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On the Contribution of Torons to the Vacuum Energy in Super-Yang-Mills Theory

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Abstract:

Toron's contribution to the path integral in Super-Yang-Mills theory is calculated. A non-zero contribution is that of 3 torons and 3 anti-torons. The vacuum energy, however, is zero if the fluctuation region of the torons is finite.

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A considerable effort was put in the past few years in an attempt to find a dynamical mechanism for supersymmetry breaking[1-3]. The reason being simple supersymmetry seems to cure the gauge hierarchy[4] problem in grand unified models[5] and it may also provide an explanation as to why the mass scales are so widely separated[1]. However, at ordinary energy scales this symmetry is not exact. Whereas perturbative quantum effects respect supersymmetry, it would be desirable if non-parturbative fluctuations were to break it. To that effect the role of instantons in supersymmetric gauge theories was extensively studied[3].

In Yang-Mills theory the ground state is infinitely degenerate and instantons provide the quantum mechanical tunnelling between these ground states, thereby contributing non-trivially to the vacuum energy density[6]. However, in the presence of massless fermions the tunnelling is completely suppressed due to the zero modes of the Dirac operator in the topologically non-trivial background (with non-zero Pontryagin index)[7]. Therefore, in supersymmetry where there are massless fermions, single instantons or single anti-instantons (or any other field configurations with non-zero Pontryagin index) do not contribute to the vacuum energy. However, configurations with zero topological charge do not have fermionic zero modes and they may contribute to the vacuum energy. An instanton-anti-instanton configuration is such an example.

Indeed it was shown in previous publications that the quantum fluctuations around this configuration induce negative vacuum energy in supersymmetric Yang-Mills theory[8] and in supersymmetric QCD[9]. The question arises whether this contribution indicates a genuine breaking of supersymmetry or it may be wiped out by other non-perturbative effects.

Non-perturbative configurations of a different type are the torons (configurations obeying twisted boundary conditions in a finite volume). Originally they were invented by 't Hooft[10] to account for the quantum mechanical tunnelling between the states having different twists, indicating the existence of electric or magnetic vortices[11]. It turns out that in a Yang-Mills theory where all fields are invariant under the center of the group (Z_N for SU(N)), periodic boundary conditions in a finite volume could be taken up to an element of the center, a twist. Such a twist defines a magnetic vortex, because it is associated with a singular gauge transformation which generates objects carrying magnetic charge. Thus we have six planes of twistings (in 4 dimensional space-time). Three of them are related to the three directions of the magnetic vortex and the other three (in the xt, yt, zt planes) when Fourier transformed are related to the electric vortices. The Fourier transform is carried out in the set of elements belonging to the center. Thus when the Wilson loop operator acts on a state carrying a twist which was Fourier transformed, it changes by one unit. Being a measuring operator for electric vortex lines, it means that the state carries such a vortex.

With these new boundary conditions (related to $\pi_1(G/Z)$), the degeneracy of the physical states is multiplied. On top of the Pontryagin index, n, which labels each state, we have the labels ($\vec{m}$, $\vec{e}$) of the magnetic and electric vortices. These are defined modulo N for SU(N). The degeneracy expected is, then, N^6. However, 't Hooft was able to show[11)] that the degeneracy is lifted because there are different phases of the theory, in which either the electric or the magnetic vortices become energetic (confinement or Higgs phase respectively). Therefore, we are left with N^3 degeneracy. Even this is still too much because Witten[2)] pointed out that for the zero energy states ($F_{\mu\nu} = 0$) with a given twist, there are only N independent gauge transformations, (that cannot be continuously deformed into each other) which generate N classical ground states. This was used to count the inequivalent zero energy states in supersymmetric Yang-Mills theory, the Witten index.

However, the question arises whether there are quantum mechanical tunnelling effects which may lift this degeneracy even further, or at least change the vacuum energy. Field configurations (torons) which may provide tunnelling between these ground states were found by 't Hooft[10]. These are finite action configurations defined in Euclidean space-time and satisfy twisted boundary conditions (The action is $4\pi^2/g^2N$ for SU(N). They carry fractional topological charge (multiples of 1/N) and they have a finite

contribution to the functional integral in the large N limit when g^2N is kept finite (unlike instantons whose contribution is suppressed by a factor e^{-CN}). Moreover, they were used by various authors[12)] to calculate fermionic condensates in SYM theory and to show the existence of chiral symmetry breaking. Their relevance to tunnelling (and vacuum energy) in a supersymmetric theory is a bit more limited due to the existence of fermionic zero modes in a topologically non-trivial background. Thus the only configurations to be considered are those having zero net topological charge.

In the following we will show that most of the contributions to the functional integral are zero but for that of 3 torons and 3 anti-torons. The vacuum energy, however, stays at zero if the fluctuation region of the torons is finite. The meaning of this result is two-fold. By itself, there is a suppression mechanism in the supersymmetric theory which makes most of the contributions zero. But it is not enough because some are still left. They do not lead to a vacuum energy and to supersymmetry breaking, though, because of a kinematical reason; the contribution to the path integral is not proportional to space-time volume (unlike that of an instanton anti-instanton). The second point to be noted is that the contribution of an instanton-anti-instanton[8)] is not wiped out and it has yet to be understood.

To be more specific we consider an SU(2) supersymmetric Yang-Mills theory. The Euclidean action is given by

$$S_E = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\lambda}^a i D_\mu \bar{\Sigma}_\mu \lambda^a \right) \quad . \qquad (1)$$

Here $F_{\mu\nu}$ is the field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\varepsilon^{abc} A^b_\mu A^c_\nu \quad ,$$

$D^{ac}_\mu = \delta^{ac}\partial_\mu + g\varepsilon^{abc} A^b_\mu$ is the covariant derivative, A^a_μ (a = 1,2,3) are the gauge potentials and λ^a are Majorana fermions. They are expressed in Euclideanized Weyl basis with Dirac matrices being

$$\gamma_\mu = \begin{pmatrix} 0 & \Sigma_\mu \\ \bar{\Sigma}_\mu & 0 \end{pmatrix} \quad , \qquad \Sigma_\mu = \bar{\Sigma}^+_\mu = (i\sigma_i , \mathbb{1})$$

and
$$\mathrm{Tr}\left(\Sigma_\mu \bar{\Sigma}_\nu\right) = 2\,\delta_{\mu\nu}$$

As was pointed out before, physical states are labelled by $|\vec{m},\vec{e}\rangle$, where $\vec{m}$ is the magnetic and $\vec{e}$ is the electric flux[11]. They are defined in the following way. We take wave functionals which depend on the gauge potentials satisfying twisted boundary conditions in the xy, yz, xz planes. This defines the magnetic charge, $\vec{m}$, with components in the z, x, y directions, respectively; $m_i = 1/2\ \varepsilon_{ijk}\, m_{jk}$, where m_{jk} is the twist in jk plane. This twist is defined up to an element of the center (Z_N for SU(N)) and it cannot be gauged away, thereby making the magnetic flux gauge invariant. Twists in the xt, yt, zt planes are implemented by the action of a gauge transformation satisfying twisted boundary conditions in these planes. The twists are once again up to elements of the center denoted by $\vec{k}$ (taking values in Z_N). We then Fourier transform $\vec{k}$ (within the center of the group) thus getting the electric flux $\vec{e}$. That is

$$|\vec{m},\vec{e}\rangle = \sum_{\vec{k}=0,1} |\vec{m},\vec{k}\rangle \exp\left(i\pi\vec{k}\cdot\vec{e}\right)^{*} \quad . \tag{2}$$

The Hamiltonian matrix for the lowest energy states is defined by:

$$H_{\vec{e},\vec{e}'}(\vec{m},\vec{m}') = \lim_{T\to\infty} \frac{1}{T} \sum_{\vec{k},\vec{k}'=0,1} \langle \vec{m}';\vec{k}'|\, e^{-HT} |\vec{m},\vec{k}\rangle \exp\left[i\pi(\vec{k}\cdot\vec{e}-\vec{k}'\cdot\vec{e}')\right], \tag{3}$$

where the transitions between the classical vacua are mediated by torons[10)]

$$\langle \vec{m}',\vec{k}'|\, e^{-HT} |\vec{m},\vec{k}\rangle \underset{T\to\infty}{\longrightarrow} \int \mathcal{D}A^a_\mu\, \mathcal{D}\lambda^a\, \mathcal{D}\bar{\lambda}^a\, e^{-S_E}$$

and the functional integral has to be evaluated over finite action confirations having twists $\vec{m}'-\vec{m}$, $\vec{k}'-\vec{k}$. In the supersymmetric model where there

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* For SU(N) the sum is over $\{k_1, k_2, k_3\} = 0,1,\ldots N-1$ modulo N and the phase is $(2\pi/N)\,\vec{k}\cdot\vec{e}$.

are massless fermions, the tunnelling effect by configurations having a net twist is completely suppressed due to the fermionic zero modes. Thus the only non-zero transitions are those by configurations having a net zero twist, i. e. by torons-anti-torons. As a result the initial and final states in (3) have the same twists: $\vec{m} = \vec{m}'$, $\vec{k} = \vec{k}'$.

Moreover, it was pointed out by Witten[2)] that the states with zero classical energy ($F_{\mu\nu} = 0$) have $\vec{k}\,||\,\vec{m}$. The gauge potential for such a state is a pure gauge $A_i = -i/g\, \partial_i\, U(x)\, U^{-1}(x)$ (i = 1,2,3 in the $A_0 = 0$ gauge), and for a given $\vec{m}$ (say in the z direction) $U(x,y,z) = P\, U(x+L, y,z)\, P^{-1} = Q\, U(x,y+L,z)\, Q^{-1} = U(x,y,z+L)$, where P,Q are constant SU(2) matrices satisfying $PQ = QP \exp(\pi i m)$. This matrix, $U(x,y,z)$, can then be written as $U(x,y,z) = P^{k_1} Q^{k_2} T_z^{k_3}$ where only $T_z(x,y,z)$ contributes non-trivially to $A_i(x)$ (because unlike P, Q it is not a constant matrix). As a result there are only 2 inequivalent gauge transformations (that cannot be deformed into each other) which define 2 independent classical ground states. For these states the Hamiltonian matrix becomes $H_{\vec{m}\cdot(\vec{e}-\vec{e}')}$. It is a Hermitian 2x2 matrix which classically has 2 zero eigenstates*. Quantum mechanically torons' (or anti-torons') contributions may lift the degeneracy of the ground states.

These finite action configurations are defined in a box of size L

$$A^a_\mu = \frac{4\pi}{g}\, \frac{\alpha^{(a)}_{\mu\nu}\,(x-z)_\nu}{L^2} \quad ; \quad \bar{A}^a_\mu = \frac{4\pi}{g}\, \frac{\bar{\alpha}^{(a)}_{\mu\nu}\,(x-\bar{z})_\nu}{L^2} \tag{4}$$

where $z_\nu(\bar{z}_\nu)$ are the locations and $\alpha^{(a)}_{\mu\nu}$ $(\bar{\alpha}^{(a)}_{\mu\nu})$ are the twist matrices of the toron (anti-toron). They are given by

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* The Hamiltonian matrix is NxN for SU(N) and has N zero energy classical states, thereby making the Witten index equal to N when the classical ground states are counted.

$$\alpha^{(1)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\quad;\quad\alpha^{(2)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}\quad;\quad\alpha^{(3)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$$

$$\bar{\alpha}^{(1)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}\quad;\quad\bar{\alpha}^{(2)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}\quad;\quad\bar{\alpha}^{(3)}_{\mu\nu}=\frac{1}{4}\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}\tag{5}$$

and they satisfy the following relations:

$$\alpha^{(a)}_{\mu\nu}=\frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\,\alpha^{(a)}_{\rho\sigma}\tag{6a}$$

$$\bar{\alpha}^{(a)}_{\mu\nu}=-\frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\,\bar{\alpha}^{(a)}_{\rho\sigma}\tag{6b}$$

$$\alpha^{(a)}_{\mu\nu}\,\alpha^{(a)}_{\nu\lambda}=\bar{\alpha}^{(a)}_{\mu\nu}\,\bar{\alpha}^{(a)}_{\nu\lambda}=-\frac{1}{16}\,\delta_{\mu\lambda}\tag{6c}$$

$$\alpha^{(a)}_{\mu\nu}\,\bar{\alpha}^{(b)}_{\nu\lambda}=0\tag{6d}$$

The action for both configurations is*

$$S_E=\frac{4\pi^2}{g^2}\tag{7}$$

We note that for a given twist, only translation invariance associated with the arbitrary choice of z_μ yields zero modes of the bosonic determinant. In particular there are no zero modes associated with dilation or orientation in group space. The reason being the finite box size and the twisted boundary conditions which do not allow constant gauge transformations to be implemented on the field configuration. Thus we have only 4 bosonic zero modes given by

$$A^{(+)a}_{\mu}(y)=-\frac{4\pi}{g}\,\frac{\alpha^{(a)}_{\mu\nu}}{L^2}\qquad;\qquad A^{(-)a}_{\mu}(y)=-\frac{4\pi}{g}\,\frac{\bar{\alpha}^{(a)}_{\mu\nu}}{L^2}\tag{8}$$

* For SU(N), $S_E = 8\,\pi^2/(g^2N)$ and it is finite when $N\to\infty$ if g^2N = finite

In the Gaussian approximation we use collective coordinates to account for these zero modes. These are given by an integration over $\left(\frac{4\pi^2}{g^2}\frac{1}{8\pi}\right)^2\mu^4 d^4z$, where the constant is just the normalization factor of the above zero modes and μ is the renormalization point, which has to be inserted due to the need to renormalize the determinant.

In the supersymmetric model, the massless fermions have zero modes in the background of the configurations (4). For the toron we have two left handed zero modes

$$\lambda^{(+)a}_{\alpha}=\frac{(\sigma^a)_\alpha^{\ \beta}\,u^{(+)}_\beta}{L^{3/2}}\tag{9}$$

and for the anti-toron two right handed zero modes

$$\bar{\lambda}^{(-)a\dot{\alpha}}=\frac{(\bar{\sigma}^a)^{\dot{\alpha}}_{\ \dot{\beta}}\,u^{(-)\dot{\beta}}}{L^{3/2}}\tag{10}$$

where $u^{(+)}$, $\bar{u}^{(-)}$ are given by either (1,0) or (0,1). In the functional integral we account for these fermionic zero modes by integrating over the Grassmann variables $\frac{d^2\beta^{(+)}}{\mu L}$ for the toron and $\frac{d^2\beta^{(-)}}{\mu L}$ for the anti-toron. This is zero because the integrand does not depend on $\beta^{(+)}(\beta^{(-)})$, thus proving that in the supersymmetric model quantum mechanical tunneling in a background of a toron or anti-toron is suppressed. In a similar way it is easily proven that tunnelling with any configuration having a net twist is suppressed. We are thus led to consider tunnellings by configurations having an equal number of torons and anti-torons twisted in the same directions. Twists in different directions will not do either, because of the existence of fermionic zero modes.

We first examine the contribution of a toron-anti-toron. We thus have the configurations (4) in two boxes L_1 and $L_{\bar{T}}$, which we take to be non-overlapping. We expand the quantum fields around this background and use the Gaussian approximation. The result of the functional integral is the inverse of the square root of the bosonic determinant where the zero modes (8) are factored out and integrated over by the collective coordinate method. For the fermions, we first double the number of degrees of freedom to get Dirac fermions and define the functional integral over the Weyl fermions as the square root of the determinant of the Dirac operator in the above background. Factoring out the contribution of the fermionic zero modes in (9) and (10), we then get the square root of the ratio of the fermionic over bosonic non-zero modes determinants. For non-overlapping boxes this ratio can be factorized into a product of ratios of determinants in a background of a toron and anti-toron and each is equal to one because of supersymmetry. We are thus left with the integrals over the bosonic collective coordinates, and the fermionic determinant in the subspace of zero modes (9) and (10). This turns out to be zero:

$$\bar{\lambda}^{(-)a}\, i D_\mu \bar{\Sigma}_\mu \lambda^{(+)a} = 0 \quad , \tag{11}$$

where D_μ is the covariant derivative in the background of a toron-anti-toron in boxes L_1 and $L_{\bar{1}}$, respectively. It is zero because both $\bar{\lambda}^{(-)}$ and $\lambda^{(+)}$ do not depend on x and they are parallel in group space to A_μ, $\bar{A}_\mu$.

We next examine the contribution of two torons-two-anti-torons. We have four boxes L_1, L_2, $L_{\bar{1}}$, $L_{\bar{2}}$, and we make the assumption that they are non-overlapping. Since we need a zero net twist each pair of toron-anti-toron should have twists in parallel directions. Without loss of generality we take 1 to be parallel to $\bar{1}$ and 2 to $\bar{2}$. As before the contribution of the non-zero mode fermionic over the non-zero mode bosonic determinant is one, and we are left with a fermionic determinant in the subspace of fermionic zero modes (9), (10) and an integration over the bosonic zero modes (8). Once again the fermionic determinant yields zero. Its matirx elements are

$$K_{ij} = \sum_s \int d^4x\, \bar{\lambda}^{(-)i}\, i D_\mu(A^s)\, \lambda^{(+)j} \tag{12}$$

where $i = \bar{1},\bar{2}$ and $j = 1,2$, and the sum is over all the boxes $1,\bar{1},2,\bar{2}$. Clearly to get a non-zero result A^S cannot be parallel in group space neither to $\bar{\lambda}^{(-)i}$ nor to $\lambda^{(+)j}$. But that is not possible because we have only two-torons and two anti-torons so it is parallel either to $\bar{\lambda}^{(-)i}$ or to $\lambda^{(+)j}$ (or to both). The result is that $K_{ij} = 0$ and there is no contribution from two torons and two anti-torons either. Note that (12) would have not been zero

if we had the freedom to orient the twisted configurations in different directions in group space. But this freedom was lost because of the twisted boundary conditions which the gauge transformation has to satisfy, and it cannot if it is associated with global gauge transformations.

The first non-zero contribution is that of three torons and three anti-torons. We have the boxes L_i, $L_{\bar{i}}$ $i = 1,2,3$, with toron i being parallel to anti-toron $\bar{i}$. The determinant to be calculated is that of a 12x12 matrix (for Dirac fermions) with matrix elements as in (12)

$$K_{ij} = \frac{4\pi}{L}\, \varepsilon_{ikj} \left(\alpha^{(k)}_{\mu\nu} z_{k\nu} + \bar{\alpha}^{(k)}_{\mu\nu} \bar{z}_{k\nu} \right) \sigma_i\, i \bar{\Sigma}_\mu \sigma_j \tag{13}$$

and its Hermitian conjugate. (Each element is a 2x2 matrix). In (13) we took all the boxes to have the same size, L, otherwise we need to keep track of the size, L_i, of each box. Thus

$$B^2 = \det \begin{pmatrix} 0 & K \\ K^+ & 0 \end{pmatrix} = \det (K K^+) \tag{14}$$

Define
$$k_{\ell\mu} = \frac{4\pi}{L}\, \varepsilon_{i\ell j} \left(\alpha^{(\ell)}_{\mu\nu} z_{\ell\nu} + \bar{\alpha}^{(\ell)}_{\mu\nu} \bar{z}_{\ell\nu} \right) \tag{15}$$

then
$$B = \det \begin{pmatrix} 0 & -k_{3\mu}\Sigma_\mu & k_{2\mu}\Sigma_\mu \\ k_{3\mu}\Sigma_\mu & 0 & -k_{1\mu}\Sigma_\mu \\ -k_{2\mu}\Sigma_\mu & k_{1\mu}\Sigma_\mu & 0 \end{pmatrix} \tag{16}$$

where we have factorized out the σ_i, σ_j and used $\det \sigma_i = 1$. The determinant in (16) is easily calculated to yield

$$B = \det \left[(k_1\cdot\Sigma)(k_3\cdot\bar{\Sigma})(k_2\cdot\Sigma) - (k_2\cdot\Sigma)(k_3\cdot\bar{\Sigma})(k_1\cdot\Sigma) \right] \tag{17}$$

and we use the notation $(k\cdot\Sigma) = k_\mu \Sigma_\mu$. To calculate the determinant in (17), we note that the matrix is a 2x2 matrix and can be expressed as $A_\alpha \Sigma_\alpha$, where

I_α is a four vector given by

$$A_\alpha = \frac{1}{2} \, Tr \, \bar{\Sigma}_\alpha \left[(k_1 \cdot \Sigma)(k_3 \cdot \bar{\Sigma})(k_2 \cdot \Sigma) - (k_2 \cdot \Sigma)(k_3 \cdot \bar{\Sigma})(k_1 \cdot \Sigma) \right] \qquad (18)$$

then

$$B = A_\alpha A_\alpha = 4 \left(\varepsilon_{\alpha\beta\gamma\delta} \, k_{1\beta} k_{2\gamma} k_{3\delta} \right)^2 \quad . \qquad (19)$$

To get (19) we use the anti-commutation relation satisfied by $\bar{\Sigma}_\mu$, Σ_ν, and the properties of the commutator matrices

$$\Sigma_{\mu\nu} = \frac{1}{2} \left(\Sigma_\mu \bar{\Sigma}_\nu - \Sigma_\nu \bar{\Sigma}_\mu \right)$$

$$\bar{\Sigma}_{\mu\nu} = \frac{1}{2} \left(\bar{\Sigma}_\mu \Sigma_\nu - \bar{\Sigma}_\nu \Sigma_\mu \right)$$

satisfying

$$\Sigma_{ij} = \bar{\Sigma}_{ij} = i \varepsilon_{ijk} \sigma_k$$

$$\Sigma_{4i} = -\bar{\Sigma}_{4i} = -i \sigma_i$$

From (19) we note that the contribution of three torons and three anti-torons vanishes if two of the three vectors k_i are parallel, i. e. if two of the three torons (or anti-torons) coincide which is, of course, consistent with the result found before that the contribution of two torons and two anti-torons vanishes.

We need now to integrate over the collective coordinates:

$$\prod_{i=1}^{3} \int d^4 z_i \, d^4 \bar{z}_i \, A_\alpha A_\alpha = 96 \left(\frac{2\pi^2 L^8}{3} \right)^3 \qquad (20)$$

where we use

$$\int d^4 z_1 \, d^4 z_2 \, z_{1\mu} z_{2\nu} = 0$$

$$\int d^4 z \, z_\mu z_\nu = \frac{L^6}{3} \delta_{\mu\nu}$$

and the relations (6). Multiplying now by the Jacobian factor $\mu^4 \left(\frac{4\pi^2}{g^2} \frac{1}{8\pi} \right)^2$ for the translational zero modes and by $(\mu L)^{-1}$ for each pair of fermionic zero modes and taking into account the classical action (7) of the twisted configurations we finally get

$$\langle e^{-HT} \rangle \simeq 96 \left(\frac{2\pi^2 L^8}{3} \right)^3 \mu^{24} \left(\frac{4\pi^2}{g^2} \frac{1}{8\pi} \right)^{12} (\mu L)^{-6} \exp\left(-\frac{24\pi^2}{g^2(\mu)} \right) .$$

Using the renormalization group invariant scale

$$\Lambda_{QCD}^{18} = \mu^{18} \exp\left(-\frac{24\pi^2}{g^2(\mu)} \right) \quad , \qquad (21)$$

we find

$$\langle e^{-HT} \rangle \simeq \frac{256}{9} \left(\frac{1}{64\pi} \right)^6 \left(\frac{4\pi^2}{g^2(L)} \right)^{12} \left(\Lambda_{QCD} L \right)^{18} \quad . \qquad (22)$$

We note that four bosonic and two fermionic zero modes for each twisted configuration yield correctly the renormalization group behavior of the coupling constant.

The contribution of n torons and n anti-torons vanishes unless $n = 3r$ where r is an integer. The reason is that there are only 3 independent directions in space. We can thus group the torons (and anti-torons) according to the direction of the twist, having n_j ($j = 1,2,3$) torons in the j'th direction and $n_1 + n_2 + n_3 = n$. The matrix elements K_{ij} then become $2n_i \times 2n_j$ matrices having equal collums, and the determinant in (14) vanishes. For the case $n = 3r$, we can group the twisted configurations into groups of three torons and three anti-torons and take the contribution (22) to the power r, thereby exponentiating it.

From this we can calculate the vacuum energy by dividing by T and taking the limit $T \to \infty$. Since the fluctuation region of the twisted configurations, L, is finite, the vacuum energy is zero, which means that supersymmetry is not broken by this type of quantum fluctuations. The result would not change if the boxes are taken to have different sizes, or to overlap. If there is such an overlap, factorizing the determinants into products of determinants in a background of a toron or anti-toron may not be justified, so some of the contributions found to be zero may not vanish.

However, vacuum energy will still vanish because, generally the contribution of a toron whose fluctuation region is finite, is not proportional to space-time volume. This is different from the case of instantons whose contribution is proportional to space-time volume. Thus torons cannot break supersymmetry, whereas instantons may. It is interesting to note, though, that there is a suppression mechanizm in the supersymmetric model where most of the contributions of configurations satisfying twisted boundary conditions to the path integral vanish. However, it is not enough and some are left. They do not lead to supersymmetry breaking only because the order parameter for such a breaking is the vacuum energy, and their contribution to the vacuum energy vanishes whether the theory is supersymmetric or not.

What can be learned from this and the previous calculations[8,9] is that supersymmetry is not protective enough and contributions of non-perturbative effects to the path integral are not necessarily zero. Thus much more work has to be put in order to understand the dynamical mechanizm of supersymmetry breaking.

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