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BOSE-EINSTEIN CORRELATIONS OBSERVED IN e^+e^- ANNIHILATION AT A CENTRE OF MASS ENERGY OF 34 GeV

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Abstract:

Bose-Einstein correlations between pairs of charged particles produced in e^+e^- annihilation into hadronic final states have been studied as a function of Q^2, the relative momentum squared of the two particles in their centre of mass, and as functions of various pairs of kinematic variables. The observed Bose-Einstein enhancement reveals correlation between the position and time of particle emission, and the space-time structure of the source is shown to differ from that of a pion fireball. While most features of the data are well accounted for in terms of the space-time structure of a simple string model, the correlations are better described by the simple function $1 + \alpha e^{-\beta Q^2}$. The implications of this result are discussed. The principal features of three particle correlations are explained in terms of the structure of the source inferred from the observed two particle correlations.

1. Introduction

In the debris of hadronic final states, the rate of production of pairs of particles having the same sign of charge is enhanced when the members of the pair have very small momentum difference. This phenomenon was first observed in $p\bar{p}$ annihilation and attributed to the Bose-Einstein statistics appropriate to identical pion pairs [1]. Just as intensity correlations between separated telescopes may be used to determine stellar dimensions [2], elementary considerations indicate that Bose-Einstein correlations between pion pairs could reveal the space-time structure of the source [3,4]. The phenomenon may be more complicated than implied by [3,4] in that the degree of enhancement depends in principle not only on the dimensions of the source but also on the extent to which it is chaotic (like a thermal source) or coherent (like a laser) [5,6]. A detailed discussion has been given in [6], but that discussion is primarily directed towards correlations between pions boiling off from nucleus-nucleus collisions. A discussion in elementary terms addressing the problem of correlations in the two jet events which dominate e^+e^- annihilation at PEP and PETRA energies has recently been given in [7].

Bose-Einstein correlations between pairs of pions produced in hadron-hadron interactions have been extensively studied, e.g. [8]. In e^+e^- annihilation, results have been presented from data taken at SPEAR [9], PEP [10] and PETRA [11,12]. A comprehensive review has been given in [13].

The Bose-Einstein correlation between pairs of identical particles is defined through the ratio C_2 of the joint probability of pairs of identical particles $P(k_1,k_2)$ to the product $P(k_1)P(k_2)$ of single particle probabilities, where k_1, k_2 are the 4-momenta of the two particles. For a chaotic source of bosons this ratio is given by

$$C_2 = \frac{P(k_1,k_2)}{P(k_1)P(k_2)} = 1 + \tilde{\rho}^2(\Delta\mathbf{k}, \Delta\omega) \tag{1.1}$$

where $\tilde{\rho}(\mathbf{\Delta k}, \Delta\omega)$ is the Fourier transform, normalised to unity as $\mathbf{\Delta k}, \Delta\omega \to 0$, of the source distribution with respect to $\mathbf{\Delta k}, \Delta\omega$; the momentum and energy difference of the identical bosons. The ratio C_2 may be regarded as the ratio of the number of identical pairs to the number that would obtain were the particles to be distinguishable in principle. In practice the denominator of (1.1) has been inferred from the number of unlike sign pairs [9,11,12] and from the number of like sign pairs constructed by drawing particles from different events [10;13].

The extracted correlation functions C_2 have been studied primarily as a function of the single variable Q^2, the square of the momentum difference of particles evaluated in the pair rest frame. For a pair of identical particles,

$$Q^2 = M^2 - 4m^2$$

where M is the mass of the pair and m the particle mass. This variable was first introduced in this context in [1]; see also [13]. The extracted correlations have been fitted with the *ad hoc* form

$$C_2 = 1 + \alpha e^{-\beta Q^2} \tag{1.2}$$

For two pion correlations, the parameter β has been found to be ~ 15 GeV^{-2}, corresponding to an effective radius ~ 0.7fm, and the parameter $\alpha \sim 0.6$. The parameter α has been interpreted as a measure of the degree of coherence of the source [9,13] but as pointed out in [7,12] this is unlikely to be justified in e^+e^- annihilation at high energies.

In order to obtain information on the space-time structure of the source, it is desirable to study the correlations as a function of the three components of the momentum difference and the energy difference of the pair, or if azimuthal symmetry is assumed, as a function of the momentum differences transverse and parallel to the event axis and the energy difference. The data are insufficient to support such an investigation.

In this paper we have studied the Bose-Einstein correlations as a function of the variable Q^2 for a number of subsets of our data and have also studied the correlations as a function of four pairs of kinematic variables. These four pairs, which are not independent, are:

(i) $|Q_L^2|$ versus Q_T^2, the variables suggested in [7], where Q_T^2 is the square of the momentum difference of the pair transverse to the sphericity axis of the event. The variable $Q_L^2 = (\Delta k_L)^2 - (\Delta\omega)^2$, where Δk_L is the momentum difference parallel to the sphericity axis and $\Delta\omega$ is the energy difference

(ii) $(\Delta k)^2$ versus $(\Delta\omega)^2$, where $(\Delta k)^2$ is the square of the momentum difference

(iii) $(\Delta k_T)^2$ versus $(\Delta\omega)^2$, the Kopylov-Podgoretsky variables [4], where $(\Delta k_T)^2$ is the square of the momentum difference perpendicular to the pair momentum

(iv) Q_L^2 versus Q_T^2.

All these variables are defined in the rest frame of the event, and were calculated assuming pion masses.

It has recently been shown [7] that the published data on Bose-Einstein correlations in high energy e^+e^- annihilation are generally in accord with expectations based on the Artru-Mennessier string model, which is closely related

[14] to the symmetric LUND model [16] and our studies were initially directed to testing further this hypothesis, which we find to account for most aspects of the data (Section 4). However, in the course of testing less plausible models of the source, we found the simple string model of [7] to be inadequate in one particular kinematic region, characterised by Q_L^2 negative and substantially larger in magnitude than $Q^2 = Q_L^2 + Q_T^2 \sim 0$. This region is populated by pairs which decay at a small angle to the line of flight in the pair centre of mass and for which the line of flight is at a large angle to the sphericity axis of the event. For such pairs

$$\Delta k^2 \simeq Q^2\gamma^2 \simeq Q_T^2 \quad ; \quad \Delta k_L^2 \simeq 0$$
$$\Delta\omega^2 \simeq Q^2\beta^2\gamma^2 \quad ; \quad Q_L^2 \simeq -Q^2\beta^2\gamma^2$$

where γ is the Lorentz factor of the pair. In such a configuration, $Q_L^2 \simeq -0.1$ GeV^2 for $Q^2 = 0.01$ GeV^2, $\gamma \simeq 3$.

The evidence that the string model fails in this particular kinematic region is discussed in Section 5, in which we also show that the expression (1.2) gives the best description of the Bose-Einstein correlations observed in our data, in all four pairs of variables. The implications of this remarkable result are addressed in Section 7.

2. Data

The data employed in this work were accumulated with the TASSO detector at PETRA at centre of mass energies between 29 and 37 GeV. The mean energy was 34.4 GeV and the bulk of the events corresponded to energies between 33 and 35 GeV. The criteria employed for the selection of these events have been described previously (see for example [15]): the sample consists of some 22400 hadronic events with charged particle multiplicity ≥ 5, and mean charged particle multiplicity 11, including the decay products of K° and Λ.

The Bose-Einstein correlation will appear only between $\pi^+\pi^+$, $\pi^-\pi^-$, K^+K^+ and K^-K^- pairs. Furthermore, there will be no detectable correlation between pairs of identical particles one of which is a decay product of a weakly decaying hadron. It is not possible to distinguish particles which are the decay products of hadrons containing b or c quarks, but the background due to particles from K or Λ decay has been reduced to some extent by imposing a cut $d_0 < 1.5$cm, where d_0 is the distance of closest approach of a track to the nominal beam position, in the plane perpendicular to the beam†. While it is desirable to remove πK, $\pi p \ldots$ pairs from the sample before studying Bose-Einstein correlations, we have made no attempt to use particle identification to reduce background from this source. In all our studies we have used Monte

† In our earlier work [12] a tighter cut, $d_0 < 0.5$cm, was imposed. In the present work we have chosen to employ $d_0 < 1.5$cm because the tighter cut reduced the number of pairs by a factor ~ 2 on average without greatly improving the ratio of prompt pairs to those containing a K or Λ decay product.

Carlo methods to estimate the background of pairs incapable of exhibiting Bose-Einstein correlation, employing for this purpose a sample of events generated using the LUND model [16]. These events were passed through a simulation of the TASSO detector and subjected to the same selection procedures as the real data, yielding some 70,000 Monte Carlo events.

In our previous paper [12] we demonstrated the existence of an enhancement in the ratio r of like sign to unlike sign pairs for $Q^2 < 0.1$ GeV^2. The ratio r is shown as a function of Q^2 for the present selection in Fig.1a. In addition to the enhancement attributed to Bose-Einstein correlation, this figure shows that there are $\sim 20\%$ more pairs of unlike sign particles than like sign, and that this ratio is dependent on Q^2 well beyond the region wherein Bose-Einstein correlation is manifest. Indeed if an event contains 5 positive and 5 negative particles, then the number of like sign pairs is 20 and the number of unlike sign pairs is 25. Structure in the intermediate region of Q^2 is attributable to a residual background of K° and the presence of ρ meson decay products in the unlike sign combinations. The distribution in r has been fitted with the form

$$r = \gamma(1 + \delta Q^2)(1 + \alpha e^{-\beta Q^2}) \tag{2.1}$$

excluding the K° and ρ region. The factor

$$1 + \alpha e^{-\beta Q^2}$$

would represent C_2, as parametrised by eq.(1.2), if all pairs consisted of identical particles. The factor γ is included to take account of the excess of unlike sign pairs over like sign pairs and the factor

$$1 + \delta Q^2$$

is included to take account of the relatively slow variation of r with Q^2 observed well beyond the low Q^2 region. The parameters obtained from the fit are listed in Table 1.

The values of α and β extracted from the data using (2.1) could in principle be subject to the following sources of bias:

(i) Ghost tracks found close to real tracks could enhance the ratio r at low Q^2. It is known from scanning events that the effect of such spurious tracks is negligible in our data.

(ii) The value of r at low Q^2 would not reflect the Bose-Einstein enhancement if track finding algorithms resolved like and unlike sign pairs with different efficiency. We have studied the ratio of the number of pairs generated in Monte Carlo events to the number remaining after simulating the TASSO detector and applying the same analysis and selection procedures as for the real data. For both like and unlike sign pairs this ratio is smooth as a function of Q^2, with no indication of either enhancement or depletion at very small Q^2. Such effects are in any event implausible, for the typical opening angle of a pair with $Q^2 = 0.01$ GeV^2 is 24° for pairs with a Lorentz factor γ of 2.9, which is the mean value of γ in our data. Such tracks are well separated in the central detector.

(iii) The ratio of like sign pairs to unlike sign pairs exhibits a slow variation with Q^2 beyond the region where a Bose-Einstein enhancement is observed. It is possible that the ratio of the number of like sign pairs that would be observed in the absence of Bose-Einstein correlation to the number of unlike sign pairs could vary more rapidly at low Q^2. This possible source of bias may be removed in principle by dividing the ratio r by the equivalent ratio r_{MC} obtained from the Monte Carlo events, which do not contain a Bose-Einstein enhancement. This normalised ratio, R, is defined by

$$R = \frac{r}{r_{MC}} = \frac{N_D^L}{N_D^U} \Big/ \frac{N_{MC}^L}{N_{MC}^U} \tag{2.2}$$

where N_D^L, N_D^U are the number of like sign and unlike sign pairs in the data and N_{MC}^L, N_{MC}^U are the corresponding numbers obtained from the Monte Carlo events. This procedure should also remove the effect of K° and ρ in unlike sign combinations, correct for the slow variation with Q^2 (or other such variables) of the ratio of like to unlike sign pairs and correct for any difference in the efficiency of finding like and unlike sign pairs when the momenta of tracks within the pair are similar.

In all our subsequent work we have employed the normalised ratio R, which is shown as a function of Q^2 in Fig.1b. It is seen that to a large extent the undesirable effects of taking the ratio of like to unlike sign pairs have been removed, and a clear enhancement attributable to Bose-Einstein correlation is visible, well fitted by the form

$$R = \gamma(1 + \delta Q^2)(1 + \alpha e^{-\beta Q^2}) \tag{2.3}$$

The parameters obtained from the fit are listed in Table 1.

Since it is possible that the size of the source, viewed from the pair rest frame in which Q^2 is the square of the momentum difference, may depend on the pair Lorentz factor γ, we have examined the ratio R as a function of Q^2 for the selections $\gamma < 2$, $2 < \gamma < 4$, $4 < \gamma < 6$, $6 < \gamma$. (Fig.2). Since the events are far from spherically symmetric we also examined the ratio R as a function of Q^2 for the selections $|\Delta\hat{\mathbf{k}} \,.\, \hat{\mathbf{S}}| > 0.7$, $|\Delta\hat{\mathbf{k}} \,.\, \hat{\mathbf{S}}| < 0.3$ (Fig.3), where $\Delta\hat{\mathbf{k}}$ is a unit vector in the direction of the momentum difference in the event frame and $\hat{\mathbf{S}}$ is a unit vector along the sphericity axis of the event. The parameters extracted from fits employing the form (2.3) may be found in Table 2. We find no significant evidence for dependence of the Bose-Einstein correlation parameters α and β on the Lorentz factor γ of the pairs, nor on the angle between the sphericity axis of the event and the momentum difference within the pair, measured in the event frame. The superimposed curves in Figs.2 and 3 are taken from the string model of [7] (see Section 4) and are in good agreement with the data.

3. Correction for background

The maximum value of the ratio C_2, eq.(1.2) is reached as $\Delta\mathbf{k}$, $\Delta\omega \to 0$ and for pairs of identical particles emitted from a chaotic source would reach a value of 2 in this limit. The maximum value of the parameter α in (2.1, 2.3) is thus 1, but in our data sample some 50% of like sign pairs are either not identical particles or are incapable of exhibiting a visible Bose-Einstein enhancement because one member of the pair originates from the decay of a long lived hadron. We have determined the proportion of like sign pairs where the two particles are not identical and the proportion of pairs where one member comes from decay of K° or Λ, and passes the cut $d_0 < 1.5$cm, from the sample of Monte Carlo events. This sample reproduces all the principal features of the data and in particular reproduces the observed number of K° and Λ: the number of charged kaons is approximately equal to the number of K°. We find that at low Q^2 15% of all like sign Monte Carlo pairs contain a K° or Λ decay product, 29% are not identical particles (excluding pairs known to contain a K° or Λ decay product) and $\sim 8\%$ of all like sign pairs contain one or both particles from decay of a hadron containing a c or b quark (but not included in the previous two classes). We have assumed that pairs in the last category cannot exhibit a visible Bose-Einstein correlation. Thus $\sim 50\%$ of all pairs of like sign will exhibit no correlation and the values of the parameter α in eq.(2.1) do not reflect the true value of the maximum visible enhancement for identical particles, but only half that value. This figure is uncertain largely because of errors on the measured proportion of kaons, Λ and protons: the proportion of background pairs in the Monte Carlo sample is $50 \pm 3\%$.

The value of α extracted from fitting to the ratio R (Table 1) is $\alpha = 0.35 \pm 0.03$. There remains some uncertainty in the determination of α,for the value extracted from fitting to the ratio r is significantly smaller and so the result is dependent on the extent to which the Monte Carlo reproduces correctly the ratio of like sign pairs to unlike sign pairs at low Q^2 in the absence of a Bose-Einstein correlation. We assign a systematic error of ± 0.04 to allow for this uncertainty. Our result for the value of the parameter α, eq.(1.2), corrected for the effects of background, is thus

$$\alpha(\text{corrected}) = 0.70 \pm 0.06 \pm 0.09$$

where the first error is statistical and the second is compounded from the systematic uncertainty on the proportion of background pairs and the assigned systematic uncertainty in the normalisation. We note that this value is in good agreement with the results of [9,10] and with the value obtained in our earlier work [12].

4. Comparison with string model predictions

The physical significance of the parameter β in (2.1) is by no means clear [7]. It is interesting that the values of the parameters α and β are consistent with little dependence on either the Lorentz factor γ of pairs or on the angle

between the event axis and the momentum difference (Table 2) and it is important to determine whether these features can emerge from a plausible space-time structure for the source.

The longitudinal structure of hadron production in e^+e^- annihilation is plausibly similar to that of the Artru-Mennessier string [14]. This model has an explicit space-time structure and the Fourier transform of meson production points in this model has recently been calculated [7].

The model admits only one space dimension and the Fourier transform $\tilde{\rho}_L$ is a function only of the longitudinal momentum difference, Δk_L, and the energy difference $\Delta\omega$ of a pair. The model cannot predict the Fourier transform as a function of the square of the momentum difference transverse to the event axis, Q_T^2. Following [7] we assumed

$$\tilde{\rho}^2 = \tilde{\rho}_L^2 \tilde{\rho}_T^2; \qquad \tilde{\rho}_T^2 = \exp[-\beta_T Q_T^2] \tag{4.1}$$

The Fourier transform squared, $\tilde{\rho}_L^2$, is given by eq.(6.19) of [7] as a function of Δk_L^2, $\Delta\omega^2$ and the mean rapidity and transverse mass of the pair. That expression can be integrated, under reasonable approximations, over the momentum spectrum of pairs to yield [7]

$$\tilde{\rho}_L^2(\Delta k_L, \Delta\omega) = A \frac{1}{1 - [\beta_L Q_L^2]^2} \ln \sqrt{\frac{1}{\beta_L |Q_L^2|}} \quad ; \quad \tilde{\rho}_L^2 < 1 \tag{4.2}$$

The most significant features of this result are that after integration over the pair momentum spectrum the longitudinal Fourier transform is a function only of the variable

$$Q_L^2 = (\Delta k_L)^2 - (\Delta\omega)^2$$

and that this function is very narrow at low Q_L^2 but has a polynomial tail. The form (4.1), (4.2) can be fitted to the data as a function of $|Q_L^2|$, Q_T^2 and the parameters A, β_L, β_T extracted directly. An alternative procedure is to weight pairs of identical particles generated by Monte Carlo methods with the square of the Fourier transform obtained before integrating over the pair momentum spectrum (eq.(6.19) of [7]), thus simulating the predicted Bose Einstein correlation in the Monte Carlo events. This has the advantage that background is easily taken into account by weighting only those pairs which can exhibit a Bose-Einstein enhancement, and the weighted events can be binned as required for obtaining the predicted enhancement in any variable or pair of variables. We have compared the predictions of the model with the data, employing both methods.

The ratio R was binned in a two-dimensional array in the variables $|Q_L^2|$, Q_T^2, in intervals of 0.01, 0.04 GeV^2 respectively and fitted with the form (4.1), (4.2), multiplied by a correction factor

$$\gamma(1 + \delta Q_L^2 + \epsilon Q_T^2) \tag{4.3}$$

The range fitted was $0 \leq |Q_L^2| \leq 0.25$ GeV^2, $0 \leq Q_T^2 \leq 0.6$ GeV^2 The fit yielded $\chi^2 = 378$ for 368 degrees of freedom and a good description of the data. The fitted parameters are listed in Table 3. The value of the parameter A, after allowance for background, is in the range considered plausible in [7] (~ 0.3 but uncertain by $\sim 50\%$). The value of β_L is smaller than suggested (5–12 GeV^{-2}), but within one standard deviation of the lower value, and so is not implausible. It is worth noting that if the bin $|Q_L^2| < 0.01$, $Q_T^2 < 0.04$ GeV^2 is excluded from the fit, the value of β_L becomes $5.4^{+5.7}_{-3.1}$ GeV^{-2}, entirely compatible with the range suggested in [7]. The fitted ratio R in the bin $|Q_L^2| < 0.01$, $Q_T^2 < 0.04$ GeV^2 only changed from 1.2 to 1.25 on excluding that bin from the fit. The value of β_T is in both cases compatible with the suggested value (12.5 GeV^{-2}), and corresponds to a flux tube radius ~ 0.7fm.

Projections of the two dimensional array are shown in Figs. 4 and 5; the superimposed curves are explained below.

The same two dimensional array was also fitted to the ratio of weighted to unweighted Monte Carlo like sign pairs, using (6.19) of [7] as a weight for pairs capable of exhibiting an enhancement. In this case the parameters in the fit were merely those of the correction factor (4.3): β_T was chosen *a priori* to be 12.5 GeV^{-2} and two values of the quantity $\mathcal{P}/2\sigma^2$ (where $\mathcal{P}$ is the string breaking probability and σ the string tension, see [7]) were tried: 0.5 and 1 GeV^{-2}. The latter gave the better representation of the data and the fit yielded $\chi^2 = 373$ for 371 degrees of freedom. The parameters are given in Table 3 and the fitted forms are shown on the projections, Figs.4 and 5. They are indistinguishable from the curves corresponding to the fitted analytic form (not shown). However, our data have not the precision needed to test the characteristic form of $\tilde{\rho}_L^2$ predicted by the string model [7]: fitting the array with the form

$$\tilde{\rho}^2 = \alpha e^{-\beta_L |Q_L^2|} e^{-\beta_T Q_T^2} \tag{4.4}$$

yielded an equally good fit ($\chi^2/dof = 370/368$) and the parameters given in Table 3 (see also Figs. 4, 5).

The weighted events were also fitted to the ratio R as a function of Q^2 alone, again employing a correction factor of the form given in (2.3). The results are shown in Figs. 1–3 and the parameters summarised in Tables 1, 2. The weighted Monte Carlo events, with $\mathcal{P}/2\sigma^2 = 1$ GeV^{-2}, $\beta_T = 12.5$ GeV^{-2}, reproduce well the enhancements observed in the four selections on the Lorentz factor γ and the two selections on the angle between the momentum within the pair and the sphericity axis.

Although there is not perfect agreement, the string model of [7] is thus found to be in good accord with those aspects of our data so far considered, describing well the Bose-Einstein enhancement as a function of $|Q_L^2|$, Q_T^2 and of Q^2 both for all pairs and for the six subsamples of data given above. This description is achieved with reasonable values of the parameters $\mathcal{P}/2\sigma^2$, β_T (which have not been fine tuned) and with no overall normalisation applied to $\tilde{\rho}^2$. The above features of Bose-Einstein correlation in e^+e^- annihilation may thus be understood in terms of a physically reasonable space-time structure of the source.

5. Further studies

To explore the extent to which the data can distinguish between different models for the hadron source, we binned the ratio R in three other two-dimensional arrays, motivated initially by two models which appear to be wholly unreasonable for hadron production in e^+e^- annihilation. In the course of these studies we found that the simple string model of [7] fails in one particular kinematic region, and that the Bose-Einstein enhancement in our data is best represented by the form

$$C_2 \sim 1 + \alpha e^{-\beta Q^2}$$

5.1 Fits employing the variables $(\Delta k)^2$ *vs* $(\Delta\omega)^2$

The first unreasonable model supposes the source to consist of a spherically symmetric assembly of oscillators, thermally excited, with mean lifetime τ. If the distribution in space in the laboratory frame is gaussian, and the assembly is largely transparent to hadrons, then [7,10]

$$\tilde{\rho}_{HS}^2 = \frac{e^{-\beta(\Delta k)^2}}{1 + (\Delta\omega)^2\tau^2} \tag{5.1}$$

where Δk is the momentum difference and $\Delta\omega$ the energy difference of the identical pair, in the event frame. This Hot Spot [13], or fireball, model might be appropriate to pion production in central collisions of light nuclei but is not consonant with our ideas about pion production in e^+e^- annihilation. The crucial distinction is that such a model contains no correlation between the position at which a pion is emitted and the time at which it is emitted.

In order to test this model, the ratio R was binned in a two dimensional array of $(\Delta k)^2$ vs $(\Delta\omega)^2$ (0.02×0.02 GeV^4) and the form

$$R = \gamma(1 + \delta(\Delta k)^2 + \epsilon(\Delta\omega)^2)(1 + \alpha\tilde{\rho}_{HS}^2) \tag{5.2}$$

was used in an attempt to fit the array, over the range $0 \le (\Delta k)^2 \le 0.5$ GeV^2, $0 \le (\Delta\omega)^2 \le 0.4$ GeV^2. No stable fit was found with physically admissible values of the parameters. In particular the best value of the inherently positive quantity τ^2 was negative, and if τ^2 was constrained to be ≥ 0 then inadmissibly small values of the parameter γ resulted, with unphysically large values of the parameter α. The best fit with physically reasonable parameters, obtained by constraining $\tau^2 \ge 0$ and $\gamma \ge 0.9$, yielded $\chi^2 = 394$ for 297 degrees of freedom (confidence level 0.01%).

Inspection of the data array and fitted arrays revealed that the principal discrepancies were found close to the diagonal $(\Delta k)^2 = (\Delta\omega)^2$†, where the ratio

† This kinematic region corresponds to pair decay close to the line of flight in the pair centre of mass and large values of γ^2, where γ is the pair Lorentz factor (see section 1).

R was ~ 1.2 (with substantial errors in individual bins.) The $(\Delta k)^2$ *vs.* $(\Delta\omega)^2$ array was therefore fitted with the form

$$R = \gamma(1 + \delta(\Delta k)^2 + \epsilon(\Delta\omega)^2)(1 + \alpha e^{-\beta(\Delta k)^2} e^{+t(\Delta\omega)^2}) \qquad (5.3)$$

and this yielded a much better fit with $\chi^2 = 350$ for 297 degrees of freedom (confidence level 2%). The fitted value of the parameter t was positive and approximately equal to β: this two dimensional array was best fitted by the form

$$\tilde{\rho}^2 \sim e^{-\beta Q^2} \qquad (5.3a)$$

The parameters obtaining in these three fits are summarised in Table 4.

The form (5.3) is merely a parametrisation: its significance lies in the unambiguous evidence that the Bose-Einstein enhancement in the data is approximately a function of $Q^2 = (\Delta k)^2 - (\Delta\omega)^2$, not only when both $(\Delta k)^2$ and $(\Delta\omega)^2$ are small, but also when both $(\Delta k)^2$ and $(\Delta\omega)^2$ are large but their difference is relatively small. This behaviour is quite different from that expected from the fireball model (5.1) in which a source volume of fixed radius is heated abruptly on a time scale short compared with the lifetime for emitting pions. It would however be expected if the source density were a function of a proper time $t^2 - r^2$ in the forward light cone [7].

Fitting the Monte Carlo events, weighted with the string model, to this array yielded a better fit than the hot spot model but not an acceptable fit: $\chi^2 = 373$ for 300 degrees of freedom (confidence level 0.2%). We note that the fit with string model weighted Monte Carlo events yielded values for the ratio R which, for fixed $(\Delta k)^2$, increased with $(\Delta\omega)^2$ as the diagonal corresponding to $Q^2 \simeq 0$ was approached. Close to the diagonal the calculated values were intermediate between the fit obtained using (5.3) and the best fit with (5.2), in which no dependence on $(\Delta\omega)^2$ obtained. This is a reflection of the fact that the string model results can be approximated by (5.3a) for $Q_L^2 \geq 0$ but not for $Q_L^2 < 0$ (see Section 7 below).

5.2 Fits employing the Kopylov-Podgoretsky variables

The second unreasonable model compared with the data is that of Kopylov and Podgoretsky [4] which corresponds to radiation of pions from thermally excited oscillators on the surface of a sphere; that is, the sphere is largely opaque to hadrons. This might approximate to pion production in collisions of heavy nuclei, but if taken literally is wholly implausible as a model of hadron production in high energy e^+e^- annihilation. The ratio R was binned in a two-dimensional array of $(\Delta k_T)^2$ versus $(\Delta\omega)^2$ (0.02×0.02 GeV^4), where Δk_T is the relative momentum within the pair transverse to the pair momentum, and the form

$$\tilde{\rho}_{KP}^2 = \frac{e^{-\beta(\Delta k_T)^2}}{1 + (\Delta\omega)^2\tau^2} \quad ; \qquad (5.4)$$
$$R = \gamma(1 + \delta(\Delta k_T)^2 + \epsilon(\Delta\omega)^2)(1 + \alpha\tilde{\rho}_{KP}^2)$$

was fitted to the data. The fit was acceptable: $\chi^2 \simeq 530$ for 494 degrees of freedom, (confidence level $\sim 13\%$,) with $\beta \sim 13$ GeV^{-2}, $\tau^2 \sim 2$ GeV^{-2}. The precise values of these parameters depended on limits imposed on γ in eq.(5.4): the best fit corresponded to $\gamma = 0.88$, $\alpha = 0.42$ ($\chi^2 = 529$) whereas for γ fixed at 0.95, $\alpha = 0.345$ and $\chi^2 = 532$. For subsequent studies we adopted the latter fit, $\beta = 16 \pm 2$ GeV^{-2}, $\tau^2 = 2.3^{+1.5}_{-1.1}$ GeV^{-2}. Fitting the Monte Carlo events, with the string weight applied, to the $(\Delta k_T)^2$ *vs.* $(\Delta\omega)^2$ array yielded a barely adequate fit, with $\chi^2 = 558$ for 497 degrees of freedom (confidence level 3%).

As a further test, the results of the constrained fit (5.2), the Q^2 fit (5.3) and the Kopylov-Podgoretsky form (5.4) were applied as weights to Monte Carlo events and the results fitted to the distributions of R in Q^2 (Figs.1–3) and to the three two-dimensional arrays $|Q_L^2|$ vs Q_T^2, $(\Delta k)^2$ vs $(\Delta\omega)^2$, $(\Delta k_T)^2$ vs $(\Delta\omega)^2$. The weight from (5.2), with the parameters given in Table 4, may be excluded. The weights from (5.3) and (5.4) fitted the Q^2 distributions well and fitted all three arrays significantly better than did the events weighted according to the string model. The fits, to all three arrays, using (5.3) and (5.4) proved to be indistinguishable, the reason being that the variable $\Delta k_T^2 \approx Q^2$ and that the fitted $(\Delta\omega)^2$ dependence is such that $\tilde{\rho}_{KP}^2$ closely approximates the form $e^{-\beta Q^2}$. This is plausible, since if the relative momentum vector makes an angle θ with the line of flight in the pair centre of mass then

$$(\Delta k_T)^2 = Q^2 \sin^2\theta; \quad (\Delta\omega)^2 = (\gamma^2 - 1)Q^2\cos^2\theta$$

and therefore

$$Q^2 = (\Delta k_T)^2 + \frac{1}{\gamma^2 - 1}(\Delta\omega)^2. \qquad (5.5)$$

where γ is the pair Lorentz factor.

A function

$$e^{-\beta Q^2}$$

will therefore be approximately represented by

$$\exp\left\{-\beta(\Delta k_T)^2 - \left\langle\frac{1}{\gamma^2 - 1}\right\rangle\beta\Delta\omega^2\right\} \qquad (5.5a)$$

where $\left\langle\frac{1}{\gamma^2-1}\right\rangle$ represents some appropriate average over pairs. There is no simple recipe for evaluating this quantity, but in data such as ours relatively large values of $\Delta\omega^2$ are important in determining the coefficient of $\Delta\omega^2$ and they must be associated with relatively small values of Q^2: $Q^2 \lesssim 0.1$ GeV^2. In our data, the average value $<\gamma>$ of the pair Lorentz factor is 2.9 (and the average value of γ^2 is approximately equal to $<\gamma>^2$). One may thus expect

$$\left\langle\frac{1}{\gamma^2 - 1}\right\rangle \approx \frac{1}{<\gamma>^2 - 1} = 0.13$$

Approximating (5.4)

$$\frac{e^{-\beta(\Delta k_T)^2}}{1 + \tau^2(\Delta\omega)^2} \approx \exp\left\{-\beta[(\Delta k_T)^2 + \frac{\tau^2}{\beta}(\Delta\omega)^2]\right\}$$

and this expression is equivalent to (5.5a) if

$$\frac{\tau^2}{\beta} \approx \left\langle \frac{1}{\gamma^2 - 1} \right\rangle$$

The fit to the data with (5.4) corresponds to $\tau^2/\beta = 0.14$ and this agrees with the estimate $\left\langle \frac{1}{\gamma^2-1} \right\rangle \approx 0.13$.

The success of the Kopylov-Podgoretsky formula in fitting the data does not necessarily imply that the source looks like a star, in our data it is consistent with the enhancement having approximately the form $e^{-\beta Q^2}$.

5.3 Comparison with PEP4-TPC results

A two-dimensional fit employing the Kopylov-Podgoretsky variables was reported in [10]: rather than fitting to an exponential in $(\Delta k_T)^2$ a form

$$1 + \lambda[2J_1(\Delta k_T \xi)/\Delta k_T \xi]^2/[1 + (\Delta\omega)^2\tau^2] \tag{5.6}$$

was assumed. The fitted parameters were

$$\lambda = 0.62 \pm 0.06 \pm 0.06$$
$$\xi = 1.27 \pm 0.07 \pm 0.08\,\mathrm{fm}$$
$$(c)\tau = 0.62 \pm 0.1 \pm 0.15\,\mathrm{fm} \quad : \quad \tau^2 = 9.9^{+3.4+5.3}_{-3.0-4.2}\mathrm{GeV}^{-2}$$

Fitting our data with (5.6) yielded as good a fit as fitting with (5.4) and parameters

$$\xi = 1.32 \pm 0.14\,\mathrm{fm}$$
$$\tau^2 = 2.1^{+1.3}_{-1.0}\,\mathrm{GeV}^{-2} : \quad (c)\tau = 0.29 \pm 0.08\,\mathrm{fm}$$

Our value of the parameter ξ is entirely consistent with the value given in [10]: the value of the parameter τ is somewhat greater in [10]. The maximum particle momentum in the data presented in [10] was 1.45 GeV/c and the centre of mass energy 29 GeV. Monte Carlo studies indicate that if the Bose-Einstein enhancement is best represented by the form $\alpha e^{-\beta Q^2}$, with $\beta \sim 13$ GeV^{-2}, then a value $\tau^2 \sim 2$ GeV^{-2} should obtain in our data, in agreement with our findings, whereas in the data of [10] a value $\tau^2 \sim 3$ GeV^{-2} ($c\tau \sim 0.34$ fm) is expected. Only further analysis of the data of [10] can determine whether or not that data is also well represented by an enhancement of the form $\alpha e^{-\beta Q^2}$.

5.4 Fits employing the variables Q_L^2 *vs* Q_T^2

To study further the behaviour of the ratio R at small Q^2, we binned the ratio in a two-dimensional array Q_L^2 vs Q_T^2, retaining the sign of Q_L^2 since Q_L^2 is not an inherently positive quantity. The bins were of dimension $0.01 \times 0.04\,\mathrm{GeV}^4$, and the range $-0.25 \leq Q_L^2 \leq 0.25$; $0 \leq Q_T^2 \leq 0.8$ GeV^2. This array was first fitted with the form

$$R = (1 + \delta Q_L^2 + \epsilon Q_T^2) \times (1 + \alpha e^{-\beta_L Q_L^2} e^{-\beta_T Q_T^2}) \tag{5.7}$$

A good fit was obtained, $\chi^2 = 952$ for 919 degrees of freedom (confidence level 22%). The fitted parameters are listed in Table 5. It will be noted that β_L and β_T have the same fitted value: the fitted enhancement once again takes the form $\alpha e^{-\beta Q^2}$ and it was noticeable that the value R was ≈ 1.2 in those bins close to the diagonal $Q_L^2 = -Q_T^2$ †, for values of Q_T^2 as large as 0.2 GeV^2. The data and exponential fits are shown in Figs. 6a), 6b) for $0.08 < Q_T^2 \leq 0.12$, $0.2 < Q_T^2 \leq 0.24$ GeV^2 respectively. While the statistical weight of individual bins is low out along the diagonal, the enhancement is systematic and the cumulative effect substantial. A fit of this two dimensional array with Monte Carlo events weighted with the string model yielded $\chi^2 = 983$ for 922 degrees of freedom (confidence level 8%). A fit with Monte Carlo events weighted according to $\alpha e^{-\beta \Delta k^2}$ (5.1, 5.2) yielded $\chi^2 = 990$ for 919 degrees of freedom (confidence level 6%): both sets of Monte Carlo events failed to reproduce the enhancement in R running along the diagonal. A fit with Monte Carlo events weighted according to the results of fits with (5.3) was, naturally, successful. Furthermore, the difference in χ^2 is accumulated almost entirely in the region $Q_L^2 < 0$, a finding we checked by fitting the weighted distributions independently to the regions $Q_L^2 < 0$; $Q_L^2 \geq 0$ (Table 5).

In the simple string model of [7] the predicted enhancement is a function of $|Q_L^2|$ (eq.4.2). This model fails for substantially negative values of Q_L^2 and we find that the best overall description of the Bose-Einstein correlations observed in our data is provided by the form (1.2). The implication is that in the pair rest frame the spatial distribution of the source is approximately spherically symmetric and that the characteristic radius is approximately independent of the Lorentz factor γ of the pair. These features are not inconsistent with a string-like space-time structure of the source: see Section 7.

6. Relevance to three particle correlations

The number of triplets of particles, all of like sign, has also been shown to exhibit an enhancement attributable to the Bose-Einstein effect [9,11,12]. This enhancement has been studied as a function of the single variable Q^2, where for three pions Q^2 is given by

$$Q^2 = M_{3\pi}^2 - 9m_\pi^2 \tag{6.1}$$

In this case the enhancement is observed in the ratio of triplets, all of the same sign, to triplets containing two particles of the same sign and one of the opposite sign. After normalisation, the ratio has been fitted with the form

$$C_3 = 1 + \alpha_3 e^{-\beta_3 Q^2} \tag{6.2}$$

and the parameter α_3 finally corrected for the effect of the Bose-Einstein correlations in like sign pairs [9,12]. The fitted values of β_3 are $\lesssim \beta/2$ and the values

† The kinematic configuration appropriate to this region has been discussed in section 1.

of α_3 are $\approx 5\alpha$, where α and β are the corresponding parameters for like sign pairs. For example, the results given in our previous paper [12] correspond to a value of $\alpha_3 = 2.8 \pm 0.5$, after all corrections have been made†, to be compared with $\alpha = 0.61 \pm 0.08$ for pairs. The value of β_3 in [12] is $6.9 \pm 1.3 \pm 1.5$ GeV^{-2}, corresponding to a radius parameter $r = 0.52 \pm 0.07$fm, to be compared with $\beta = 15.3 \pm 2.7 \pm 3.0$ GeV^{-2}, $r = 0.76 \pm 0.12$fm, for pairs. A similar difference in the two and three particle scale parameters was reported in [9], and this effect has not been explained hitherto [13].

We have found that these results can be explained in terms of the structure of the source already revealed by two particle correlations and that in particular the difference between the two and three particle scale parameters has a trivial origin.

For a chaotic source, a simple extension of the calculations of [3,4,6,7] shows that the ratio C_3, defined in a way analogous to C_2, eq.(1.1), is given by

$$\begin{aligned} C_3 = 1 &+ 2\,\mathrm{Re}[\tilde{\rho}(k_{12})\tilde{\rho}(k_{23})\tilde{\rho}(k_{31})] \\ &+ \tilde{\rho}^2(k_{12}) + \tilde{\rho}^2(k_{23}) + \tilde{\rho}^2(k_{31}) \end{aligned} \tag{6.3}$$

where $\tilde{\rho}(k_{ij})$ is the normalised Fourier transform of the source with respect to the 4-momentum difference of the pair (ij).

The last three terms of (6.3) are known from studies of two particle Bose-Einstein correlations, and as we have shown may be represented by the form

$$\tilde{\rho}^2(k_{ij}) \approx \alpha e^{-\beta Q_{ij}^2}$$

Knowledge of these terms does not determine the triple product in (6.3), but to the extent that phase factors may be neglected,

$$C_3 \approx 1 + 2\alpha^{3/2} e^{-\frac{\beta}{2}Q^2} + \alpha \sum_{\text{pairs}} e^{-\beta Q_{ij}^2} \tag{6.4}$$

where α, β are appropriate to the two particle correlations,

$$Q^2 = \sum_{\text{pairs}} Q_{ij}^2.$$

and the appropriate average of Q_{ij}^2 will be $\sim Q^2/3$. Thus it is to be expected that

$$\alpha_3 \approx 5\alpha$$

$$\beta_3 \approx \frac{\beta}{3} \quad \text{to} \quad \frac{\beta}{2}$$

and these expectations are in accord with such data as exist [9,12].

† The value of α_3 given in Table 2 of [12] was not corrected for the effect of two particle correlations, in order that the most direct comparison with other results could be made.

We have verified that such results are obtained when $\tilde{\rho}(k_{ij})$ is taken from the string model [7] (which does contain phase factors) and triplets are weighted with (6.3).

It is not to be expected that the Bose-Einstein enhancement exhibited by triplets of like sign particles will reveal information independent of that contained in like sign pairs.

7. Discussion

While the string model of [7] accounts reasonably well for the Bose-Einstein correlations observed as a function of Q^2, both for all pairs and for various subsamples of the data, and for R as a function of $|Q_L^2|$, Q_T^2 it fails in one particular kinematic region, along the diagonals $\Delta k^2 \simeq \Delta\omega^2$; $Q_L^2 \simeq -Q_T^2$. The observed enhancement is better represented by the form

$$\alpha e^{-\beta_L Q_L^2} e^{-\beta_T Q_T^2} \tag{7.1}$$

with the parameters β_L and β_T approximately equal, namely the form

$$\alpha e^{-\beta Q^2} \tag{7.2}$$

with $\beta \approx 13$ GeV^{-2} corresponding to a scale parameter ~ 0.7fm. This simple result is at first sight astonishing, for as pointed out in [7] the result

$$\tilde{\rho}^2(\Delta\mathbf{k}, \Delta\omega) \to \tilde{\rho}^2(Q^2)$$

is obtained if the source density in the event frame takes the spherically symmetric form $f(t^2 - r^2)$ in the forward lightcone of the annihilation point. Nonetheless, our results can probably be understood in terms of a physically reasonable picture of the pion source. To this end, it is illuminating to consider how it comes about that the string model of [7] is fairly successful in reproducing the Bose-Einstein correlations observed, despite the fact that in any string model in which the string tension is given by the slope of the Regge trajectories the maximum extension of the string at a centre of mass energy of 34 GeV is ~ 34fm. In the Artru-Mennessier model the space-time coordinates at which a meson is produced are strongly correlated with the transverse mass and rapidity: mesons of given momentum are produced within local regions of space-time of area $\sim 1/\mathcal{P}$ [7], where the probability of cutting the string by creation of a quark-antiquark pair within a space-time area $\Delta x \Delta t$ is given by $\mathcal{P}\Delta x \Delta t$ [14]. The structure of the model is such that the longitudinal range of production points of a meson of given momentum is Lorentz contracted on transforming to the meson rest frame and

$$< \Delta x_{cm}^2 > \sim 1/\mathcal{P}$$

where Δx_{cm} is the longitudinal range in the rest frame. This result is independent of rapidity. Thus the scale for the dependence of the Bose-Einstein enhancement on Q_L^2 is set by $\sim 1/\mathcal{P}$, which for $\frac{\mathcal{P}}{2\sigma^2} = 1$ GeV^{-2} has a value ~ 16

GeV^{-2}, and this scale is largely independent of the momentum of the particles in a pair characterised by low Q^2.

In a purely one dimensional model, with the particle masses identical (that is, transverse mass not introduced) $Q_L^2 = Q^2 \geq 0$. In [7] the continuous mass spectrum resulting from the classical nature of the string model is interpreted in terms of transverse mass: Q_L^2 may then be negative and it was suggested in [7] that the Fourier transform of the transverse dimensions of the source be represented by an exponential in Q_T^2. Then for positive Q_L^2, approximating the string model function $\tilde{\rho}_L^2(Q_L^2)$ in the resolved region by a constant multiplied by an exponential, the string model prediction may be represented by

$$C_2 \approx 1 + \alpha e^{-\beta_L Q_L^2} e^{-\beta_T Q_T^2} \tag{7.3}$$

where the value of β_L is expected to be $\sim 1/\mathcal{P} \sim 16$ GeV^{-2}. The fitted value of β_L is ~ 13 GeV^{-2} (eq.5.7) and is approximately equal to β_T (Table 5). Thus (7.3) takes on the form

$$C_2 \approx 1 + \alpha e^{-\beta Q^2} \tag{7.4}$$

for Q_L^2 positive, although the model is far from being spherically symmetric in the event frame. In the pair frame the Lorentz contracted longitudinal scale is approximately equal to the transverse size of the source.

The string model of [7] is constructed from the Fourier transform of the Artru-Mennessier source, in the longitudinal variable x and time t, multiplied by an exponential representing the Fourier transform of a Gaussian flux tube profile transverse to the tube or string. Because of the assumed factorisation, eq.(4.1), the approximation (7.3) to the model of [7] cannot hold in the region of negative Q_L^2. The reason is that neither $\tilde{\rho}_L^2$ nor $\tilde{\rho}_T^2$ can exceed unity if factorisation is assumed. The more general restriction is merely that $\tilde{\rho}^2$ cannot exceed unity and thus the success of the form (7.3), (7.4) in representing our data in all kinematic regions implies that the factorisation assumed in (4.1) is not adequate.

In a classical flux tube model, the transverse dimensions of the flux tube could not expand away from the annihilation point at a speed greater than that of light. Given that the flux tube has radius ~ 0.7fm, it is unrealistic to represent the transverse profile as being independent of the longitudinal variable x and time t and it is entirely plausible that the source density is a function of the proper time, $(t^2 - r^2)$, rather than of $(t^2 - x^2)$. It should be remembered that the string model fails only for negative Q_L^2 with magnitude $\gg Q^2$. These pairs are produced at a large angle to the event axis, and so have low rapidity. In the string model such pairs are produced at small values of the longitudinal variable x and a correlation between time t and the transverse coordinate of meson production points, as implied by our data, is entirely plausible. The simple string model of [7] thus fails in a natural way for such pairs.

Given that the contracted longitudinal scale in the string model is of the order of the measured transverse dimensions of the source, it is to be expected that a more realistic flux tube model would yield approximately the form (7.4), corresponding to an approximately spherically symmetric source in the pair

rest frames, without any requirement of spherical symmetry in the event frame. There is marginal evidence [10] that the source as seen from the pair frames is more extended in the direction of the sphericity axis than transverse to the axis, although those data are also consistent with spherical symmetry in the pair frames.

Our data require a value of $\mathcal{P}/2\sigma^2$ which is at the upper end of the range considered plausible in [7]. Furthermore, resonance production and decay have not been taken explicitly into account in our work. While essentially the same functional form for the quantity $\tilde{\rho}_L^2$ results when resonance decay is taken into account [7], the effect is to decrease somewhat the effective value of $\mathcal{P}/2\sigma^2$, thereby shrinking the Bose-Einstein correlations. This effect is not expected to dominate the scale of the correlations, since the proper lifetime of all but the narrowest resonances lies in the range 5–10 GeV^{-1}, and in the string model the root mean square value of the proper time separating first generation meson production points from the origin is $\sim \sqrt{2/\mathcal{P}}$ and lies in this range. It is clear however that if the underlying dynamics can indeed be represented by a string, then not only are the correlations between transverse mass, rapidity and production point contained in the model of [7] necessary to produce the magnitude and scale of the Bose-Einstein correlations we observe, but also, since the quantity $\mathcal{P}/2\sigma^2$ is constrained by our data to be $\gtrsim 1$ GeV^{-2}, there is little room for any dilution of the effect by coherence. The source must be highly chaotic.

The string model as formulated in [7] does not contain any predictions for the transverse Fourier transform of the string or flux tube. The characteristic transverse dimension determined by our data is ~ 3.6 GeV^{-1}, 0.7fm. This result must be compounded from the transverse size of the hadron source and the propagation of resonances before decay, and is of the same order as hadron dimensions. One would not expect to find a value significantly smaller.

8. Conclusions

We summarise our conclusions as follows:

(i) The TASSO data on Bose-Einstein correlation of like sign pairs of charged particles are well represented in two dimensional arrays by the simple function

$$C_2 \approx 1 + \alpha e^{-\beta Q^2} \tag{8.1}$$

with $\beta \approx 13$ GeV^{-2}.

(ii) To the extent that the observed Bose-Einstein correlation is a function only of Q^2, the source must be spherically symmetric when viewed from the rest frame of any pair the members of which are close in momentum. This does not imply a spherically symmetric source in the event frame.

(iii) The principal features of Bose-Einstein correlation exhibited by triplets of like sign particles are readily explained in terms of the results obtained for pairs.

(iv) The string model of [7] accounts well for most features of the data. Since this model assumes a chaotic source, the parameter α in (8.1) should not be interpreted as a measure of the coherence of the source.

(v) The string model of [7] fails in a natural way in the kinematic region of negative Q_L^2, $Q^2 \approx 0$. It is expected that a more realistic flux tube model could account for the data.

(vi) Within the context of a string model, the size and scale of the Bose-Einstein correlations observed require both a highly chaotic source and correlations between transverse mass, rapidity and position at which a meson originates of the kind which appear naturally in the Artru-Mennessier model [7].

These results are not wholly devoid of interest, but to obtain more detailed information about the space-time structure of the hadron source in e^+e^- annihilation requires at least an order of magnitude more data, of high precision, than is available at present.

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Table 1

Fitted Bose-Einstein Correlation as a function of Q^2, all data
K° ρ region excluded from fits

	α	β GeV^{-2}	γ	δ GeV^{-2}	χ^2	*dof*	*cl*	Comments
(i)	0.27 $\pm$0.03	21.2 $^{+3.3}_{-2.8}$	0.78 $\pm$0.01	0.11 $\pm$0.01	80	70	20%	Fit to ratio r, eq.(2.1)
(ii)	0.35 $\pm$0.03	16.5 $^{+2.5}_{-2.1}$	0.96 $\pm$0.01	0.04 $\pm$0.01	57	70	87%	Fit to ratio R, eq.(2.3)
(iii)			0.990 $\pm$0.01	0.01 $\pm$0.01	73	72	45%	Fit to R with string model weighted Monte Carlo events, $P/2\sigma^2 = 1$ GeV^{-2}, $\beta_T = 12.5$ GeV^{-2}

Table

Fits to the ratio R as a function of Q^2, various selections
K° ρ region excluded from fits

	Selection	α	β GeV^{-2}	γ	δ GeV^{-2}	χ^2	*dof*	*cl*	Comments
(i)	$\gamma < 2$	0.38 $\pm$0.08	24.7 $\pm$7.7	0.97 $\pm$0.02	0.03 $\pm$0.01	64	70	68%	Analytic form (2.3)
				0.96 $\pm$0.01	0.05 $\pm$0.010	67	72	65%	String model weighted MC events
(ii)	$2 < \gamma < 4$	0.31 $\pm$0.05	10.3 $^{+3.1}_{-2.3}$	0.93 $\pm$0.03	0.05 $\pm$0.03	73	70	38%	Analytic form (2.3)
				0.98 $\pm$0.01	0.05 $\pm$0.03	83	72	18%	String model weighted MC events
(iii)	$4 < \gamma < 6$	0.54 $\pm$0.10	29.2 $^{+9.7}_{-7.7}$	1.00 $\pm$0.03	-0.02 $\pm$0.03	52	70	94%	Analytic form (2.3)
				1.02 $\pm$0.02	-0.04 $\pm$0.02	60	72	84%	String model weighted MC events
(iv)	$6 < \gamma$	0.25 $\pm$0.07	16.1 $^{+8.4}_{-5.6}$	1.01 $\pm$0.04	-0.09 $\pm$0.04	76	70	38%	Analytic form (2.3)
				1.03 $\pm$0.01	-0.11 $\pm$0.02	79	72	27%	String model weighted MC events
(v)	$\Delta\hat{\mathbf{k}}.\hat{\mathbf{S}} > 0.7$	0.32 $\pm$0.04	17.9 $^{+3.5}_{-2.9}$	0.97 $\pm$0.02	0.03 $\pm$0.01	55	70	90%	Analytic form (2.3)
				0.99 $\pm$0.01	0.02 $\pm$0.01	67	72	51%	String model weighted MC events
(vi)	$\Delta\hat{\mathbf{k}}.\hat{\mathbf{S}} < 0.3$	0.57 $\pm$0.09	15.8 $^{+8.1}_{-4.8}$	0.90 $\pm$0.05	0.05 $\pm$0.05	68	70	45%	Analytic form (2.3)
				0.97 $\pm$0.02	-0.02 $\pm$0.02	77	72	35%	String model weighted MC events

Table 3

Fits to the ratio R as functions of Q_T^2 v. $|Q_L^2|$

	A, α	β_L GeV^{-2}	β_T GeV^{-2}	γ	δ GeV^{-2}	ϵ GeV^{-2}	χ^2	*dof*	*cl*	Comments
(i)	A 0.14 $\pm$ 0.02	2.8 $^{+2.7}_{-1.6}$	10.5 $^{+2.8}_{-2.3}$	0.96 $\pm$0.03	0.02 $\pm$0.08	−0.04 $\pm$0.06	378	368	35%	Analytic form eqs.(4.1, 4.2)
(ii)				0.98 $\pm$0.01	0.00 $\pm$0.06	−0.08 $\pm$0.03	373	371	47%	String model weighted MC events
(iii)	α 0.33 $\pm$ 0.04	15.8 $^{+4.9}_{-3.9}$	11.1 $^{+2.8}_{-2.3}$	0.96 $\pm$0.02	0.09 $\pm$0.1	−0.06 $\pm$0.05	370	368	47%	Analytic form eqs.(4.3, 4.4)

Note: The two values of β_L are parameters in different formulae, (4.2) and (4.4).

Table 4

Fits to the ratio R as functions of $(\Delta k)^2$ v. $(\Delta\omega)^2$

	α	β GeV^{-2}	τ^2, t GeV^{-2}	γ	δ GeV^{-2}	ϵ GeV^{-2}	χ^2	*dof*	*cl*	Comments
(i)	0.39 $^{+0.03}_{-0.07}$	6.7 $\pm$0.9	τ^2 0 $^{+0.18}_{-*}$	0.9 $^{+0.05}_{-*}$	−0.08 $^{+0.04}_{-0.10}$	0.38 $\pm$0.07	394	297	0.01%	Analytic form (5.1, 5.2) (Hot Spot) * Parameter at limit
(ii)				0.99 $\pm$0.01	−0.20 $\pm$0.04	0.18 $\pm$0.07	373	300	0.2%	String model weighted MC events
(iii)	0.40 $\pm$0.04	10.5 $^{+1.6}_{-1.3}$	t 10.7 $^{+1.8}_{-1.4}$	0.90 $^{+0.03}_{-*}$	0.06 $\pm$0.04	−0.26 $\pm$0.13	350	297	2%	Parameterisation (5.3) * $\gamma \geq 0.9$ imposed

Table 5

Fits to the ratio R as functions of Q_L^2 v. Q_T^2

	α	β_L GeV^{-2}	β_T GeV^{-2}	γ	δ GeV^{-2}	ϵ GeV^{-2}	χ^2	dof	cl	Comments
(i)	0.31 $\pm$0.03	13.4 $^{+2.4}_{-2.1}$	13.5 $^{+2.3}_{-1.9}$	0.96 $\pm$0.02	0.04 $\pm$0.05	-0.11 $\pm$0.03	952	919	22%	Analytic form (5.7) $-0.25 \leq Q_L^2 \leq +0.25$ GeV^2
(ii)				0.99 $\pm$0.01	-0.08 $\pm$0.04	-0.15 $\pm$0.02	983 $Q_L^2 < 0$ 435 $Q_L^2 > 0$ 548	922 425 bins 500 bins	8%	String weighted MC events $-0.25 \leq Q_L^2 \leq +0.25$ GeV^2
(iii)				0.98 $\pm$0.01	-0.07 $\pm$0.07	-0.17 $\pm$0.02	541	497	8%	String weighted MC events $0 < Q_L^2 \leq 0.25$ GeV^2
(iv)				1.00 $\pm$0.01	-0.08 $\pm$0.10	-0.15 $\pm$0.03	433	422	35%	String weighted MC events $-0.25 \leq Q_L^2 < 0$ GeV^2
(v)				0.96 $\pm$0.01	0.04 $\pm$0.04	-0.11 $\pm$0.02	948 $Q_L^2 < 0$ 406 $Q_L^2 > 0$ 542	922 425 bins 500 bins	27%	MC events weighted with (5.3) fit $-0.25 \leq Q_L^2 \leq +0.25$ GeV^2
(vi)				0.96 $\pm$0.01	0.13 $\pm$0.07	-0.14 $\pm$0.02	538	497	10%	MC events weighted with (5.3) fit $0 < Q_L^2 \leq 0.25$ GeV^2
(vii)				0.96 $\pm$0.01	0.21 $\pm$0.10	-0.04 $\pm$0.04	400	422	77%	MC events weighted with (5.3) fit $-0.25 \leq Q_L^2 < 0$ GeV^2

Figure Captions

1. a) The ratio r of like sign pairs to unlike sign pairs in the data. The superimposed curve is the result of fitting with (2.1). The $K^\circ - \rho$ region was excluded from the fit.
   b) The ratio R of like sign pairs to unlike sign pairs in the data, divided by the corresponding ratio for Monte Carlo generated events. The solid curve shows the result of fitting with Monte Carlo events weighted with the string model; the broken curve is the result of fitting with the form of (2.3). The $K^\circ - \rho$ region, indicated in a), was excluded from the fits.

2. The ratio R as a function of Q^2 for a) $\gamma < 2$ b) $2 < \gamma < 4$ c) $4 < \gamma < 6$ d) $6 < \gamma$. The superimposed curves are the results of fitting with Monte Carlo events weighted with the string model. The $K^\circ - \rho$ region was excluded from the fits.

3. The ratio R as a function of Q^2 for a) $|\Delta\hat{\mathbf{k}} \cdot \hat{\mathbf{S}}| > 0.7$ b) $|\Delta\hat{\mathbf{k}} \cdot \hat{\mathbf{S}}| < 0.3$. The superimposed curves are the results of fitting with Monte Carlo events weighted with the string model. The $K^\circ - \rho$ region was excluded from the fits.

4. The ratio R as a function of $|Q_L^2|$ for a) $Q_T^2 < 0.04\,\mathrm{GeV}^2$ b) $Q_T^2 < 0.6\,\mathrm{GeV}^2$. The solid curves are the result of fitting the array with Monte Carlo events weighted with the string model, and are indistinguishable from the fit using (4.1, 4.2, 4.3). The broken curves represent an exponential fit (4.4).

5. The ratio R as a function of Q_T^2 for a) $|Q_L^2| < 0.01\,\mathrm{GeV}^2$ b) $|Q_L^2| < 0.25\,\mathrm{GeV}^2$. The curves are the results of fitting with Monte Carlo events weighted with the string model and are indistinguishable from the results of fitting with (4.1, 4.2, 4.3; 4.4).

6. The ratio R as a function of Q_L^2 for a) $0.08 < Q_T^2 \leq 0.12\,\mathrm{GeV}^2$ b) $0.20 < Q_T^2 \leq 0.24\,\mathrm{GeV}^2$. The curves are the results of fitting the array with the form (5.7). The ratio $R \rightarrow \sim 1.2$ as $Q_L^2 \rightarrow -Q_T^2$.

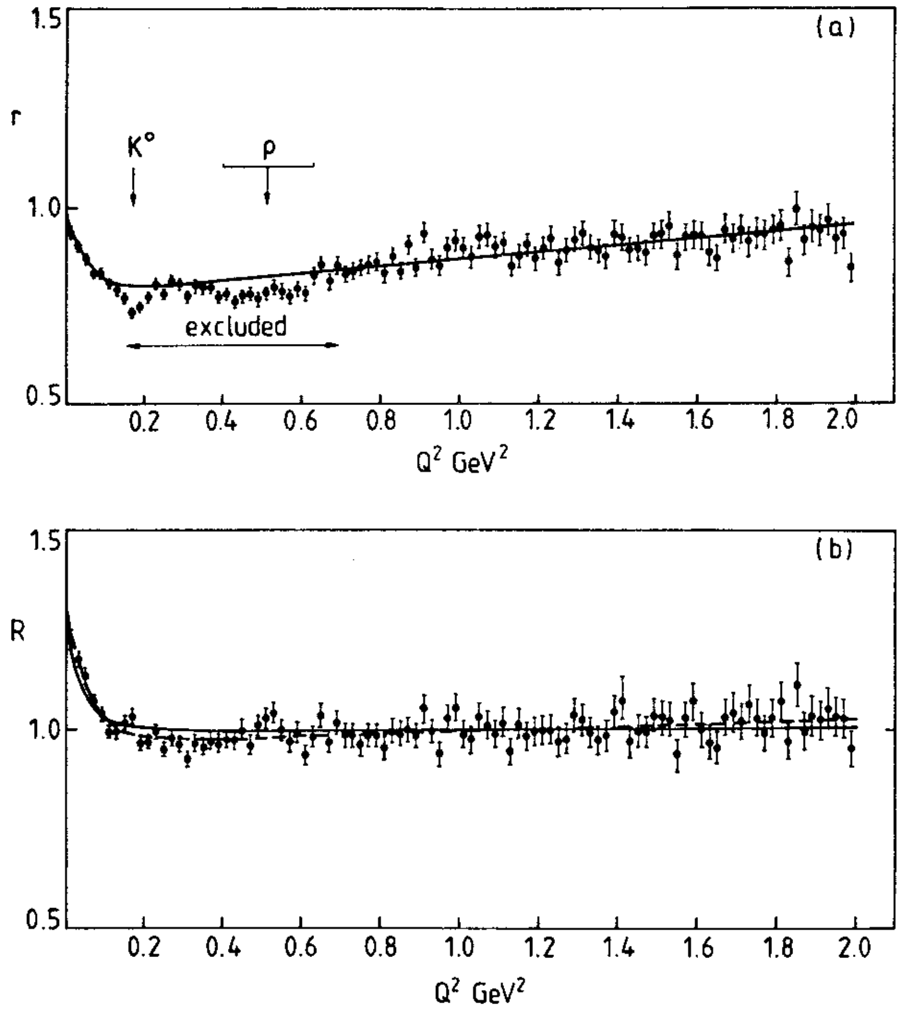
(a)
r
K°
ρ
excluded
Q^2 GeV^2
(b)
R
Q^2 GeV^2

Fig. 1.

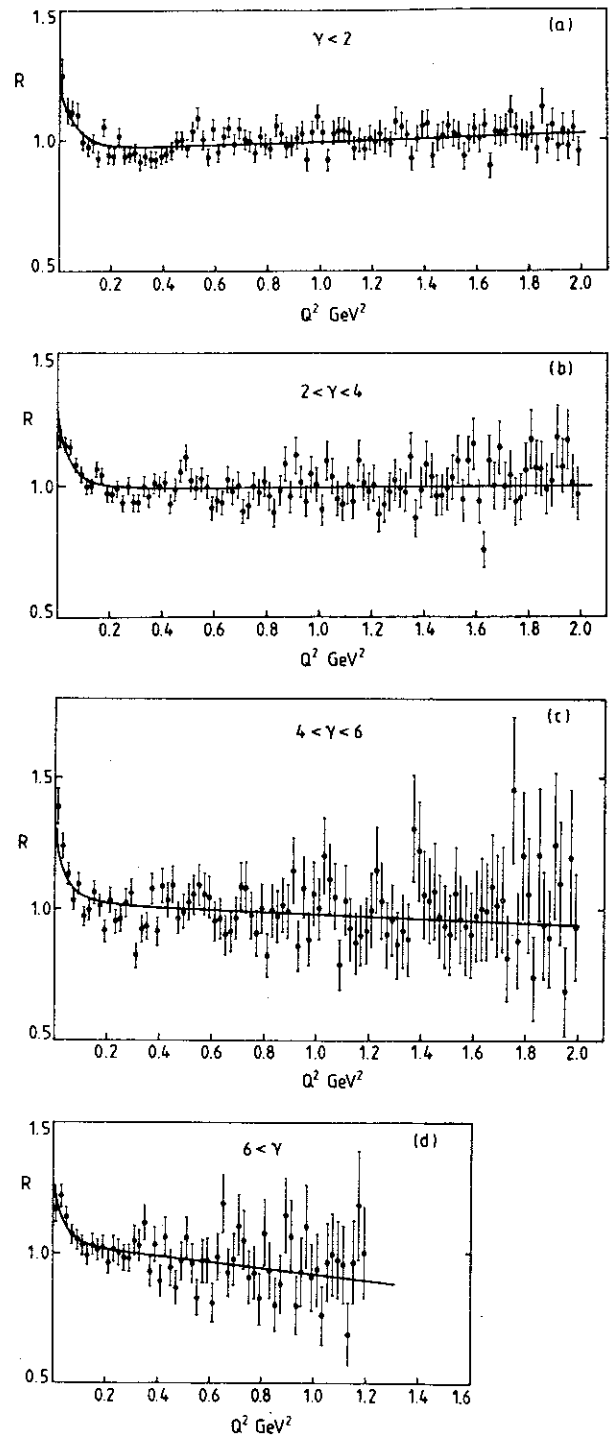
(a)
Y < 2
R
Q^2 GeV^2
(b)
2 < Y < 4
R
Q^2 GeV^2
(c)
4 < Y < 6
R
Q^2 GeV^2
(d)
6 < Y
R
Q^2 GeV^2

Fig. 2.

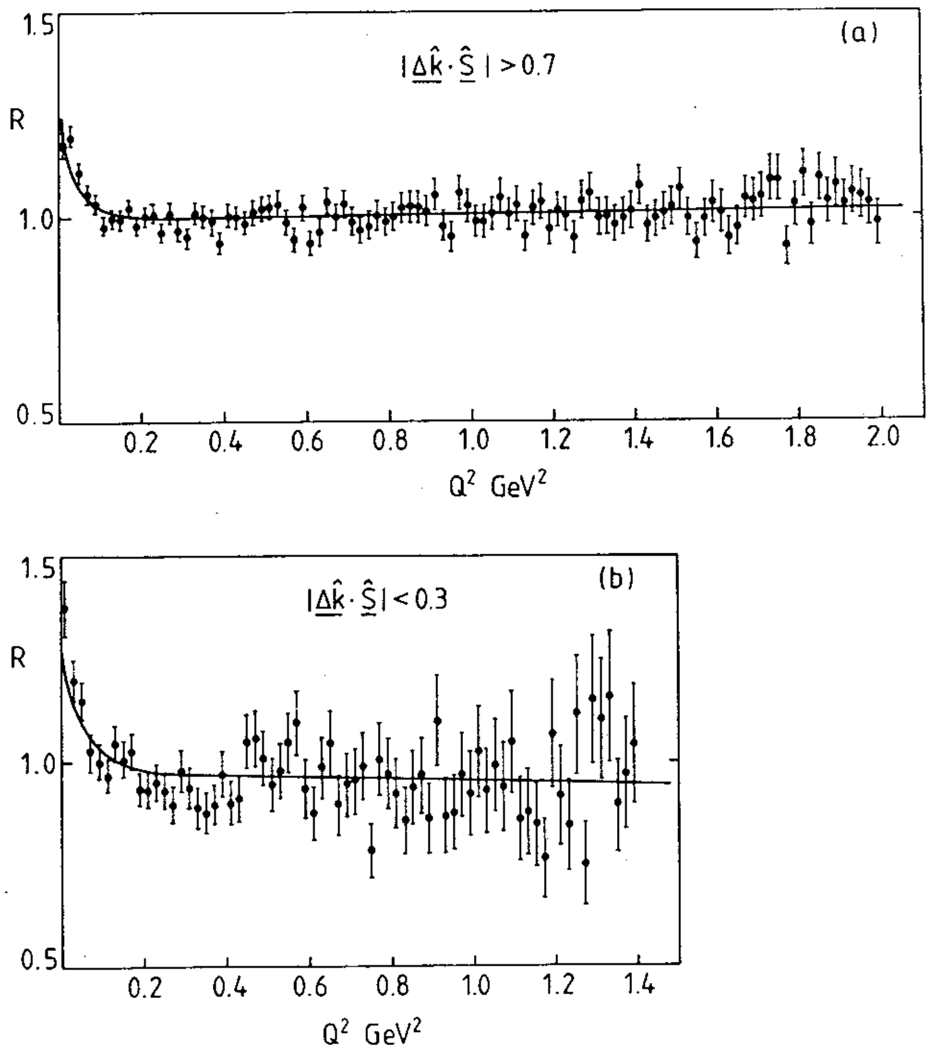
(a)
|Δk̂ · Ŝ| > 0.7
R
1.5
1.0
0.5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Q^2 GeV^2
(b)
|Δk̂ · Ŝ| < 0.3
R
1.5
1.0
0.5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Q^2 GeV^2

Fig. 3.

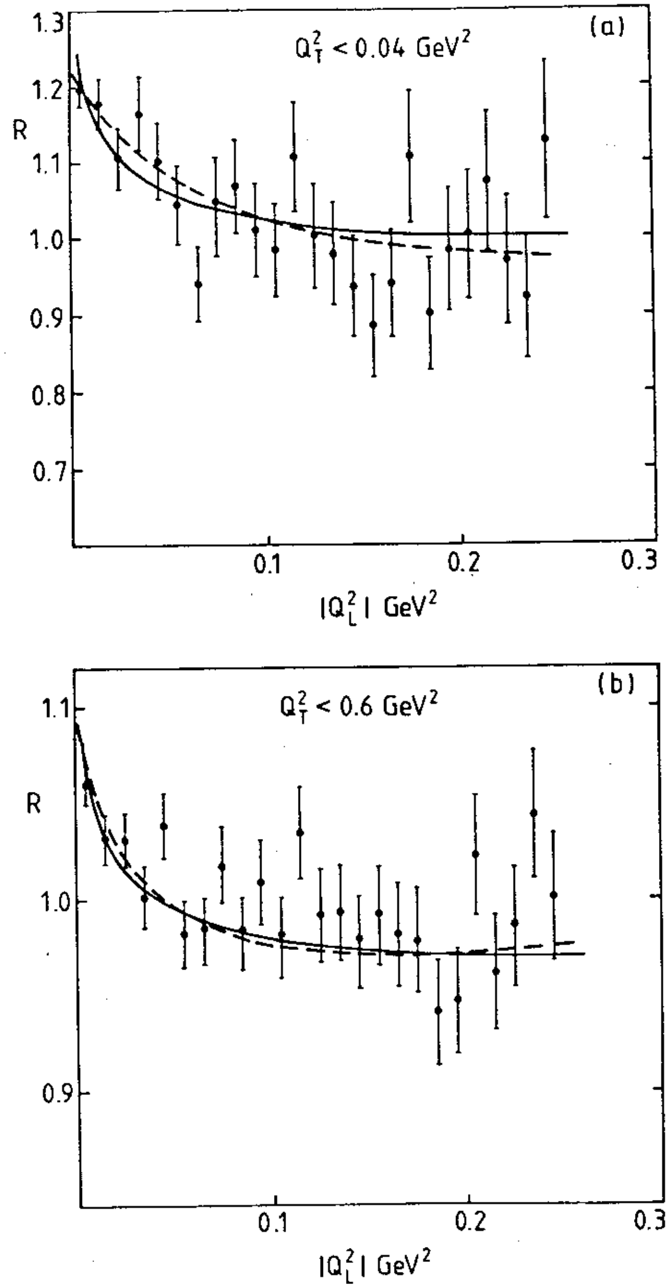
(a)
$Q_T^2 < 0.04$ GeV^2
R
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.1
0.2
0.3
$|Q_L^2|$ GeV^2
(b)
$Q_T^2 < 0.6$ GeV^2
R
1.1
1.0
0.9
0.1
0.2
0.3
$|Q_L^2|$ GeV^2

Fig. 4.

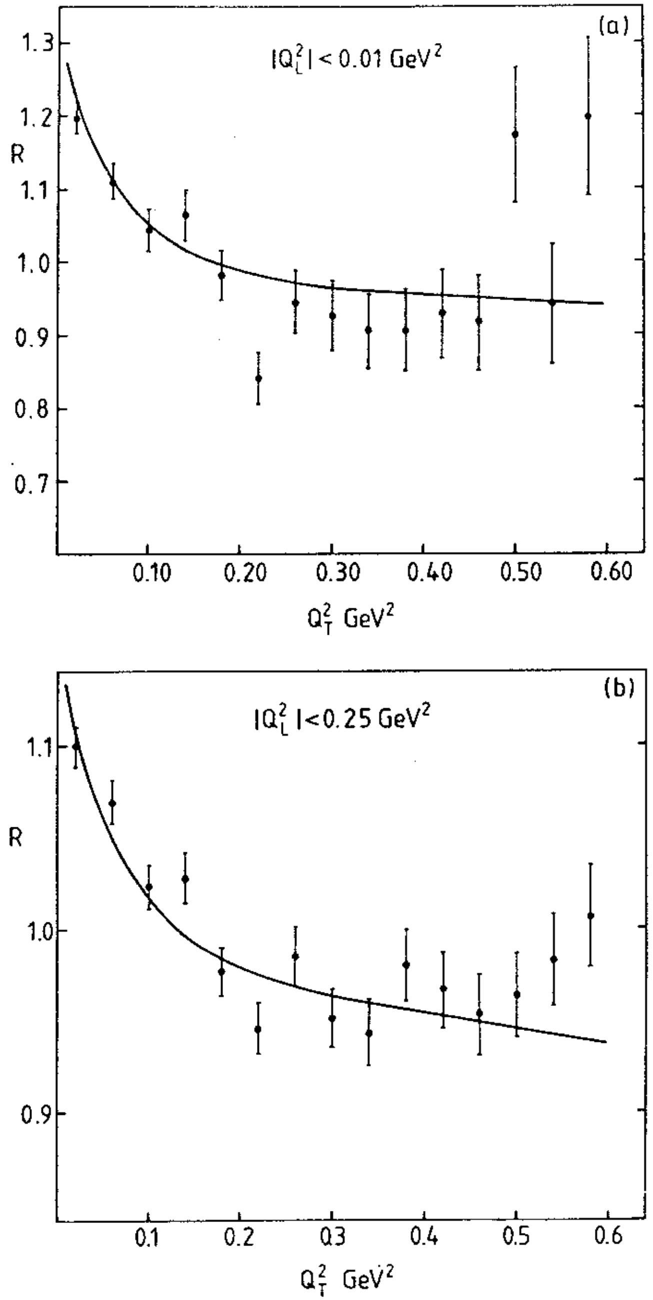
(a)
$|Q_L^2| < 0.01\ GeV^2$
R
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.10
0.20
0.30
0.40
0.50
0.60
Q_T^2 GeV^2
(b)
$|Q_L^2| < 0.25\ GeV^2$
R
1.1
1.0
0.9
0.1
0.2
0.3
0.4
0.5
0.6
Q_T^2 GeV^2

Fig. 5.

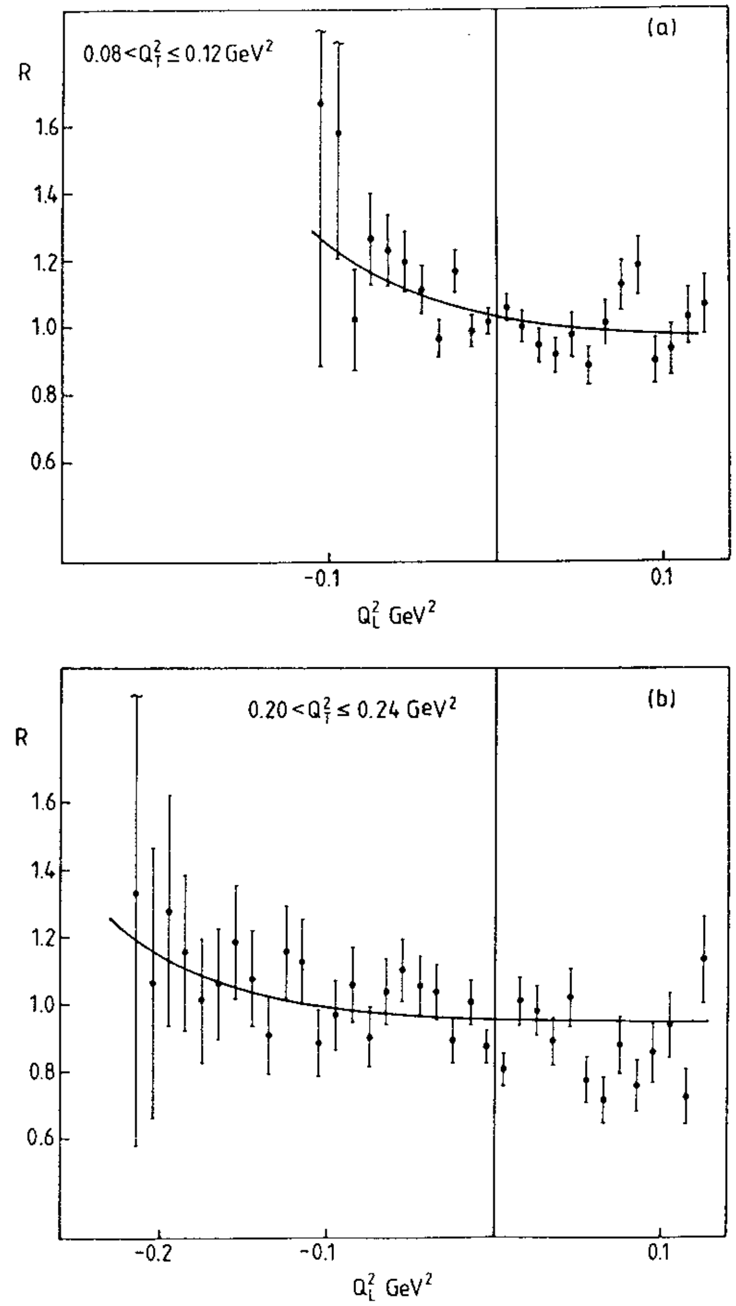
(a)
$0.08 < Q_T^2 \leq 0.12\ GeV^2$
R
1.6
1.4
1.2
1.0
0.8
0.6
-0.1
0.1
Q_L^2 GeV^2
(b)
$0.20 < Q_T^2 \leq 0.24\ GeV^2$
R
1.6
1.4
1.2
1.0
0.8
0.6
-0.2
-0.1
0.1
Q_L^2 GeV^2

Fig. 6.