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R.D. Peccei

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MASS GENERATION IN COMPOSITE MODELS

R. D. Peccei *

Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

I discuss aspects of composite models of quarks and leptons connected with the dynamics of how these fermions acquire mass. Several issues related to the protection mechanisms necessary to keep quarks and leptons light are illustrated by means of concrete examples and a critical overview of suggestions for family replications is given. Some old and new ideas of how one may actually be able to generate small quark and lepton masses are examined, along with some of the difficulties they encounter in practice.

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1. Mass Protection Mechanisms and Dynamical Issues

The idea that quarks and leptons may not be elementary has been pursued with vigor in the last few years[1]. However, it has become clear that, if quarks and leptons are composite, the dynamics of the underlying theory which produces these states as bound states is quite different than any other bound state dynamics we know. To appreciate this point, it is convenient to consider the typical dynamical momentum scale (the compositeness scale Λ_c) connected with the bound state process. For ordinary bound states, like atoms or hadrons, this typical dynamical scale is smaller than, or of the same order of the typical bound state masses. For quarks and leptons, on the other hand, one knows that

$$\Lambda_c \gg m_{q,\ell} \tag{1}$$

Indeed, already the most conservative bounds on Λ_c, coming from an analysis of the electron and muon (g-2) and from Bhabha scattering at high energies, give $\Lambda_c \gtrsim 1$ - 2 TeV [2]. Flavor changing transitions, like $K \to \mu e$, if they proceeded unhindered in the underlying theory, could indicate even larger values of Λ_c [2] - $\Lambda_c \gtrsim 10^2$ - 10^3 TeV.

Before one can take the idea that quarks and leptons are composite seriously, one must be able to offer an explanation for Eq. (1). The current thinking is that this obtains through some protective symmetry. The usual presumption made is that the underlying theory (preon theory) is some kind of confining non Abelian gauge theory. Such theories, in the absence of preon masses, have as their only scale the scale Λ_{preon} where the running coupling constant becomes of O(1). Hence $\Lambda_c \sim \Lambda_{preon}$. However, since Λ_{preon} is really the only scale in the theory, one then expects that also all bound state masses should be proportional to Λ_{preon}: $M \sim \Lambda_{preon} \sim \Lambda_c$. Therefore, <u>to obtain light bound states, the quarks and leptons with $m_{q,e} \ll \Lambda_c$, require some protection mechanism to decouple their masses from the dynamical scale $\Lambda_{preon} \sim \Lambda_c$.</u>

In this talk I will pursue this "conventional" point of view. However, before beginning my detailed discussion I want to make four general remarks:

i) It could be that the preon theory is not a non Abelian gauge theory. Then the dynamics could well be different and perhaps light bound states could originate naturally. Unfortunately, no one know how to proceed in this direction. Even a toy example of an "alternative" preon model would be very welcome[3].

ii) Models of composite quarks and leptons, based on an underlying non Abelian gauge theory, which do not have a built in protective mechanism to keep the quarks and leptons light should not be considered too seriously. Unfortunately, many such models exist in the literature![4] I would not advocate totally ignoring these models, as some yet undiscovered physical principle - as was the case of quark confinement for QCD - may one day make them viable. However, from the standpoint of our present understanding, they are dynamically inconsistent.

iii) Protective mechanisms to keep quarks and leptons light, in practice, produce these states as massless bound states of the underlying theory. Furthermore, it usually turns out that to make sure that there are really these massless states in the spectrum, one needs to make additional assumptions besides those of the protective mechanism. Clearly these additional assumptions need cross checks, whenever possible. In what follows I shall discuss certain classes of models where one can perform the cross checks, thereby confirming the presence of massless states in the bound state spectrum.

iv) To make contact with reality the protection mechanisms, and related assumptions, imposed to get massless quarks and leptons need, eventually, to be violated weakly. It is this last step, that of going from $m_{q,e} = 0$ to small quark and lepton masses, which is the most difficult. I shall address this issue in the closing part of this paper.

To date, two mass protection mechanisms have been suggested to get massless fermionic bound states from a confining non Abelian preon theory. The first of

these mechanisms, suggested by 't Hooft[5], employs chiral symmetry; the second mechanism, suggested by Buchmüller, Yanagida and myself[6], employs supersymmetry and a broken global symmetry. It is possible to combine both mechanisms[7] and, indeed, one of the examples I shall discuss has this feature.

Using chirality as a protection mechanism is very appealing and easy to understand. If the preon theory has a global chiral symmetry G, which is preserved in the binding, then one naturally expects to have chirally unpaired massless bound states and chirally paired heavy states. The quarks and leptons are then to be identified as the massless bound states of these theories. The, so called, Quasi Goldstone Fermion (QGF) mechanism of Ref. 6 is also rather simple. Imagine a supersymmetric preon theory with some global symmetry G. If in the binding G suffers a spontaneous breakdown to another symmetry H, then necessarily there ensue dim G - dim H Nambu Goldstone massless bosons in the theory. Because of supersymmetry, these bound state bosons are accompanied by massless fermion partners - the QGF. Quarks and leptons in these models are identified as these QGF excitations.

It is clear from the above that these protective mechanisms guarantee dynamically that there are massless fermions in the bound state spectra. Nevertheless, for the protective mechanisms really to apply requires that some additional dynamical assumptions hold. For the chirality case, it is necessary that the chiral symmetry G be preserved in the binding. In the case of the QGF mechanisms, on the other hand, the global symmetry G need to break down in the binding. Furthermore, in this case, the particular pattern of QGF is not just purely fixed by group theory. It depends on other conditions besides the breakdown $G \rightarrow H$.

t'Hooft[5], in his classic lectures, has spelled out the conditions which are necessary for chirality preservation in the binding. These are that the anomalies connected with the chiral currents of G, computed at the preon level, match those computed at the bound state level, using the unpaired $m = 0$ fermions in the spectrum. The necessity, but not sufficiency, of this anomaly matching condition can be appreciated as follows. The global chiral currents of G have, in general, an Adler-Bell-Jackiw[8] anomaly. That is, the three point function of these currents, although classically conserved since G is a symmetry, actually has a non zero divergence

$$q_3^\lambda \; \Gamma_{\mu\nu\lambda}(q_1 q_2 q_3) = A_{\rm Preon}\, \epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \qquad (2)$$

The coefficients $A_{\rm preon}$ can be computed, algebraically, from a knowledge of the chiral charges of the preons. It follows from (2) - which is basically a short distance result - that $\Gamma_{\mu\nu\lambda}$ must have a singular behaviour in the momentum squared q_i^2 - which reflects long distance properties of the theory. It is easy to show[9] that, at the symmetric point $q_1^2 = q_2^2 = q_3^2 = q^2$,

$$\Gamma_{\mu\nu\lambda}(q_1 q_2 q_3)\Big|_{\substack{\rm Sym.\\ \rm pt.}} = \frac{A_{\rm Preon}}{q^2}\left\{\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta q_{3\lambda} + {\rm cycl.}\right\} + \text{non sing. terms} \qquad (3)$$

Since $\Gamma_{\mu\nu\lambda}$ is a Green's function of the theory this singular behaviour must be reproduced by computing $\Gamma_{\mu\nu\lambda}$ in terms of the possible bound states of the theory. One possibility is that the singularity in $\Gamma_{\mu\nu\lambda}$ is due to the presence of massless fermionic states. The structure of $\Gamma_{\mu\nu\lambda}$ that emerges from this computation is precisely that of Eq. (3), except that the residue of the q^2 pole now will involve the coefficent A_{BS}, computed algebraically from a knowledge of the chiral charges of the $m = 0$ bound states. Consistency necessarily requires

$$A_{\rm preon} = A_{BS} \qquad (4)$$

which is 't Hooft condition. However, the singularity in $\Gamma_{\mu\nu\lambda}$ could also be due to the presence of zero mass bound state bosons coupled to the chiral currents. These Goldstone bosons signal the breakdown of the chiral symmetries. Since, in general, it is not possible to exclude this latter possibility, it is clear that finding a solution of the anomaly matching equations is no guarantee that the dynamics of the model really gives rise to $m = 0$ fermions. Although one can show[9] that only zero mass bosons or massless bound state spin 1/2 fermions can reproduce the singularity in Eq. (4), to actually determine, in practice, which option applies for a given model requires more dynamical input. It is here that cross checks of the dynamics are important.

Similar dynamical queries exist for the QGF mechanism. In general for a given global symmetry G, many possible breakdown patterns exist

$$G \rightarrow H_i \qquad i = 1,2,\ldots \qquad (5)$$

Which of these patterns obtains is a dynamical question which requires investigation. A connected issue in that, given a breakdown pattern $G \rightarrow H_i$, the spectrum of QGF is not fixed immediately just by group theoretical considerations. Supersymmetry only tells one that the number of bosonic degrees of freedom and the number of fermionic degrees of freedom must match. One has that the number of QGF is given by[10]

$$n_{QGF} = \frac{1}{2}\left\{ n_{GB} + n_{QGB}\right\} \qquad (6)$$

Here n_{QGB} is the number of possible additional bosonic partners of the Goldstone bosons, which dynamically occur in the breakdown. On general principles one only knows that[11]

$$1 \leq n_{QGB} \leq n_{GB} \qquad (7)$$

To determine what is the number of these bosonic excitations requires a dynamical calculation, which, in general, is beyond our present capabilities. However, for some models, by looking at different dynamical aspects, one can at times infer with a certain degree of reliability what is the most likely pattern of QGB (and therefore of QGF) which should emerge.

For the remaining of this section I would like to discuss two examples where one can perform some dynamical cross checks which provide evidence that the desired protection mechanism really works. These examples provide the best evidence I know that it is indeed dynamically possible to obtain massless bound state fermions in the spectrum of a confining non Abelian gauge theory.

In the first example, chirality is the protection mechanism. The underlying theory is an SU(N) gauge theory and one has two kinds of fermionic Weyl preons. A preon A_{ij} transforming according to the two index antisymmetric representation of SU(N), and (N-4) preons $\bar{F}^i_\alpha$ ($\alpha = 1,\dots,N-4$), transforming as the fundamental-bar representation of SU(N). (The reason that there are (N-4) $\bar{F}$ is so that the model is free of SU(N) gauge anomalies.) This model has been studied, some time ago, by Georgi[12] and by Dimopolous, Raby and Susskind[13]. It has been recently reexamined by Eichten, Preskill, Zeppenfeld and myself[14], particularly in regard to the properties at large N. Classically, the model has a global SU(N-4) x $U_F(1)$ x $U_A(1)$ chiral symmetry. However, the preon number symmetries have gauge anomalies $U_A(1)SU(N)^2$, $U_F(1)SU(N)^2$ and are not, individually, good quantum symmetries. At the quantum level, the global symmetry of the model is

$$G = SU(N-4) \times U_Q(1) \tag{8}$$

where

$$Q = \frac{1}{N}\left\{ (N-4)\, Q_A - (N-2)\, Q_F \right\} \tag{9}$$

is the combination of the preon number symmetries which has no gauge anomalies.

The global SU(N-4) and Q currents have non trivial anomalies at the preon level. These anomalies must be matched at the bound state level by either having massless fermions or, in case the symmetry is broken, by having massless Goldstone bosons in the spectrum. For this model it is easy to find candidate bound state fermions for which the anomalies match. For instance, consider the $U_Q(1)^3$ anomaly. At the preon level one has

$$A_{\rm Preon}[U_Q(1)^3] = \left[\frac{N-4}{N}\right]^3 \frac{N(N-1)}{2} + \left[-\left(\frac{N-2}{N}\right)\right]^3 N(N-4) = -\frac{(N-4)(N-3)}{2} \tag{10}$$

where the factors in the square brackets above are the $U_Q(1)^3$ charges of the preons, while the other factors are their SU(N) and SU(N-4) multiplicity in the triangular loop. The simplest candidate fermion bound states are

$$(B_S)_{\alpha\beta} \sim (F^{Ti}_\alpha\, \sigma_2\, \sigma_\mu\, F^j_\beta)\, \sigma^\mu A_{ij} \tag{11a}$$

$$(B_A)_{\alpha\beta} \sim (F^{Ti}_\alpha\, \sigma_2\, F^j_\beta)\, A_{ij} \tag{11b}$$

which both have Q = -1 and transform, respectively, as the two rank symmetric and two rank antisymmetric representations under the global SU(N-4). Since the number of components of B_S is precisely $\frac{1}{2}$(N-4) (N-3), it is clear that the $U_Q(1)^3$ anomaly computed with these states matches the preonic anomaly

$$A_{B_S}[U_Q(1)^3] = (-1)^3 \frac{(N-4)(N-3)}{2} = A_{\rm Preon}[U_Q(1)^3] \tag{12}$$

By direct calculation, it is not hard to show that all the other global anomalies, computed with B_S, also match those at the preon level. Hence, the states B_S are natural candidates for being massless bound state of the model, protected by the chiral symmetry G. The important question to ask, however, is whether this is the way the dynamics works. For this model, in fact, there are two separate dynamical arguments[13,14] which support the presumption that the B_S states are really massless. I shall briefly discuss these arguments here, since they are very instructive.

Dimopolous, Raby and Susskind[13] used "Complementarity" to argue for having the B_S states massless. The notion of complementarity, which is borrowed from results obtained in lattice gauge theories[15], is that, if a gauge theory suffers spontaneous breakdown via some Higgs fields transforming according to the fundamental representation of the gauge group, there should be no phase boundary between the Higgs phase and the phase where the gauge theory confines. This has two consequences:

i) The global symmetries in the Higgs and confining phases should be the same.

ii) The massless excitations in both phases should agree.

Hence, if one can apply complementarity to the model, one can learn about the massless fermions in the confining phase by studying the model in the much simpler Higgs phase. This is what was done in Ref. 13.

The Higgs phase of the model occurs if condensates of the $\bar{F}$ and A preons form, which break the SU(N) gauge symmetry. It is easy to check that the channel in which condensation is most attractive (MAC)[13] is the one involving $\bar{F}$ and A. Writing an "effective" Higgs field

$$\Phi_i^{\alpha} = A_{ij} F^{j\alpha} \tag{13}$$

and assuming that the vacuum expectation value of Φ takes its most symmetric form

$$\langle \Phi_i^{\alpha} \rangle = v\, \delta_i^{\alpha} \qquad \alpha, i = 1, \ldots, N-4 \tag{14}$$

one sees that, in the Higgs phase, the gauge and global symmetry of the model breaks as follows.

$$SU(N)_{\text{Gauge}} \times [SU(N-4) \times U(1)_Q] \rightarrow SU(4)_{\text{Gauge}} \times [SU(N-4)_{\text{diag}} \times U(1)_{\bar{Q}}] \tag{15}$$

The remaining global symmetry, in this phase, is obviously the same as the global symmetry in the confining phase. Since Φ indeed transforms according to the fundamental representations of the gauge group, this is precisely what we would expect from complementarity.

Because of the breakdown, certain of the $\bar{F}$ and A preons now acquire mass, but some still remain massless. It is straightforward to check that the vacuum expectation value (14) leaves the states of Table I massless.

Table I: Massless Fermions in the Higgs Phase

$SU(4)_{\text{gauge}}$	$SU(N-4)$	$U_Q(1)$
1	$\square\!\square$	-1
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	0
$\bar{\square}$	$\bar{\square}$	$-1/2$
$\square$	$\square$	$+1/2$

I note that the gauge non singlet states are in either a real ($\begin{smallmatrix}\square\\\square\end{smallmatrix}$) or in vector-like ($\square + \bar{\square}$) combinations. One expects, therefore, that the dynamics of the SU(4) gauge theory is such that these states are confined and give rise to no physical massless states. The gauge singlet states are, therefore, the only relevant massless states in the Higgs theory. By complementarity, these states must have the same quantum numbers as the massless states in the confining phase. From Table I it is clear that these states indeed have the quantum numbers of the B_S fermions. Hence complementarity makes it plausible that the B_S fermions appear as massless bound states in the spectrum and that the G symmetry is unbroken.

Complementarity is not the only dynamical argument for the B_S fermions to appear in the theory. One can argue their existence also from large N considerations[14]. The point is that the large N limit of this model ($N \to \infty$, $g^2 \to 0$ with g^2N fixed) is quite different than in QCD. In QCD, in the large N limit, one can show[16] that the chiral flavor symmetry $SU(m)_L \times SU(m)_R$ breaks down completely. Here, because the number of flavors grows with N, it no longer follows that all the chiral symmetries break down. Indeed, one can show[14] that at least the $U_Q(1)$ chiral symmetry must survive. Since the $U_Q(1)^3$ anomaly is nonvanishing (c.f. Eq. (10)), it follows that there must exist massless fermion bound states in the spectrum to match this anomaly. Clearly the B_S states with $Q = -1$ fit the bill.

Let me enlarge slightly on the above discussion. In QCD in the large N limit planar graphs without quark loops dominate. Thus the only bound states which survive are mesons and these are stable since decay amplitudes are down by $1/N$. Since the chiral $SU(m)_L \times SU(m)_R$ symmetry has non vanishing anomalies at the quark level, it follows that the anomaly matching must be done by the appearance of Goldstone excitations in the bound state spectrum. (These are the only ones which survive at large N!). Hence the chiral symmetry is broken.

For the model at hand, it is no longer true that planar graphs with $\bar{F}$ loops inside are down by $1/N$. Because there are N possible $\bar{F}$ states going

around the loop, these graphs are also of O(1). Furthermore, the insertion of an A loop also does not give a factor of $1/N$, since the two index nature of A_{ij} compensates the purely kinematical $1/N$ factor from having an extra loop. Hence, at large N, both meson and nonmeson states survive, as well as contributions involving BB^+ states. Because all of the contributions are of the same order, the q^2 singularity in the three point function of the global currents can arise from the presence of Goldstone bosons and/or the existence of massless fermionic bound states. However, for the case of the $U_Q(1)^3$ anomaly one can argue[14] that this singularity must be produced by massless fermions, rather than by Goldstone bosons. This is because in the large N limit all condensates which survive (i. e. whose relevant operators appear in cut planar diagrams) have $Q = 0$. To break $U_Q(1)$ requires condensates involving ϵ tensors, and these objects are down by powers of $1/N$. Thus $U_Q(1)$ remains unbroken, and the $U_Q(1)^3$ anomaly informs us that there must be massless fermions in the spectrum to provide the matching. As I said earlier, the B_S states appear as the most likely candidates for the matching of anomalies. However, at large N, it is not possible to argue that the SU(N-4) symmetry really survives. So the $N^2/2$ massless fermions with $Q = -1$ could well be appropriate linear combinations of B_S and B_a. Only complementarity, with the most symmetric breaking of Eq. (14) is an argument for SU(N-4) to remain unbroken.

The second example I want to discuss was developed some time ago in collaboration with Buchmüller and Yanagida[17]. It uses the QGF mechanism to keep some bound state fermions massless and its dynamics is also subject to a variety of consistency checks. The model is based on a supersymmetric preon theory with an SU(2) gauge interaction. There are in total 6 preon superfields (containing both complex scalars and Weyl fermions) which transform as SU(2) doublets: Φ^P_α ($P = 1,\ldots,6$; $\alpha = 1,2$). The global symmetry of the model, at the quantum level is

$$G = SU(6) \times U_X(1) \qquad (16)$$

where $U_x(1)$ is an $SU_{gauge}(2)$ anomaly free linear combination of preon number and of R symmetry[17].

This model can provide one generation of left-handed quarks and leptons - plus an additional fermion, the novino - if one assumes that the symmetry in (16) breaks down as

$$G \rightarrow H = SU(4) \times SU(2) \times U_{X'}(1) \tag{17}$$

Obviously from such a breakdown there ensue 17 Goldstone bosons, transforming under H as

$$GB \sim (4,2) + (\bar{4},2) + 1,1) \tag{18}$$

The desired QGF pattern to get physics out of the model is that these states transform only as a (4,2) but not as a $(\bar{4},2)$. That is, one needs to obtain chiral fermions. So it is necessary to assume that the dynamics is such that the pattern of supersymmetric partners of the GB is:

$$\begin{aligned} QGF &\sim (4,2) + (1,1) \\ QGB &\sim (1,1) \end{aligned} \tag{19}$$

If indeed (19) obtains, then by assigning color and charge properly to the preons, the massless bound states contain the 4 left handed doublets of one generation of quark and leptons plus an extra singlet. Right handed states can be obtained by a slight extension of the model[18]. However, the question that needs to be answered here, is whether Eqs. (17) and (19) obtain.

For the breakdown $G \rightarrow H$ of (17) to occur one needs the condensate

$$v_{12} = \langle \epsilon^{\alpha\beta} \Phi^1_\alpha \Phi^2_\beta \rangle \tag{20}$$

to form. One can ask, however, why do not the condensates v_{34}, v_{56}, etc. form? Or why do any condensates form at all? Even granting that (20) is the only condensate that forms in the theory, besides the QGF, QGB pattern in (19),

one could have instead:

$$QGF \sim QGB \sim (4,2) + (\bar{4},2) + (1,1) \tag{21}$$

Why does (19) and not (21) obtain in the theory?

For this model there are two separate arguments which support the breakdown given in (17) and the pattern of QGF of Eq. (19):

(i) The group H is chiral and therefore, if it is to be preserved in the binding, there must be massless fermions in the spectrum which match the preon anomalies. It is easy to check[17] that the QGF given in Eq. (19) precisely match the preon anomalies. Thus their masslessness could be attributed directly to chirality, without appealling to supersymmetry! The QGF given in Eq. (21) do not suffice to match the H anomalies.

(ii) The model has a complementary Higgs phase in which the scalar components of the superfield Φ^p_α condense as

$$\langle \Phi^p_\alpha \rangle = v\, \delta^p_\alpha \tag{22}$$

Obviously such a vaccum expectation value breaks the gauge and global symmetry to:

$$SU(2)_{gauge} \times SU(6) \times U(1)_x \rightarrow SU(4) \times SU(2)_{diag} \times U(1)_{x'} \tag{23}$$

The remaining global symmetry is precisely the one assumed to hold in the confining phase in Eq. (17). Furthermore, it is easy to check that the massless fermions in this Higgs phase have precisely the quantum numbers of the QGF of Eq. (19).

2. Family Replication - a Critical Overview

Besides having dynamics which allow for the appearance of m = 0 fermions, preon models must be able to generate family replications. After all, this is one of the prominent and unexplained features of quarks and leptons. Two routes have been followed here: either one puts in, more or less by hand, an underlying family structure, or one tries to generate family repetitions through the dynamics. The first approach is somewhat inelegant and unsatisfactory, but could after all be right. It has happend before. The strangeness of the kaons is due to the presence of a strange quark in the bound state! The second approach is more ambitious and needs quite clever dynamics. The most usual strategy is to try to convert gauge degrees of freedom in the underlying theory into some sort of bound state repetitions.

As an example of how one can mechanically generate families, one can think of the supersymmetric preon model of the last section with $4\,n_f + 2$ preons Φ, instead of just 6. The condensate (20) then breaks the natural global $SU(4n_f+2)$ symmetry into $SU(4n_f) \times SU(2)$. The relevant QGF representing the left handed quarks and leptons transform as $(4n_f,2)$ which can be interpreted as $n_f(4,2)$replications. There are some problems with this way of generating families - besides its ad hoc nature. For instance, for the model at hand, if $n_f > 2$ then the SU(2) gauge theory ceases to be asymptotically free. Furthermore, and this is a more generic problem, the model possess a very large family symmetry $SU(4n_f)$, which must at some stage be broken. Clearly one must do better. An attempt in this direction is to study special coset spaces which gives rise naturally to family replications among the QGF. Yanagida[19] has discussed in this symposium some examples of coset spaces involving exceptional groups where this happens. A very general investigation of this type of coset spaces has also been performed recently by Buchmüller and Napoly[20]. Although these examples are very interesting, the underlying physics cannot be just a simple non Abelian gauge theory, since typical global symmetries for these theories do not involve exceptional groups.

As I mentioned earlier, attempts at getting families out of the underlying dynamics, in general, try to trade gauge degrees of freedom for a family structure. The easiest way to see how this may happen is by studying the 't Hooft anomaly matching conditions for chiral models. I will illustrate this with a nice example of Marshak and Li[21], although in the end I shall also criticize this example dynamically. The model studied by Marshak and Li is quite analogous to the SU(N) model discussed in the last section. However, instead of having a preon A_{ij} they have a preon S_{ij}, which transforms as the two rank symmetric tensor under SU(N). This then requires one to have (N+4) $\bar{F}$'s to avoid any SU(N) gauge anomalies. They choose particularly $N = 8$, so their model has a $SU(12)\times U_Q(1)$ natural global symmetry. Marshak and Li, However, assume that the only chiral symmetry that survives the binding is

$$H = SU(4) \times SU(4) \times SU(2) \times SU(2) \qquad (24)$$

under this symmetry the 12 $\bar{F}$ preons transform as $(\Box,1,1,1) + (1,\bar{\Box},1,1) + (1,1,\Box,1) + (1,1,1,\bar{\Box})$. The only anomalies to worry about are the $(SU(4))^3$ anomalies, which at the preon level just count the dimension of the preons under the SU(8) gauge group

$$A_{\text{preon}}(SU(4)^3) = 8 \qquad (25)$$

A possible solution to the anomaly matching equations, which is advocated by Marshak and Li[21], is provided by the massless bound states

$$\begin{aligned} B_1 &\sim (\Box,1,\Box,1) \\ B_2 &\sim (1,\bar{\Box},1,\bar{\Box}) \end{aligned} \qquad (26)$$

<u>only if</u> these states are 4 fold degenerate.

$$A_{\text{bound states}}((SU(4))^3) = 2 \cdot 4 = 8 \qquad (27)$$

The SU(8) underlying gauge theory therefore forces the repetition of states at the bound state level.

Although the above illustrates nicely how one may get family repetitions (even with realistic quantum numbers, as in Eq. (26)!) out of the dynamics, one must really try to check that the dynamics works this way. In fact, I do not believe that the Marshak-Li model is realized dynamically in the nice way they want. One can study this model in a complementary picture by using as an effective Higgs field

$$\Phi_i^{\alpha} = S_{ij} F^{j\alpha} \tag{28}$$

By choosing

$$\langle \Phi_i^{\alpha} \rangle = v_\alpha \, \delta_i^{\alpha} \tag{29}$$

with

$$v_\alpha = (v_1 v_1 v_1 v_1 ; v_2 v_2 ; v_3 v_3) \tag{30}$$

the $SU(8)_{gauge} \times SU(12)_{global}$ symmetry is reduced to that of H (apart from U(1) factors - see below). However, it is easy to check that the massless states in the Higgs phase are not 4 repetitions of states with the quantum numbers of B_1 and B_2. For instance, in the Higgs phase, there is a state transforming as $(\square, \square, 1, 1)$. To be fair, the breakdown caused by (29) does not precisely leave H as a global symmetry, but there are some additional U(1)'s. However, these U(1)'s can be eliminated by having some additional condensates, with different expectation values. I believe it unlikely that the presence of these extra condensates spoils complementarity. So, although the Marshak and Li solution (26) matches anomalies, the dynamics is more likely to be realized by having as massless fermionic bound states those with the quantum numbers obtained in the Higgs phase.

There are toy models where one can actually check, via complementarity, that the dynamics lead to repetitions. These class of models were orignally investigated by Preskill[22] - who actually studied them for this reason.

They have been recently reexamined in some detail by Goity, Zeppenfeld and myself[23]. The models are built by subtracting the SU(N) models with an antisymmetric preon A_{ij} from that with a symmetric preon S_{ij}. Hence, the total preon content for the model is

$$S^{ij} \sim \overline{\square\square} \;;\; A_{ij} \sim \begin{smallmatrix}\square\\ \square\end{smallmatrix} \;;\; 8\; F_i \sim 8\,\square \tag{31}$$

The quantum global chiral symmetry of the model is thus

$$G = SU(8) \times U_1(1) \times U_2(1) \tag{32}$$

where the two U(1)'s are linear combinations of the $\bar{S}$, A and F numbers that do not have SU(N) anomalies.

This global symmetry is not totally preserved in the binding. By studying the Higgs phase of the model, which has a tumbling complementarity to the confining phase[23], one finds that G → H and that furthermore the chiral symmetry H that remains depends on what N is[22,23]. Let me write N as N = 8 n+k, so that k distinguishes different N, modulo 8. Then the breakdown of G is different, depending on what k is. One has

$$G \to H = SU(k) \times SU(8-k) \times \bar{U}_1(1) \times \bar{U}_2(1) \tag{33}$$

The massless fermionic states which match the preonic H anomalies can be easily determined from the Higgs phase. Remarkably, they appear in repetitive patterns which depend on n. So the larger the gauge group SU(N), the more families of massless fermions!

Table II gives the massless fermionic bound states in the model, which emerge from studying the Higgs phase of the theory[23]

Table II: Massless Fermions in the SU(8n+k) Model

SU(k)	SU(8-k)	$\bar{U}_1(1)$	$\bar{U}_2(1)$	Number of states
$\boxminus$	1	2(8-k)	N-2-4s	s = 0,1,...,2n+1
$\square$	$\bar{\square}$	8-2k	N-2-4s	s = 0,1,...,2n
1	$\bar{\boxminus}$	-2k	N-2-4s	s = 0,1,...,2n-1

One sees that the only thing that distinguishes the 2n+2 ($\boxminus$,1), 2n+1 ($\square$, $\bar{\square}$) and 2n (1, $\bar{\boxminus}$) states is the $\bar{U}_2$ quantum number, which thus acts as a generation number for the model. One can actually well understand in this model why the repetitions in the spectrum occur by studying the Higgs phase[23]. In this phase, the original gauge and global symmetry SU(8n+k) x SU(8) goes through a process of multiple tumbling[24] down to $SU(k)_{gauge}$ x SU(8):

$$SU(8n+k)_{gauge} \times SU(8) \rightarrow SU(8(n-1)+k)_{gauge} \times SU(8) \rightarrow \cdots \rightarrow SU(k)_{gauge} \times SU(8) \tag{34}$$

Finally the $SU(k)_{gauge} \times SU(8) \rightarrow SU(k)_{diag} \times SU(8-k)$

At each stage in the tumbling one precisely gets as gauge singlet massless fermions two SU(8) $\boxminus$ which correspond in term of H to

$$2\,\boxminus = 2\left\{(\boxminus,1) + (\square,\bar{\square}) + (1,\bar{\boxminus})\right\} \tag{35}$$

So the multiple tumbling explains the 2n repetitions in Table II. The last 3 states come from the last step in which the $SU(k)_{gauge}$ finally also breaks down.

3. Trying to Generate Mass

In the last two sections, I discussed two of the requirements needed for a realistic composite model (besides that of getting the correct quantum numbers for the quarks and leptons!). Namely, having a dynamically consistent mass protection mechanism and having a believable family replication mechanism. As we saw, these requirements are not easy to satisfy in practice. In this

section, I want to address myself to what is probably an even harder point to achieve: the generation of small quark and lepton masses.

There are really two separate difficulties. One is how to generate small masses, $m \ll \Lambda_c$, and the other is how to generate a hierarchy among these small masses. (After all $m_b/m_e \sim 10^4$!). The most ambitious hope would be to be able to generate both small masses and a hierarchy in a given model. Less ambitious, but still very hard, is to input a small mass parameter m in the theory and then generate a hierarchy. Or, alternatively, forgetting about generations just to try to generate $m \ll \Lambda_c$ in a one family model. These attempts are, in general, further complicated because in most models which try to be at least semirealistic, quarks and leptons are not the only massless states in the theory. Thus, when one then tries to give quarks and leptons some mass, one must also check that this same mass generation mechanism is clever enough to produce a much larger mass for the unwanted extra states.

This last point is particularly crucial for models in which the massless fermions were obtained as QGF. Obviously, in this case, one has accompanying the wanted fermions, unwanted massless bosons. Indeed, since the bosons arose dynamically as Golstone excitations from the G$\rightarrow$H breakdown and dragged by supersymmetry the fermions to zero mass, their dynamical status is more secure. For instance, breaking supersymmetry but not G would generate mass for the QGF but leave the Goldstone bosons massless - a phenomenologically disastrous situation!. Even breaking supersymmetry and G at the same time in general leads to trouble, since it is difficult not to get $m_{fermion} \sim m_{boson}$, after both breakdowns[10,25]. Only if the QGF have an additional chiral protection[7], it will be possible to separate sufficiently in mass the unwanted bosons from the fermions, in the process of mass generation. The model of Ref. 17, which I briefly discussed in Section 1, has this property. The QGF in this model are massless because they are the supersymmetric partners of the 17 Goldstone bosons from the G$\rightarrow$H breakdown. However, these fermions are also massless because they are the fermions required to match the H anomalies. They are, thus, doubly protected.

In a recent paper, Masiero, Pettorino, Roncadelli and Veneziano[26] have examined how mass can be generated in a one family QGF model with double protection. Their nice discussion shows many of the intricacies and difficulties that one must face in a semirealistic situation. I will summarize some of their results since they are very instructive, even though in some aspects disappointing. The model is based on a supersymmetric preon theory with an SU(6) gauge group, which has a global $SU(6)_L \times SU(6)_R \times U(1)^3$ symmetry. This symmetry is broken by condensates to $SU(6)_{diag} \times U_X(1)$, where the $U_X(1)$ is chiral. In the breakdown there emerge as QGF the usual 16 quarks and lepton states of one generation (including ν_R), 6 leptoquarks, 8 colored states, a triplet of states with the quantum numbers of the W's and some singlets.

To give mass to these states and their bosonic counterparts, the authors of Ref. 26 proceed in two stages:

Stage I: Explicit supersymmetry and G breaking soft mass terms for the scalars are introduced, which preserve R symmetry. Because of this remaining chiral protection, all QGF remain massless. All scalars, except the 8, 3 and some singlets, acquire mass.

Stage II: Gaugino mass terms for a subgroup of G are introduced, which break the R symmetry. As a result all fermions now acquire mass and the remaining massless scalars acquire radiative masses, although one axion-like state remains.

By this two step process, in fact, it is possible to split the scalars from the fermions sufficiently, so that the scalars are not a phenomenological embarrassment. However, within the fermions themselves no hierarchy

is generated. The fermion masses depend both on the soft scalar mass terms and on the gaugino mass terms

$$m_f \sim m_{\tilde g} \, F(\mu^{i}_{soft}) \tag{36}$$

but are being driven by the gaugino masses (stage II). As a result, Masiero et al.[26] obtain the unwanted near equality of electron and neutrino masses. Further it also appears difficult to separate the leptoquark fermions sufficiently from the ordinary quarks and leptons. Given that their mass generation was essentially induced from the "outside", this is rather disappointing.

The model discussed by Masiero, Pettorino, Roncadelli and Veneziano[26] is an example of imputing some outside mass parameter (s) (here the scalar soft breaking masses and the gaugino masses) and trying to get a hierarchy. In this case these authors successfully got a hierarchy between the fermions and the bosons, by doubly protecting the fermions, but failed to get enough of a hierarchy among the fermions. Furthermore, all light masses were decoupled from Λ_c since they are driven by the outside mass parameters. It is interesting to ask whether it is possible at all to get $m \ll \Lambda_c$ without putting a small seed mass in? I want to end this section by qualitatively describing two new ideas which may allow this to happen.

The first idea is a suggestion of H. Georgi[27] connected with the composite Higgs scenario, developed by him and collaborators[28]. One imagines that in a first stage, a confining preon theory has a global symmetry G broken to some chiral symmetry H. As a result, both massless fermions and some Goldstone bosons, arising from $G \to H$ breakdown, appear in the bound state spectrum. In a second stage, the usual SU(3) x SU(2) x U(1) gauge interactions are turned on at the preon level. These gauge interactions actually produce additional terms in the effective potential among the Goldstone bosons, since now these states are no longer really Goldstone excitations, because G is broken by the gauge interactions. It may so happen, and this is the contention of the composite Higgs mechanism[28] that these extra terms cause some of these erstwhile Goldstone states to acquire vacuum expectation value, thereby precipitating a spontaneous breakdown of SU(2) x U(1). The scale of the SU(2) x U(1) breakdown, because it occurs only after gauging the preon theory, is effectively rather decoupled from the dynamical scale $\Lambda_{preon} \sim \Lambda_c$. Georgi[27] argues that if this composite Higgs scenario obtains, and provided that after the gauging no exact chiral symmetries remain, then it is possible for the massless fermions to get mass. Obviously the mass so generated will be given by the weak breaking scale times powers of the gauge coupling constants. Since the weak scale is mostly decoupled from Λ_c, so are the fermion masses. Although this is a rather intricate scenario, it is clear that it has promise and deserves very thorough study.

The second idea, to try to generate small masses, uses a suggestion of Mizrachi and myself[29] that in confining theories there may exist condensates (irrelevant condensates) whose scale is much smaller than the natural scale of the theory. To explain this idea, it is useful to consider a QCD analogy. In QCD one knows that bilinear quark condensates form, which break chiral symmetry. These condensates have a size which is of the order of the typical dynamical scale of QCD, Λ_{QCD}. That is, one finds

$$\langle \bar{u} u \rangle = c\, \Lambda^3_{QCD} \qquad (37)$$

with $c \sim O(1)$. Such a condensate, I shall denote as <u>relevant</u>. An <u>irrelevant</u> condensate, on the other hand, would be a condensate in QCD whose size would <u>not</u> be Λ_{QCD}. An example of a possible irrelevant condensate would be a "2 pion" condensate like

$$\langle \underbrace{\bar{u} d|}_{\text{Color singl.}} \; \underbrace{\bar{d} u|}_{\text{Color singlet}} \rangle = c' \Lambda^6_{QCD} \qquad (38)$$

if $c' \ll 1$. Although (38) must kinematically scale as Λ^6_{QCD}, if $c' \ll 1$, then the effective scale of this condensate would be much less, thereby making it irrelevant.

Is it really possible that c' in QCD is much less than unity? In principle, I see three possible logical values for c'. Either it vanishes absolutely, or it is of order unity, or $c' \ll 1$. I would find it difficult to believe that (38) never formed in QCD. On the other hand, I would also find it very strange that $c' \sim O(1)$, since the color is already screened in (38). Hence the idea that $c' \ll 1$ is not totally outlandish. It, of course, would be very good if one could test for this suggestion by means of some lattice calculation, although the fact of having four fermion operators makes this probably too hard to attempt, at the present moment.

If irrelevant condensates existed, then one can well imagine being able to generate small fermion masses, $m \ll \Lambda_c$[29]. Schematically, one can think of the fermion mass generation this way - although this is not really the way one would go about calculating these masses in practice. If quarks and leptons are composite, there should exist effective residual interactions among the massless fermions. These interactions should be chiral, but may well involve both left and right components of the same fermions. Let me focus on such a term for a generic fermion f, which should scale as, on purely dimensional grounds:

$$\mathcal{L}^{eff} \sim \frac{1}{\Lambda_c^2} (\bar{f}_L \gamma^\mu f_L)(\bar{f}_R \gamma_\mu f_R) \qquad (39)$$

The fermions f are preon composites which are themselves gauge singlets, with respect to the underlying confining group. To a first approximation they are massless, due to some protection mechanism. However, it may well be that some irrelevant condensates among these gauge singlets form (cf. Eq. (38)), as a result of the underlying theory. That is, one could have

$$\langle \bar{f}_L f_R \rangle = c_f' \Lambda_c^3 \tag{40}$$

with $c_f' \ll 1$. Such a condensate, in conjunction with Eq. (39), would imply a mass term for the fermion f of order

$$m_f \sim \frac{1}{\Lambda_c^2} \langle \bar{f}_L f_R \rangle \sim c_f' \Lambda_c \tag{41}$$

which would be small with respect to Λ_c because $c_f' \ll 1$. Note that if $c_f' \ll 1$ it is really consistent to neglect dynamically these condensates at first and have therefore the fermion masses vanishing by the built in protection mechanism.

This mass generation mechanism is quite similar to that in extended technicolor (ETC) models[30]. As such it may run into some of the troubles that ETC encountered. However, because this mechanism is not as constrained as ETC, it may be able to avoid some of these difficulties. At any rate one should expect that:

i) one will generate flavor changing neutral current FCNC at a dangerous rate, unless the model has very special residual interactions. Typically, to avoid problems, these interactions should be flavor diagonal and universal <u>before</u> mass generation[29,31].

ii) there should exist pseudo-Goldstone excitations connected with the formation of the irrelevant condensates (40). These pseudo Goldstone excitations may be very weakly coupled, since they must decouple from the spectrum when the irrelevant condensates are neglected. Nevertheless, they constitute one of the most telling (damning?) signals of this mechanism of mass generation. For instance, one would expect quite naturally to have pseudo-Goldstone bosons with both lepton and quark number, which could provide, in fact, the most accessible traces of compositeness.

iii) neutrinos to be again problematic in models where a right-handed neutrino binds. Then, as in Ref. 26, it appears to be almost impossible to avoid obtaining neutrino masses of the order of the electron mass.

Even given all of the above potential problems, I feel that it is reasonable to pursue this idea a bit further.

4. Concluding Remarks

I have tried to indicate here some of the dynamical challenges posed by composite models and describe some of the attempts that have been made to generate light quarks and leptons as preonic bound states. As it should be clear, we are still very far away from generating an even semirealistic model. However, the field has matured to a considerable extent. It no longer suffices to have a purely <u>algebraic</u> model of composite quarks and leptons, without dynamics. Now real dynamical questions are being asked and some progress is being made in understanding the structure of chiral and supersymmetric gauge theories, which might bind (nearly) massless fermions. Nevertheless, the difficulties and frustrations of composite models may call into question the wisdom of pursuing this line of speculation, especially if one could find an alternative way to "understand" families and mass.

The suddenly very popular superstring theories[32] have a potential for doing so. I list some of the putative advantages of superstrings over what I have tried to describe:

i) The group structure is fixed by anomaly freedom. In particular the $E_8 \times E_8$ superstring fits well with the quark and lepton quantum numbers - although it also predicts some exotic excitations.

ii) The number of families can be related to the Euler Characteristic of the compactified manifold, and hence has a geometrical meaning.

iii) In principle, the effective Higgs couplings from which the quark and lepton masses eventually originate can be calculated by integrating out in the superstring theory the degrees of freedom that depend on the compact directions.

These advantages, however, may be illusions. All the physics of relevance is happening at the Planck, or compatifications, scale but it must then be connected with what is happening at the scale of the weak interaction. This hides an enormous amount of dynamics, which I suspect is going to be considerably more difficult than the relatively "simple" composite dynamics I have been talking about.

To conclude, what is real important, in superstrings as in composite models, is to find some experimetal hints that we are on the right track. In the meanwhile theorists should continue sharpening their theoretical tools by at least trying to address some of the dynamical issues. I have tried to do a bit of that here.

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