = 4\lambda f \quad ; \quad m_A^2 = 2g^2 f$$

shows no explicit traces of the original U(1) symmetry, except that certain of the parameters in the interactions have particular interrelations.

Let me close this section by discussing the two versions of the model (Wigner-Weyl ($f < 0$) and Nambu Goldstone ($f > 0$)) in terms of the degrees of freedom present in the theory. In the Wigner Weyl case, one has in the theory a complex scalar field ϕ (2 degrees of freedom) plus a massless gauge field (again 2 degrees of freedom, corresponding to the two transverse polarizations). In the Nambu Goldstone case, there is in the theory a real scalar field ρ (1 degree of freedom) plus a massive spin one field B_μ (3 degrees of freedom). Clearly both versions of the theory have the same total number of degrees of freedom. However, the spectrum of excitations is totally different.

II. The SU(2) x U(1) Model of the Electroweak Interactions

At first sight weak and electromagnetic interactions seem to have little in common. Electromagnetic interactions are responsible for the binding of atoms. Weak interactions, on the other hand, cause rather long lived nuclear disintegrations, like neutron beta decay. However, there are at least two phenomenological similarities that hint at the unification of these forces:

(1) In both weak and electromagnetic interactions, currents are involved. In the electromagnetic case the interaction

$$\mathcal{L}_{em} = e\, A^\mu(x)\, J_\mu^{em}(x) \tag{II.1}$$

gives rise to long range forces between charged particles. Charged particles, according to (II.1), interact due to the exchange of a photon. The $1/q^2$ propagator for the photon is what is responsible for the $1/r$ potential between charged particles. For the weak interactions, which are responsible for the neutron instability, it has been known for a long time that they can be described by an effective current-current theory

$$\mathcal{L}_{Fermi} = \frac{G_F}{\sqrt{2}}\, J_+^\mu(x)\, J_{-\mu}(x) \tag{II.2}$$

Here G_F is the Fermi constant, which has dimension of $(\text{mass})^{-2}$. If one imagined that the contact nature of the above interaction was due to the exchange of a very heavy "weak-boson", then the resemblance between weak and electromagnetic processes would be greater. At a deeper level the weak (charged current) interactions could be written as in (II.1) by introducing charged massive spin one fields W_+^μ and W_-^μ:

$$\mathcal{L}_{weak} = \tilde{g}\left(J_+^\mu(x)\, W_{-\mu}(x) + J_-^\mu(x)\, W_{+\mu}(x) \right) \tag{II.3}$$

Eq. (II.1) would reemerge in the limit in which any momentum dependence in the W-propagators was neglected in comparison to the mass and one would identify:

$$\frac{G_F}{\sqrt{2}} = \frac{\tilde{g}^2}{M_W^2} \tag{II.4}$$

(2) The charged currents that enter in weak decays are not unrelated to the electromagnetic current, at least as far as the strongly interacting particles are concerned. This interrelation was discovered a long time ago /8/. The vector piece of the weak currents $J_\pm^\mu$ is identical to the $1 \pm i2$ component of the strong isospin current. In turn, the isovector piece of the electromagnetic current is nothing but the 3rd component of the strong isospin current. This connection between weak and electromagnetic properties at the phenomenological level leads one to predict subtle weak effects - like weak magnetism /8/ - entirely in terms of known electromagnetic properties.

Although the above remarks make it attractive to try to unify weak and electromagnetic interactions, they are by no means compelling. The dominant reason for attempting an unification is theoretical. While pure QED is a renormalizable theory, the Fermi theory of the weak interactions is not. The contact interaction in (II.3) gives rises to incurable divergences in higher order in perturbation theory. These divergences are not ameliorated if the Fermi interaction (II.2) is replaced by interactions mediated by a heavy vector boson (cf Eq. II.3). The propagator for such a spin one boson as $q \to \infty$ is as badly behaved, as if it did not exist:

$$\Delta_{\mu\nu}(q) = \frac{\eta_{\mu\nu} + q_\mu q_\nu / q^2}{q^2 + M^2} \xrightarrow[q \to \infty]{} O(1) \tag{II.5}$$

Thus it is not possible to just add "by hand" an interaction like (II.3) to the electromagnetic interactions and hope to obtain a renormalizable interaction. If, however, Eq. (II.3) were to arise as the result of making a global symmetry local - so that $W_\pm^\mu$ are gauge fields - then it is possible to obtain a renormalizable theory. In these circumstances, as first shown by 't Hooft /9/, the mass of the vector bosons arises because of a spontaneous breakdown, but the gauge structure effectively allows one to calculate with propagators which vanish as $1/q^2$ for large momenta.

If one wants to construct models of the weak interactions based on a symmetry group G, whose currents must at least include J_+^μ and J_-^μ, one in general finds that G also contains a $U(1)_{em}$ symmetry, which can be associated with electromagnetism. When the theory spontaneously breaks down, only this $U(1)_{em}$ symmetry survives with its massless photon. At the broken stage, however, there is always some mixing of the photon with some of the weak bosons of G. Hence renormalizability forces one to think directly of unified weak and electromagnetic interactions.

The simplest unified model of electroweak interactions which contains J_+^μ, J_-^μ and J_{em}^μ clearly would be based on the group O(3) /10/. However, the discovery of weak neutral currents /11/ argues for a 4 parameter group. The suggestion of Glashow, Salam and Weinberg /4/, made well before the discovery of the neutral currents, was that electroweak interactions could be described if the symmetry of the theory was SU(2) x U(1). This suggestion has been phenomenologically extremely successful. In this section, I would like therefore to detail the structure of the GSW model and examine some of its predictions in the light of experiment.

The GSW model was built to reproduce the known structure of the charged current weak interactions. It predicted then particular neutral current interactions, whose experimental verification provided a direct test of the model. From a long series of experiments in the 1950's and 1960's one knew that the weak currents in Eq. (II.2) had a (V-A) form. That is, that only the left-handed fermionic fields appear to participate. For instance, J_-^μ was known to contain a neutrino-electron term

$$J_-^\mu = \bar{\nu}_e \gamma^\mu (1-\gamma_5) e = 2 \bar{\nu}_{eL} \gamma^\mu e_L$$

where the projections $\psi_{L,R}$ are just

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi \quad ; \quad \psi_R = \tfrac{1}{2}(1+\gamma_5)\psi \tag{II.6}$$

No terms containing the right-handed fields, however, enter in J_-^μ.

To detail the structure of the GSW model one has to specify how the matter degrees of freedom transform under the SU(2) x U(1) group. The fundamental matter entities presently known are the quarks and leptons, which appear in a repetitive generation pattern. Each generation of quarks and leptons has the same SU(2) x U(1) quantum numbers. To date we know of the existence of three generations: the electron family (e, ν_e ; d, u); the muon family (μ, ν_μ ; s, c); and the τ -lepton family (τ, ν_τ ; b, t). Each of the leptons is accompanied by its own neutrino and a pair of quarks. The quarks in the pair actually are comprised each of three states, since each quark carries a color index i, i = 1,2,3. The strong interactions of quarks, which binds them into hadrons, have to do with their color properties. The weak interactions act on their flavor properties; u,d,s, etc.

I will detail now the properties of the electron family under SU(2) x U(1) - the μ and τ families having identical quantum numbers. In view of the preceding discussion, it is clear that only the left projections of these fields carry SU(2) quantum numbers and that $(\nu_e, e)_L$ and $(u, d)_L$ are doublets under SU(2). Furthermore, since SU(2) x U(1) $\supset$ $U(1)_{em}$ the electromagnetic charge Q must be the sum of the U(1) generator Y and the diagonal SU(2) generator T_3:

$$Q = T_3 + Y \tag{II.7}$$

Hence the U(1) properties of the fields in the electron family follow from their charge. These considerations lead to the following table

Table I: SU(2) x U(1) properties of the states in the electron family

states	$\binom{u}{d}_L$	u_R	d_R	$\binom{\nu_e}{e}_L$	e_R	ν_R
SU(2)	2	1	1	2	1	1
U(1)	1/6	2/3	-1/3	-1/2	-1	0

The right handed neutrino is usually not included as a real excitation. It is a total singlet under SU(2) x U(1).

Knowing the transformation properties of the quarks and leptons under SU(2) x U(1), one may use the results of the previous section to write down immediately a Lagrangian for the theory that is locally SU(2) x U(1) invariant, by replacing in the kinetic energy terms for the quarks and leptons all derivatives by SU(2) x U(1) covariant derivates. Let W_i^μ i = 1,2,3 and Y^μ be the gauge fields corresponding to the SU(2) and U(1) symmetries. Then, from Table I, it follows that the covariant derivatives for the quarks and leptons are

$$D_\mu \binom{u}{d}_L = [\partial_\mu - i g' \tfrac{1}{6} Y_\mu - i g \tfrac{\tau_i}{2} W_{\mu i}] \binom{u}{d}_L$$

(cont.)

$$D_\mu u_R = [\partial_\mu - i g' \tfrac{2}{3} Y_\mu] u_R$$

$$D_\mu d_R = [\partial_\mu + i g' \tfrac{1}{3} Y_\mu] d_R$$

$$D_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = [\partial_\mu + i g' \tfrac{1}{2} Y_\mu - i g \tfrac{\tau_i}{2} W_{\mu i}] \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \text{(II.8)}$$

$$D_\mu e_R = [\partial_\mu + i g' Y_\mu] e_R$$

Here g and g' are the SU(2) and U(1) coupling constants and the matrices $1/2\, \tau_i$ – with τ_i the Pauli matrices – are those appropriate for fields that transform as SU(2) doublets:

$$[\tfrac{1}{2}\tau_i, \tfrac{1}{2}\tau_j] = i\, \epsilon_{ijk}\, \tfrac{1}{2}\tau_k \qquad \text{(II.9)}$$

For one generation of quarks and leptons, the Lagrangian of the GSW model is thus

$$\begin{aligned}\mathcal{L}^{f-g}_{GSW} = &- (\bar u\, \bar d)_L \gamma^\mu \tfrac{1}{i} D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L - \bar u_R \gamma^\mu \tfrac{1}{i} D_\mu u_R - \bar d_R \gamma^\mu \tfrac{1}{i} D_\mu d_R \\ &- (\bar\nu_e\, \bar e)_L \gamma^\mu \tfrac{1}{i} D_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L - \bar e_R \gamma^\mu \tfrac{1}{i} D_\mu e_R \\ &- \tfrac{1}{4} W_i^{\mu\nu} W_{i\mu\nu} - \tfrac{1}{4} Y^{\mu\nu} Y_{\mu\nu}\end{aligned} \qquad \text{(II.10)}$$

In the above, the field strengths $W_i^{\mu\nu}$ and $Y^{\mu\nu}$ are given by

$$W_i^{\mu\nu} = \partial^\mu W_i^\nu - \partial^\nu W_i^\mu + g\, \epsilon_{ijk} W_j^\mu W_k^\nu \qquad \text{(II.11)}$$

$$Y^{\mu\nu} = \partial^\mu Y^\nu - \partial^\nu Y^\mu$$

Note that the Lagrangian (II.10) includes no mass terms for the fermion fields. Mass terms involve a Left-Right transition:

$$\mathcal{L}_{mass} = - m \bar\psi \psi = - m (\bar\psi_L \psi_R + \bar\psi_R \psi_L) \qquad \text{(II.12)}$$

However, as Table I shows, ψ_L and ψ_R for the quarks and leptons transform respectively as doublets and singlets of SU(2). Hence the symmetry forbids adding these terms. As I will show later, masses can be generated when SU(2) x U(1) is spontaneously broken down.

From the Lagrangian (II.10) one can read out explicit interaction terms of the gauge fields with the fermionic currents. One has

$$\mathcal{L}_{int} = g' J^\mu_Y Y_\mu + g J^\mu_i W_{i\mu} \qquad \text{(II.13)}$$

where J^μ_Y and J^μ_i are the "weak" U(1) and SU(2) currents of the fermions:

$$J^\mu_i = (\bar u\, \bar d)_L \gamma^\mu \tfrac{\tau_i}{2} \begin{pmatrix} u \\ d \end{pmatrix}_L + (\bar\nu_e\, \bar e)_L \gamma^\mu \tfrac{\tau_i}{2} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$\begin{aligned}J^\mu_Y = &\tfrac{1}{6} (\bar u\, \bar d)_L \gamma^\mu \begin{pmatrix} u \\ d \end{pmatrix}_L + \tfrac{2}{3} \bar u_R \gamma^\mu u_R - \tfrac{1}{3} \bar d_R \gamma^\mu d_R \\ &- \tfrac{1}{2} (\bar\nu_e\, \bar e)_L \gamma^\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L - \bar e_R \gamma^\mu e_R\end{aligned} \qquad \text{(II.14)}$$

I note that in the model, since

$$J^\mu_{em} = J^\mu_3 + J^\mu_Y \qquad \text{(II.15)}$$

the phenomenological observation that the vector piece of the weak charged currents and the isovector piece of J^μ_{em} are related, is built in already.

It is convenient to rewrite (II.13) in terms of physical fields. Clearly if the model is to reproduce the weak interactions, the symmetry SU(2) x U(1) must suffer a spontaneous breakdown. As I showed in the preceding section, in this circumstance the gauge fields get a mass. The exchange of massive gauge fields can then reproduce the short range weak interactions. Of course, since the photon field must remain massless, the breakdown of SU(2) x U(1) cannot be complete. One expects SU(2) x U(1) $\rightarrow$ $U(1)_{em}$. Then, of the four gauge fields W_i^μ and Y^μ, three will acquire a mass and one will remain massless. The photon field will in general be a linear combination of the two neutral gauge bosons W_3^μ and Y^μ, with the orthogonal combination being associated with a massive excitation - the Z^0 boson. It has become convenient to parametrize these linear combinations in terms of an angle - the Weinberg angle θ_W. One has:

$$\begin{aligned} W_3^\mu &= \cos\theta_W Z^\mu + \sin\theta_W A^\mu \\ Y^\mu &= -\sin\theta_W Z^\mu + \cos\theta_W A^\mu \end{aligned} \tag{II.16}$$

It is also useful to rewrite W_1^μ and W_2^μ in terms of fields of definite charge

$$W_\pm^\mu = \frac{1}{\sqrt{2}} \left(W_1^\mu \mp i W_2^\mu \right) \tag{II.17}$$

In terms of the physical fields Z^μ, A^μ, $W_\pm^\mu$ the interaction (II.13) becomes

$$\begin{aligned} \mathcal{L}_{int} = & \frac{1}{2\sqrt{2}} g \left[W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu} \right] \\ & + \left\{ (g\cos\theta_W + g'\sin\theta_W) J_3^\mu - g'\sin\theta_W J_{em}^\mu \right\} Z_\mu \\ & + \left\{ g'\cos\theta_W J_{em}^\mu + (g'\cos\theta_W - g\sin\theta_W) J_3^\mu \right\} A_\mu \end{aligned} \tag{II.18}$$

In the above, the charged current $J_\pm^\mu$ are defined in terms of J_1^μ and J_2^μ as

$$J_\pm^\mu = 2 \left(J_1^\mu \mp i J_2^\mu \right) \tag{II.19}$$

where the factor of 2 is introduced so that the currents $J_\pm^\mu$ are precisely those that appear in the Fermi theory, Eq. (II.2). Furthermore, I have eliminated throughout the weak U(1) current J_Y^μ in favor of the electromagnetic current and of J_3^μ, by using Eq. (II.15). This rewriting has an important consequence. Namely, if A^μ is to be the photon field, then its interaction can only be with J_{em}^μ, with coupling strength e. That is (II.18), as far as the photon part goes, must reduce to (II.1). This informs one that:

$$e = g'\cos\theta_W = g\sin\theta_W \tag{II.20}$$

Using this information and eliminating g and g' in Eq. (II.18) in favor of e and $\sin^2\theta_W$ yields for $\mathcal{L}_{int}$ of the GSW model the expression:

$$\begin{aligned} \mathcal{L}_{int} = & \frac{e}{2\sqrt{2}\sin\theta_W} \left(W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu} \right) \\ & + \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu J_\mu^{NC} \end{aligned} \tag{II.21}$$

where the neutral current J_μ^{NC} is given by

$$J_{NC}^\mu = 2 \left[J_3^\mu - \sin^2\theta_W J_{em}^\mu \right] \tag{II.22}$$

I note that the parameter $\tilde{g}$, given in Eq. (II.3), is identified here as

$$\tilde{g} = \frac{e}{2\sqrt{2}\sin\theta_W} \tag{II.23}$$

Whence, the comparison with the Fermi theory for the charged currents, Eq. (II.4), identifies the Fermi constant G_F as

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 M_W^2 \sin^2\theta_W} \tag{II.24}$$

Knowing the Weinberg angle, the above gives direct information on the mass of the heavy weak boson which is supposed to mediated the charged current weak interactions. As I shall show, low energy neutral current experiments give $\sin^2\theta_W \simeq 1/4$. Using this value in Eq. (II.24), along with the experimental value for the Fermi constant $G_F \simeq 10^{-5}$ GeV^{-2} and $\alpha = e^2/4\pi \simeq 1/137$, yields for M_W a mass of around 80 GeV. This prediction has been spectacularly confirmed at the CERN collider, by discovering a particle of this mass with the experimental characteristics of the W /12/. I will return later on in these lectures to discuss these matters in more detail.

If the weak charged bosons have a mass of around 80 GeV, one can well understand why the Fermi theory was such a good approximation. In typical weak interaction experiments, the momentum transfer q^2 is very much smaller than M_W^2. Thus the exchange of W-bosons can be well approximated by the contact interaction of Eq. (II.2). This is shown graphically in Fig. 1

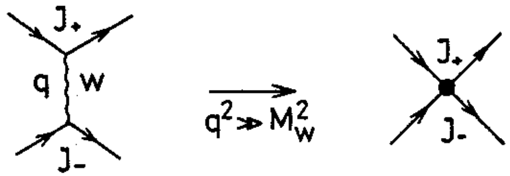

Fig. 1: Recovery of the Fermi theory from W exchange, for $q^2 \ll M_W^2$

Neutral current interactions, involving Z-boson exchange, ought also therefore to lead to an effective current-current theory for momentum transfers $q^2 \ll M_Z^2$. Using Eq. (II.21), one predicts for this effective theory an interaction Lagrangian:

$$\mathcal{L}^{\text{Neutral}}_{\text{Fermi}} = \frac{1}{2}\left[\frac{e}{2\sin\theta_W\cos\theta_W}\right]^2 \frac{1}{M_Z^2}\, J^{NC}_\mu J^\mu_{NC} \tag{II.25}$$

Using in the above equation * the identification (II.24) gives

$$\mathcal{L}^{\text{Neutral}}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}\left[\frac{M_W^2}{\cos^2\theta_W\, M_Z^2}\right] J^{NC}_\mu J^\mu_{NC} = \frac{G_F}{\sqrt{2}}\,\rho\, J^{NC}_\mu J^\mu_{NC} \tag{II.26}$$

The ratio

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} \tag{II.27}$$

gives, therefore, the relative strength of neutral to charged current weak processes, at low momentum transfers, in the GSW model.

* The factor of 1/2 in (II.25) comes from doing 2nd order perturbation theory. This factor is cancelled for charged current interactions because there are two terms: $W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu}$.

To summarize, the weak interactions in the Glashow-Salam-Weinberg model, for $q^2 \ll M_W^2,\ M_Z^2$, can be written in a current-current form:

$$\mathcal{L}_{\text{Weak}} = \frac{G_F}{\sqrt{2}}\left[J_+^\mu J_{-\mu} + \rho\, J^\mu_{NC} J^{NC}_\mu \right] \tag{II.28}$$

The charged currents, by construction, agree with experiment. Neutral weak interactions test the model, since the only free parameters are ρ and the Weinberg angle θ_W - which enters in the definition of J^μ_{NC} of Eq. (II.22). All neutral current experiments (some of which I will discuss here) can be fitted with a common value of $\sin^2\theta_W \simeq 1/4$ and of $\rho \simeq 1$, thereby providing strong support for the validity of the GSW model. Furthermore, given ρ and $\sin^2\theta_W$ one can determine the mass of the Z^0_+ from Eq. (II.27). The discovery at the CERN Collider /13/, soon after that of the W^-, of a neutral heavy particle of mass around 90 GeV, in agreement with the value predicted by the GSW model, provided a further confirmation of the model.

Before I discuss in more detail some of the neutral current experiments used to test the GSW model and to extract ρ and $\sin^2\theta_W$, and discuss the properties of the W and Z bosons found at the collider, I want to comment on the meaning of $\rho \simeq 1$. Roughly speaking this parameter measures the ratio of the W and Z masses and, therefore, it is connected to the mass generating mechanism of the GSW model. The masses of the weak bosons arise because the local SU(2) x U(1) symmetry is broken down to $U(1)_{em}$. The mechanism employed in the model to cause this breakdown is precisely that which I discussed in detail in the last section. One introduces scalar fields, which transform nontrivially under SU(2) x U(1), into the theory and assumes that their self interactions lead them to acquire a non zero vacuum expectation value, which causes the breakdown.

Since the SU(2) group must be broken down, it is necessary that the scalar field introduced into the theory carry SU(2) quantum numbers. The simplest useful possibility, thus, is that this field be an SU(2) doublet. Since it must also carry U(1) quantum numbers, one must - at the minimum - introduce a complex doublet. Consider

$$\Phi = \begin{pmatrix}\phi^0 \\ \phi^-\end{pmatrix} \tag{II.29}$$

where ϕ^0 and ϕ^- are complex fields. Φ therefore has U(1) charge Y = - 1/2. If one assumes that Φ has self interactions given by the potential (cf. Eq. I.37)

$$V = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 \tag{II.30}$$

it is clear that SU(2) x U(1) will be broken down. The choice of the vacuum expectation value:

$$\langle \Phi \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} v \\ 0 \end{pmatrix} \tag{II.31}$$

guarantees that SU(2) x U(1) is broken down to $U(1)_{em}$ *.

Because Φ carries SU(2) x U(1) properties, to maintain the SU(2) x U(1) local symmetry, the Φ piece of the GSW Lagrangian must involve the covariant derivative:

$$D_\mu \Phi = [\partial_\mu + i \frac{g'}{2} Y_\mu - i g \frac{\tau_i}{2} W_{\mu i}] \Phi \tag{II.32}$$

One has

$$\mathcal{L}^{H-g}_{GSW} = -(D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi^+, \Phi) \tag{II.33}$$

The seagull piece in the first term above will generate masses for the $W^\mu_\pm$ and Z^μ fields, when the field Φ is replaced by its expectation value. With Φ an SU(2) doublet, it turns out that the W and Z masses are such that $\rho = 1$. Hence the experimental determination of $\rho \simeq 1$ gives information on the symmetry breakdown. In this sense, it is satisfying that the simplest possibility seems to be favored.

Let me demonstrate this assertion. The masses of the gauge fields arise from the seagull term

$$\mathcal{L}_{mass} = -[(g \frac{\tau_i}{2} W^\mu_i - \frac{g'}{2} Y^\mu) \langle \Phi \rangle]^+ [(g \frac{\tau_j}{2} W_{\mu j} - \frac{g'}{2} Y_\mu)]$$

However,

* For one scalar doublet one can always define $U_{em}(1)$ as that U(1) that is left unbroken. The choice (II.31) is dictated by the definition (II.7); any other choice would do, but it would change the definition of Q

$$g \frac{\tau_i}{2} W^\mu_i - \frac{g'}{2} Y^\mu = \begin{bmatrix} \frac{g}{2} W^\mu_3 - \frac{g'}{2} Y^\mu & \frac{g}{\sqrt{2}} W^\mu_+ \\ \frac{g}{\sqrt{2}} W^\mu_- & -\frac{g}{2} W^\mu_3 - \frac{g'}{2} Y^\mu \end{bmatrix}$$

$$= \begin{bmatrix} \frac{g}{2\cos\theta_W} Z^\mu & \frac{g}{\sqrt{2}} W^\mu_+ \\ \frac{g}{\sqrt{2}} W^\mu_- & \frac{g}{2\cos\theta_W} [\sin^2\theta_W - \cos^2\theta_W] Z^\mu - A^\mu \end{bmatrix}$$

where the second line follows by using Eq. (II.16) and the identification (II.20). Since $\langle \Phi \rangle$ has only an upper component, it follows immediately that

$$\mathcal{L}_{mass} = -\left(\frac{gv}{2}\right)^2 W^\mu_+ W_{-\mu} - \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z^\mu Z_\mu \tag{II.34}$$

so that

$$M^2_W = \frac{1}{4}(gv)^2$$
$$M^2_Z = \frac{1}{4\cos^2\theta_W}(gv)^2 = \frac{M^2_W}{\cos^2\theta_W} \tag{II.35}$$

This proves the contention that $\rho = 1$, with this simple doublet breaking. One can show that if the breaking of SU(2) x U(1) is done by a scalar field carrying SU(2) quantum number I, with a component carrying I_3 with non zero vacuum expectation value, $\langle \Phi(I, I_3) \rangle \neq 0$, then /14/:

$$\rho = \frac{I^2 - I_3^2 + I}{2 I_3^2} \tag{II.36}$$

Obviously, from this formula, if $I = I_3 = 1/2$ as was assumed, then $\rho = 1$.

The parameter v which enters in the potential (II.30) - Higgs potential - sets the entirely by the value of the Fermi constant. Using Eqs. (II.20), (II.24) and (II.35), it follows that

$$v = (\sqrt{2}\, G_F)^{-1/2} \simeq 250 \text{ GeV} \qquad \text{(II.37)}$$

Questions concerning the origin and the dynamics associated with this scale will be addressed in the last part of these lectures. It is time now to look at experiment.

The presence of neutral current weak interactions can be detected in purely leptonic experiments. These experiments are somewhat easier to analyze theoretically, because one does not have to worry about strong interaction effects. The GSW model is written at the level of leptons and quarks and if one considers experiments involving hadrons one has to be able to translate the quark level predictions into hadronic predictions. This step is obviously unnecessary for purely leptonic processes. Unfortunately neutrino interactions off electrons are a factor of m_e/M, where M is the proton mass, weaker. Hence these purely leptonic experiments suffer from a lack of statistics.

The process $\nu_\mu e \to \nu_\mu e$, and its companion involving antineutrinos $\bar\nu_\mu e \to \bar\nu_\mu e$, tests precisely the additional neutral current piece in the weak interaction effective Lagrangian (II.28). I have written the currents J_i^μ and J_Y^μ in Eq. (II.14) for one family of quarks and leptons, but they can be trivially extended to the case of more than one family of states. Because these currents only connect fields within the same generation *, obviously the process $\nu_\mu e \to \nu_\mu e$ cannot be caused by a charged current weak interaction. There is no current involving ν_μ and an electron which can cause this transition. The neutral current, however, has both a $\nu_\mu - \nu_\mu$ and an e-e piece and therefore the $J^\mu_{NC} J^{NC}_\mu$ term in Eq. (II.28) can cause ν_μ e scattering. Since for an electron one has

$$\begin{aligned}(J^\mu_{NC})_{electron} &= 2\,[\,J_3^\mu - \sin^2\theta_W\, J^\mu_{em}\,]_{electron}\\ &= 2\,[\,\bar e_L \gamma^\mu (-\tfrac{1}{2})\, e_L - \sin^2\theta_W (-1)\, \bar e \gamma^\mu e\,]\\ &= \bar e\,[\,\gamma^\mu(1-\gamma_5)\, Q_L + \gamma^\mu(1+\gamma_5)\, Q_R\,]\,e\end{aligned}$$

* I will show later that no intragenerational mixing is caused by the SU(2) x U(1) breakdown, essentially because the neutrinos are massless, for the leptonic sector.

where

$$Q_L = \sin^2\theta_W - \tfrac{1}{2} \quad ; \quad Q_R = \sin^2\theta_W \, , \qquad \text{(II.38)}$$

while for a neutrino one has

$$\begin{aligned}(J^\mu_{NC})_{\nu_\mu} &= 2\,[\,J_3^\mu - \sin^2\theta_W\, J^\mu_{em}\,]_{\nu_\mu}\\ &= 2\,[\,\bar\nu_{\mu L}\, \gamma^\mu\, \tfrac{1}{2}\, \nu_{\mu L}\,]\\ &= \tfrac{1}{2}\,[\,\bar\nu_\mu \gamma^\mu (1-\gamma_5)\, \nu_\mu\,]\end{aligned}$$

the effective Lagrangian for $\nu_\mu e$ and $\bar\nu_\mu e$ scattering is simply *

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}}\, \rho\, [\,\bar\nu_\mu \gamma^\mu (1-\gamma_5)\, \nu_\mu\,]\,[\,\bar e\,(\gamma_\mu(1-\gamma_5)\, Q_L + \gamma_\mu (1+\gamma_5)\, Q_R)\, e\,] \qquad \text{(II.39)}$$

It is straightforward to compute the scattering cross section for the $\nu_\mu e \to \nu_\mu e$ and $\bar\nu_\mu e \to \bar\nu_\mu e$ processes from (II.39). The relevant kinematics for these reactions is shown in Fig. 2

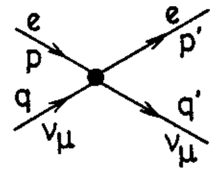

Fig. 2: Kinematics for ν_μ- e scattering

It proves convenient to define the scalar invariant

$$y = \frac{p \cdot p'}{p \cdot q} \qquad \text{(II.40)}$$

Thus y in the laboratory frame, where the incident electron is at rest, is a measure of the outgoing electron's energy to that of the incident neutrino

* Remember that there are two cross terms, that is why no factor of 1/2 appears in (II.39)

$$y_{Lab} = \frac{E'_e}{E_\nu}$$

In the CM frame, on the other hand, y is a measure of the angle by which the electron is scattered, as shown in Fig. 3.

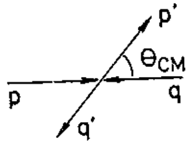

Fig. 3: Kinematics in the CM system

One has, neglecting the electron mass,

$$y_{CM} = \frac{1}{2}(1 - \cos\theta_{CM})$$

An elementary calculation gives for $\frac{d\sigma}{dy}$ for neutrino and antineutrino scattering off electrons

$$\left(\frac{d\sigma}{dy}\right)^{\nu_\mu e} = \frac{2G_F^2 m_e E_\nu \rho^2}{\pi}\left[Q_L^2 + Q_R^2(1-y)^2\right]$$

$$\left(\frac{d\sigma}{dy}\right)^{\bar{\nu}_\mu e} = \frac{2G_F^2 m_e E_\nu \rho^2}{\pi}\left[Q_R^2 + Q_L^2(1-y)^2\right] \qquad \text{(II.41)}$$

The structure of these formulas is actually rather easy to understand. The interaction which causes the scattering, given in Eq. (II.39), is a vector interaction which conserves the handedness (or helicity *) of the fields. For neutrino scattering off electrons one has two contributions: the left handed neutrinos scatter off left handed electrons ($\sim Q_L^2$) or the left-handed neutrinos scatter off right handed electrons ($\sim Q_R^2$). For antineutrinos, since the helicity reverses, the right-left scattering is proportional to $Q_L{}^2$ while the right-right scattering is proportional to $Q_R{}^2$. I show this pictorially in Fig. 4 for both the cases of LL (or RR) and LR scattering in the CM of the neutrino electron system.

* The helicity is just the projection of the spin along the direction of motion

(a) (b)

Fig. 4: Scattering of LL (a) and LR (b) fermions in the CM system with vector interactions

For LL or RR scattering the initial state has J = 0 and so one expects that $\left(\frac{d\sigma}{d\Omega}\right)_{CM}$ be isotropic. But since cos θ_{CM} is simply related to y one sees that this implies

$$\left(\frac{d\sigma}{dy}\right)_{LL} = \left(\frac{d\sigma}{dy}\right)_{RR} \sim 1 \qquad \text{(II.42)}$$

For LR scattering, on the other hand, it is clear that a configuration where the final particles are emitted backwards is forbidden since it corresponds to $\Delta J = 2$. Thus $\left(\frac{d\sigma}{d\Omega}\right)_{CM} \sim (1 + \cos\theta_{CM})^2$ which implies

$$\left(\frac{d\sigma}{dy}\right)_{LR} = \left(\frac{d\sigma}{dy}\right)_{RL} \sim (1-y)^2 \qquad \text{(II.43)}$$

Because the electron mass enters in Eq. (II.41), the cross sections are very small - typically of order 10^{-42} E (GeV) cm^2. Nevertheless, both $\nu_\mu e$ and $\bar{\nu}_\mu e$ processes have been measured at CERN and Brookhaven recently, with about 100 events in each experiment - which is high statistics in this difficult field! Because both $\nu_\mu e$ and $\bar{\nu}_\mu e$ are measured their ratio can be determined, which gets rid of the ρ parameter (and of various systematic errors). Integrating Eqs. (II.41) over y and taking the ratio gives

$$R = \frac{\sigma_{\nu_\mu e}}{\sigma_{\bar{\nu}_\mu e}} = \frac{3Q_L^2 + Q_R^2}{3Q_R^2 + Q_L^2} = \frac{3 - 12\sin^2\theta_W + 16\sin^4\theta_W}{1 - 4\sin^2\theta_W + 16\sin^4\theta_W} \qquad \text{(II.44)}$$

which is a pure function of the Weinberg angle.

The CHARM experiment at CERN /15/ gives for the ratio

$$R = 1.37 \begin{smallmatrix} +\,0.65 \\ -\,0.44 \end{smallmatrix} \qquad \text{(CHARM)} \qquad \text{(II.45)}$$

which implies a value of the Weinberg angle:

$$\sin^2\theta_W = 0.215 \pm 0.032 \pm 0.013 \qquad \text{(CHARM)} \qquad \text{(II.46)}$$

where the first error above is statistical and the second systematic. The BNL experimental results, obtained very recently /16/, are in complete agreement with the CHARM results. They report a value of the ratio

$$R = 1.38 \begin{smallmatrix} +\,0.40 \\ -\,0.31 \end{smallmatrix} \pm 0.17 \qquad \text{(BNL)} \qquad \text{(II.47)}$$

which gives a value for $\sin^2\theta_W$, from their analysis, of

$$\sin^2\theta_W = 0.209 \pm 0.029 \pm 0.013 \qquad \text{(BNL)} \qquad \text{(II.48)}$$

The absolute values of these cross sections can then be used to infer a value for ρ. I quote below the results given for the cross section slope with neutrino energy by both experiments

$$\begin{aligned} \sigma_{\nu_\mu e}/E_\nu &= (1.90 \pm 0.40 \pm 0.40) \times 10^{-42}\ \text{cm}^2/\text{GeV} && \text{(CHARM)} \\ \sigma_{\nu_\mu e}/E_\nu &= (1.60 \pm 0.29 \pm 0.26) \times 10^{-42}\ \text{cm}^2/\text{GeV} && \text{(BNL)} \\ \sigma_{\bar{\nu}_\mu e}/E_\nu &= (1.50 \pm 0.30 \pm 0.40) \times 10^{-42}\ \text{cm}^2/\text{GeV} && \text{(CHARM)} \\ \sigma_{\bar{\nu}_\mu e}/E_\nu &= (1.16 \pm 0.20 \pm 0.14) \times 10^{-42}\ \text{cm}^2/\text{GeV} && \text{(BNL)} \end{aligned} \qquad \text{(II.49)}$$

The CHARM collaboration extracts from their absolute cross section measurements a value for ρ

$$\rho = 1.09 \pm 0.09 \pm 0.11 \qquad \text{(II.50)}$$

which is consistent with the prediction $\rho = 1$ from doublet symmetry breaking. The BNL collaboration gives no value for ρ, from their analysis. A cursory glance at Eq. (II.49) would suggest that if such an analysis were performed, the central value for ρ from the Brookhaven data would just be slightly below one.

A second, purely leptonic test, of the GSW neutral current interaction is afforded by the process $e^+e^- \to \mu^+\mu^-$, measured at DESY and at SLAC. In QED this process occurs to lowest order in α by one photon exchange. In the GSW model there is an additional contribution due to Z^0 exchange, as shown in Fig. 5

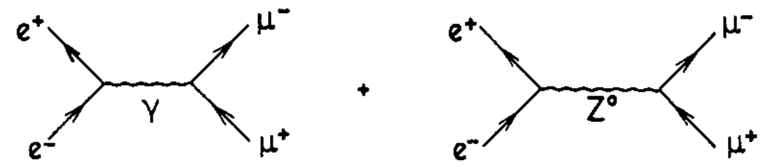

Fig. 5: Electromagnetic and weak contributions to $e^+e^- \to \mu^+\mu^-$

For energies much below the Z^0 mass, the second term in this figure can be approximated by the contact interaction of Eq. (II.28). One can readily estimate how important is the neutral current contribution relative to the purely electromagnetic term. One has for the ratio of the amplitudes

$$\frac{T_{weak}}{T_{em}} \sim \frac{G_F}{e^2/q^2} \sim 10^{-4}\, q^2\,(\text{GeV}^2) \qquad \text{(II.51)}$$

Since for the experiments at DESY and SLAC, typically $q^2 \sim O(10^3\ \text{GeV}^2)$, the interference between the electromagnetic and the weak amplitudes should give rise to effects of the order of 10 %.

To measure this interference effect experimentally, one studies the asymmetry between the number of μ^- produced in the direction of the incoming electron in the CM system and those produced against this direction. If $(d\sigma/d\Omega)_{cm}$ is the differential cross section for $e^+e^- \to \mu^+\mu^-$ in the CM system, then this forward-backward asymmetry is defined by

$$A_{F-B} = \frac{\int_0^1 d\cos\theta_{cm} \left(\frac{d\sigma}{d\Omega}\right)_{cm} - \int_{-1}^0 d\cos\theta_{cm} \left(\frac{d\sigma}{d\Omega}\right)_{cm}}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad \text{(II.52)}$$

In the absence of the neutral current interaction this asymmetry vanishes *. Hence A_{F-B} tests directly the presence of the Z^o coupling. Unfortunately, as I will show, it does not provide any information on $\sin^2\theta_W$, although its magnitude provides information on ρ.

I will quote the result of the calculation and then describe qualitatively why it has this structure. In the GSW model one finds

$$A_{F-B} = -\left[\frac{3G_F s}{4\pi\sqrt{2}\alpha}\rho\right] g_A^\ell \tag{II.53}$$

Here $s = -q^2$ is the square of the total energy in the CM system and g_A is an axial coupling constant, to be defined below. In the GSW model $g_A = 1/2$, so that the asymmetry is

$$A_{F-B} \simeq -\frac{3G_F s}{16\pi\sqrt{2}\alpha}\rho \simeq -7\times10^{-5}\, s(\mathrm{GeV}^2)\,\rho$$

To understand this result, let me write down the effective neutral current Lagrangian relevant for this case

$$\mathcal{L}^{NC}_{eff} = \frac{2G_F\rho}{\sqrt{2}}\left[\bar{e}(\gamma^\lambda g_V + \gamma^\lambda\gamma_5 g_A)e\right]\left[\bar{\mu}(\gamma_\lambda g_V + \gamma_\lambda\gamma_5 g_A)\mu\right] \tag{II.54}$$

The coupling constants g_V and g_A can be gotten directly from the structure of J^μ_{NC} and one has

$$g_V = Q_R + Q_L = 2\sin^2\theta_W - \tfrac{1}{2}$$
$$g_A = Q_R - Q_L = \tfrac{1}{2} \tag{II.55}$$

The amplitude obtained from (II.54) is to be added to the purely electromagnetic amplitude and the absolute square gives the desired cross section. The claim is that only the cross term between the weak and the em amplitude gives the asymmetry.

* This is only strictly correct to $O(\alpha)$. There are $O(\alpha^3)$ contributions to A_{F-B} which come from higher order QED processes

It is easy to check that the purely electromagnetic contribution is symmetric in $\cos\theta_{CM}$. The fact that one has a vector interaction means that the scattering will occur only between left handed electrons and right-handed positrons or viceversa, producing $\mu^+\mu^-$ pairs also in these two configurations. This is shown pictorially in Fig. 6.

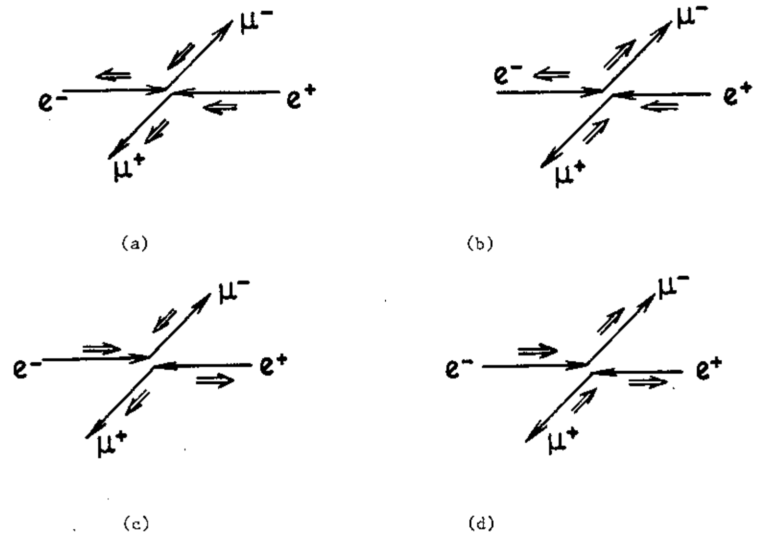

Fig. 6: Possible configurations in $e^+e^- \to \mu^+\mu^-$ scattering due to vector interactions

If θ_{CM} is the angle between the outgoing μ^- and the incoming electron, then considerations analogous to those presented for $\nu_\mu e$ scattering give

$$\frac{d\sigma}{d\Omega}(a) = \frac{d\sigma}{d\Omega}(d) \sim (1+\cos\theta_{cm})^2$$
$$\frac{d\sigma}{d\Omega}(b) = \frac{d\sigma}{d\Omega}(c) \sim (1-\cos\theta_{cm})^2 \tag{II.56}$$

For unpolarized scattering and not measuring final state polarizations one just sums over (a) - (d). Hence

$$\left(\frac{d\sigma}{d\Omega}\right)_{em} \sim (1+\cos\theta_{cm})^2 + (1-\cos\theta_{cm})^2 \sim (1+\cos^2\theta_{cm}) \tag{II.57}$$

which establishes the contention that A_{F-B} needs to arise from the weak-electromagnetic interference.

The differential cross section arising from the interference can be infered by using the diagrams of Fig. 6 and the form of $\mathcal{L}^{NC}_{eff}$ of Eq. (II.54). The point is that γ_5 for a left-handed particle is (-1), but it is (+1) for a right-handed particle. Thus terms in the electromagnetic-weak interference proportional to g_A contribute with a (-1) in each of the diagrams (a) - (d) whenever one has a left-handed electron or muon, but (+1) otherwise. I give below the relevant signs of the four diagrams for the four possible interference terms. The first $g_{V,A}$ below corresponds to the electrons, the second to the muons:

$$
\begin{aligned}
g_V\, g_V &: \quad (a) + (b) + (c) + (d) \sim 2\,(1 + \cos^2\theta_{CM}) \\
g_V\, g_A &: \quad -(a) + (b) - (c) + (d) = 0 \\
g_A\, g_V &: \quad -(a) - (b) + (c) + (d) = 0 \\
g_A\, g_A &: \quad (a) - (b) - (c) + (d) \sim 4 \cos\theta_{CM}
\end{aligned}
\tag{II.58}
$$

Clearly, therefore, only the $g_V^{\,2}$ and the $g_A^{\,2}$ terms contribute and it is only this latter term that gives the asymmetry. The sign of the asymmetry given in Eq. (II.53) is negative because it involves also the photon propagator $1/q^2$, and $q^2 = -s < 0$. So all the qualitative features of Eq. (II.53) are understood.

The forward-backward asymmetry has been measured at SLAC and at DESY showing good agreement with the value expected from the GSW model. Because A_{F-B} involves g_A^2, the asymmetry gives no direct information on the Weinberg angle but its magnitude constrains ρ. I give below the asymmetry for the process $e^+e^- \to \mu^+\mu^-$ obtained at DESY at two (average) energies: $\sqrt{s} = 34.5$ GeV and $\sqrt{s} = 41.6$ GeV /17/. These results are themselves averages of the asymmetries obtained by all the four running experiments (five in the case of the lower energy asymmetry)

$$
\begin{aligned}
A_{F-B}(e^+e^- \to \mu^+\mu^-) &= -10.8 \pm 1.1\ \% \qquad \sqrt{s} = 34.5\ \text{GeV} \\
A_{F-B}(e^+e^- \to \mu^+\mu^-) &= -14.7 \pm 3.1\ \% \qquad \sqrt{s} = 41.6\ \text{GeV}
\end{aligned}
\tag{II.59}
$$

The predictions from Eq. (II.53) taking $\rho = 1$ are respectively - 8.1 % and 11.7 %, which are somewhat below the values of Eq. (II.59). In fact, however, the effect of a finite Z^o mass is <u>not</u> totally negligible here. Including a Z^o propagator effect changes s in Eq. (II.53) by

$$
s \to \frac{s M_Z^2}{s - M_Z^2} \tag{II.60}
$$

Taking the experimental value for M_Z /13/, $M_Z = 93.2 \pm 1.5$ GeV, the substitution of Eq. (II.60) increases the predictions for A_{F-B} at the two energies in questions by 14 % and 20 %, respectively. Hence, the values to compare (II.59) with are, for $\rho = 1$

$$
\begin{aligned}
A_{F-B}(GSW) &= -9.4\ \% \qquad \sqrt{s} = 34.5\ \text{GeV} \\
A_{F-B}(GSW) &= -14.5\ \% \qquad \sqrt{s} = 41.6\ \text{GeV}
\end{aligned}
\tag{II.61}
$$

These are in excellent agreement and speak for the validity of the model and for the simple choice $\rho = 1$.

The most precise values for $\sin^2\theta_W$ and ρ come from deep inelastic scattering of neutrinos *. In these experiments one scatters neutrinos off a nuclear target and sums over all possible final states. The scattering process can occur, in the GSW model, either mediated by W-exchange (Charged Current or CC process) or mediated by Z-exchange (Neutral Current or NC process). This is shown pictorially in Fig. 7

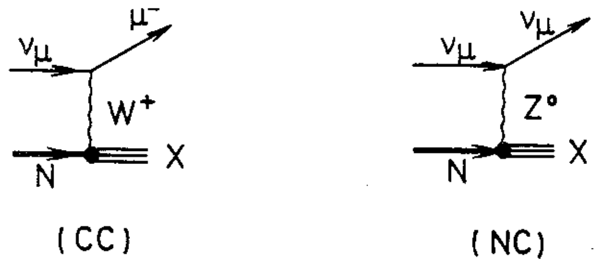

Fig. 7: CC and NC deep inelastic scattering of neutrinos

By comparing the ratio of the charged current processes to the neutral current processes one can obtain information on ρ and $\sin^2\theta_W$. Doing this both for neutrino and antineutrino scattering then fixes these parameters, much in the same way that this happened in the case of scattering off electrons. However, here the relevant

* A very precise value for $\sin^2\theta_W$ comes also from polarized electron deuteron deep inelastic scattering /18/

mass parameter is the nucleon mass and not the electron mass, and so one is dealing with much bigger cross sections. As a result both $\sin^2\theta_W$ and ρ are determined more accurately than in neutrino-electron scattering.

The advantage of considering an inclusive process - that is a process where one sums over all possible hadronic final states - is that one can avoid most of the theoretical issues connected with how quarks bind among each other to become hadrons. The excitation of the initial hadronic state N by the virtual W or Z^o of Fig. 7 must proceed through the coupling of the W or Z^o to one of the quarks in N. The scattered quark then recombines with the spectator constituents of the initial hadronic state to form the final state X. I illustrate this process in Fig. 8

Fig. 8: Decomposition of the vertex $N(W/Z^o)X$

To the extent that one is summing over all possible final states X, it is clearly irrelevant how the scattered quark combines with the spectators to make a particular final state. Hence the deep inelastic scattering cross section should be given by the convolution of the probability of finding a quark with a certain fraction of the initial hadron momentum, with the cross section for the scattering of this quark by the initial neutrino or antineutrino. This is the parton model of Feynman /19/ and of Bjorken and Paschos /20/. I shall not elaborate further on this topic since it is covered by the lectures of Close and Brodsky in this school. For my purposes here, all that is relevant is that the deep inelastic processes are proportional to the corresponding quark scattering processes.

To simplify my discussion, I shall suppose that the only important constituents of nucleons are the "valence" quarks. That is, a proton is made up of 2 u quarks and a d quark and the neutron is made up of 2 d and a u quark. All "sea" contributions of virtual $q\bar{q}$ pairs in this approximation are neglected. Let $f_u(x)$ and $f_d(x)$ be the probabilities of finding a u quark or a d quark in a proton, carrying a momentum fraction x of the initial proton momentum. Then, by isospin symmetry, $f_d(x)$ and $f_u(x)$ are the respective probabilities of finding a u quark or a d quark in the neutron. Charged current deep inelastic scattering of neutrinos acts only on the d quarks, through the reaction $\nu_\mu + d \to \mu^- + u$. For antineutrinos the charged current scattering involves only u quarks: $\bar{\nu}_\mu + u \to \mu^+ + d$. Neutral current

scattering, since it is charge diagonal, will involve scattering of both the u and d valence quarks. Hence in general, even knowing how to calculate the scattering at the quark level is not enough to interrelate the CC and NC processes, since different combinations of probability functions enter. For isoscalar targets, however, everything simplifies very nicely.

An isoscalar target - like ^{12}C - is made up of an equal number of protons and neutrons. Hence scattering off isoscalar targets is like scattering from an average of a proton and a neutron: $N = 1/2(p+n)$. Consider then the CC process $\nu_\mu N \to \mu^- X$. In the parton model it is given by the convolution (denoted by $\otimes$ below):

$$d\sigma_{cc}(\nu_\mu N \to \mu^- X) = \frac{1}{2}(f_d + f_u) \otimes d\sigma(\nu_\mu d \to \mu^- u) \tag{II.62a}$$

since f_d is the probability of finding a d quark in a proton but f_u is the probability of finding a d quark in a neutron. Similarly, for antineutrino scattering one has

$$d\sigma_{cc}(\bar{\nu}_\mu N \to \mu^+ X) = \frac{1}{2}(f_u + f_d) \otimes d\sigma(\bar{\nu}_\mu u \to \mu^+ d) \tag{II.62b}$$

For neutral current scattering, on the other hand, one has

$$d\sigma_{NC}(\nu_\mu N \to \nu_\mu X) = \frac{1}{2}(f_u + f_d) \otimes [d\sigma(\nu_\mu d \to \nu_\mu d) + d\sigma(\nu_\mu u \to \nu_\mu u)]$$

$$d\sigma_{NC}(\bar{\nu}_\mu N \to \bar{\nu}_\mu X) = \frac{1}{2}(f_u + f_d) \otimes [d\sigma(\bar{\nu}_\mu d \to \bar{\nu}_\mu d) + d\sigma(\bar{\nu}_\mu u \to \bar{\nu}_\mu u)] \tag{II.63}$$

Clearly therefore - in the valence approximation - the ratio of CC to NC deep inelastic scattering off isoscalar targets can be computed entirely from a knowledge of the elementary quark-ν_μ or quark-$\bar{\nu}_\mu$ processes.

The effective Lagrangian for NC scattering of neutrinos or antineutrinos off quarks in the GSW model has the same form as (II.39), except that the coupling constants Q_L and Q_R are replaced by those appropriate to the quarks in question:

$$\mathcal{L}^{NC}_{eff} = \frac{G_F}{\sqrt{2}}\,\rho\,[\bar{\nu}_\mu \gamma^\mu(1-\gamma_5)\nu_\mu]\cdot[\bar{q}\,(\gamma_\mu(1-\gamma_5)Q^q_L + \gamma_\mu(1+\gamma_5)Q^q_R)\,q] \tag{II.64}$$

Here $q = \{u,d\}$ and the coupling constants Q^q_L, Q^q_R which follow from the structure of the neutral current of the GSW model are:

$$\begin{aligned} Q^u_R &= -2/3\sin^2\theta_W & \quad Q^u_L &= -2/3\sin^2\theta_W + 1/2 \\ Q^d_R &= 1/3\sin^2\theta_W & \quad Q^d_L &= 1/3\sin^2\theta_W - 1/2 \end{aligned} \tag{II.65}$$

The charged current effective Lagrangian, of course, is just that of the Fermi theory:

$$\begin{aligned}\mathcal{L}^{CC}_{eff} = \frac{G_F}{\sqrt{2}}\Big\{&[\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu]\cdot[\bar{d}\gamma_\mu(1-\gamma_5)u] \\ &+ [\bar{\mu}\gamma^\mu(1-\gamma_5)\nu_\mu]\cdot[\bar{u}\gamma_\mu(1-\gamma_5)d]\Big\}\end{aligned} \tag{II.66}$$

The differential cross sections $d\sigma/dy$ that one must calculate to compute the ratios of NC and CC processes have precisely the same structure as Eq. (II.41) for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. For CC processes, however, there are no right-handed couplings, the left-handed couplings are unity and $\rho = 1$:

$$Q^u_R = Q^d_R = 0; \quad Q^u_L = Q^d_L = 1; \quad \rho = 1 \qquad \text{CC} \tag{II.67}$$

Let me define the ratios

$$R_\nu = \frac{\sigma^{NC}(\nu_\mu N \to \nu_\mu X)}{\sigma^{CC}(\nu_\mu N \to \mu^- X)} = \frac{\int_0^1 dy\, \frac{d\sigma^{NC}}{dy}(\nu_\mu N \to \nu_\mu X)}{\int_0^1 dy\, \frac{d\sigma^{CC}}{dy}(\nu_\mu N \to \mu^- X)} \tag{II.68a}$$

and

$$R_{\bar{\nu}} = \frac{\sigma^{NC}(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma^{CC}(\bar{\nu}_\mu N \to \mu^+ X)} \tag{II.68b}$$

Then it is easy to see that

$$R_\nu = \rho^2 \left\{(Q^u_L)^2 + (Q^d_L)^2 + \tfrac{1}{3}[(Q^u_R)^2 + (Q^d_R)^2]\right\}$$

$$R_{\bar{\nu}} = 3\rho^2 \left\{(Q^u_R)^2 + (Q^d_R)^2 + \tfrac{1}{3}[(Q^u_L)^2 + (Q^d_L)^2]\right\}$$

which, using Eqs. (II.65), can be rewritten in terms of the Weinberg angle as

$$\begin{aligned} R_\nu &= \rho^2\left[\tfrac{1}{2} - \sin^2\theta_W + \tfrac{20}{27}\sin^4\theta_W\right] \\ R_{\bar{\nu}} &= \rho^2\left[\tfrac{1}{2} - \sin^2\theta_W + \tfrac{20}{9}\sin^4\theta_W\right] \end{aligned} \tag{II.69}$$

The ratios R_ν and $R_{\bar{\nu}}$ have been very accurately determined by the CHARM and the CDHS experiments at CERN. The results of these two collaborations are in very good agreement with each other, as the values below show /21/ /22/:

$$\begin{aligned} R_\nu &= 0.300 \pm 0.007 && \text{(CDHS)} \\ R_\nu &= 0.320 \pm 0.010 && \text{(CHARM)} \\ R_{\bar{\nu}} &= 0.357 \pm 0.015 && \text{(CDHS)} \\ R_{\bar{\nu}} &= 0.377 \pm 0.020 && \text{(CHARM)} \end{aligned} \tag{II.70}$$

Using the formula (II.69) and taking $\rho = 1$ and a value of $\sin^2\theta_W = 0.23$ gives $R_\nu = 0.31$ and $R_{\bar{\nu}} = 0.39$, which fits very nicely with Eq. (II.70). To actually extract the best value for ρ and $\sin^2\theta_W$ one needs to correct the theoretical formula (II.69) for the contribution of the quark sea and other small effects. This has been done in the analysis of their data by the CHARM and CDHS collaborations. The value of $\sin^2\theta_W$ one obtains depends on whether ρ is kept fixed at 1 or not. For instance, the CHARM collaboration /21/ gives for $\rho = 1$ a value

$$\sin^2\theta_W = 0.220 \pm 0.014 \tag{II.71}$$

If ρ is also varied the best fits are

$$\begin{aligned} \rho &= 1.027 \pm 0.023 \\ \sin^2\theta_W &= 0.247 \pm 0.038 \end{aligned} \quad \text{(CHARM)}$$

$$\begin{aligned} \rho &= 0.996 \pm 0.026 \\ \sin^2\theta_W &= 0.221 \pm 0.030 \end{aligned} \quad \text{(CDHS)} \tag{II.72}$$

I should remark that the values of ρ and $\sin^2\theta_W$ extracted from neutrino-electron scattering are in total agreement with the above. This remark is non trivial, since a priori different vertices are tested in these two experiments. In the GSW model, of course, they are related and the nice agreement found in these two different experiments is a further confirmation of the model.

Because the neutral current interactions in the GSW model are parity violating, they give rise to small parity violation effects in atoms. These effects have been studied experimentally in the last 7-8 years and, after an initial period of confusion, have now been definitely established. I want to make a few remarks on this subject because it illustrates so nicely the unity of physics. These atomic parity violating experiments deal with transitions in the electron volt energy range. Yet, because they are looking for effects of the parity violating neutral current, they are testing physics at the Fermi scale of O(100 GeV)!

The parity violating interaction between electrons and quarks (hence nucleons) will induce small admixtures of opposite parity components into the atomic levels. The presence of these opposite parity admixtures can then be detected by studying transitions induced by using incident polarized light and measuring an asymmetry in the photon absorption cross section when the polarization is flipped. This parity violating asymmetry, nominally, should again be of the order * $\frac{G_F q^2}{e^2} \sim 10^{-4}\, q^2$ (GeV^2). If this were the case, then the effect is really too small to measure. In atomic physics $q^2 \sim \frac{1}{\langle r^2 \rangle_{atom}} \sim (m_e \alpha)^2$, which predicts an asymmetry: $A \sim 10^{-14}$.

* The asymmetry in $e^+e^- \to \mu^+\mu^-$ which I discussed before was not parity violating, since it involved g_A^2. However, the estimate there of $G_F q^2/e^2$ is really an estimate of the magnitude of the interference.

Fortunately this estimate turns out to be a bit too crude and there are methods to enhance this number considerably, by studying parity violating transitions in heavy atoms /23/. Effectively, in the case of heavy atoms, an extra enhancement of Z^3 - where Z is the number of protons in the atom - arises. For Bismuth, for instance where Z = 81, this is a substantial enhancement and asymmetries of the order $A \sim 10^{-8}$ are to be expected.

I want to derive the parity violating Hamiltonian applicable in atomic physics from the fundamental neutral current interaction of Eq. (II.28). For parity violations I need to focus on the terms in $\mathcal{L}^{NC}_{eff}$ which involve a vector vertex at the quark level and an axial vector vertex at the electron level, and vice versa. A simple calculation gives

$$\mathcal{L}^{eff}_{PV} = \sqrt{2}\, G_F\, \rho \Big\{ g_A\, \bar e \gamma_\mu \gamma_5 e \left[g_V^u (\bar u \gamma^\mu u) + g_V^d (\bar d \gamma^\mu d) \right] + g_V\, \bar e \gamma_\mu e \left[g_A^u (\bar u \gamma^\mu \gamma_5 u) + g_A^d (\bar d \gamma^\mu \gamma_5 d) \right] \Big\} \tag{II.73}$$

Here the couplings g_A and g_V are those given in Eq. (II.55). Further

$$\begin{aligned} g_V^u &= Q_R^u + Q_L^u = \tfrac{1}{2} - \tfrac{4}{3}\sin^2\theta_W \\ g_V^d &= Q_R^d + Q_L^d = -\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W \\ g_A^u &= Q_R^u - Q_L^u = -\tfrac{1}{2} \\ g_A^d &= Q_R^d - Q_L^d = \tfrac{1}{2} \end{aligned} \tag{II.74}$$

Since the nucleus in an atom is essentially static, the quark currents that enter in (II.73) can be considerably simplified by making a non relativistic reduction. For that purpose the Dirac representation of the γ -matrices is convenient:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad ; \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Retaining only the upper conponents, since the lower components are down by v/c, means that for the vector piece of the quark current only the γ^0 term survives, while for the axial piece only the $\vec{\gamma}\gamma_5 \sim \vec{\sigma}$ term remains. In this non relativistic limit thus, the first term in Eq. (II.73) is proportional to the sum of the u and d quark number densities in the nucleus ($g_v^u u^\dagger u + g_v^d d^\dagger d$), while the second term is proportional to the nuclear spin. For heavy nuclei, obviously the first term dominates since the contributions add coherently. In the spin term there will be substantial cancellations and so I shall neglect it from now on.

The dominant quark contribution, proportional to $g_v^u u^\dagger u + g_v^d d^\dagger d$, acts as static source for the electrons which is proportional to $Q_w\, \delta^3(\vec{x})$. Here Q_w just measures the "weak charge" of the nucleus:

$$Q_w = g_v^u N_u + g_v^d N_d \tag{II.75}$$

where N_u and N_d are the number of u and d quarks, respectively, in the nucleus. In a valence approximation, these numbers are simply related to the number of protons, Z, and neutrons, N, in the nucleus:

$$N_u = 2Z + N \quad ; \quad N_d = 2N + Z \tag{II.76}$$

Thus

$$Q_w = (2g_v^u + g_v^d)Z + (2g_v^d + g_v^u)N = (1/2 - 2\sin^2\theta_W)\, Z - 1/2\, N \tag{II.77}$$

Since the Weinberg angle experimentally is near $\sin^2\theta_W \simeq 1/4$, one sees that $Q_w \simeq -1/2$ N. Thus for heavy nuclei Q_w is large, being essentially proportional to the number of neutrons in the nucleus.

It remains to compute the contribution of electrons to the parity violating Hamiltonian. The surviving term in Eq. (II.73) involves $\bar{e}\,\gamma^0\gamma_5\, e$. I will again consider this contribution in the non relativistic limit. In the Dirac representation $\gamma^0\gamma_5$ connects the upper and lower components of the electron wavefunction. Since this wavefunction in the non relativistic limit is given by

$$e \underset{NR}{\simeq} \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}_e}{2m_e}\chi \end{pmatrix} \tag{II.78}$$

it is clear that the electron contribution to the parity violating potential is just $\frac{\vec{\sigma}\cdot\vec{p}_e}{m_e}$ *. Putting all the pieces together, one sees that the GSW model predicts a parity violating potential:

$$H_{PV} = \frac{G_F}{\sqrt{2}}\, \rho\, Q_W\, \frac{\vec{\sigma}\cdot\vec{p}_e}{m_e}\, \delta^3(\vec{x}) \tag{II.79}$$

A few comments are in order on this result:

(1) Obviously this potential is parity violating, since it is proportional to the pseudoscalar interaction $\vec{\sigma}\cdot\vec{p}_e$. Thus it will have non zero matrix elements only between atomic states of opposite parity. Furthermore, it is short range because of the $\delta^3(\vec{x})$, so it will measure essentially the electron wavefunctions at the origin.

(2) For heavy atoms one can understand qualitatively where the Z^3 enhancement factor, mentioned earlier, comes from. A factor of Z comes from $Q_W \simeq -1/2$ N (which is roughly Z). A second factor of Z comes from the electron's momentum, since $\langle p_e \rangle \sim m_e \alpha Z$ in an atom. The final factor of Z arises from the $\delta^3(\vec{x})$ factor, which tells one that the parity violating matrix element is proportional to the electron wavefunction at the origin. This factor can be shown to vary as Z also for heavy atoms.

(3) Parity violating experiments in principle can serve to give yet another determination of $\sin^2\theta_W$ and ρ, since these quantities enter in Eq. (II.79). However, since the feasible experiments are done in heavy atoms, there is considerable theoretical atomic physics to be done before one can extract the relevant particle physics information from the data.

To date three kind of different atomic parity violation experiments have been performed: optical rotation in Bismuth atoms /24/, circular dichroism in Thallium vapor /25/, parity violating polarization in Cesium atoms /26/. I shall not go into the precise details of what is measured, as that would take me too far afield. I summarize, however, the present status of theory and experiments in Table II. The theoretical numbers use $\rho = 1$, $\sin^2\theta_W = 1/4$, but the atomic uncertainties seem to

* This contribution comes with a (−) sign since what emerges from (II.73) is really $\bar{e}\gamma_0\gamma_5\, e$. However, going from $\mathcal{L}$ to the Hamiltonian this sign is again changed.

dominate the theoretical spread - especially for Bismuth.

Table II: Summary of Parity Violating Experiments

1) Optical Rotation in Bismuth ($R \times 10^8$)

Experiment /24/			Theory:	λ = 648 mm	λ = 876 mm
Oxford	λ = 648 mm	-9.3 ± 1.5	Ref. 27	- 17	- 13
Novosibirsk	λ = 648 mm	-20.2 ± 2.7	Ref. 28	- 18.8	- 8
Moscow	λ = 648 mm	-7.8 ± 1.3	Ref. 29	- 13	- 11
Seattle	λ = 876 mm	-10.4 ± 1.7	Ref. 30	- 11.1	- 8.3

2) Circular Dichroism in Thallium ($A \times 10^3$)

Experiment /25/	Theory /31/
Berkeley $-1.73 \pm 0.26 \pm 0.07$	-1.31 ± 0.26

3) Electronic Polarization in Cs

Experiment /26/	Theory /32/
Paris $-1.56 \pm 0.17 \pm 0.12$	$-1.61 \pm 0.07 \pm 0.20$

Although both experimental and theoretical uncertainties are too big to get a good determination of ρ and $\sin^2\theta_W$, is is clear that the GSW model predicts effects of the order of those observed, in these very difficult experiments.

I have left for last, rather fittingly, a discussion of the properties of the W and Z bosons. Their recent discovery at the CERN collider /12/ /13/ is the crowning glory of the GSW model. The data at the moment is still quite limited - less than 100 W events, reconstructed through their decays into $e\,\nu_e$ or $\mu\,\nu_\mu$; less than 20 Z^o events, reconstructed through their e^+e^- or $\mu^+\mu^-$ decay modes. Nevertheless, it seems to be in very good agreement with the prediction of the theory. Obviously, however, a much more detailed study of the properties of the $W^\pm$ and the Z^o is warranted, as any small departure from what is predicted by the GSW model may signify new physics. Hopefully more information will become available soon as the new data taken in Fall 1984 at the CERN collider is analyzed.

The production of the W and Z^o in $p\bar{p}$ collisions can be computed by applying the same parton model ideas I discussed for the case of deep inelastic scattering. In this case the W or Z^o get produced by "fusing" a quark from the proton with an antiquark from the antiproton, as indicated schematically in Fig. 9

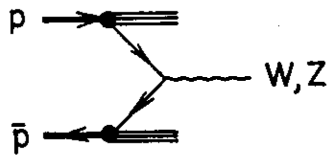

Fig. 9: Production of W/Z^o by q-$\bar{q}$ fusion

The cross section for the process $p\bar{p} \to W/Z\ X$ therefore can be computed by convoluting the distribution functions for finding the quark (antiquark) in the proton (antiproton) with the elementary cross section $q\bar{q} \to W/Z$. One may use deep inelastic scattering information to determine the quark distribution functions in the proton. By charge symmetry the antiquark distributions in the antiproton are the same as those of quarks in protons. Therefore one has a direct prediction for the expected cross sections.

I illustrate the above discussion by considering specifically the production of W^- in $p\bar{p}$ collisions. In the valence approximation I am considering, the W^- gets produced by fusing a d quark in the proton with a $\bar{u}$ in the antiproton. If ξ (ξ') is the fraction of the initial proton (antiproton) momentum carried by the d($\bar{u}$) quark, then the cross section for W^- production implied by the diagram in Fig. 9 is just

$$\sigma(p\bar{p} \to W^- x) = \int d\xi\, d\xi'\, f_d(\xi)\, f_u(\xi')\, \frac{1}{3}\, \sigma(d+\bar{u} \to W^-) \tag{II.80}$$

The factor of 1/3 above follows since for the W^- to be produced the d and $\bar{u}$ quarks must have the same color and the probability of that is 1/3. A simple calculation, using for $\mathcal{L}_{int}$ the expression given in Eq. (II.21) for the GSW model, yields

$$\begin{aligned} \sigma(d+u \to W^-) &= \frac{\pi e^2}{4\sin^2\theta_W}\, \delta(\xi\xi' s - M_W^2) \\ &= \sqrt{2}\,\pi\, G_F M_W^2\, \delta(\xi\xi' s - M_W^2) \end{aligned} \tag{II.81}$$

Here $\sqrt{s}$ is the total energy of the $p\bar{p}$ system in the CM and the δ-function above informs one that the W can only be produced if the squared mass of the virtual q-$\bar{q}$ system is M_W^2. Therefore one finds for the production cross section the formula

$$\sigma(p\bar{p}\to W^- x) = \frac{\sqrt{2}\pi}{3}\, G_F \left(\frac{M_W^2}{s}\right) \int_{M_W^2/s}^{1} \frac{d\xi}{\xi}\, f_d(\xi)\, f_u\!\left(\frac{M_W^2}{\xi s}\right) \tag{II.82}$$

An analogous formula, involving now $f_d \otimes f_d$ and $f_u \otimes f_u$, applies for Z^o production.

Note that the result in Eq. (II.82) is just a function of M_W^2/s. As the energy increases, contrary to the naive expectation one may get by looking at this equation, the production cross section increases since the integral gets more contributions nearer to zero. I show in Fig. 10 predictions for the production cross section for W and Z bosons, calculated sometime ago by Paige /33/. These curves underestimate the actual production cross section by perhaps a factor of 1.5, which is certainly within the experimental and theoretical errors.

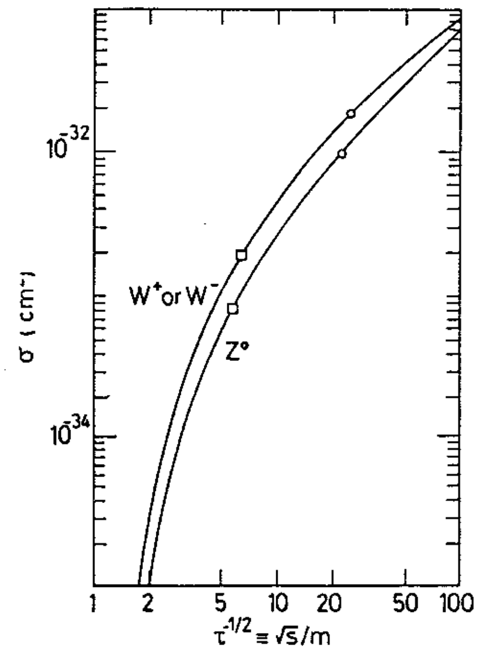

Fig. 10: W and Z production, from Ref. 33

The branching fractions of the W and Z^o into various channels is straightforward to compute from the Lagrangian (II.21):

$$\mathcal{L}_{WZ} = \frac{e}{2\sqrt{2}\sin\theta_W}\left[W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu}\right] + \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu J_{NC\mu}$$

Let me consider first W decays. The currents $J_\pm^\mu$ just involve pairwise each of the two members of the fermion doublets in the theory. If one neglects the masses of the quarks and leptons relative to the W *, then the branching fraction of W^- decays into $e\bar{\nu}_e$ is simply

$$B(W^-\to e\bar{\nu}_e) = \frac{\Gamma(W^-\to e^-\bar{\nu}_e)}{\Gamma(W^-\to \text{all})} \simeq \frac{1}{n_g[1+3]} \simeq 8.3\,\% \tag{II.83}$$

In the above $n_g = 3$ is the number of generations of quarks and leptons known and I have used that for each lepton pair there are three pairs of quarks, since quarks carry the additional color index. I will show in the next section that because of the SU(2) x U(1) symmetry breakdown, quarks of different generations mix with each other. However, because this mixing is caused by a unitary matrix the result (II.83) still applies. A very straightforward calculation gives for the actual rate

$$\Gamma(W^-\to e\bar{\nu}_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \simeq 260 \ \text{MeV} \tag{II.84}$$

where the numerical value follows by using for M_W a value of 83 GeV. Hence from (II.83) we expect the total width of the W to be around 3 GeV. Using the result (II.83) and the calculation of Paige /33/ for the production rate (see Fig. 10):

$$\sigma(W^+) + \sigma(W^-) \simeq 4\times 10^{-33}\ \text{cm}^2 = 4\ \text{nb} \tag{II.85}$$

one expects

$$\sigma\cdot B(W^\pm\to e\nu) \simeq 0.33\ \text{nb} \tag{II.86}$$

* This is reasonable except for the case of the top quark, whose mass is in the range of 40 GeV

The value quoted for this branching fraction by the UA1 and UA2 collaborations is /34/

$$\begin{aligned} \sigma \cdot B(W^{\pm} \to e\nu) &= (0.53 \pm 0.08 \pm 0.09)\ \mathrm{nb} \quad (\mathrm{UA1}) \\ \sigma \cdot B(W^{\pm} \to e\nu) &= (0.53 \pm 0.10 \pm 0.10)\ \mathrm{nb} \quad (\mathrm{UA2}) \end{aligned} \qquad \text{(II.87)}$$

which is slightly higher than (II.86).

For Z^0 decays one can proceed analogously. However, because of the structure of the neutral current there will be a difference in the branching fraction of the Z^0 to different fermion pairs. Since

$$J^{\mu}_{NC} = 2\left(J^{\mu}_{3} - \sin^2\theta_W J^{\mu}_{em}\right)$$

it is easy to see that the effective coupling of the Z^0 to the various fermions is given by (I write this for one generation, for brevity)

$$\mathcal{L}^{NC}_{int} = \frac{e}{2\cos\theta_W \sin\theta_W} Z^{\mu}\left\{ \bar{\nu}_e\left[\tfrac{1}{2}\gamma_\mu - \tfrac{1}{2}\gamma_\mu\gamma_5\right]\nu_e + \bar{e}\left[g_V\gamma_\mu + g_A\gamma_\mu\gamma_5\right]e \right.$$

$$\left. + \bar{u}\left[g^u_V\gamma_\mu + g^u_A\gamma_\mu\gamma_5\right]u + \bar{d}\left[g^d_V\gamma_\mu + g^d_A\gamma_\mu\gamma_5\right]d \right\} \qquad \text{(II.88)}$$

The various couplings g in Eq. (II.88) are detailed in Eqs. (II.55) and (II.79). In the partial rates the contributions of the vector and axial couplings contribute equally and there is no vector-axial interference term. This is because one can compute the decay of the Z^0 into states of given handedness, and obviously each of these two configurations will not interfere. The projections into given handedness configurations in (II.88) involve the couplings: $(g_V \pm g_A)$. Whence the total rate is proportional to $(g_V^2 + g_A^2)$. The rate $Z^0 \to \bar{\nu}_e \nu_e$ is easily scaled from Eq. (II.84). One finds

$$\Gamma(Z^0 \to \nu_e\bar{\nu}_e) = \frac{G_F M_Z^3}{6\sqrt{2}\,\pi}\left[\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2\right] = \frac{G_F M_Z^3}{12\sqrt{2}\,\pi} \qquad \text{(II.89)}$$

The relative branching ratios into the other channels can be read off from Eq. (II.88) and the formulas for the coupling constants in terms of the Weinberg angle. One finds, remembering the factor of 3 for color for the quarks,

$$\Gamma(Z^0 \to \bar{\nu}_e\nu_e) : \Gamma(Z^0 \to ee) : \Gamma(Z^0 \to \bar{u}u) : \Gamma(Z^0 \to \bar{d}d) =$$

$$2 : (1 - 4\sin^2\theta_W)^2 + 1 : 3\left\{(1 - \tfrac{8}{3}\sin^2\theta_W)^2 + 1\right\} : 3\left\{(1 - \tfrac{4}{3}\sin^2\theta_W)^2 + 1\right\} \qquad \text{(II.90)}$$

Using for the Z^0 mass $M_{Z^0} \simeq 94$ GeV, Eq. (II.89) gives a partial width of $Z^0 \to \bar{\nu}_e\nu_e$ of about 180 MeV. Using Eq. (II.90), extended to three generations and taking account of some kinematical suppression for the decay $Z^0 \to t\bar{t}$, gives a total Z^0 width of about 3 GeV - quite similar to that of the W. Since $\sin^2\theta_W \simeq \frac{1}{4}$, the width $\Gamma(Z^0 \to e^+e^-) \simeq 90$ MeV. Thus the branching fraction of Z^0 into lepton pairs is smaller than the corresponding branching ratio for the W's (Eq. II.83):

$$B(Z^0 \to e^+e^-) \simeq 3\ \% \qquad \text{(II.91)}$$

This result coupled with a somewhat smaller production cross section, as shown in Fig. 10, explains why the number of W's detected experimentally is much above that of the Z^0's.

The (V-A) nature of the coupling of the W to fermions predicts a correlation between the direction of the electrons (positrons) coming from W^- (W^+) decay and that of the proton (antiproton). Preferentially, electrons will be produced in the direction of the proton, positrons along that of the antiproton. This feature can be easily understood by focusing on the handedness of the particles involved in the process. Consider, for instance a W^- being produced in the CM system of $p\bar{p}$ by the fusion of a d and a $\bar{u}$. Because of the (V-A) interaction the collision occurs only if the d is left handed, as shown in Fig. 11a

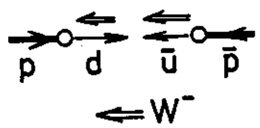

Fig. 11a: Production of W^- in $p\bar{p}$ collision

and the W^- is produced in a particular helicity state. Since the W^- decays into a left handed e^-_L, the direction of the e^- is correlated to that of the proton, as shown in Fig. 11b

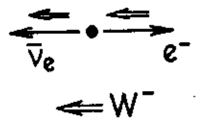

Fig. 11b: Decay correlation in W^- decay

In the rest frame of the W^-, the e^- angular distribution with respect to the direction of the proton beam should have a maximum for collinearly produced electrons and vanish in the backward direction. This is just the familiar $(1 + \cos\theta_{CM})^2$ distribution which I discussed before. The situation is precisely the same as that depicted in Fig. 6(a). I show in Fig. 12 the angular distribution for the electron emission angle in the rest frame of the W^-, determined by the UA1 collaboration /35/. As can be seen the data fits very well the $(1 + \cos\theta_{CM})^2$ distribution. This is positive evidence that the heavy particle seen in these experiments really has something to do with the mediator of the parity violating weak interactions.

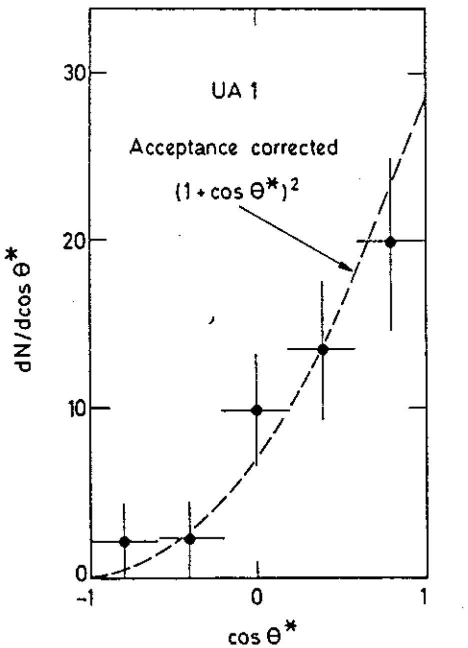

Fig. 12: Electron angular distribution in W^- decay, from Ref. 35.

The masses of the W and Z measured at the CERN collider by the UA1 and UA2 collaborations are: (These are averages of the results of the two experiments) /36/

$$\begin{aligned} M_W &= 82.2 \pm 1.8 \text{ GeV} \\ M_Z &= 93.2 \pm 1.5 \text{ GeV} \end{aligned} \tag{II.92}$$

The GSW model predicts (see Eqs. II.24 and II.27)

$$\begin{aligned} M_W &= \left[\frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_W}\right]^{1/2} \simeq \frac{37.28}{\sin\theta_W} \text{ GeV} \\ M_Z &= \frac{M_W}{\sqrt{\rho}\cos\theta_W} \end{aligned} \tag{II.93}$$

Taking $\sin^2\theta_W = 0.23$ and $\rho = 1$ gives

$$\begin{aligned} M_W &= 77.8 \text{ GeV} \\ M_Z &= 88.7 \text{ GeV} \end{aligned} \tag{II.94}$$

These numbers are in fairly good agreement with the experimental results (II.92), but perhaps a little on the low side. In fact, this is not totally unexpected. The results given in Eq. (II.93) are computed <u>without</u> worrying about possible radiative corrections. Because the GSW model is a renormalizable theory, radiative effects are computable and in general change the lowest order predictions by corrections of $O(\alpha)$. For the case of the relations in Eq. (II.93), however, it turns out that the change, although of $O(\alpha)$, is numerically big. Roughly speaking, this is because all the parameters that enter in Eq. (II.93) for the W and Z masses (α, G_F, $\sin^2\theta_W$ and ρ) are determined at very low energy scales. The radiative corrections to Eq. (II.93) contain logarithmic terms of $O(\alpha \ln M_W^2/\langle q^2\rangle)$ where $\langle q^2\rangle$ is this typical mass scale.

I quote below the predicted values for the W and Z^o masses in the GSW model, including radiative corrections /37/:

$$\begin{aligned} M_W(\text{Theory}) &= 83 \pm 2.5 \text{ GeV} \\ M_Z(\text{Theory}) &= 93.8 \pm 2.5 \text{ GeV} \end{aligned} \tag{II.95}$$

These values are in very good agreement with the collider results. The rather large error in the theoretical numbers is mostly a reflection of the experimental error in determining $\sin^2\theta_W$. An error in $\sin^2\theta_W$, $\Delta\sin^2\theta_W = \pm 0.006$ is reflected in a 1 GeV uncertainty in M_W. Indeed, the value of the W mass found at the collider provides, at the moment, the most accurate determination of $\sin^2\theta_W$!

III. Structural and Open Problems of the GSW Model

Undoubtedly, as the discussion of the previous section showed, the GSW model is phenomenologically very successful. There are, however, parts of the model which are unsatisfactory. The part of the model which is really tested experimentally concerns the fermion-gauge sector, given by the interaction Lagrangian of Eq. (II.21). What is untested and, as I shall discuss, in some ways theoretically troubling is the whole symmetry breaking sector of the theory. To be sure, the idea that $SU(2) \times U(1) \rightarrow U(1)_{em}$ has received dramatic confirmation at the collider, with the discovery of the W and Z with the predicted properties. However, there is no evidence yet for a scalar field - the Higgs field - which is supposed to be associated with the breakdown.

Recall that in the GSW model, to trigger the breakdown of $SU(2) \times U(1) \rightarrow U(1)_{em}$, it was necessary to add scalar fields with an appropriate self interaction. To reproduce the experimentally successful prediction that $\rho = 1$, these scalar fields had to be doublets under SU(2), and the simplest possibility is thus to add just one complex doublet Φ. The potential

$$V = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 \tag{III.1}$$

forces Φ to have a non zero vacuum expectation value

$$\langle\Phi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}v \\ 0 \end{pmatrix} \tag{III.2}$$

and leads to the breakdown $SU(2) \times U(1) \rightarrow U(1)_{em}$. Of the four real fields in Φ, three are really absorbed to give the longitudinal degrees of freedom of the W and the Z - necessary for massive spin 1 particles. There remains, however, one excitation left over, which will be associated with a massive spin zero state - the Higgs boson.

The irrelevant fields in Φ are those that would correspond to Goldstone excitations, if the $SU(2) \times U(1)$ symmetry had not been gauged. They may be eliminated from the theory by adopting the same kind of exponential parametrization for Φ as that discussed in Section I, in connection with the U(1) model (cf. Eq. (I.86)). Let me write

$$\Phi(x) = e^{i\vec{\tau}\cdot\vec{\xi}(x)/v}\begin{pmatrix} \frac{1}{\sqrt{2}}(v + H(x)) \\ 0 \end{pmatrix} \tag{III.3}$$

Obviously all dependence on the three fields $\vec{\xi}(x)$ disappear in V. Furthermore, one can show that $\vec{\xi}$ also can be eliminated from the covariant-derivative term in $\mathcal{L}^{H-g}_{GSW}$ of Eq. (II.33) - basically by picking a definite gauge for the $\vec{W}_\mu$ fields, essentially in the same way as it was done in Sect. I. After this gauge choice, the Lagrangian for the scalar field will only contain H. A simple calculation gives

$$\begin{aligned}\mathcal{L}^{H-g}_{GSW} = & -\frac{1}{2}\partial^\mu H\,\partial_\mu H - \lambda\left(vH + \frac{1}{2}H^2\right)^2 \\ & -\frac{1}{4}g^2(v+H)^2 W_+^\mu W_{-\mu} - \frac{1}{8}(g^2+g'^2)(v+H)^2 Z^\mu Z_\mu\end{aligned} \tag{III.4}$$

where of course the coupling constants g and g' are related to e and $\sin^2\theta_W$ by Eq. (II.20).

From (III.4) certain properties of the Higgs field H can be immediately read off. The quadratic term in the field H in Eq. (III.4) identifies the mass of H as

$$m_H^2 = 2\lambda v^2 \tag{III.5}$$

Although one knows the magnitude of v - the Fermi scale - since it sets the scale of the weak interactions (cf Eq. II.37):

$$v = (\sqrt{2}\, G_F)^{-1/2} \simeq 250 \text{ GeV}, \tag{III.6}$$

the mass of the Higgs field is arbitrary since it is proportional to the coupling constant λ, of which one has no information yet. Putting it the other way around, once H is discovered (if it exists!), then Eq. (III.5) will give a measure of the scalar field self coupling .

There are both trilinear and quadrilinear couplings of the field H to the W and Z bosons. I give below the trilinear couplings, which can be directly read off Eq. (III.4). These couplings, I note, are proportional to the mass of the gauge bosons:

$$\mathcal{L}_{HZZ} = -\frac{e}{\sin 2\theta_W} M_Z Z^\mu Z_\mu H \qquad \text{(III.7)}$$

$$\mathcal{L}_{HWW} = -\frac{e}{\sin \theta_W} M_W W_+^\mu W_{-\mu} H$$

If the mass of the Higgs boson is lighter than that of the W or Z then these particles can decay into a Higgs plus a lepton pair, by virtue of the couplings given in (III.7). These processes occur through the graphs shown in Fig. 13, where the virtual W or Z then transmute themselves into lepton pairs.

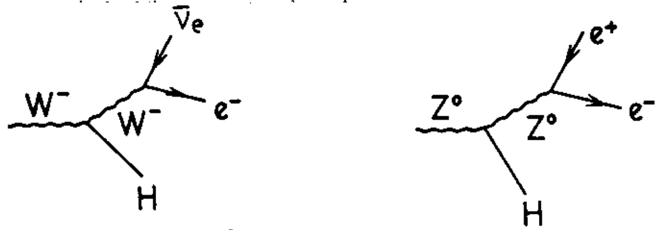

Fig. 13: Decays of a W or Z^0 into a Higgs boson plus a lepton pair.

The actual magnitude of these decays is very small, however. Bjorken /38/ estimates that for a 20 GeV Higgs boson

$$\frac{\Gamma(Z^0 \to H\mu^+\mu^-)}{\Gamma(Z^0 \to \mu^+\mu^-)} \simeq 8 \times 10^{-4} \qquad \text{(III.8)}$$

Since the branching ratio of the Z^0 into μ-pairs is only 3 %, one sees that the detection of the Higgs boson at the collider, as a byproduct of W or Z decay, is essentially hopeless. Furthermore, since λ is arbitrary it could also well be that the Higgs boson mass is above that of the gauge bosons. Clearly, the Higgs boson appears rather elusive!

The introduction of the doublet field Φ in the GSW model, which precipitates the spontaneous breakdown of SU(2) x U(1) into $U(1)_{em}$, has a further utility. It can be

used to generate masses for the quarks and leptons! Recall that explicit mass terms for the fundamental fermionic fields were forbidden in the GSW model, since the left and right handed fields transform differently under SU(2) x U(1). However, couplings between Φ and two fermion fields can be built, which are SU(2) x U(1) invariant. When Φ acquires a vacuum expectation value, these couplings are the source of the mass terms for the fermions.

Let me illustrate this for the specific case of the u-quark. The Yukawa interaction

$$\mathcal{L}_{Yukawa} = -h\left\{ (\bar{u}\,\bar{d})_L \begin{pmatrix}\phi^0\\ \phi^-\end{pmatrix} u_R + \bar{u}_R (\phi^{0*}\ \phi^+) \begin{pmatrix}u\\ d\end{pmatrix}_L \right\} \qquad \text{(III.9)}$$

is obviously SU(2) invariant. It is U(1) invariant also, since $Y_u = 2/3$, $Y_\Phi = -1/2$, $Y_{\binom{u}{d}_L} = 1/6$. Clearly, when one reparametrizes Φ as in Eq. (III.3) and absorbs the unphysical Goldstone fields $\vec{\xi}$ in a redefinition of the $\binom{u}{d}_L$ doublet *:

$$\begin{pmatrix}u\\ d\end{pmatrix}_L \to e^{i\vec{\tau}\cdot\vec{\xi}/v} \begin{pmatrix}u\\ d\end{pmatrix}_L \equiv \begin{pmatrix}u\\ d\end{pmatrix}_L \qquad \text{(III.10)}$$

the Yukawa Lagrangian contains a mass for the u quark:

$$\begin{aligned}\mathcal{L}_{Yukawa} &= -\frac{h}{\sqrt{2}}\left[\bar{u}_L u_R + \bar{u}_R u_L\right](v+H) \\ &= -m_u \bar{u}u - \frac{m_u}{v}\bar{u}uH\end{aligned} \qquad \text{(III.11)}$$

where

$$m_u = \frac{h}{\sqrt{2}} v \qquad \text{(III.12)}$$

* To simplify the notation, I use the same symbols for the redefined fields in Eq. (III.10)

The u quark mass is proportional to the Fermi scale and arises only because SU(2)xU(1) is broken down spontaneously. Because the Yukawa coupling h is unknown, Eq. (III.12) is not a prediction for the quark mass. Rather, the quark mass parametrizes the Yukawa coupling. This is an analogous situation to what happened for the Higgs field. Note that the Higgs field through the Yukawa interaction now couples to the u quarks, with a coupling which is proportional to the mass of the u quark.

One can generalize this analysis so as to give masses to all quarks and leptons in the GSW model. Beside Φ, which has $Y=-1/2$ one also needs another doublet that has $Y = +1/2$. This can be constructed from Φ itself. One can check that the charge-conjugate field $\tilde{\Phi}$ defined by

$$\tilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi^+ \\ -\phi^{0*} \end{pmatrix} \qquad \text{(III.13)}$$

is an SU(2) doublet and has $Y = 1/2$. Just as $\langle\Phi\rangle$ gave mass to the u quark, $\langle\tilde{\Phi}\rangle$ will give mass to the lower members of the fermion doublets. It is clear, however, that none of the fermion masses will be predicted since they will all depend on unknown Yukawa couplings – the analog of Eq. (III.12).

Although the fermion mass generating mechanism I outlined is full of arbitrariness, and as I shall discuss shortly is one of the mysteries to be solved, it does make an interesting structural prediction. Namely, that as a result of the SU(2) x U(1) breakdown, the currents $J^{\mu}_{\pm}$ which couple to the W bosons are not generation diagonal for the quarks. The neutral currents, however, remain diagonal. To understand this point I need a small amount of notation. Let

$$Q_{iL} = \left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L ; \begin{pmatrix} c \\ s \end{pmatrix}_L ; \begin{pmatrix} t \\ b \end{pmatrix}_L ; \cdots \right\} \quad ; \quad L_{iL} = \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ; \cdots \right\}$$

$$u_{iR} = \{ u_R ; c_R ; t_R ; \cdots \} \quad ; \quad d_{iR} = \{ d_R ; s_R ; b_R ; \cdots \}$$

$$\ell_{iR} = \{ e_R ; \mu_R ; \tau_R ; \cdots \} \qquad \text{(III.14)}$$

(I will not introduce any right-handed neutrino fields, since neutrinos are massless*).

* Even if neutrinos had a small mass, it is so much smaller than that of the charged fermions that it can be neglected for the purposes of this discussion.

Then the most general SU(2) x U(1) invariant coupling one can write involving these fields is

$$\mathcal{L}_{\text{Yukawa}} = -\left\{ h^u_{ij}\, \bar{Q}_{iL} \Phi\, u_{jR} - h^d_{ij}\, \bar{Q}_{iL} \tilde{\Phi}\, d_{jR} - h^\ell_{ij}\, \bar{L}_{iL} \tilde{\Phi}\, \ell_{jR} + \text{h.c.} \right\} \qquad \text{(III.15)}$$

The coupling constants h^f_{ij} with $f = \{u, d, \ell\}$ do not need to be generation diagonal. The spontaneous breakdown of SU(2) x U(1) will generate mass matrices for the fermions. These matrices will only connect fields of the same charge, but otherwise they are arbitrary, since the couplings in (III.15) are. It is easy to convince oneself that for each charged fermion species:

$$M^f_{ij} = \frac{1}{\sqrt{2}}\, h^f_{ij}\, v \qquad \text{(III.16)}$$

Obviously, if the matrices M^f are not diagonal one must make a basis change to deal with states which are diagonal in mass. This basis change will in general cause mixing among generations in the fermionic currents of the GSW model. It is easy to check that only the quark charged currents are affected by this mixing. To prove this assertion let me organize all the charged fermions of a given type f, $f = \{u, d, e\}$, into column vectors ψ^f_L and ψ^f_R. For instance:

$$\psi^u_L = \begin{pmatrix} u_L \\ c_L \\ t_L \\ \vdots \end{pmatrix}$$

The matrices M^f can always be diagonalized by bi-unitary transformations

$$(U^f_L)^\dagger M^f U^f_R = M^f_{\text{diag}}$$

where $U^f_L = U^f_R$ if M^f is Hermitean. This diagonalization is effected by making a basis change in the fermion fields

$$\psi^f_L \to U^f_L\, \psi^f_L \quad ; \quad \psi^f_R \to U^f_R\, \psi^f_R \qquad \text{(III.17)}$$

This transformation will not affect the neutral currents since these are charge diagonal and $U^\dagger U = 1$. For the charged currents, on the other hand, one has two different unitary matrices entering and there will be a non zero generation mixing.

Consider, for instance, J_-^μ. In terms of the compact notation I am using, before the basis change

$$J_-^\mu = 2\left[\bar\psi^u_L \gamma^\mu \psi^d_L + \bar\psi^\nu_L \gamma^\mu \psi^e_L\right]$$

where $\psi^\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \vdots \end{pmatrix}$. After the basis change J_-^μ becomes:

$$J_-^\mu = 2\left[\bar\psi^u_L \gamma^\mu (U^u_L)^\dagger (U^d_L)\psi^d_L + \bar\psi^\nu_L \gamma^\mu (U^e_L)\psi^e_L\right] \qquad \text{(III.18)}$$

Hence there is a general mixing matrix $\tilde C = (U^u_L)^\dagger(U^d_L)$ between the quarks of different generations and a matrix U^ℓ_L in the lepton sector. The effect of U^ℓ_L, however, is illusory. Since the neutrinos are massless, any linear combination of neutrino fields is still a neutrino field. The redefinition $\psi^\nu_L \rightarrow U^e_L \psi^\nu_L$ gets rid of the matrix U^ℓ_L in Eq. (III.18) and defines the physical ν_ℓ field as that which is coupled in the weak current to the charged lepton ℓ.

There is physics, however, in the matrix $\tilde C$. For n generations of quarks $\tilde C$ is a unitary n x n matrix, which is characterized by $\frac{n(n-1)}{2}$ real angles an $\frac{n(n+1)}{2}$ phases. Not all these phases are physical since one can rotate (2n-1) of the phases away by redefinition of the quarks fields ψ^u_L and ψ^d_L, still retaining the mass matrices M^u and M^d diagonal *. Let me call C the matrix obtained from $\tilde C$ after these redefinitions. Then C has $\frac{n(n-1)}{2}$ real angles and 1/2(n-1)(n-2) phases. The matrix C is called the Cabibbo-Kobayashi-Maskawa matrix. For the case of three generations of quarks, it is characterized by three angles θ_1, θ_2 and θ_3 and a phase δ. These parameters can be measured experimentally by looking at charged current weak interaction processes involving the quarks of the 2nd and 3rd generation. The presence of a non zero phase δ allows for the violation of CP in the GSW model.

Introducing Yukawa couplings in the GSW model allows for mass generation for the fermions and predicts that in the quark sector there should be mixing in the charged

* 2n-1 not 2n since an overall phase has no meaning.

currents. Although this is observed, as the Cabibbo Kobayashi Maskawa matrix is non trivial experimentally, one is left in a rather unsatisfactory situation. All the quark and lepton masses are input parameters in the model, as are the angles θ, θ_1, θ_3 and δ. In principle, all these parameters are arbitrary, since they are related to the arbitrary Yukawa couplings of Eq. (III.15). In practice, however, the pattern of the fermion masses is so bizarre, and the amount of mixing so "obviously" related to the mass pattern, that one is naturally led to ask what underlying physics determines all of this. To answer these questions one must go beyond the GSW model, since in the model, as it is presently formulated, any pattern of masses and mixing are allowed. This conundrum is not a theoretical inconsistency but a mystery, whose resolution will require new physics. A probably related mystery of the GSW model is the question of why are there generations of quarks and leptons, anyway? The number of families is, in principle, a free parameter in the model and there is no understanding of why there are (apparently) only three in nature.

I would like to quantify these statements a bit by detailing some of the information one has on the fundamental fermions. As far as one knows, neutrinos are massless, although positive evidence for $m_{\nu_e} \simeq 30$ eV has recently been reported /39/. The particle data book /40/ gives the bounds:

$$m_{\nu_e} \lesssim 46\ \text{eV} \quad ; \quad m_{\nu_\mu} \leq 500\ \text{KeV} \quad ; \quad m_{\nu_\tau} \leq 164\ \text{MeV} \qquad \text{(III.19a)}$$

The charged leptons have the masses /40/:

$$m_e = 0.511\ \text{MeV} \ ; \quad m_\mu = 105.66\ \text{MeV} \ ; \quad m_\tau = 1.784\ \text{GeV} \qquad \text{(III.19b)}$$

The masses of the quarks are inferentially determined. For the light quarks using current algebra techniques and for the heavy quarks from the study of quarkonia /41/. The top quark mass has very recently been established from W decay ($W^+ \rightarrow t\bar b$) at the collider, by trying to reconstruct this decay process /42/. One finds

$$\begin{array}{lll} m_u \simeq 5\ \text{MeV} & m_c \simeq 1.5\ \text{GeV} & m_t \simeq 40\ \text{GeV} \\ m_d \simeq 10\ \text{MeV} & m_s \simeq 200\ \text{MeV} & m_b \simeq 5\ \text{GeV} \end{array} \qquad \text{(III.19c)}$$

The elements of the Cabibbo Kobayashi Maskawa matrix are equally "random", although the randomness appears to be definitely correlated with the large mass splittings. Essentially the mixing becomes less and less as the quarks get more separate in mass and as they get heavier. For instance, from a comparison of μ decay and nuclear β decay, including radiative corrections, one infers a small departure from unity for

C_{ud} /43/

$$|C_{ud}| = 0.9737 \pm 0.0025 \tag{III.20a}$$

A new careful study of hyperon and kaon decays yields /44/

$$|C_{us}| = 0.221 \pm 0.002 \tag{III.20b}$$

From the shape of the lepton spectra in decays of B mesons /45/ and from new measurements of their lifetimes /46/ one infers

$$|C_{ub}| \lesssim 0.005 \tag{III.20c}$$

and

$$|C_{cb}| = 0.044 \pm 0.005 \tag{III.20d}$$

It is hard to believe that all these numbers should be taken as free parameters. The introduction of the Yukawa couplings in Eq. (III.15) mechanically makes it possible for masses and mixing to arise, once SU(2) x U(1) is broken down. It fails to explain, however, at the basic level why they really arise. I should note an incidental result, which may be of some phenomenological importance. If the fermion do get their mass from the Lagrangian (III.15), one has an immediate prediction of how the Higgs field H couples to fermions. Since, according to Eq. (III.3), the Higgs field H enters in Φ always in the combination (v+H), it is clear that the basis change which diagonalizes the fermion mass matrices (III.16) will also diagonalize the coupling of H to the fermions. Hence one predicts that H couples diagonally to fermions with a coupling proportional to the fermion mass:

$$\mathcal{L}_{Hff} = - \frac{m_f}{v} \bar{f} f H \tag{III.21}$$

where f is any of the fermions in the theory. According to this equation, therefore one expects the Higgs boson to decay to the heaviest pair of fermions kinematically allowed. Unfortunately Eq. (III.21) also suggests that direct production of the Higgs boson by fusion of light quarks in $p\bar{p}$ collisions, or directly in e^+e^- collisions, will be greatly hindered.

The symmetry breakdown caused by the Higgs potential (III.1) gives masses to all the fundamental fields in the GSW model in terms of the scale v of the breakdown. Of

course, in the fermion sector, since one knows the masses of the fermions, one prefers instead to eliminate the unknown Yukawa couplings and replace them by $^{m}f/_{v}$, e.g. as done in Eq. (III.21) Nevertheless, if there were no symmetry breakdown, all particles would be massless in the theory. Since v is the only mass parameter in the model, it is certainly silly to ask what sets its scale to approximately 250 GeV. It has to be obviously an input parameter. However, if one imagines that the GSW model is not a complete theory, the question of the scale of v becomes meaningful.

Wilson /46/ was one of the earliest to remark that a fixed value for v $\simeq$ 250 GeV, is not particularly natural. His arguments can be well understood by focusing on the Higgs mass and working in a theory with a cutoff. The cutoff represents the scale at which physics beyond the GSW model emerges. The value of the Higgs mass, given in Eq. (III.5), gets shifted by radiative corrections due to the interaction of the Higgs field with itself and with the other fields in the theory (gauge bosons and fermions). These radiative shifts are quadratically divergent and in a theory without a physical cutoff have to be renormalized away. That is, one has to imput the Higgs value as a parameter. Here, however, one wants to see what influence the underlying theory has and one ought to compare these effects to the lowest order value. The Higgs mass, including the radiative effects, is schematically:

$$m_H^2 = 2\lambda v^2 + \alpha \Lambda^2 \tag{III.22}$$

where Λ is the cut off. Clearly if one wants the Higgs mass to be light with respect to the cut off, one has to adjust the initial value carefully to effect a cancellation. Saying it another way, the theory is unstable under perturbations. There is no relation between the input and output parameters that one can control.

To restore naturalness in the theory there are clearly two options, which in fact can be read off from Eq. (III.22):

Option (1): The cut off Λ could be so small that indeed the radiative corrections are small corrections. But this means that the cut off is really of the same size as the mass of the Higgs boson - or the Fermi scale. Clearly this can only happen if the Higgs boson is not elementary but composite! There is no elementary Higgs field, but some underlying strong interaction theory which in some way triggers the spontaneous breakdown of SU(2) x U(1).

Option (2): There are extra interactions in the theory which force cancellations in the radiative corrections, so that these corrections are no longer quadratically dependent on Λ but only dependent logarithmically on Λ'. This means that the final value of m_H^2 depends on the initial value. Obviously, unless there is some symmetry reason it will be impossible to obtain the desired cancellation. Naturalness is restored because the corrections are protected by some symmetry.

Attempts to enlarge the GSW theory with the view of making the Higgs sector of the theory natural have been vigorously pursued in the last few years and there exists an ever expanding literature on the subject. Suggestions following the first option above go under the generic name of Technicolor /47/ and attempt to find a nonperturbative, strong coupling, scenario to understand the breakdown of SU(2) x U(1) dynamically. The followers of the second option, instead, introduce a Bose-Fermion symmetry - supersymmetry - to stabilize the GSW model perturbatively. As supersymmetry is the subject of J. Ellis lectures in this school, I shall not discuss in any detail how it stabilizes the theory. Suffice it to say that given the complete symmetry between bosons and fermions, there is cancellation among the graphs because fermions obey the Pauli principle and introduce appropriate factors of (-1). Rather, I shall discuss a bit here how Technicolor theories are supposed to work and tie this in with the even more speculative idea that the fundamental excitations of the GSW model are themselves composite.

I begin my discussion of Technicolor - or dynamical symmetry breakdown - by considering a numerically unrealistic, but physically cogent, source for the masses of the W and Z bosons, which has to do with the strong interactions of quarks. The quarks interact among themselves strongly through their color interactions. These interactions are also governed by a gauge theory, based on the group SU(3). This theory, Quantum Chromodynamics (QCD) is amply discussed in the lectures of Quigg and Brodsky at this school. So I shall only focus on the few properties of QCD which I need for my purposes here. QCD, in contrast to the electroweak theory, suffers no spontaneous breakdown. Because the gauge group is unbroken the theory confines. Thus, one does not see the elementary excitations of QCD, the quarks and the gauge bosons (gluons), in the spectrum but only their bound states, the hadrons.

In the limit in which one neglects the masses of the quarks, QCD has a large global symmetry group. One can rotate all the left handed quark fields - of different flavors: u,d,s,... - among themselves and all right-handed quark fields among themselves and the QCD Lagrangian remains the same. This is easy to see. The QCD Lagrangian is, neglecting quark masses

$$\mathcal{L}_{QCD} = -\sum_{\text{flavors}} \bar{q}^f \left(\gamma^\mu \frac{1}{i} D_\mu\right) q^f - \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} \qquad \text{(III.23)}$$

where $F_a^{\mu\nu}$ is the gluon field strength and D_μ is the appropriate SU(3) covariant derivative for the quarks. Since the interaction is vectorial, the kinetic energy term is just the sum of LL and RR contributions

$$\sum_f \bar{q}^f \gamma^\mu \frac{1}{i} D_\mu q^f = \sum_f \bar{q}^f_L \gamma^\mu \frac{1}{i} D_\mu q^f_L + \sum_f \bar{q}^f_R \gamma^\mu \frac{1}{i} D_\mu q^f_R$$

and this is obviously invariant under the $SU(n_f)_L \times SU(n_f)_R$ transformations

$$q^f_L \to U_L^{ff'} q^{f'}_L \;. \qquad\qquad q^f_R \to U_R^{ff'} q^{f'}_R \qquad \text{(III.24)}$$

The dynamical scale Λ_{QCD}, associated with the energy scale at which the binding of the quarks and gluons into hadrons happens, is of the order of several hundred MeV. On this scale, only the masses of the u and d quarks are negligible. So, as far as the strong interaction dynamics goes, only $SU(2)_L \times SU(2)_R$ is a good (approximate) symmetry. For the heavier quarks, the neglect of their masses is a bad dynamical approximation, so the transformation in (II.24) is not even (approximately) an invariance of the theory *. Whenever there is a global symmetry, as I discussed in Section I, there is either a Wigner Weyl realization or a Nambu-Goldstone realization, depending on whether there are degenerate multiplets or Goldstone bosons. The spectrum of hadrons one knows shows no multiplet structure corresponding to an $SU(2)_L \times SU(2)_R$ symmetry, but there is a clear (approximate) isospin symmetry. The isospin symmetry corresponds to the diagonal subgroup of $SU(2)_L \times SU(2)_R$. That is, one rotates u and d quarks into each other the same way for both left and right helicities. Furthermore the π -mesons appear to be considerably lighter than any other hadronic excitations. This pattern suggests that the global $SU(2)_L \times SU(2)_R$ symmetry in QCD is broken down to $SU(2)_{L+R}$. As a result of the breakdown $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ one expects to have three Goldstone bosons, associated with the generators of $SU(2)_{L-R}$, which do not annihilate the vacuum. These Goldstone bosons are the pions, which in reality acquire a small mass, since m_u and m_d are not strictly zero.

One may reasonably ask, what this nice piece of physics has to do with the W and Z^o masses? To answer this I must add one piece of dynamical information. The breakdown of $SU(2)_L \times SU(2)_R$ in QCD happens because the QCD vacuum allows the formation of $SU(2)_L \times SU(2)_R$ variant condensates. Although the color singlet combinations $\bar{u}_i(x)u_i(x)$, $\bar{d}_i(x)d_i(x)$ (Here i is a color index i = 1,2,3) transform non trivially under $SU(2)_L \times SU(2)_R$ they have a non zero vacuum expectation value

* For the strange quark, s, the situation is marginal.

$$\langle \bar{u}(x)u(x) \rangle = \langle \bar{d}(x)d(x) \rangle \neq 0 \qquad \text{(III.25)}$$

This is analogous to what happens in superconductivity. Although fermion number is a good symmetry of the theory, there exist Cooper pairs in the BCS ground state - which carry fermion number two. The dynamical condensates in (III.25) are the precise analogue of the Cooper pairs in a superconductor. The existence of these non-zero vacuum expectation values is what causes the breakdown of:

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$$

and the formation of the Goldstone pions.

I am now ready to return to weak interactions. The condensates in (III.25) not only break the global $SU(2)_L \times SU(2)_R$ symmetry. If weak interactions are given by the GSW $SU(2) \times U(1)$ model, clearly the condensates in (III.25) will also cause a spontaneous breakdown of the local $SU(2) \times U(1)$ symmetry. Indeed $\bar{u}u$ and $\bar{d}d$ carry both $SU(2)$ and $U(1)$ quantum numbers, but not charge. Hence their condensates will break $SU(2) \times U(1) \rightarrow U(1)_{em}$. The breakdown of a local symmetry always gives mass to the gauge fields coupled to the broken generators. Therefore the $W^\pm$ and the Z^0 will get mass. Of course, since the condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$ are typically of the scale given by the strong interactions $O(\Lambda^3_{QCD})$, this mechanism provides a tiny W and Z mass. Furthermore, there is the further drawback that the "Goldstone" pions - which do exist in the real world! - would have to disappear to give the longitudinal degrees of freedom for the gauge bosons.

Despite these obvious drawbacks, it is instructive to compute the expected W and Z masses from pure QCD effects, as Technicolor /47/ is just a scaled version of this analysis. In section I, I demonstrated the Higgs mechanism of mass generation at the Lagrangian level. The existence of a nontrivial vacuum expectation value $\langle \phi \rangle$ implies that "seagull" term $-g^2 \phi^\dagger \phi A^\mu A_\mu$ generates a mass for the gauge field A^μ : $m_A^2 = 2g^2 \langle \phi^\dagger \rangle \langle \phi \rangle$ Here there is no scalar fields coupled to the $SU(2) \times U(1)$ gauge fields and what gets vacuum expectation value are the composite operators $\bar{u}_i(x)u_i(x)$, $\bar{d}_i(x)d_i(x)$. How does one go about computing a mass? The key to the answer to this question are the Goldstone pions. The existence of these massless particles causes a non trivial shift in the propagators of the gauge fields, which results in a mass term.

Let's see this in detail. The propagator for a gauge field is proportional to $\frac{1}{q^2}$. This is familiar from QED, and just reflects the fact that the gauge field is massless. In the presence of interactions, the propagator structure changes. For instance, the vacuum polarization graphs shown in Fig. 14, changes the photon propagator in QED to - for $q^2 \gg m_e^2$:

$$D_{QED}(q^2) = \frac{1}{q^2\left[1 - \frac{3\alpha}{\pi} \ln q^2/m_e^2\right]} \qquad \text{(III.26)}$$

Fig. 14: Vacuum polarization graphs in QED

The effect of interactions, as seen in Eq. (III.26), is to give an overall change in the denominator structure of the gauge propagator. However, gauge invariance, still demands that this denominator structure be proportional to q^2. In general, therefore, including interactions one expects for gauge propagators the replacement

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2\left[1 + \Pi(q^2)\right]} \qquad \text{(III.27)}$$

This equation makes it clear how a gauge field can acquire a mass when there is spontaneous symmetry breaking. In these cases there are zero mass particles in the theory to which the gauge field can couple (through the currents of the theory) - the Goldstone bosons. The contribution of $\Pi(q^2)$, because the Goldstone bosons have zero mass, is proportional to $\frac{1}{q^2}$, and this gives a finite mass shift in Eq. (III.27).

I want to compute $\Pi(q^2)$ in QCD. For that purpose I need to know how do the weak isospin current J_i^μ and the weak hypercharge current J_Y^μ couple to the Goldstone pions. According to Eq. (II.14), focusing only on the quark piece, one has

$$J_i^\mu = (\bar{u}\,\bar{d})_L \gamma^\mu \frac{\tau_i}{2} \begin{pmatrix} u \\ d \end{pmatrix}_L = \frac{1}{4} (\bar{u}\,\bar{d}) \gamma^\mu (1-\gamma_5) \tau_i \begin{pmatrix} u \\ d \end{pmatrix} \qquad \text{(III.28)}$$

$$J_Y^\mu = \frac{1}{6} (\bar{u}\,\bar{d})_L \gamma^\mu \begin{pmatrix} u \\ d \end{pmatrix}_L + \frac{2}{3} \bar{u}_R \gamma^\mu u_R - \frac{1}{3} \bar{d}_R \gamma^\mu d_R$$

$$= \frac{5}{12} \bar{u} \gamma^\mu u - \frac{1}{12} \bar{d} \gamma^\mu d + \frac{1}{4} (\bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d) \qquad \text{(III.29)}$$

The pions are the Goldstone bosons of the breakdown of $SU(2)_L \times SU(2)_R$ into $SU(2)_{L+R}$. Hence the matrix element of the broken $SU(2)_{L-R}$ currents between the pions and the vacuum is non zero (cf. Eq. I.29):

$$\langle 0 | (J^\mu_{L-R})_i | \pi_j ; p \rangle = i f_\pi p^\mu \delta_{ij} \tag{III.30}$$

The $(J^\mu_{L-R})_i$ currents of QCD are easily computed. They are precisely identical with the purely axial vector piece of the weak SU(2) currents. That is for the QCD Lagrangian

$$(J^\mu_{L,R})_i = (\bar u \; \bar d)_{L,R} \, \gamma^\mu \frac{\tau_i}{2} \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \tag{III.31}$$

Hence

$$(J^\mu_{L-R})_i = -\frac{1}{2} (\bar u \; \bar d) \, \gamma^\mu \gamma_5 \tau_i \begin{pmatrix} u \\ d \end{pmatrix} \tag{III.32}$$

From these equations one can compute the matrix element of the weak currents between the vacuum and the appropriate one pion states:

$$\begin{aligned} \langle 0 | J^\mu_i | \pi_j ; p \rangle &= i \left(\tfrac{1}{2} f_\pi \right) p^\mu \delta_{ij} \\ \langle 0 | J^\mu_Y | \pi_3 ; p \rangle &= i \left(-\tfrac{1}{2} f_\pi \right) p^\mu \end{aligned} \tag{III.33}$$

I note in particular that $J^\mu_{em} = J^\mu_3 + J^\mu_Y$ has no matrix element between the pions and the vacuum.

The masses of the W and Z bosons arise because the contribution of the vacuum polarization tensor is singular due to the presence of the massless pions. I show this schematically in Fig. 15

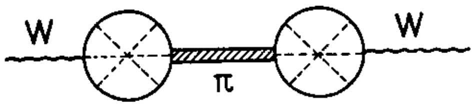

Fig. 15: Origin of the $\frac{1}{q^2}$ terms in the vacuum polarization $\Pi(q^2)$ due to the one pion intermediate states

For the Z bosons, for example, according to Eq. (II.21), the relevant coupling is

$$\mathcal{L}_{int} = \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu J^{NC}_\mu = \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu \left(J_{3\mu} - \sin^2\theta_W J_{em\,\mu} \right) \tag{III.34}$$

For the graph of Fig. 15, the $J_{em\,\mu}$ term does not contribute since this term has no coupling to the Goldstone pion. The $J_{3\mu}$ term, however, contributes a mass shift

$$(M_Z^2)_{QCD} = \left[\frac{e}{\cos\theta_W \sin\theta_W} \cdot \frac{1}{2} f_\pi \right]^2 = \left[\frac{g f_\pi}{2\cos\theta_W} \right]^2 \tag{III.35}$$

A similar calculation for the $W^\pm$ bosons gives the same answer except for the $\cos\theta_W$ factor, since one has a coupling for i = 1,2

$$\mathcal{L}_{int} = g W^\mu_i J_{i\mu} \tag{III.36}$$

Thus

$$(M_W^2)_{QCD} = \left[\frac{g f_\pi}{2} \right]^2 \tag{III.37}$$

Remarkably, QCD predicts the same ratio for the W and Z masses as the GSW model with one Higgs doublet

$$\frac{(M_W^2)_{QCD}}{(M_Z^2)_{QCD} \cos^2\theta_W} = 1 \tag{III.38}$$

The actual value for these masses is, however, ridiculously small. The parameter f_π can be calculated from the lifetime of the pions, since the matrix element (III.33) is precisely that relevant for the two body pion decay $\pi \to \mu\nu$. From this calculation one obtains

$$f_\pi \simeq 95 \text{ MeV} \tag{III.39}$$

Hence

$$(M_W)_{QCD} = \frac{e f_\pi}{2\sin\theta_W} \simeq 30 \text{ MeV} \tag{III.40}$$

The Technicolor idea /47/ developed by Susskind and Weinberg, uses the dynamical ingredients of the above calculation to generate an "appropriate" breakdown of SU(2) x U(1). Eq. (III.37) fails in obtaining the observed W mass because the parameter f_π is too small. Indeed f_π has a scale characteristic of that of QCD: $f_\pi \sim \Lambda_{QCD}$. To get the "right" W mass, one must replace in Eq. III.40 $f_\pi \to v$, the Fermi scale (cf. Eq. (II.35). The suggestion of Susskind and Weinberg is that there exist an underlying QCD-like theory, Technicolor, which has a dynamical scale $\Lambda_{TC} \simeq 1$ TeV = 10^3 GeV. If in such a theory a global $SU(2)_L \times SU(2)_R$ symmetry broke down to $SU(2)_{L+R}$ diagonal, the W and Z^o could acquire mass by absorbing the "Technipions" of that theory, precisely in the way this happened in the QCD case. The relevant parameter f_π for the breakdown in the Technicolor theory is nothing else but the Fermi scale v.

I should comment on the necessity of an $SU(2)_L \times SU(2)_R$ structure to get the correct ρ parameter. This may appear as a miracle, and it is worthwhile to ask how this can be connected to the simple Higgs example I discussed in Sect. II. The reason one gets $\rho = 1$ in QCD and, by repeating this procedure, in Technicolor, has to do with the breakdown $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$. It is the remaining global symmetry that forces the parameter f_π to be common to all three pion states in Eq. (III.30). In turn this gives $\rho = 1$. All that the mixing of Y^μ and W_3^μ causes is the appearance of a $\cos\theta_W$ between the scales of the W and Z masses. Without any mixing, it is the residual $SU(2)_{L+R}$ symmetry that guarantees the equality of the f_π's and therefore of the masses of the gauge bosons.

One can check that in the Higgs case, actually, this same phenomena happens. The point is that the potential

$$V = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$

has a larger symmetry than SU(2) x U(1). The field Φ is a complex doublet, which I can write, if I wish, in terms of 4 real fields

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{III.41}$$

It is obvious, therefore, that V is really O(4) invariant. But O(4) is isomorphic to SU(2) x SU(2), so really V has an SU(2) x SU(2) symmetry also. When Φ gets a vacuum expectation value $\langle\Phi\rangle \neq 0$, really this corresponds to giving one of the fields in (III.41) a vacuum expectation value. Hence one breaks O(4) $\to$ O(3). But O(3) is really isomorphic to SU(2). Hence the spontaneous breakdown occurring in the Higgs case is also

$$SU(2) \times SU(2) \to SU(2)$$

Therefore one sees why one should get the same results in the two cases.

Technicolor is a very nice idea, since it replaces the imput parameter v of the Higgs potential by a much more dynamical scenario. Roughly speaking, the difference between the Technicolor theory and the GSW Higgs sector, is the difference there is between the BCS theory of superconductivity, where the Cooper pair condensate is a dynamical order parameter, and the Landau-Ginzburg effective theory, where the order parameter is again the expectation value of some scalar excitation. However, Technicolor suffers from two severe drawbacks, one technical and one more aesthetic. The technical problem, which is unsurmountable at least in the simplest version of Technicolor, is that there appears to be no way to generate fermion masses in this way. The more aesthetic objection is that to accomplish the breakdown of SU(2) x U(1) one has replaced one doublet of scalar fields by a whole strong interaction field theory, whose only raison d'être is to allow SU(2) x U(1) to break down!

To be sure, ultimately experiment will decide what is correct. The simple Higgs potential predicts the existence of a single excitation - the Higgs boson H. In a Technicolor theory, there is a plethora of Technihadrons, which are the analogue of hadrons for the Technicolor theory. In absence of better guidance, one would scale the Technihadron spectrum from that of ordinary hadrons by the ratio

$$\xi_{TC} = \frac{v}{f_\pi} = \frac{(\sqrt{2}\, G_F)^{-1/2}}{f_\pi} \simeq 2500 \tag{III.42}$$

Hence one predicts an extraordinarily rich phenomenology in the TeV energy scale.

To pursue the Technicolor scenario at all, one has to find a way to generate masses for the quarks and leptons. In pure Technicolor this appears impossible. This theory is built in complete analogy to 2 flavor QCD. The two techniquarks T_u and T_d have the same SU(2) x U(1) properties as the u and d quarks, and the assumption is that the condensates $\langle\bar{T}_u T_u\rangle = \langle\bar{T}_d T_d\rangle$ form, breaking SU(2) x U(1) dynamically. This gives the W and Z^o a mass, but does not affect the ordinary fermions, since there is no coupling of Techniquarks to ordinary quarks. To generates quark masses one needs to introduce some interaction between quarks and Techniquarks. Soon after the invention of Technicolor, the quark mass problem spawned the idea of Extended Technicolor (ETC) /43/ by which one can generate fermion masses.

In ETC theories one supposes that the Techniquarks and quarks are connected by (yet) another gauge interaction, which suffers a spontaneous breakdown at a scale $\Lambda_{ETC} \gg \Lambda_{TC}$. The exchange of these supermassive ETC gauge bosons gives an effective current $\otimes$ current interaction between ordinary quarks and Techniquarks. Schematically one has:

$$\mathcal{L}_{eff} \sim \frac{1}{\Lambda^2_{ETC}} (\bar{T} \gamma^\mu Q)(\bar{Q} \gamma_\mu T) \qquad \text{(III.43)}$$

where Q and T are the quarks and Techniquarks, respectively. It is clear that when the Techniquarks are allowed to condense: $\langle \bar{T}T \rangle \sim (\Lambda_{TC})^3 \neq 0$, the effective interaction (III.43) will generate quark masses of the order of

$$m_Q \sim \frac{\Lambda^3_{TC}}{\Lambda^2_{ETC}} \qquad \text{(III.44)}$$

ETC theories run into a variety of very difficult problems, which have discouraged people - after a period of initial great enthusiasm - to pursue this line of attack to the mass problem. The most pressing difficulties have to do with the presence of unwanted interactions among ordinary fermions, induced by having introduced the additional ETC interaction /49/. In addition, to construct models which are at least semirealistic, it is necessary to have more than one doublet of Technifermions. In such theories, the global group is larger than $SU(2)_+ \times SU(2)$ and in the breakdown more than just three Technipions emerge. Since the W^- and Z^0 can absorb only three Goldstone states, these models always have some pseudogoldstone excitations in the spectrum *. These states have masses of $O(\alpha_i \Lambda^2_{TC})$ where α_i is one of coupling constant squared of the standard SU(3) x SU(2) x U(1) model, and some states are in definite conflict with known bounds /50/.

* Pseudogoldstones since, after turning on the ordinary strong and electroweak interactions, these states gain a small mass.

I would like to end these lectures by discussing a line of speculations with which I am actively engaged. As I have mentioned earlier, a more (aesthetic) drawback of Technicolor is that one has invented a whole other strong interaction theory, whose sole purpose is really to provide condensates to break SU(2) x U(1). The idea that I want to air here is that, if one thinks of quarks and leptons as composite objects, then the underlying theory whose bound states are the quarks and leptons may serve also at the same time as a Technicolor theory. The same dynamics which is responsible for producing the quarks and leptons as bound state is also able to form condensates which break SU(2) x U(1) $\rightarrow$ $U(1)_{em}$. Furthermore, if quark and leptons are bound states, in principle their mass spectrum should be calculable. Hence this approach looks, potentially at least, very interesting. In practice, however, there are many difficulties, some of which I will try to explain here.

Perhaps one of the principal stumbling blocks against the idea that quarks and leptons are composite is that all the experimental evidence one has points towards their elementarity! Naturally, this evidence is stronger for leptons, but one can also infer that quarks are "elementary" down to certain minimum sizes. The apparent elementarity of quarks and leptons implies that, if they are really composite, then their characteristic size $\langle r \rangle$ is smaller than that which can be reached with present experiments. It has become conventional, instead of discussing a size $\langle r \rangle$ to put bounds on a so called, compositeness scale Λ_c, with $\langle r \rangle \sim 1/\Lambda_c$. To the extent that one has only evidence for elementarity, this means that Λ_c is much larger than the energies presently being explored.

Let me be a bit more specific and discuss two bounds on Λ_c:

(1) The anomalous magnetic moments of electrons and muons have been measured with incredible accuracy. They have also been computed theoretically on the basis of QED * - which assumes that the electrons and muons are point like. Any discrepancy between theory and experiment can then be a signal of a finite size for these leptons. To date the possible discrepancy is only at an extremely low level, which could be well due to experimental uncertainty. One has /51/

$$\delta a = \frac{1}{2}\left[(g-2)_{exp} - (g-2)_{QED}\right] = \begin{cases} 3.2 \times 10^{-10} & e \\ 1.5 \times 10^{-8} & \mu \end{cases} \qquad \text{(III.45)}$$

* Small weak and strong interaction corrections are also included.

This discrepancy could be due to effects of substructure. If leptons had an intrinsic size one would expect an additional magnetic interaction, scaled by Λ_c

$$\mathcal{L}^{eff} = \frac{e}{\Lambda_c} \bar{\ell} \sigma_{\mu\nu} \ell F^{\mu\nu} \tag{III.46}$$

just as it happens, for instance, in nucleons. This effective interaction gives an extra contribution to $g-2$ and one predicts

$$\delta a \simeq \frac{m_\ell}{\Lambda_c} \tag{III.47}$$

This equation sets a very high bound for Λ_c

$$\Lambda_c(e) \gtrsim 1.5 \times 10^6 \text{ GeV}$$
$$\Lambda_c(\mu) \gtrsim 7 \times 10^5 \text{ GeV} \tag{III.48}$$

If the compositeness scale of leptons (and therefore of quarks) is as high as that given in the above equation, it will be essentially impossible to establish this fact by direct experimentation. However, the bound in Eq. (III.48) may be an over estimate. The effective interaction in Eq. (III.46) connects again ℓ_R with ℓ_L, since $\sigma_{\mu\nu}$ contains 2 γ-matrices and $\{\gamma_\mu, \gamma_5\} = 0$. One may argue that operators in which there is a R-L transition ought to vanish as the mass of the fields involved vanishes. I shall make this argument more precise below. If this is indeed the case, then the effective interaction in (III.46) ought to contain a further factor of m_ℓ/Λ_c. That is, no anomalous interaction appears until whatever mechanism is responsible for getting mass is also turned on. If /52/

$$\mathcal{L}^{eff}_{chiral} = \frac{e}{\Lambda_c}\left(\frac{m_\ell}{\Lambda_c}\right) \bar{\ell} \sigma_{\mu\nu} \ell F^{\mu\nu} \tag{III.49}$$

is the effective residual interaction, instead of (III.46), then

$$\delta a \simeq \left(\frac{m_\ell}{\Lambda_c}\right)^2 \tag{III.50}$$

In this case the bounds on Λ_c are substantially reduced

$$\Lambda_c(e) \gtrsim 28 \text{ GeV}$$
$$\Lambda_c(\mu) \gtrsim 850 \text{ GeV} \tag{III.51}$$

(2) One may also obtain bounds on Λ_c by asking how well do cross sections for a given process, computed with elementary quarks and leptons, agree with experiment. Deviations could signal the presence of new contact interactions induced by compositeness. If the quarks and leptons have some nontrivial internal structure, then one necessarily has effective interactions among these fields scaled by the scale Λ_c. In the limit as $\Lambda_c \to \infty$, these additional interactions vanish. Schematically, therefore, one expects additional interactions among the quarks and leptons of a generic structure as that given below.

$$\mathcal{L}_{contact} = \frac{c^{ab}_{ijke}\, g^2_{eff}}{\Lambda_c^2} \bar{\psi}_i \Gamma_a \psi_j \bar{\psi}_k \Gamma_b \psi_e \tag{III.52}$$

Here the ψ_i are quark or lepton fields; g^2_{eff} is an effective coupling constant, and the coefficients c^{ab}_{ijke} are Clebsch Gordon coefficients of O(1), which give the relative strength of the various "current-current"-interactions in (III.52). The matrices Γ_a, in general can span the whole set of possible interactions. Because the quarks and leptons are presumed to be bound states of a strongly interacting theory, the residual interactions in (III.52) are not "weak". Rather one expects $g^2_{eff}/4\pi \sim O(1)$.

Eichten, Lane and Peskin /53/ analyzed e^+e^- interactions with a view of determining Λ_c this way. As they found no deviation from the behaviour of these scattering amplitudes from the predictions of the electroweak theory, they set a bound:

$$\Lambda_c \gtrsim 750 - 1000 \text{ GeV} \tag{III.53}$$

This bound is very comparable to Eq. (III.51). More recent reanalyses of the high precision PETRA data have moved this bound closer to 2 TeV.

I should note that the scale Λ_c defined through the effective interactions (III.46) or (III.49) and (III.52) is not precisely the same. However, since both these interactions reflect the same phenomena - a non zero effective size for the leptons (and quarks) - the scale Λ_c defined in these two equations ought to be approximately the same. Eqs. (III.51) and (III.53) give a minimum scale $\Lambda_c \gtrsim 1$ TeV. This scale can be much increased if one does not "protect" the interaction (III.46) by an additional mass factor, as is done in (III.49). Also, considerations of flavor changing processes (like $\mu \to e\gamma$) can give substantially larger bounds for Λ_c. Each of these bounds, however, involves more assumptions than those discussed here. I shall be conservative (or is it, perhaps, radical?) and only bound $\Lambda_c \gtrsim 1$ TeV.

Even a "low" bound for Λ_c, $\Lambda_c \gtrsim 1$ TeV, already signals that the dynamics of composite quarks and leptons is peculiar. This scale is certainly much bigger than that of the masses of the bound states

$$\Lambda_c \gg m_q, m_e \tag{III.54}$$

Such a circumstance is novel in physics. It means that the bound states (quarks and leptons) have a size which is substantially smaller than their Compton's wavelength!

$$\langle r \rangle_{q,e} \sim \frac{1}{\Lambda_c} \ll \lambda_{q,e} \sim \frac{1}{m_{q,e}} \tag{III.55}$$

In atomic physics one is used to the fact that the sizes of atoms are substantially bigger than the relevant Compton wavelength. For instance in positronium $\langle r \rangle \sim \frac{1}{m_e \alpha}$ but $\lambda_e \sim \frac{1}{m_e}$. Thus

$$\langle r \rangle_{\text{positronium}} \gg \lambda_e \tag{III.56}$$

essentially because $\alpha \ll 1$. In the case of hadronic physics one has

$$\langle r \rangle_{\text{hadrons}} \sim \lambda_{\text{hadrons}} \tag{III.57}$$

This equation is again easy to intuit. Hadronic physics - at least for hadrons made up of the light quarks - is determined entirely by the dynamical scale Λ_{QCD}. Since this is the only scale, it is clear that both $\langle r \rangle_{\text{hadrons}}$ and M_{hadrons} should be related. Namely $M_{\text{hadrons}} \sim \Lambda_{QCD}$; $\langle r \rangle_{\text{hadron}} \sim \frac{1}{\Lambda_{QCD}}$ and hence Eq. (III.57) follows

If one is to consider the idea that quarks and leptons are composite seriously, one must construct a dynamics where Eq. (III.55) hold. This causes substantial difficulties if the dynamics of the underlying theory, which bounds the quarks and leptons, is analogous to QCD. The existence of a unique dynamical scale always forces the bound states to have roughly this mass scale. The only way to avoid this conclusion is to build in some protective symmetry which forces certain states to decouple from this scale and go to zero mass. In the case of QCD this actually happens - at least in the limit of zero m_u and m_d masses. The pions, since they are Goldstone bosons actually are forced to zero mass - irrespective of the value of Λ_{QCD}.

Two suggestions have been put forth for producing (essentially) massless quarks and leptons in theories where these states are dynamical bound states:

(1) Quarks and leptons are (approximately) massless on the scale of Λ_c because the underlying theory - called in the literature a preon theory - has a protective chiral symmetry. This symmetry furthermore is not broken in the binding /54/.

(2) Quarks and leptons are essentially massless because the underlying theory is supersymmetric and a global symmetry in this theory is spontaneously broken down /55/. Because there is a spontaneous breakdown there exist necessarily Goldstone bosons in the spectrum of the theory. Because of the supersymmetry these states are necessarily accompanied by fermionic excitations - which one then associates with the quarks and leptons.

Two remarks are in order. Chirality protection - which is 't Hooft /54/ mechanism - is a non trivial dynamical assumption. Clearly if a chiral symmetry <u>survives</u> the binding process then fermions with non zero chirality cannot have mass. However, in QCD - which is a prototype for these kind of theories - chirality does not survive in the binding. That is, $\mathcal{L}_{QCD}$ is $SU(2)_L \times SU(2)_R$ symmetric. This is a chiral symmetry, since I can rotate the right and left handed underlying fields independently. However, $SU(2)_L \times SU(2)_R$ does not survive the binding process. Condensates form which break $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$, which is no longer a chiral symmetry. Indeed in QCD there are no massless fermions as bound states. The protons and neutrons are massive even in the limit of m_d, $m_u \to 0$, since the chiral symmetry is spontaneously broken.

't Hooft /54/ spells out precise conditions which are necessary in a theory to allow chirality to survive in the binding. These anomaly matching conditions of 't Hooft are somewhat technical and I shall not attempt to explain them in detail. Suffice it to say that if one wants an underlying chiral symmetry to be preserved in the binding then one must match the behaviour of certain Green's functions (scattering amplitudes) computed at the underlying level with that computed at the bound state level.

If one uses supersymmetry and spontaneous breakdown to get massless fermions in the theory /55/, then one can be sure that dynamically these states always ensue. Nevertheless, since eventually one wants to remove the "Goldstone" partners of the quarks and leptons to some sufficiently heavy mass, so as not to be in conflict with experiment ($M \gtrsim 20$ GeV?), the supersymmetry must at some stage be broken. Once the supersymmetry is broken, however, one must again appeal to some chirality to keep the bound states light. Models with both the supersymmetric Goldstone mechanism and some chiral protection are therefore quite attractive /56/.

Although many fundamental details are still missing, and many dynamical issues are unclear, I want to illustrate with a particular model these ideas. This model will tie together the ideas of composite quarks and leptons with Technicolor, thus illustrating the connection that I alluded to earlier. I will simplify some of the technical details of the discussion, which are not relevant here. For a more detailed exposition, see Ref. 57. The model has an underlying SU(2) x SU(2)' gauge structure and is supersymmetric. This SU(2) x SU(2)' is an unbroken symmetry and provides the strong binding for the theory just as in QCD. There are in the theory 6 fundamental doublets of SU(2), 6 fundamental doublets of SU(2)' and an object which is a doublet under both groups. Each of these doublets consists of a two component (left handed) fermion and its two supersymmetric scalar partners. The global symmetry of the model is thus SU(6) x SU'(6) x U(1).

The dynamics of the model is assumed to be such that condensates form to break this global symmetry.* Since the object $\epsilon_{ab}\chi^a\chi^b$, with χ^a and χ^b SU(2) doublet fields, is an SU(2) singlet, it is clear that the natural condensates

$$\langle \epsilon_{ab}\chi_1^a \chi_2^b \rangle = \tilde{v}$$
$$\langle \epsilon_{a'b'}\chi_1'^{a} \chi_2'^{b} \rangle = \tilde{v}' \qquad \text{(III.58)}$$

with 1,2 being "flavor" indices for the fundamental fields will lead to the breakdown of SU(6) $\rightarrow$ SU(4) x SU(2) and SU(6)' $\rightarrow$ SU(4)' x SU(2)'. If $\chi_{aa'}$ is the fundamental field which carries both SU(2) and SU(2)' quantum numbers, then the condensate

$$\langle \chi_1^a \chi_{aa'} \chi_1^{a'} \rangle = \langle \chi_2^a \chi_{aa'} \chi_2^{a'} \rangle \sim v_\pi \qquad \text{(III.59)}$$

will break the U(1) x SU(2) x SU(2') $\rightarrow$ SU(2)$_{\mathrm{diag}}$. Hence as a result of these con-

---
* There are actually some checks that one can perform which indicate that the symmetry breakdown assumed is the one that likely happens

densates the global symmetry of the original theory has broken down to SU(4) x SU(4)' x SU(2)$_{\mathrm{diag}}$.

It is easy to check - by counting generators - that in the breakdown

$$SU(6) \times SU(6)' \times U(1) \rightarrow SU(4) \times SU(4') \times SU(2)_{\mathrm{diag}} \qquad \text{(III.60)}$$

there ensue 17 + 17 + 4 = 38 Goldstone bosons. The supersymmetry forces these bosonic states to be accompanied by a number of fermionic partners (Quasi Goldstone fermions: QGF) and also some bosonic partners (Quasi Goldstone bosons: QGB) so that

$$n_G + n_{QGB} = 2n_{QGF} \qquad \text{(III.61)}$$

where n_G, n_{QGB}, n_{QGF} is the number of the appropriate bound states. One can check in the model that for each of the three spontaneous breakdowns there ensue 9+9+4 = 22 QGF and 1+1+4 = 6 QGB.

These states are very simple to construct in terms of the fundamental fields. If I denote by $\psi(\chi)$ the fermionic (bosonic) component of the fundamental fields, then for example, the nine QGF which emerge from the breakdown SU(6) $\rightarrow$ SU(4) x SU(2) transform as

$$\psi_{pq} \sim \epsilon_{ab}(\psi_p^a \chi_q^b - \psi_q^a \chi_p^b) \qquad p=1,2 \quad q=3,4,5,6$$
$$\psi \sim \epsilon_{ab}(\psi_1^a \chi_2^b - \psi_2^a \chi_1^b) \qquad \text{(III.62)}$$

It is clear that the fields ψ_{pq} are 4 doublets of SU(4) x SU(2). By assigning the charge and color of the fundamental field appropriately, one can obtain precisely the 4 left-handed doublets of one generation of quarks and leptons:

$$\psi_{pq} \equiv \left\{ \begin{pmatrix} u_1 \\ d_1 \end{pmatrix}_L \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}_L \begin{pmatrix} u_3 \\ d_3 \end{pmatrix}_L \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right\} \qquad \text{(III.63)}$$

The corresponding fields in the breakdown of the other SU(6) (SU(6)') give rise to the right handed fields - which are also organized in doublets of another SU(2) group. In this model there is a right handed neutrino and a few extra states, like ψ in Eq. (III.62).

If one wants to introduce the gauged SU(2) x U(1) of the GSW model here this can be done. Because of the identification of (III.63), it is clear that one has to consider the first <u>two</u> components of the fundamental fields as doublets under the SU(2) of GSW. The condensate v_π of Eq. (III.59) which forms in the theory breaks SU(2) x U(1) down since it involves χ_1 and χ_2, which are the fields that carry SU(2) quantum numbers.* In fact the W and Z bosons (and their supersymmetric partners) get their mass by absorbing three out of the four excitations which emerged as Goldstone fields and partner fields in the last stage breakdown $U(1) \times SU(2) \times SU(2) \rightarrow SU(2)_{diag}$. Thus the underlying theory not only binds quarks and leptons as massless bound states but also provides condensates which can break SU(2) x U(1). <u>The preon theory acts as technicolor.</u>

The idea that the spontaneous breakdown of SU(2) x U(1) may occur dynamically, in theories of composite quarks and leptons is extremely exciting. Since the breakdown of SU(2) x U(1) must occur at a scale of $\sim$ 250 GeV, this tells one that the scale of compositeness cannot be arbitrarily far away. Indeed since one is producing both the binding and the breakdown by the same theory, there is an intimate relationship between Λ_c and v. Of course, Λ_c must be somewhat bigger than v, so that the residual interactions of Eq. (III.52) do not affect the success of the GSW theory, and $\Lambda_c \gtrsim 10$ v appears to suffice phenomenologically.

Unfortunately, to construct realistic models incorporating this idea has proven very tough. Two stumbling blocks have stood in the way:

(1) Families of quarks and leptons have no simple dynamical realization. They can be put in somewhat mechanically, but then one does not really gain a correct understanding of their roles in the dynamics.

(1) The problem of how to generate mass for the quarks and leptons remains. The underlying theory is constructed with protective symmetries to give massless quarks and leptons. How does one relax these protective symmetries?

Both these issues are extremely complex and I do not know what really is the correct answer, especially with respect to families. I have a small hunch on the issue of mass generation, which I am pursuing, because I think it might not be a totally

---

* These fields do not carry U(1) quantum numbers. But because χ_1', χ_2' do carry U(1) numbers, this symmetry is also broken by the condensate v_π

unreasonable dynamics. The idea that I am trying to follow is that the residual interactions due to compositeness may themselves act as seeds for the masses of the quarks and leptons. That is, in the theory there are certain "irrelevant" condensates - corresponding to condensates of quarks field bilinears or lepton field bilinears - which one can ignore as a first approximation. These condensates are small on the scale of Λ_c, because they are condensates of objects which are themselves singlets under the gauge group. Including these condensates, however, can turn the residual interactions (III.52) into mass terms for the quark and leptons. Time will tell whether this idea has any merit.

<u>References</u>

/1/ P. Langacker, Proceedings of the XXII International Conference of High Energy Physics, Leipzig, East Germany, July 1984.

/2/ R.D. Peccei, Proceedings of the Advanced Study Institute on Techniques and Concepts in High Energy Physics, St. Croix, Virgin Islands, August 1984.

/3/ G. Altarelli, Proceedings of the XII GIFT Winter School in Theoretical Physics.

/4/ S.L. Glashow, Nucl. Phys. <u>22</u>, (1961) 579; A. Salam, in <u>Elementary Particle Theory</u>, ed. by N. Svartholm (Almquist and Wiksells, Stockholm 1969); S. Weinberg, Phys. Rev. Lett <u>19</u> (1967) 1264.

/5/ See for example, C. Itzykson and J.B. Zuber <u>Quantum Field Theory</u>, (McGraw Hill, NY 1978).

/6/ J. Goldstone, Nuovo Cim. <u>19</u> (1966) 156; J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. <u>127</u> (1962) 965.

/7/ P.W. Higgs, Phys. Rev. Lett <u>12</u> (1964) 132; F. Englert and R. Brout, Phys. Rev. <u>13</u> (1964) 321; G.S. Guralnik, C.R. Hagen and T.W. Kibble, Phys. Rev. Lett <u>13</u> (1964) 585.

/8/ R.P. Feynman and M. Gell Mann, Phys. Rev. <u>109</u> (1958) 193; M. Gell Mann, Phys. Rev. <u>111</u> (1958) 362.

/9/ G. 't Hooft, Nucl. Phys. <u>B35</u> (1971) 167.

/10/ H. Georgi and S.L. Glashow, Phys. Rev. Lett <u>28</u> (1972) 1494.

/11/ F.J. Hasert et al., Phys. Lett <u>46B</u> (1973) 138.

/12/ G. Arnison et al., Phys. Lett <u>122B</u> (1983) 103; M. Banner et al., Phys. Lett. <u>122B</u> (1983) 496.

/13/ G. Arnison et al., Phys. Lett <u>126B</u> (1983) 393; P. Bagnaia et al., Phys. Lett <u>129B</u> (1983) 130.

/14/ D. Ross and M. Veltman, Nucl. Phys. <u>B95</u> (1975) 135.

/15/ F. Bergsma et al., Phys. Lett. <u>147B</u> (1984) 481.

/16/ L.A. Ahrens et al., Phys. Rev. Lett <u>51</u> (1963) 1516; <u>54</u> (1984) 18.

/17/ H.U. Martyn, Proceedings of the Symposium on High Energy e^+e^- Interactions, Vanderbilt University, Nashville, TN, April 1984.

/18/ C. Prescott et al., Phys. Lett 77B (1978) 347; 84B (1979) 524.

/19/ R.P. Feynman, Photon Hadron Interactions (Benjamin, Reading Mass. 1972).

/20/ J.D. Bjorken and E. Paschos, Phys. Rev. 135 (1969) 1978; D1 (1970) 3151.

/21/ M. Jonker et al., Phys. Lett 102B (1981) 67.

/22/ C. Geweniger, Proceedings of the International Europhysics Conference on High Energy Physics, Brighton, U.K., July 1983.

/23/ M.A. Bouchiat and C. Bouchiat, Phys. Lett 47B (1974) 1111.

/24/ For a thorough review see M.A. Bouchiat and L. Pottier in Atomic Physics 9, R.V. Van Dyck Jr. and E.N. Fortson ed. (World Scientific, Singapore, 1985).

/25/ P.S. Drell and E. Commins, Berkeley preprint. For older results, see P. Bucksbaum, E. Commins and L. Hunter, Phys. Rev. Lett. 46 (1981) 640.

/26/ M.A. Bouchiat, J. Guena, L. Pottier and L. Hunter, Phys. Lett 136B (1984) 463.

/27/ N. Novikov et al., JETP 46 (1976) 420.

/28/ I.B. Kriplovich, JETP Lett 20 (1984) 315; S.L. Carter and H.P. Kelly, Phys. Rev. Lett 42 (1979) 965.

/29/ P.G.H. Sandars, Phys. Scripta 21 (1980) 284.

/30/ A. Martensson et al., Phys. Rev. A 24 (1981) 308.

/31/ D. Neuffer and E. Commins, Phys. Rev. A16 (1977) 844.

/32/ C. Bouchiat and C.A. Piketty, Phys. Lett 128B (1983) 73; C. Bouchiat, C.A. Piketty and D. Pignon, Nucl. Phys. B221 (1983) 68.

/33/ F.E. Paige, Brookhaven report BNL - 27006 (1979). For a more recent calculation, see for example G. Altarelli et al., Nucl. Phys. B246 (1984) 12.

/34/ G. Arnison et al., Phys. Lett 129B (1983) 273; P. Bagnaia et al., Zeit fur Physik C24 (1984) 1.

/35/ C. Rubbia, Proceedings of the Advanced Study Institute on Quarks and Leptons, Munich, West Germany, September 1983.

/36/ H.D. Wahl, Proceedings of the 15th International Symposium on Multiparticle Dynamics, Lund, Sweden, June 1984.

/37/ W. Marciano, Proceedings of the Fourth Topical Workshop on Proton Antiproton Collider Physics, Bern, Switzerland, March 1984.

/38/ J.D. Bjorken, Proceedings of the 1976 SLAC Summer Institute in Particle Physics, (SLAC - 193).

/39/ V.A. Lyubimov et al., Phys. Lett 94B (1981) 266.

/40/ Particle Data Group, Rev. Mod. Phys. 56 1 1984.

/41/ J. Gasser and H. Leutwyler, Phys. Rept C 87 (1982) 77.

/42/ G. Arnison et al., Phys. Lett. 147B (1984) 493.

/43/ R. Schrock and L.L. Wang, Phys. Rev. Lett 41 (1978) 1692.

/44/ J. Lee Franzini, Proceedings of the Europhysics Conference on Flavour Mixing in Weak Interactions, Erice, Italy, March 1984.

/45/ For futher details, see P. Langacker Ref. 1

/46/ K. Wilson as quoted in L. Susskind, Phys. Rev. D 20 (1979) 2619.

/47/ L. Susskind, Phys. Rev. D 20 (1979) 2619; S. Weinberg, Phys. Rev. D 16 (1976) 974; D 19 (1979) 1277.

/48/ S. Dimopolous and L. Susskind, Nucl. Phys. B155 (1979) 237; E. Eichten and K.D. Lane, Phys. Lett 90B (1980) 125.

/49/ S. Dimopolous and J. Ellis, Nucl. Phys. B182 (1981) 505.

/50/ See for example, M. Althoff et al., Phys. Lett. 122B (1983) 95.

/51/ For a discussion, see L. Lyons Prog. in Part. and Nucl. Phys. 10 (1983) 227.

/52/ S.J. Brodsky and S.D. Drell, Phys. Rev. D 22 (1980) 2236.

/53/ E. Eichten, K. Lane and M. Peskin, Phys. Rev. Lett. 50 (1983) 811.

/54/ G. 't Hooft, Proceedings of the Cargese Summer School, Cargese, France, August 1979.

/55/ W. Buchmuller, R.D. Peccei and T. Yanagida, Phys. Lett. 124B (1983) 67; Nucl. Phys. B 227 (1983) 503.

/56/ R. Barbieri, A. Masiero and G. Veneziano, Phys. Lett 128B (1983) 179.

/57/ W. Buchmuller, R.D. Peccei and T. Yanagida, Nucl. Phys. B244 (1984) 186.