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AN ESTIMATE FOR THE HIGGS MASS

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Abstract:

The high energy behaviour of weak interactions is assumed to be softened (by some unspecified strong interactions) to the extent that the $W_LW_L \longrightarrow W_LW_L$ amplitude with "exotic" t-channel isospin $I_t=2$ satisfies a superconvergence sum rule. At low energies, $E \lesssim (\sqrt{2}\,G_F)^{-1/2}$, its saturation in terms of the standard model tree amplitude turns out to work amazingly well, determining the only unknown parameter, the Higgs mass, to $m_H \simeq 2.3\ m_W \simeq 190$ GeV.

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The Higgs sector, which is as yet unconfirmed, continues to be an intriguing aspect of the standard Glashow-Salam-Weinberg model[1)] of electroweak interactions. In particular, the question of the Higgs mass is of prime importance, both theoretically and from an experimental point of view. Although the mass of the Higgs scalar formally appears as a free parameter in the classical Lagrangian of the standard model, a number of bounds or even estimates exist in the literature. Of course, at the present level of understanding, any result on the Higgs mass requires some kind of additional assumption.

Common to a first category of constraints for the Higgs mass is the assumption that perturbation theory is to remain valid at high energies. Additional ingredients are e.g. tree-level unitarity[2)] or, more stringently, the renormalization group equations combined with the (almost established) triviality of $\lambda\phi^4$ theory[3)], general searches for parameter reduction via renormalization[4)] or ideas of grand unification and vacuum stability[5)].

A second category of estimates for the Higgs mass rests on the more heretical philosophy that the high energy behaviour of the theory may actually be governed by some kind of non-perturbative effects. Let us mention here the study of the non-perturbative limit of a strongly interacting Higgs sector by means of the 1/N expansion[6)], composite Higgs models[7)] or first results from lattice Monte Carlo calculations[8)].

Adopting the "non-perturbative" philosophy inherent in the latter category as a working hypothesis, in this letter we follow a different, in some sense pedestrian line of thought to arrive at an estimate for the Higgs mass.

We concentrate on elastic scattering of longitudinally polarized W bosons, $W_LW_L \longrightarrow W_LW_L$, since the high energy behaviour of this process is known to crucially depend on the interplay between W and Higgs exchanges. While taking it for granted that a perturbative evaluation of the standard model is a good approximation in the "low"-energy regime,

$$\sqrt{s} \lesssim (\sqrt{2}\,G_F)^{-1/2} \sim 250\ \text{GeV}, \qquad (1)$$

we envisage a possible softening of the high energy behaviour due to some kind of (largely unspecified) strong interaction. In fact, in order to arrive at an estimate for the Higgs mass, we only need to invoke a minimal amount of input about the nature of these strong interactions.

To be specific, we shall only be concerned with the $W_L W_L \longrightarrow W_L W_L$ amplitude corresponding to "exotic" t-channel (weak) isospin $I_t=2$, i.e. to $W_L^+ W_L^+ \longrightarrow W_L^+ W_L^+$ scattering in the t-channel. The aim of this paper is then to explore the consequences of the assumption that this amplitude -unlike the tree approximation- vanishes sufficiently fast at high energies, such as to satisfy a superconvergence sum rule of the kind

$$\int_0^\infty \nu \, d\nu \, \mathrm{Im} \, A^{I_t=2}_{W_L W_L \to W_L W_L}(\nu,t) = 0, \qquad (2)$$

for some interval of $t \lesssim 0$. Here ν is the familiar energy variable

$$\nu = \frac{s-u}{4} \qquad \text{(with } s+t+u = 4m_W^2) \; . \qquad (3)$$

The interesting point is that a constraint on the high energy behaviour of an "exotic" WW amplitude thus translates, via analyticity, into a strong restriction on the spectrum at low energies, the "perturbative regime" of the GSW standard model.

It is, therefore, tempting to proceed by trying a semi-local, minimal saturation of the superconvergence sum rule (2) in the low energy regime (1) of the integration variable ν : in terms of the $Z°$ $(+\gamma)$ and Higgs pole contributions in the s-channel.

Let us emphasize that this saturation of the sum rule (2) is highly non-trivial. With only one free parameter, m_H/m_W, the ratio of the Higgs mass to the W mass, the saturation of Eq. (2) has to hold over a whole range of spacelike t values. In other words, the ratio m_H/m_W, determined from Eq. (2) as a function of t, has to be (roughly) constant for consistency. As we shall see below, m_H/m_W indeed turns out to be surprisingly t independent over a convincingly large range of t: $0 \lesssim -t \lesssim 5\, m_W^2$. We feel that this result strongly supports *a posteriori* the internal consistency of the assumptions made. At the same time it increases the credibility of the resulting value for the Higgs mass

$$m_H \simeq 2.3\, m_W \simeq 190 \text{ GeV}. \qquad (4)$$

Next, we shall present the derivation of these results and fill in the necessary technical details. At the end of the paper we shall attempt to further illuminate the origin and model insensitivity of the superconvergence relation (2) in the light of a more specific framework for how weak interactions could become strong at high energies.

To be precise, we formulate the superconvergence sum rule for the $I_t=2$ W W $\longrightarrow$ W W scattering amplitude $A^{I_t=2}_{\{\lambda_t\}=0}$ corresponding to longitudinally polarized W's in the c.m. system of the crossed t-channel+), i.e. to W's with t-channel helicity 0. This amplitude is distinguished by two properties: i) absence of kinematical singularities[9] in s (or ν), a necessary prerequisite in the context of dispersion sum rules, and ii) s-u crossing symmetry.

As announced earlier, we attempt to saturate the superconvergence sum rule

$$\int_0^\infty \nu \, d\nu \, \mathrm{Im} \, A^{I_t=2}_{\{\lambda_t\}=0}(\nu,t) = 0 \qquad (5)$$

by the tree approximation of the standard Glashow-Salam-Weinberg model. In a first step, let us switch off the electromagnetic gauge interactions, i.e. treat the case $\sin\Theta_W=0$. The $I_t=2$ amplitude then only involves the s- and u-channel Z° $(=W^3)$ and Higgs contributions which are straightforwardly calculated from the standard model Lagrangian to be

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+) In case of external particles with spin, t-channel helicity amplitudes are well known to represent a suitable choice in the context of fixed t dispersion relations and sum rules derived thereof.

$$A^{I_t=2}_{\{\lambda_t\}=0}(WW\rightarrow WW) = -g^2\Big\{\Big(\frac{s}{2m_W^2}\cdot\frac{t}{4m_W^2-t}-1\Big)^2-\Big(\frac{t}{2m_W^2}-1\Big)^2$$
$$+\frac{1}{s-m_W^2}\Big[\Big(\frac{s}{2m_W^2}\cdot\frac{t}{4m_W^2-t}-1\Big)^2(u-t)-\frac{2ts}{m_W^2}\cdot\frac{s-2u}{4m_W^2-t}\Big]$$
$$+\frac{m_W^2}{s-m_H^2}\Big(\frac{s}{2m_W^2}\cdot\frac{t}{4m_W^2-t}-1\Big)^2\Big\} + (s\leftrightarrow u)_{t\ \mathrm{fixed}}. \quad (6)$$

(This expression does, of course, not involve any of the frequently used high energy approximations).

We evaluate the integral (5) in the usual zero width approximation with the a posteriori justification that also the Higgs width turns out to be of the order of a few GeV and thus both, the Z° and the Higgs widths are very small as compared to their masses. The superconvergence sum rule (5) then turns into the following equation

$$\Big(\frac{1}{2}-\frac{\tau}{4}\Big)\Big\{\Big(\frac{1}{2}\,\frac{\tau}{4-\tau}-1\Big)^2(3-2\tau)+\frac{2\tau}{4-\tau}(5-2\tau)\Big\}$$
$$+\Big(1-\frac{\mu^2}{2}-\frac{\tau}{4}\Big)\Big(\frac{\mu^2}{2}\,\frac{\tau}{4-\tau}-1\Big)^2 = 0, \quad (7)$$

involving the dimensionless variable

$$\tau = \frac{t}{m_W^2} \quad (8)$$

and the dimensionless parameter

$$\mu = \frac{m_H}{m_W}. \quad (9)$$

In Fig.1 we plot the solution of Eq. (7), i.e. $\mu=m_H/m_W$ as a function of $\sqrt{-\tau}=\sqrt{-t}/m_W$. Clearly and most non-trivially, μ turns out to be almost t independent over a surprisingly large t interval, $0\lesssim -t\lesssim 5\ m_W^2$!

A further test for the stability of the solution of the sum rule is near at hand. It is obvious that the saturation of Eq. (7) arises due to a subtle interplay between the Z° and Higgs contributions, which each are strongly

t dependent. If we now leave the ratio g_H/g_W of the WW-Higgs to the WWW coupling as a free parameter in addition to μ, it is very interesting to ask, how close the best-fit value for g_H/g_W comes to the GSW standard model value $g_H/g_W=1$. As may be seen from Fig.2, the χ^2, introduced to measure the t dependence of the resulting $\mu(t,g_H/g_W)$, is a rapidly varying function of g_H/g_W. Indeed, it exhibits a pronounced minimum at $g_H/g_W=0.82$, which is satisfactorily close to 1 (with $\langle m_H/m_W\rangle\sim 2.3$ also for $g_H/g_W=0.82$).

Next, let us ask what happens for $\sin\Theta_W\neq 0$. We do not really intend to enter theoretical speculations on whether or not the s- and u-channel photon pole terms should be included in a saturation of the sum rule (5). This is largely a question of the dynamics responsible for the softening of the high energy behaviour of WW amplitudes. Nevertheless, it is reassuring that a straightforward inclusion of the photon pole terms changes the result displayed in Fig.1 only on the few percent level. The superconvergence Eq. (7) in this case has to be replaced by

$$\cos^2\Theta_W\Big(1-\frac{1}{2\cos^2\Theta_W}-\frac{\tau}{4}\Big)\Big\{\Big(\frac{1}{2\cos^2\Theta_W}\,\frac{\tau}{4-\tau}-1\Big)^2\Big(4-\frac{1}{\cos^2\Theta_W}-2\tau\Big)$$
$$+\frac{2}{\cos^2\Theta_W}\,\frac{\tau}{4-\tau}\Big(8-\frac{3}{\cos^2\Theta_W}-2\tau\Big)\Big\}+\sin^2\Theta_W\Big(1-\frac{\tau}{4}\Big)(4-2\tau)$$
$$+\Big(1-\frac{\mu^2}{2}-\frac{\tau}{4}\Big)\Big(\frac{\mu^2}{2}\,\frac{\tau}{4-\tau}-1\Big)^2 = 0, \quad (10)$$

where

$$\frac{m_Z^2}{m_W^2} = \frac{1}{\cos^2\Theta_W} \quad \text{and} \quad \sin^2\Theta_W \simeq 0.22. \quad (11)$$

At t=0, for instance, Eq. (10) has the exact solution

$$\mu = \frac{m_H}{m_W} = \sqrt{4+\frac{1}{\cos^2\Theta_W}} = 2.30 \quad \text{for } \sin^2\Theta_W = 0.22, \quad (12)$$

which is very close to

$$\mu = \sqrt{5} = 2.24 \quad \text{for } \sin^2\Theta_W = 0. \quad (13)$$

Both, Figs. 1 and 2 remain unchanged within drawing accuracy.

It is well conceivable that a superconvergence sum rule for the I_t=2 WW amplitude is a general property of a large class of schemes for weak interactions becoming "strong" at high energies. To back up this conjectured model insensitivity of the sum rule, let us conclude with an exercise (which is, however, in no way essential for our results). We take recourse to a more specific framework, that of dual string or dual resonance models, tentatively adapted to weak interactions. Such a picture would imply that weakly interacting particles like the W, Higgs etc. are to be viewed as the lowest string excitations or the lowest particles on Regge trajectories, respectively, with

$$\sqrt{2\pi}\cdot\text{string tension} \sim \sqrt{\alpha'} \sim O\left((\sqrt{2}\,G_F)^{1/2}\right). \qquad (14)$$

The framework of dual models provides a suitable toy laboratory in this context for three reasons:

1) For vanishing string tension or, equivalently, in the limit of vanishing Regge slope, $\alpha' \rightarrow 0$, dual models are known to "collapse" to Yang-Mills gauge models[10]. Thus, in the low energy regime, $\alpha' s \ll 1$, dual models may presumably[11] be taylored to reproduce the perturbative amplitudes of the standard weak gauge Lagrangian.

2) At high energies, $\alpha' s \gg 1$, they explicitly provide a "non-perturbative softening" of the asymptotic behaviour[12,13] compared to that of the tree amplitudes from the limiting Yang-Mills theory.

3) Dual model amplitudes, "V(s,u)", corresponding to an exotic t-channel, are exponentially suppressed for $s \rightarrow \infty$, t fixed. This strong asymptotic decrease follows from the absence of any exotic exchanges (like particles with isospin=2) and $\alpha' \neq 0$, independently of any further details of the model[12,13]. Thus, sum rules of type (5) hold in any conceivable realization of dual models (with non-exotic trajectories).

So far, we used the language of dual models for illustrational purposes only. One should, however, envisage that this language might well become relevant, in particular in schemes involving some kind of compositeness,

like composite Higgs scalars[7,14] and/or W's, quarks, leptons[15].

It is tempting to pursue the implications of a dual model ansatz for weak interactions a little bit further. One could e.g. supplement the superconvergence sum rule (5) by finite-energy sum rules for non-exotic channels. Clearly, this involves more commitment in form of dynamical assumptions. The most interesting issue in this context is the question of the size of α'. The most clearcut determination of α' is by means of a finite-energy sum rule for $Z^\circ_L Z^\circ_L \rightarrow Z^\circ_L Z^\circ_L$ scattering (for $\sin\theta_W = 0$, say). Its minimal saturation is in terms of a Higgs trajectory $\alpha_H(t) = \alpha'(t-m_H^2)$ on the high energy side and in terms of the Higgs pole term on the low energy side. The familiar[13] matching of zeros in the t-dependences of both sides then straightforwardly leads to

$$\frac{1}{\alpha'} = 3m_H^2 - 4m_W^2 \underset{\text{Eq.(13)}}{\approx} 11\, m_W^2 \gg m_W^2 . \qquad (15)$$

For phenomenological reasons it is gratifying to see that α' comes out much smaller than $1/m_W^2$. Its value is $\approx$ (275 GeV)$^{-2}$ from Eq. (15), i.e. of the order of the inverse square of the Fermi scale

$$\alpha' \sim O(\sqrt{2}\,G_F). \qquad (16)$$

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References

1) S.L.Glashow, Nucl. Phys. 22 (1961) 579;
S.Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A.Salam, Proc. 8th Nobel Symposium, ed. N.Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367

2) D.A.Dicus and V.S.Mathur, Phys. Rev. D7 (1973) 3111
B.W.Lee,C.Quigg and H.B.Thacker, Phys. Rev. Lett. 38 (1977) 883; Phys. Rev. D16 (1977) 1519;
M.Veltman, Acta Phys. Pol. B8 (1977) 475

3) R.Dashen and H.Neuberger, Phys. Rev. Lett. 50 (1983) 1897;
D.J.E.Callaway, Nucl. Phys. B233 (1984) 189;
M.A.B.Beg, C.Panagiotakopoulos and A.Sirlin, Phys. Rev. Lett. 52 (1984) 883

4) W.Zimmermann, Comm. Math. Phys. 97 (1985) 211;
J.Kubo, K.Sibold and W.Zimmermann, Max Planck-Institut für Physik und Astrophysik -preprint MPI-PAE/PTh 19/85, Munich (1985), to be published

5) N.Cabibbo, L.Maiani, G.Parisi and R.Petronzio, Nucl. Phys. B158 (1979) 295

6) M.B.Einhorn, Nucl. Phys. B246 (1984) 75

7) D.B.Kaplan and H.Georgi, Phys. Lett. 136B (1984) 183;
D.B.Kaplan, H.Georgi and S.Dimopoulos, Phys. Lett. 136B (1984) 187;
H.Georgi, D.B.Kaplan and P.Galison, Phys. Lett. 143B (1984) 152

8) I.Montvay, Phys. Lett. 150B (1985) 441; DESY-report 85-005, Hamburg (1985); DESY-report 85-050, Hamburg (1985)

9) L.L.Wang, Phys. Rev. 142 (1966) 1187;
G.Cohen-Tannoudji, A.Morel and H.Navelet, Ann. Phys. 46 (1968) 111

10) J.Scherk, Nucl.Phys. B31 (1971) 222;
A.Neveu and J.Scherk, Nucl. Phys. B36 (1972) 155

11) K.Bardacki and M.B.Halpern, Nucl. Phys. B73 (1974) 295;
J.Scherk and J.H.Schwarz, Nucl. Phys. B81 (1974) 118

12) G.Veneziano, Nuovo Cim. 57A (1968) 190

13) V.De Alfaro,S.Fubini,G.Furlan and C.Rossetti, "Currents in Hadron Physics", North-Holland Publishing Comp., 1973, p. 571 ff

14) L.Susskind, Phys. Rev. D20 (1979) 2619

15) H.Harari and N.Seiberg, Phys. Lett. 98B (1981) 269;
O.W.Greenberg and J.Sucher, Phys. Lett. 99B (1981) 339;
L.F.Abbott and E.Farhi, Phys. Lett. 101B (1981) 69; Nucl. Phys. B189 (1981) 547;
H.Fritzsch and G.Mandelbaum, Phys. Lett. 102B (1981) 319;
B.Schrempp and F.Schrempp, Nucl. Phys. B231 (1984) 109; B242 (1984) 203;
W.Buchmüller, R.Peccei and T.Yanagida, Nucl. Phys. B231 (1984) 53; B244 (1984) 186

Figure captions

Fig.1: The solid line displays the ratio m_H/m_W, of the Higgs mass to the W mass, as determined from the superconvergence sum rule (5), resp. (7) for a range of $\sqrt{-t}/m_W$ values. The near constancy of m_H/m_W is a direct measure for its validity. The dashed line denotes the t average for the only parameter in the sum rule, $\langle m_H/m_W \rangle \sim 2.3$.

Fig.2: A further test of the non-triviality and stability of the superconvergence sum rule (5), resp. (7). The ratio g_H/g_W, of the WW-Higgs to the WWW coupling, is left as an additional free parameter besides m_H/m_W. The displayed quantity

$$\chi^2\left(\frac{g_H}{g_W}\right) = \sum_{0<-t\leq 5m_W^2} \left(\frac{m_H}{m_W}\left(t,\frac{g_H}{g_W}\right) - \left\langle\frac{m_H}{m_W}\left(\frac{g_H}{g_W}\right)\right\rangle\right)^2$$

measures the t dependence of the solution $\frac{m_H}{m_W}\left(t,\frac{g_H}{g_W}\right)$ from Eq. (7) for different values of g_H/g_W. Of significance are the strong variation of χ^2, the pronounced minimum and, in particular, the fact that the minimum (at $g_H/g_W = 0.82$) is close to the GSW standard model value $g_H/g_W=1$.

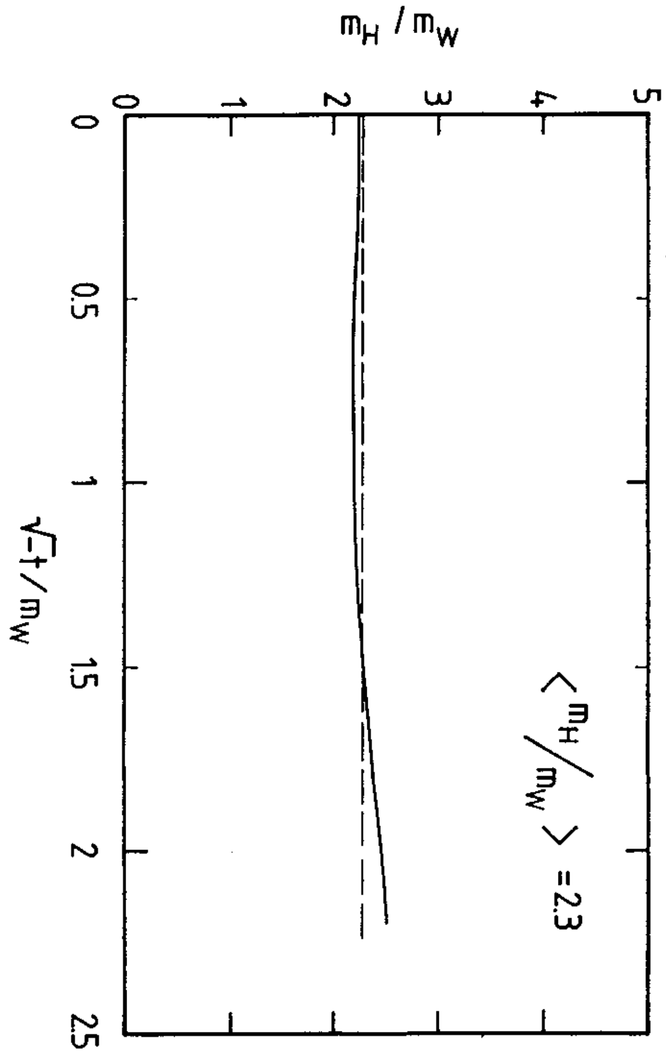

Fig. 1

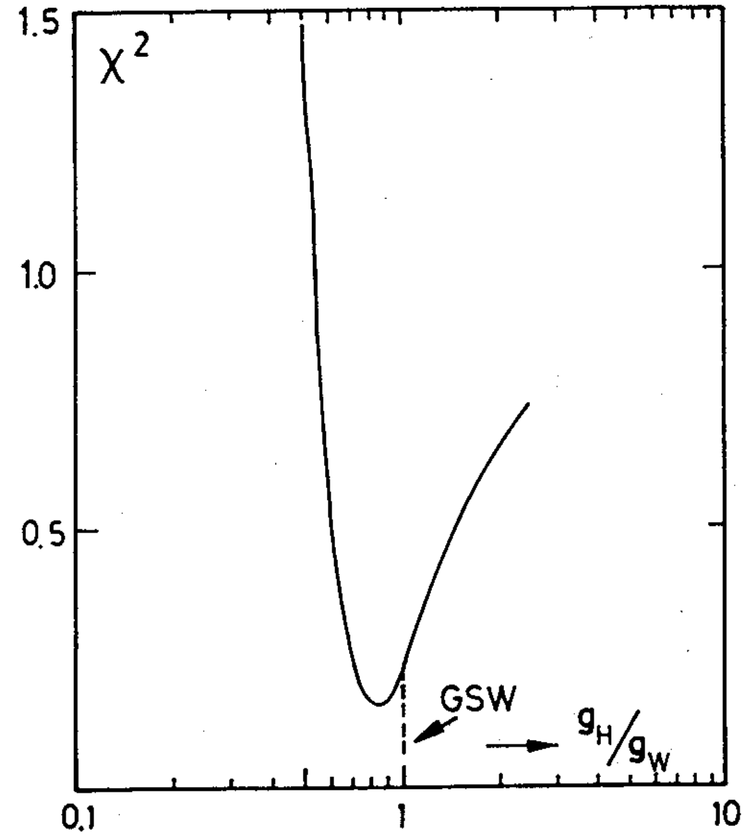

Fig.2