DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 85-066
MZ-TH/85-12
July 1985

γ_5 AND INFRARED DIMENSIONAL REGULARIZATION

by

J.G. Körner, G. Schuler

Institut f. Physik, Johannes-Gutenberg-Universität, Mainz

G. Kramer, B. Lampe

II. Institut f. Theoretische Physik, Universität Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 85-066
MZ-TH/85-12
July 1985

ISSN 0418-9833

γ_5 AND INFRARED DIMENSIONAL REGULARIZATION

J.G. Körner, G. Schuler
Institut für Physik
Johannes Gutenberg-Universität
D-6500 Mainz, West Germany

G. Kramer, B. Lampe
II. Institut für Theoretische Physik
Universität Hamburg
Luruper-Chaussee 149
D-2000 Hamburg 50, West Germany

Abstract:

We carefully discuss how to deal with γ_5 in the dimensional regularization scheme when IR singularities are present. The dimensional continuation of the Dirac algebra in odd-parity fermion traces leads to anomalous IR axial and charge conjugation contributions. In an explicit calculation, namely the $O(\alpha_s^{\ 2})$ parity violating contributions to $e^+e^- \xrightarrow{\gamma,Z^0} q\bar{q}g$, we demonstrate that the IR charge conjugation anomaly vanishes and that the finite IR axial anomalies are spurious in the sense that they cancel among the real and virtual contributions to the p.v. cross sections as do the IR singular contributions.

It is well-known that the union of γ_5 (or the fully antisymmetric tensor $\varepsilon_{\alpha\beta\gamma\delta}$) and the dimensional regularization scheme is a problematic one [1-7]. In the case of odd-parity fermion loops, the n-dimensional Dirac traces generate $O((n-4)^m;m>0)$ anomalous contributions, which in turn lead to finite anomalous terms when multiplied with ultraviolet or infrared divergent integrals.

Spurious UV anomalies can be and must be cancelled by taking the appropriate renormalization scheme [4-6]. In the IR case one encounters axial and charge conjugation anomalies. These cannot be removed by renormalization. One expects these to be spurious in the sense that they cancel when the appropriate real and virtual cross sections are added together just as the IR singular terms cancel among the two contributions.
To our knowledge the occurrence of IR anomalies in odd-parity fermion loops in the dimensional regularization scheme and their cancellation among real and virtual contributions has never been discussed before. In order to demonstrate the compatibility of γ_5 and the IR dimensional regularization scheme we calculate through a sufficiently complex process containing both UV and IR singularities, namely the $O(\alpha_s^{\ 2})$ parity violating contributions to $e^+e^- \xrightarrow{\gamma,Z^0} q\bar{q}g$.

Let us first present an argument that an anticommuting γ_5, cif.

$$\{\gamma_\mu, \gamma_5\} = 0 \tag{1}$$

is not compatible with a n-dimensional Dirac algebra, cif.

$$\gamma_\mu \gamma^\mu = g_\mu^{\ \mu} = n \tag{2}$$

Consider the trace $\mathrm{Tr}\ \gamma_5\gamma_\alpha\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}$ and anticommute γ_α once around the trace. This leads to the identity

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\, g_{\mu_5\alpha} + \text{cycl.}(\mu_1 \ldots \mu_5) = 0 \tag{3}$$

where we have introduced the totally antisymmetric ε-tensor via

$\mathrm{Tr}\ \gamma_5\ \gamma_\alpha\ \gamma_\beta\ \gamma_\gamma\ \gamma_\delta = 4i\ \varepsilon_{\alpha\beta\gamma\delta}$.

Contracting (3) with $g^{\alpha\mu_5}$ gives

$$(n-4)\ \varepsilon_{\mu_1\mu_2\mu_3\mu_4} = 0 \qquad (4)$$

which shows that (1) and (2) prevent one from analytically continuing $\mathrm{Tr}\ \gamma_5\ \gamma_\alpha\ \gamma_\beta\ \gamma_\gamma\ \gamma_\delta$ or $\varepsilon_{\alpha\beta\gamma\delta}$ from 4 to $n \neq 4$.

There exist several proposals to escape the impalatable conclusion (4) [1,2,3].

The authors of Ref.[2] decided to work with an anticommuting γ_5 (1) but dropped (2). In order to give a meaning to the trace $\mathrm{Tr}\ \gamma_5\ \gamma_\mu\ \gamma^\mu\ \gamma_a\ \gamma_b\ \gamma_c\ \gamma_d$ in the absence of (2), they assigned a one-parameter ambiguity to its value cf. $\mathrm{Tr}\,\gamma_5\ \gamma_\mu\ \gamma^\mu\ \gamma_a\ \gamma_b\ \gamma_c\ \gamma_d = (\alpha n + (1-\alpha)(8-n))\ 4i\ \varepsilon_{abcd}$. In their calculation of the VVA triangle anomaly the free parameter was then fixed by postulating the "correct" value for the anomaly.

It is not clear whether the scheme of Ref.[2] can be consistently formulated in more general situations. In particular, it is not clear whether the prescription of Ref.[2] is applicable to higher order IR calculations with their multiple γ-contractions inside parity-odd traces. These multiple γ-contractions would necessitate the introduction of multifold parameter ambiguities and it is not clear whether one could find enough physical conditions to fix them. Also dropping (2) implies $\not\ell\not\ell \neq \ell^2$ (ℓ is an n-dimensional integration momentum) which spoils the validity of higher order field equations.

Siegel in his dimensional reduction scheme [3] attempts to keep the Dirac algebra in four dimensions, whereas the integrations are in n dimensions. Thus, γ_5 is anticommuting (1), but (2) is changed to $\gamma_\mu\ \gamma^\mu = 4$. As necessarily $\not\ell\not\ell = \ell^2$ (ℓ is an n-dimensional integration momentum) this implies

$g_{\mu\alpha}\ \hat{g}^{\alpha}{}_{\nu} = g_{\mu\nu}$, where $\hat{g}_{\mu\nu}$ and $g_{\mu\nu}$ are the "4-dimensional" and the "n-dimensional" metric tensors. Latter condition can be shown to lead to $(n-4)(n-3)(n-2)(n-1) = 0$ which shows that altering (2) to $\gamma_\mu\ \gamma^\mu = 4$ is not a consistent procedure. *)

A third possibility is to drop (1), as originally proposed by 't Hooft and Veltman [7] and later systematized by Breitenlohner and Maison (BM) [1]. The main points of the BM scheme are the following:

i) in addition to the "n-dimensional" metric tensor $g_{\mu\nu}$ ($g_\mu{}^\mu = n$) introduce a "4-dimensional" metric tensor $\hat{g}_{\mu\nu}$ ($\hat{g}_\mu{}^\mu = 4$) such that

$$g_{\mu\alpha}\ \hat{g}^{\alpha}{}_{\nu} = \hat{g}_{\mu\nu} \qquad (5)$$

ii) Eq.(1) is replaced by

$$\{\gamma_\mu,\ \gamma_5\} = 2(\gamma_\mu - \hat{\gamma}_\mu)\ \gamma_5 \qquad (6)$$

where $\hat{\gamma}_\mu = \hat{g}_\mu{}^\alpha \gamma_\alpha$ is a "4-dimensional" γ-matrix

iii) Traces involving γ_5 are calculated by substituting γ_5 by its definition

$$\gamma_5 = \frac{i}{4!}\ \varepsilon_{\alpha\beta\gamma\delta}\ \gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta \qquad (7)$$

In particular, one has

$$\mathrm{Tr}\ \gamma_5\ \gamma_\alpha\ \gamma_\beta\ \gamma_\gamma\ \gamma_\delta = 4i\ \varepsilon_{\alpha\beta\gamma\delta}$$
$$\mathrm{Tr}(\{\gamma_\lambda,\ \gamma_5\}\ \gamma^\lambda\ \gamma_\alpha\ \gamma_\beta\ \gamma_\gamma\ \gamma_\delta) = 8(n-4)i\ \varepsilon_{\alpha\beta\gamma\delta} \qquad (8)$$

iv) identities involving ε-tensors are valid only for "4-dimensional" covariants, i.e.

a) $\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\ \varepsilon_{\nu_1\nu_2\nu_3\nu_4} = -\det(\hat{g}_{\alpha\beta}) \quad \alpha = \mu_1\ldots\mu_4,\ \ \beta = \nu_1\ldots\nu_4$ (9)

b) $\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\ \hat{g}_{\mu_5\alpha} + \mathrm{cycl.}(\mu_1\ldots\mu_5) = 0$ (10)

*) D. Maison, private communication.

Eq.(10) is the "4-dimensional" version of (3) and will be referred to as the Schouten identity. In contrast to the scheme of [2] the BM scheme guarantees well-defined and unique γ-matrix trace results.

That the BM scheme provides a consistent treatment of the singularities in the UV-realm (correct anomalies, all order renormalizability) has been demonstrated in Refs.[1,6,8].

That the BM scheme also provides a consistent and practicable calculational scheme for the treatment of IR-infinities will be demonstrated in the following by calculating through an example.

First note that in parity-odd traces there is no need to use the "ugly" anticommutation relation (6) since all trace manipulations can be performed without commuting by γ_5 because of the cyclic property of a trace. Second, the γ_5-substitution (7) yielding long traces need not be done explicitly if one works with suitable parity-odd projection operators. The action of the parity-odd projection operators finally bring into play "4-dimensional" scalars via (9) . These have to be treated separately from the "n-dimensional" scalars resulting from the trace manipulations. As will be clear in a moment, there will be only one relevant "4-dimensional" scalar for every infrared region, which the computer can easily handle. Even though the BM-scheme as formulated in (5-10) looks formidable at first sight from a calculational point of view, we found it to be no more difficult to implement on a computer than the corresponding parity-even IR-problem.

We shall now turn to our specific example, the $O(\alpha_s^2)$ corrections to the p.v. structure functions in $e^+e^- \to q\bar{q}g$.

First we discuss the real tree-graph contributions. Some of the contributing 4-parton diagrams are drawn in Fig.1. To bring out the general features of our calculation we concentrate on the 3-4 IR-regions, i.e. when the two gluons with p_3 and p_4 (see Fig.1a), and quark and antiquark with momenta p_3 and p_4 (see Fig.1b), become infrared unresolvable. As

infrared cut-off we choose an invariant mass cut-off $s_{ij} = 2p_ip_j \leq y\, q^2$ $(q = p_1 + p_2 + p_3 + p_4)$. *)

In the 3-4 CM-system, the 4-parton momenta have the components

$$p_1 = \frac{s_{134} - s_{34}}{2\sqrt{s_{34}}}\ (1, \ldots, \sin\beta\,, \cos\beta\,)$$

$$p_2 = \frac{s_{234} - s_{34}}{2\sqrt{s_{34}}}\ (1, \ldots, 0, 1) \tag{11}$$

$$p_{\substack{3\\4}} = \frac{1}{2}\sqrt{s_{34}}\ (1, \ldots, \pm \sin\Theta\cos\Theta', \pm\cos\Theta)$$

where $s_{ijk} = (p_i + p_j + p_k)^2$ and the dots in p_1 and p_2 denote (n-3) zeros, and (n-3) equal and opposite unspecified angular factors in p_3 and p_4.

The IR-integration involves the angular integral in "(n-1) dimensions"

$$\int d\,\Omega_{3-4}^{(n-1)}\ H_{\mu\nu}^{(4)pv}\ (p_1,p_2,p_3,p_4) = H_{\mu\nu}^{(3)pv}\ (q_1,q_2,q_3) \tag{12}$$

where $H_{\mu\nu}^{(i)pv}$ is the i-parton p.v. hadron tensor. We identify $q_1 = p_1$, $q_2 = p_2$ and $q_3 = p_3 + p_4$. The r.h.s. of (12) can be expanded as

$$H_{\mu\nu}^{(3)pv} = H_6\ \varepsilon(\mu\nu\ qq_1) + H_7\ \ (\mu\nu\ qq_2) + H'\ (\mu\nu\ q_1q_2) \tag{13}$$

Use has been made of the $\mu \leftrightarrow \nu$ antisymmetry of

$$H_{\mu\nu}^{pv} = \frac{1}{2}\,(H_{\mu\nu}^{VA} + H_{\mu\nu}^{AV}) \tag{14}$$

where V and A denote the vector and axial vector parts in the current-current contributions to the hadron tensor. Also the Schouten identity (10) has been used to limit (13) to three terms which is justified since we have choosen our coordinate system (11) such that the q_i are 4-dimensional. Note that the nonconserved axial contributions appear in H'.

Since $H_{\mu\nu}^{(3)pv}$ in (13) carries only "4-dimensional" tensor indices μ and ν it is clear that the anomalous charge conjugation pieces in $H_{\mu\nu}^{(4)pv}$ resulting from $C(\gamma_\mu\gamma_5)\,C^{-1} \neq -\gamma_\mu\gamma_5$ vanish after the IR angular integration (12).

*) We work with massless quarks and gluons, $p_i^2 = 0$, and use the Feynman gauge.

Next we scalarize the integrand in (12) by contracting with the odd-parity projection tensors $q_{2\nu}\ \varepsilon(\mu\ q_1q_2q_3)$, $q_{1\nu}\varepsilon(\mu\ q_1q_2q_3)$ and $q_\nu\ \varepsilon(\mu\ q_1q_2q_3)$ which leads to "4-dimensional" scalar contributions via Eq.(9). However, these are not hard to handle, since they all can be expressed in terms of e.g. $\hat{p}_4^{\ 2}$, since $\hat{p}_3\ \hat{p}_4 = p_4(p_3+p_4) - \hat{p}_4^2$, $\hat{p}_3^{\ 2} = \hat{p}_4^{\ 2}$ and $\hat{p}_1\ \hat{p}_3 = p_1\ \hat{p}_3 = p_1p_3$ etc. (see (5) and (11)). The scalar integrand can then be expressed as $I_s = A(s_{ij}) + \hat{p}_4^{\ 2}\ B(s_{ij})$. The first term can be integrated conventionally as in [9], whereas the second term requires the "(n-1)-dimensional" integral over the "4-dimensional" scalar $\hat{p}_4^{\ 2}$, cif.

$$\frac{1}{N}\int d\,\Omega_{34}^{n-1}\ \hat{p}_4^{\ 2} = \frac{4-n}{4(1-n)}\ s_{34} \tag{15}$$

where $N = \int d\,\Omega_{34}^{n-1}$. *)

We emphasize that the inclusion of contribution (15) is of crucial importance in obtaining the correct final result. However, due to the appearence of the explicit s_{ij} factor in (15) these contributions need to be taken into account only for diagrams with true s_{ij}^{-2} double pole contributions as depicted explicitly in Fig.1 as long as one is working only to $O(y^0)$.

Finally, after adding up the IR contributions from all $O(\alpha_s^{\ 2})$ tree diagrams we obtain to $O(y^0)$

$$H_{\mu\nu}^{(3)pv}(\text{real}) = g^4 N_c C_F (C_F H^c + \frac{N_c}{2}\ H^N + (\frac{N_f}{3} - \frac{11}{6}\ N_c)\ H^f)\ A_{\mu\nu}^{(3)pv}(\text{Born}) \tag{16}$$

where

$$A_{\mu\nu}^{(3)pv}(\text{Born}) = -8\Big[\left(\frac{1-t_{23}}{t_{13}t_{23}} - \varepsilon\ \frac{1}{t_{23}}\right)\varepsilon(\mu\nu\ qq_1) - (1\leftrightarrow 2) - \varepsilon\,\frac{1-t_{12}}{t_{13}t_{23}}\ \varepsilon(\mu\nu q_1 q_2)\Big] \tag{17}$$

is the p.v. Born term hadron tensor. We define $\varepsilon = \frac{1}{2}(4-n)$ and $q^2\ t_{ij} = 2q_iq_j$, where the q_i are the three-parton momenta. N_c and N_f are the numbers of colours and flavours, and $C_F = (N_c^{\ 2} - 1)/2N_c$. The strong coupling constant is denoted by g. The IR-structure is given by 3 functions

*) Note that integral (15) is of O(n-4) in accordance with $\hat{p}_4^{\ 2} \to p_4^{\ 2} = 0$ for $n \to 4$.

$$\begin{aligned} H^c &= C(\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon}(2\ln t_{12} - 3) + 7 - 2\ln^2 y + 4\ln y\ \ln t_{12} \\ &\qquad - \ln^2 t_{12} - 3\ln y + \frac{\pi^2}{3} \\ H^N &= C(\frac{2}{\varepsilon^2} - \frac{2}{\varepsilon}\ln\frac{t_{13}\ t_{23}}{t_{12}} + \frac{4}{3} - \ln^2 t_{13} - \ln^2 t_{23} + \ln^2 t_{12} \\ &\qquad - 2\ln^2 y + 4\ln y\ \ln\frac{t_{13}\ t_{23}}{t_{12}} + \frac{\pi^2}{3}) \\ H^f &= C(-\frac{1}{\varepsilon} - \frac{5}{3} + \ln y) \end{aligned} \tag{18}$$

and where

$$C = \frac{1}{2\pi^2}\ (\frac{4\pi\mu^2}{q^2})^\varepsilon\ \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\ (1 - \frac{\pi^2}{3}\ \varepsilon^2)$$

One notes from (16) that the $O(y^0)$ IR result factorizes into a Born term contribution and an universal IR factor as in the p.c. case discussed in [10]. The universal IR factor is the same for the p.c. and p.v. contributions.

We now turn to the virtual IR-contributions. The $O(\alpha_s^{\ 3/2})$ one-loop contributions to the vector current amplitude $J_\mu^V \to q\bar{q}g$ have been calculated in [11]. We fix the axial vector renormalization constant such that the corresponding axial vector current contributions to $J_\mu^A \to q\bar{q}g$ are obtained by substituting $\gamma_\mu \to \gamma_\mu\gamma_5$ in the vector current result [11].

In [11] it was shown that the IR-divergent parts of the vector current one-loop contributions are proportional to the Born term amplitude. In order to demonstrate the cancellation of IR singularities and axial anomalies we limit our attention only to these IR singular pieces.*) Their contributions to the hadron tensor are given by **)

$$H_{\mu\nu}^{(3)pv}(\text{virtual;sing.}) = g^4 N_c C_F (C_F H_V^c + \frac{N_c}{2}\ H_V^N + (\frac{N_f}{3} - \frac{11}{6}N_c)H_V^f)A_{\mu\nu}^{(3)pv}(\text{Born}) \tag{19}$$

where

*) A more detailed account of our work will be presented in [12].

**) g is the strong coupling constant in the $\overline{MS}$ scheme.

$$H_v^c = C(-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(2\ell n\, t_{12} - 3))$$

$$H_v^N = C(-\frac{2}{\varepsilon^2} + \frac{2}{\varepsilon}\ell n \frac{t_{13}\, t_{23}}{t_{12}}) \tag{20}$$

$$H_v^f = C\,\frac{1}{\varepsilon}$$

Comparing the singular virtual $O(\alpha_s^2)$ contributions (19,20) to the real contributions in (16-18) one sees that IR singular contributions cancel, as well as the finite contribution to the anomalous structure function $\epsilon(\mu\nu\, q_1 q_2)$.

In conclusion we summarize our results by writing out the various contributions in a symbolic notation:

$$\text{(real):} \quad \frac{a^n}{\varepsilon^2} + \frac{b^n}{\varepsilon} + \frac{b^a}{\varepsilon} + c^n(y) + c^a$$

$$\text{(virtual):} \quad -\frac{a^n}{\varepsilon^2} - \frac{b^n}{\varepsilon} - \frac{b^a}{\varepsilon} + c'^n - c^a$$

The normal IR singular pieces proportional to (a^n, b^n) as well as the IR anomalous singular and finite pieces proportional to (b^a, c^a) cancel among the real and virtual contributions. The charge conjugation anomaly vanishes after IR integration. To $O(y^o)$ the real contributions factor into the Born term and an universal IR factor.

It would be interesting to find out whether the same mechanism that leads to the cancellation of the normal IR singularities is also responsible for the cancellation of the IR singular and finite anomalous contributions.

It goes without saying that the expertise gained from this first explicit calculation of a higher order QCD correction to a p.v. cross section will be quite valuable for the many higher order QCD calculations that have to be done for the interpretation of p.v. experiments at the high energy machines to be completed in the following years.

Acknowledgement:

We would like to thank D. Maison for some clarifying discussions. Part of this work was done while J.G.K. and G.S. were visitors at the DESY Theory Group. They would like to thank F. Gutbrod and R. Peccei for hospitality and DESY for support.

References

[1] P. Breitenlohner and D. Maison, Commun.Math.Phys. 52(1977)11

[2] M. Chanowitz, M. Furman and I. Hinchliffe, Phys.Lett. 78B(1978)285; Nucl.Phys. B159(1979)225

[3] W. Siegel, Phys.Lett. 84B(1979)193

[4] G. Bonneau, Phys.Lett. 96B(1980)147

[5] D. Maison, Phys.Lett. 150B(1985)139

[6] T.L. Trueman, Phys.Lett. 88B(1979)331

[7] G. 't Hooft and M. Veltman, Nucl.Phys. B44(1972)189

[8] P. Breitenlohner and D. Maison, Commun.Math.Phys. 52(1977)39 and 55

[9] R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl.Phys. B178(1981)421; K. Fabricius, G. Kramer, G. Schierholz and I. Schmitt, Z.Phys. C11(1982)315

[10] G. Kramer and B. Lampe, Comm.Math.Phys. 97(1985)257

[11] J.G. Körner and G. Schuler, Z.Phys. C26(1985)559

[12] J.G. Körner, G. Kramer, B. Lampe and G. Schuler, DESY preprint (to be publ.)

Figure Caption

Figure 1: Sample 4-parton diagrams a) $e^+e^- \to q(p_1)\, q(p_2)\, g(p_3)\, g(p_4)$, b) $e^+e^- \to q(p_1)\, \bar{q}(p_2)\, q(p_3)\, \bar{q}(p_4)$. The depicted diagrams have true $1/s_{34}^2$ double pole singularities. The dots in a) stand for 63 (plus 4 ghost) additional $q\bar{q}gg$ diagrams and in b) for 63 additional $q\bar{q}q\bar{q}$ diagrams.

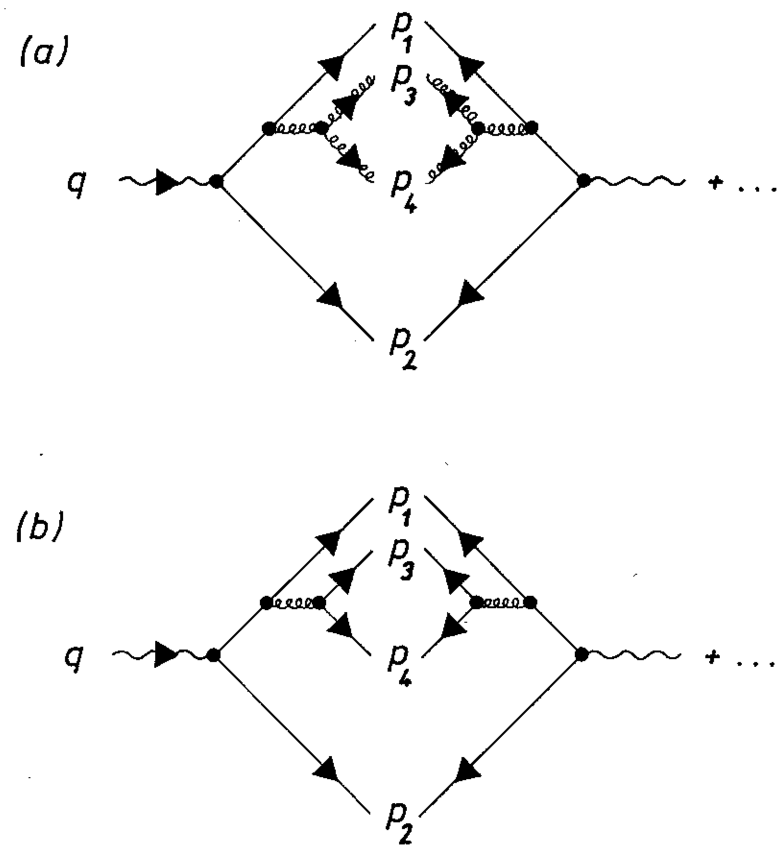

Fig.1