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Blocking Transformations for Lattice Fermions

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ABSTRACT. We introduce a class of chiral-symmetry breaking real space renormalization transformations, intended for renormalization group studies of lattice theories involving fermions. In massless free fermion theory (for a sensible choice of a certain parameter of the transformation) the scheme yields an acceptably local, Wilson-fermion-like fixed point action. We attempt to calculate a certain critical exponent in the two-flavour Schwinger model via a cumulant expansion based on our scheme. Possibilities for Monte Carlo renormalization group calculations are briefly mentioned.

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1. Introduction.

Several groups of workers have recently reported numerical results from the application of Monte Carlo Renormalization Group techniques to lattice gauge theories [1]. This work has so far been restricted to the pure gauge sector, but presumably one may eventually wish to apply similar techniques to theories involving matter fields, and in particular fermions.

Real space renormalization methods [2] were originally developed for studying the critical properties of statistical mechanical systems. In this case the renormalization group (RG) structures of primary interest were the fixed points governing the "critical surface" (in the space of conceivable hamiltonians, a subspace on which the associated correlation length diverges.) The RG behaviour of the hamiltonian in the vicinity of these fixed points determines the "universal" features (such as the critical exponents) of the phase transition.

In four dimensional QCD the emphasis is rather different. There is then (one sincerely hopes!) no phase transition at finite inverse-coupling (in the simple Wilson gauge theory, with link variables in the fundamental representation). The application there is to study the more general, non-critical scaling properties of the theory. The RG structure of primary interest is then the so-called Renormalized Trajectory, and the most interesting quantity to calculate is the non-perturbative beta function.

Once fermions enter the picture the situation is somewhat changed [3] . QCD with fermions does have a critical structure; the critical surface is the subset of couplings for which the pion mass vanishes. Thus one application of MCRG methods to QCD + quarks might be in the study of this "phase transition". An example of a critical exponent associated with this transition is the exponent relating the pion and quark masses near the chiral limit. This exponent is not particularly controversial in four dimensions — it is equal to $\frac{1}{2}$ (this is a "mean-field" result [4], not valid in lower dimensions [5]). Perhaps a more important application would be to the study of the general scaling behaviour of the theory, as has been done with the pure gauge sector.

The present paper will not give any detailed scheme for doing such calculations (although we will make one or two suggestions in this direction). We will, however make some general remarks about the kind of blocking transformation that might be required for such work, and illustrate these considerations in the context of free-fermion theory and two dimensional lattice QED. Such a study is clearly a useful preliminary in assessing the feasibility of Monte

where

$$b_\mu = 2u_\mu - 1. \tag{2.2a}$$

The sum is over the sites of the block; the vector b is (twice) the displacement of the site from the centre of the block. The constant ζ is an essential ("wave function") renormalization factor, and the only other parameter of the transformation is the constant ρ.

If ρ is set equal to zero the new block variable, up to wave function renormalization, is just the average of the 2^d old variables in the block. In the next section we look at this most naive blocking scheme ($\rho = 0$) in the context of free field theory, and establish in fact that this particular scheme is probably not very useful. Before doing this we derive the general form of the recursion relations for a free field action under the transformation (2.1).

Take a bilinear form

$$A(\overline{\psi}, \psi) = \sum_{x,y} \overline{\psi}(x)\, K(x-y)\, \psi(y), \tag{2.3}$$

for the action. In matrix notation,

$$A(\overline{\psi}, \psi) = \overline{\psi} K \psi. \tag{2.3a}$$

K should satisfy the constraints required by symmetry under the lattice group, but is otherwise essentially arbitrary, involving, *a priori*, couplings between arbitrarily remote sites. Because of the discrete rotation symmetry we may make the decomposition

$$K(u) = K_0(u) + \sum_{\mu=1}^{d} K_\mu(u)\, \gamma_\mu. \tag{2.4}$$

(In a gauge theory one could have extra $\sigma_{\mu\nu}$ terms, but we will not be including gauge field dependence in K, or P, in the next few sections).

The renormalized action, A', can be defined by

$$e^{-A'[\overline{\psi}', \psi']} = \int d\overline{\psi} d\psi\, \delta(\overline{\psi}' - \overline{\psi}\,\overline{P})\, \delta(\psi' - P\,\psi)\, e^{-A[\overline{\psi}, \psi]}. \tag{2.5}$$

A convenient representation for the fermionic delta-function is

$$\delta(y - x) = \int d\alpha\, e^{i\alpha(y-x)}, \tag{2.6}$$

where x, y and α are anticommuting variables. (This representation only makes complete sense if these delta functions come in conjugate pairs, which they do). Substituting (2.6) in (2.5), and doing some gaussian integrals, one easily finds for the effective action after one RG transformation

$$A' = \overline{\psi}' K' \psi', \tag{2.7a}$$

$$K' = [P\, K^{-1}\, \overline{P}]^{-1}. \tag{2.7b}$$

Equation (2.7b) is the desired recursion relation for the "coupling constant" matrix K. The recursion relations take a more transparent form in terms of the propagator, $S = K^{-1}$, namely

$$S' = P\, S\, \overline{P}, \tag{2.8}$$

a result which is essentially obvious if we note $S = \langle \psi\, \overline{\psi} \rangle$, and

$$\langle \psi'\, \overline{\psi}' \rangle = \langle P\, \psi\, \overline{\psi}\, \overline{P} \rangle = P \langle \psi\, \overline{\psi} \rangle \overline{P}. \tag{2.9}$$

3. Naive Blocking of Free Fermions.

Let us start from an initial Wilson-type nearest-neighbour action [9]:

$$A = \sum_x \left(\overline{\psi}\psi(x) + k \sum_u \overline{\psi}(x)[\gamma_u - r]\psi(x+u) \right), \tag{3.1}$$

where the sum on u is over positive and negative unit lattice vectors, and $\gamma_u = u.\gamma$. Notice that (3.1) is the most general nearest-neighbour bilinear action compatible with the lattice symmetries, in the absence of gauge fields. We will suppose that the starting action is on its "critical surface", $1 - 2dkr = 0$, that is we are dealing with a massless fermion. Of course, after applying the RG transformation the action will will no longer have the form (3.1), but it will still have the general form (2.3), and it will still lie on the critical surface,

$$\sum_u K'_0(u) = 0, \tag{3.2}$$

associated with (2.3), where now the sum on u goes over the entire lattice.

We now successively apply a large number, l, of iterations of the naive ($\rho = 0$) transformation to (3.1). The new fermion variables, $\psi^{(l)}(x^{(l)})$, are

$(\zeta^{(l)})^{-2} S^{(0)}(0) \sim 2^{l(d-1)}$. For $d > 1$, $S^{(l)}(0)$ grows without limit for large l; there is no fixed point propagator, and therefore no fixed point action.

The propagator (3.8,9) can easily be transformed into momentum space. First note that the lattice Fourier transform of a function $F(x)$ can be written as

$$F(k) = \sum_{x_1,\ldots,x_d=-\infty}^{+\infty} e^{-ik.x} F(x) = \sum_{n_1,\ldots,n_d=-\infty}^{+\infty} F_{cont}(k + 2\pi n) \qquad (3.10)$$

where

$$F_{cont}(k) = \int_{-\infty}^{+\infty} d^d x \, e^{-ik.x} F(x) \qquad (3.11)$$

is just the continuum Fourier transform of the same function (continued to non-integer x). But the continuum Fourier transform of the convolution (3.8) is just

$$\prod_{\mu=1}^{d} \frac{2(1-\cos k_\mu)}{(k_\mu + 2\pi n_\mu)^2} \cdot \frac{1}{\sum_\mu k_\mu \gamma_\mu} \qquad (3.12)$$

(The Fourier transform of a convolution of two functions is the product of the transforms of the individual functions). Thus

$$S_{fix}(k) = \left(\prod_{\mu=1}^{d} 2(1-\cos k_\mu) \right) \times \sum_{n_1,\ldots,n_d=-\infty}^{+\infty} \frac{1}{\prod_\mu (k_\mu - 2\pi n_\mu)^2} \, \frac{1}{\sum_\mu (k_\mu - 2\pi n_\mu)\gamma_\mu}. \qquad (3.13)$$

Now, the momentum space form of the matrix, K, of coupling constants in the fixed point action is just the reciprocal of (3.13). The problem is that (3.13) has zeros at the points $(\pi, 0, \ldots, 0), (0, \pi, \ldots, 0), (\pi, \pi, \ldots, 0)$, etc., on the edges of the Brillioun zone, so that $K_{fix}(k)$ has poles at these points. This means that the real space form of the fixed point action will be highly non-local. In fact the momentum space poles give oscillatory terms in the real-space matrix K falling off only like $\sim |x-y|^{-(d-1)}$, (see equation (2.3)).

With hindsight one can see that this is a consequence of general lore about lattice fermions [9, 10]. The transformation we have used preserves any chiral symmetry present in the starting action. Thus its fixed point is,

not surprisingly, chirally symmetric. Although we started from the chirally non-invariant Wilson action we therefore ended up with a chirally invariant fixed point action, and since (like the original action) it could only represent a single species of fermion, it had to be non-local.

A transformation which generates a non-local fixed point action is unlikely to be successful in MCRG work, which inevitably demands truncation to a small number of dominant operators. Thus we have to go to a more complicated transformation.

4. An Improved Tranformation.

The terms proportional to ρ in (2.2) break any chiral symmetry present in the starting action, so there are no longer any general grounds for expecting a non-local fixed-point action. Notice that the extra terms are proportional to a lattice derivative, thus in some sense chiral symmetry should only be damaged in the short wavelength regime — one can hope that the blocking transformation will not compromise the restoration of chiral symmetry in the continuum limit. This is akin to the usual situation with Wilson fermions.

It is more difficult to carry out the analysis of the previous section for this more general transformation, and we have not found any analytic result for the fixed point action. However in the free field case it is easy to do the blocking numerically by performing the inversions required for (2.7) by discrete Fourier analysis on some finite lattice. For reference we quote the following form for (2.7b) in momentum space

$$K'(k) = \left[\frac{1}{2^d} \sum_{v_1,\ldots,v_2=0}^{1} K^{-1}_{P\overline{P}}(\tfrac{1}{2}k + \pi v) \right]^{-1},$$

$$K^{-1}_{P\overline{P}}(l) = P(-l)\, K^{-1}(l)\, \overline{P}(l),$$

$$P(l) = \sum_u e^{-il.u} P(u), \qquad \overline{P}(l) = \sum_u e^{-il.u} \overline{P}(u) \qquad (4.1)$$

(see (2.1)). It is easy to arrange one's programs so that the "lattice-size" does not vary from iteration to iteration of the blocking transformation, unlike Monte Carlo "matching" type calculations, and also to ensure that one stays on the exact critical surface if one starts off on it.

We have experimented with the transformation (2.2) for the case of two-dimensional free fermions. Applying several iterations of (2.2) with parameter

can be written as

$$A'(\Phi') = -\ln\left[\int d\Phi\, T(\Phi',\Phi)\, e^{-A_0(\Phi)}\right] + \langle V\rangle_{\Phi'} + O(V^2). \qquad (5.2)$$

where the intra-block average of an operator O is

$$\langle O\rangle = Z_{\Phi'}^{-1} \int d\Phi\, T(\Phi',\Phi)\, O\, e^{-A_0(\Phi)}, \qquad (5.3a)$$

$$Z_{\Phi'} = \int d\Phi\, T(\Phi',\Phi)\, e^{-A_0(\Phi)}. \qquad (5.3b)$$

Neglecting the $O(V^2)$ terms in (5.2) gives the linear cumulant approximation, which is the only case we will consider.

For illustration consider again the two-dimensional free fermion theory. If we split the matrix K into an intracell part K_0, and an intercell part K_1, then

$$\begin{aligned} Z_{\overline{\psi}'\psi'} &= \int d\overline{\psi} d\psi\, \delta(\overline{\psi}' - \overline{\psi}\,\overline{P})\, \delta(\psi' - P\,\psi)\, e^{-\overline{\psi} K_0 \psi} \\ &= \det(K_0)\, \det(P\,K_0^{-1}\,\overline{P})\, e^{-\overline{\psi}'(P\,K_0^{-1}\,\overline{P})^{-1}\psi'}, \end{aligned} \qquad (5.4)$$

so the zeroth order part of (5.2) is

$$-\ln(Z_{\overline{\psi}'\psi'}) = const. + \overline{\psi}'(P\,K_0^{-1}\,\overline{P})^{-1}\psi'. \qquad (5.5)$$

The matrix K_0 does not connect different blocks. Neither, of course, do P or $\overline{P}$. So this zeroth order term does not connect block variables at different sites — it just yields $K'(0)$. The linear term in the cumulant expansion, on the other hand is

$$\langle \overline{\psi}\, K_1\, \psi\rangle_{\overline{\psi}'\psi'} = \langle\overline{\psi}\rangle_{\overline{\psi}'\psi'}\, K_1\, \langle\psi\rangle_{\overline{\psi}'\psi'}, \qquad (5.6)$$

where the factorization in the averaging occurs because K_1 *only* connects different blocks. Thus, in addition to (5.5) we need the average

$$\begin{aligned} \langle\overline{\psi}\rangle_{\overline{\psi}'\psi'} &= Z_{\overline{\psi}'\psi'}^{-1} \int d\overline{\psi} d\psi\, \delta(\overline{\psi}' - \overline{\psi}\,\overline{P})\, \delta(\psi' - P\,\psi)\, \overline{\psi}\, e^{-\overline{\psi} K_0 \psi}, \\ &= (P\,K_0^{-1}\,\overline{P})^{-1} P\,K_0^{-1}, \end{aligned} \qquad (5.7)$$

and the corresponding average for ψ. Now, in two dimensions, K_0 is just an 8 by 8 matrix (there are four sites in a block, and the Dirac matrices are 2 by

2) with various symmetries, and it is easy to invert explicitly. For example, one obtains

$$P\,K_0^{-1}\,\overline{P} = \frac{\zeta^{-2}}{4}\, \frac{1 + 4\rho k - 2\rho^2(1-2kr)}{1 + 2k^2 - 2kr}. \qquad (5.8)$$

Substituting (5.7) and the conjugate expression in (5.6) we find that the matrix $P\,K_0^{-1}K_1K_0^{-1}\,\overline{P}$ is also required. This matrix just gives the nearest neighbour terms in the renormalized action. After some algebra we get the following results for the recursion relations in the linearized approximation

$$\begin{aligned} k' &= \\ &\frac{k}{2}\, \frac{(1 + 2k^2 + 2kr) - 2\rho r(1 - 6k^2 - 2kr) + 2\rho^2(1 + 2k^2 - 6kr + 8k^2r^2)}{(1 + 2k^2 - 2kr)[1 + 4\rho k - 2\rho^2(1-2kr)]}, \\ k'r' &= \frac{k}{2}\, \frac{(1+2\rho k)[(2k + r) - 2\rho(1 - 3kr)]}{(1 + 2k^2 - 2kr)[1 + 4\rho k - 2\rho^2(1-2kr)]}, \\ \zeta^2 &= \frac{4(1 + 2k^2 - 2kr)}{1 + 4\rho k - 2\rho^2(1-2kr)}. \end{aligned} \qquad (5.9)$$

One now solves these recursion relations for their fixed points. In general this has to be done numerically, but there is one fixed point of (5.9) which has a particularly simple form, namely

$$\begin{aligned} r = \frac{1}{2\rho}, \qquad k = \frac{\rho}{2}, \\ \zeta^2 = \tfrac{1}{2}. \end{aligned} \qquad (5.10)$$

Calculating the matrix of derivatives of the recursion relations (5.9) at this fixed point (this is a somewhat tedious piece of algebra), one finds that it has eigenvalues

$$2 \qquad \text{and} \qquad \frac{3 - 2\rho^2}{2(1+\rho^2)}.$$

For $\rho > \frac{1}{2}$, the second eigenvalue is less than 1, and therefore this is the stable fixed point governing the critical surface . In this case the linear approximation reproduces the exact values for the two universal quantities, ζ and the largest eigenvalue.

This is obviously a fluke, but it suggests that the linear cumulant approximation might be reasonable in two dimensions.

(For smaller values of ρ one has to find the stable fixed point numerically. This gives results which vary slowly with ρ, and match onto the above results at $\rho = \frac{1}{2}$. This does not matter in principle, because one is free to optimize with respect to ρ)

homogeneously as the (unoccupied) site in the centre of the block. For a given density of matter variables we need four times as many gauge variables as the conventional lattice theory, but the continuum limit should be the same as the conventional theory.

The prescription we will actually adopt is slightly more economical in gauge variables, but in the same spirit. We take a conventional two-dimensional lattice gauge theory, and put fermion fields on the *odd* sites ($(-1)^{x+y} = -1$) of this lattice. The nearest-neighbour fermionic action is then

$$A = \sum_{\text{odd } x} \left(\overline{\psi}\psi(x) + \frac{\tilde{k}}{2} \sum_{\substack{u,v \\ u \cdot v = 0}} \overline{\psi}(x)[\gamma_u + \gamma_v - \tilde{r}]U(x, x+u+v)\psi(x+u+v) \right),$$

$$U(x, x+u+v) = \tfrac{1}{2}[U(x, x+u)U(x+u, x+u+v) + U(x, x+v)U(x+v, x+u+v)], \quad (6.2)$$

where the sum on u, v is over orthogonal pairs of positive and negative unit vectors. In fact there is another term which could be included in this action, proportional to σ_{12}, with the antisymmetric gauge factor, but we will neglect this term.

Note that apart from the gauge sector this is just a two-dimensional fermion theory on a square lattice, rotated by 45^o relative to the gauge-bearing lattice. The blocking transformation for the fermions is just

$$\psi'(x') = \frac{\zeta^{-1}}{4} \sum_u [1 + \tilde{\rho}\gamma_u]\psi(2x' + u),$$

$$\overline{\psi}'(x') = \frac{\zeta^{-1}}{4} \sum_u \overline{\psi}(2x' + u)[1 - \tilde{\rho}\gamma_u], \quad (6.3)$$

where, as usual, the sum is over positive and negative unit vectors, and x' is an odd site of the new lattice, $(-1)^{x'_1 + x'_2} = -1$. The transformation is illustrated pictorially in figure 3. We have not yet specified the transformation for the gauge fields, but we intend to use just "naive decimation",

$$U'(x', x' + u) = U(2x', 2x' + u)U(2x' + u, 2x' + 2u). \quad (6.4)$$

Hopefully this transformation should be adequate in two dimensions (the cumulant expansion would be difficult to implement with any more elaborate

blocking rule). In the case $U = 1$ one makes contact with the free-field calculations of the previous two sections by rotating the lattice and putting

$$\tilde{k} = 2^{-\frac{1}{2}} k, \qquad \tilde{r} = 2^{\frac{1}{2}} r, \qquad \tilde{\rho} = 2^{\frac{1}{2}} \rho. \quad (6.5)$$

Now we are in a position to calculate recursion relations in the linear cumulant approximation. We will restrict ourselves to the infinite coupling, $\beta = 0$, case (it is easy to convince oneself that this is the exact fixed point of the linearized recursion relations). Thus (6.2) is our complete action. Two adjacent blocks or cells are illustrated in figure 4, with a convenient labelling of sites. Each block contains four fermion spinor variables (at sites 1, 2, 3 and 4), plus sixteen link variables. The matrix of the fermion action is now gauge-field dependent, but as in the previous section it can be split into intracell and intercell pieces K_0 and K_1. Using an obvious shorthand for the four delta functions for the gauge fields associated with a single block, the intra-block "partition function" becomes

$$Z_{\overline{\psi}'\psi'U'} = \int dU\, \delta(U' - UU) \int d\overline{\psi} d\psi\, \delta(\overline{\psi}' - \overline{\psi}\,\overline{P})\, \delta(\psi' - P\,\psi)\, e^{-\overline{\psi} K_0(U) \psi}, \quad (6.6)$$

where according to (6.3) the matrices P and $\overline{P}$ also now have some gauge field dependence. Consider the block consisting of sites 0, 1, 2, ..., 12 in figure 4. The four outermost link variables $U(1,9), \ldots, U(4,12)$ only appear in the delta functions, so they can be integrated out. In the process the U' dependence of (6.6) goes away. Then we can use the symmetry under local gauge rotations at the sites 1,2, ...,8 to set $U(0,1) = \cdots = U(0,4) = 1$, and to replace the four products $U(1,5)U(5,2)$, etc., by four single variables $U_{12}, U_{23}, U_{34}, U_{41}$ associated with the four outside corner paths of the block, since these are the only form in which $U(1,5)$, etc., appear in the integrand (in K_0). We cannot, of course, fix the gauge with respect to symmetry under the local rotations at the sites 0, 9, 10, 11, 12, because these are part of the gauge symmetry of the renormalized action. So now the intra-block gauge integration is reduced to a four-fold group integral, rather than the original sixteen-fold form. Doing the fermion integration, we get now (c.f. (5.7))

$$Z_{\overline{\psi}'\psi'} = \int dU_{12} dU_{23} dU_{34} dU_{41}\, \Delta(U)\, e^{-\overline{\psi}' [P\, K_0^{-1}(U)\, \overline{P}]^{-1} \psi'}$$

$$\Delta(U) = \left(\det[K_0(U)]\, \det[P\, K_0^{-1}(U)\, \overline{P}] \right)^{N_f}, \quad (6.7)$$

The integrals in (6.12) and (6.13) look messy, but for the case of a U(1) gauge group, K_0 is only an 8 by 8 matrix, whose determinant and inverse one can trivially calculate numerically (in fact there are certain properties of K_0 which make this particularly simple), and the integral is only four dimensional. In other words it is easy to calculate the recursion relations on a computer. Another approach we have tried is calculating the coefficients A, B and C in a hopping parameter expansion. We made a third order expansion, which worked reasonably well, but we prefer to concentrate on the more definitive numerical results.

Unfortunately we have to report that taken as they stand the recursion relations (6.14) have no fixed point! Since the theory they purport to describe presumably indeed has a critical point, at least for $N_f > 1$, we assume that this result is a consequence either of our fairly drastic truncation of the action, or of our simple linear approximation. We do not want to abandon our approximation scheme too quickly, because it has some quite nice features, so we will instead invent an *ad hoc* way of getting predictions for the critical point out of our recursion relations anyhow. In fact what we will do is allow the coupling constants to become complex, whereupon we can always find a fixed point (although it is non-trivial for this fixed point to have the proper stability properties). We will make two excuses for this procedure. Firstly, in our experience, from looking at simple lattice gaussian models, it is not uncommon for *truncated* recursion relations not to have any fixed points, even when one can actually calculate the exact fixed point of the untruncated system. If this happens one can often get a decent approximation to the true fixed point action by finding the complex fixed point of the truncated recursion relations. Secondly, it happens that when A, B and C are calculated approximately in a third order hopping parameter expansion, one *does* get a sensible real fixed point, close to the exact numerical, complex fixed point. This somehow suggests that there is *almost* a fixed point in our recursion relations. One would hope that refining the approximations would reduce or eliminate the imaginary parts.

The results on the fixed point and eigenvalues in the $N_f = 2$ theory, for various values of $\tilde{\rho}$, are given in table 2. The prediction for ν is $(\log_2 \lambda)^{-1}$, where λ is the leading eigenvalue. Taking λ here to be the modulus of its complex value, or about 2.47 ($\tilde{\rho} = 0.5$), we get $\nu \approx 0.768$, to be compared with the true value for this theory of $\frac{2}{3} \approx 0.667$. This is not a bad result, by the standards of such calculations.

We get very similar results for the $N_f = 1$ case, which in the continuum

has no massless states. This is rather worrying, and is presumably another shortcoming of our approximations. (The $N_f = 0$ case is even worse. We expected that this case would mimic the $N_c \to \infty$, mean-field case, with $\nu = \frac{1}{2}$. In fact it gives ν *closer* to the free fermion $\nu = 1$ than $N_f = 2$ does).

Whether the problems we have encountered in this calculation can be removed or alleviated by including more terms in the action, or going beyond the leading order in the cumulant approximation, is an interesting open question.

7. Conclusions.

The indications from free-field theory are that a blocking transformation like (2.2), with an appropriately chosen value for the parameter ρ, should be suitable for RG calculations. In particular when applied to a massless fermion action it leads to a well-defined fixed point action, and it appears that errors induced by truncation of the effective action to a finite number of operators ought to be controllable.

As it stands the transformation (2.2) is not very convenient for the incorporation of gauge fields, but we saw that this problem could be overcome in the two-dimensional example of section 6. In four dimensions an approach similar to the $\sqrt{3}$-transformation of [12] might be more suitable.

The original motivation of this paper was to suggest a scheme intended for *Monte Carlo* renormalization group work, so we should say a few words about how this might be done. Firstly we assume that we are probably restricted to calculating fermion correlation functions in quenched approximation. Whether one can really carry out a self-consistent RG study in this approximation, we do not know. Assuming one can, one then needs some method of calculating the complicated correlation functions required by most MCRG methods. Because the kind of expectation values one looks at in MCRG calculations tend to be rather non-local, calculating the propagator by purely numerical methods like Relaxation or Conjugate Gradients may not be very practical, but perhaps one can adapt some "random" method (such as Pseudofermions) for calculating the propagator (or even directly calculating the required expectation values) in the quenched background field.

One needs to define some concrete goal for these calculations, analogous to calculating the beta function in the pure gauge sector. Presumably one can construct similar quantities which control the scaling of the fermionic sector of the theory.

$\tilde{\rho}$	$\tilde{k}$	$\tilde{r}$	ζ^2
0.1	$0.504 + 0.277i$	$1.129 - 0.143i$	$0.505 - 0.156i$
0.3	$0.490 + 0.255i$	$1.154 - 0.142i$	$0.673 - 0.126i$
0.5	$0.478 + 0.237i$	$1.173 - 0.138i$	$0.836 - 0.075i$
0.7	$0.467 + 0.221i$	$1.190 - 0.133i$	$0.994 - 0.011i$
0.9	$0.458 + 0.205i$	$1.205 - 0.126i$	$1.143 + 0.062i$

$\tilde{\rho}$	λ_1	λ_2	$\lvert\lambda_1\rvert$
0.1	$1.653 - 1.688i$	$0.107 - 0.071i$	2.363
0.3	$1.689 - 1.835i$	$0.119 + 0.036i$	2.494
0.5	$1.657 - 1.825i$	$0.144 + 0.011i$	2.465
0.7	$1.615 - 1.717i$	$0.177 - 0.008i$	2.357
0.9	$1.577 - 1.554i$	$0.214 - 0.023i$	2.214

Table 2. Fixed point in 2-flavour QED_2

FIGURE CAPTIONS

FIG. 1: Coupling constant flow in m, r plane in truncated free theory: a) $\rho = 0.25$, b) $\rho = 0.50$, c) $\rho = 0.75$.

FIG. 2: "Universal" quantities in truncated free theory (solid lines).

FIG. 3: Old and new lattices. $\times$ = site occupied by matter field.

FIG. 4: Two "unit cells". NB. $5 \equiv \hat{7}$, $9 \equiv \hat{12}$, $10 \equiv \hat{11}$; *the* x_1*-direction is horizontal, the* x_2*-direction is vertical.*

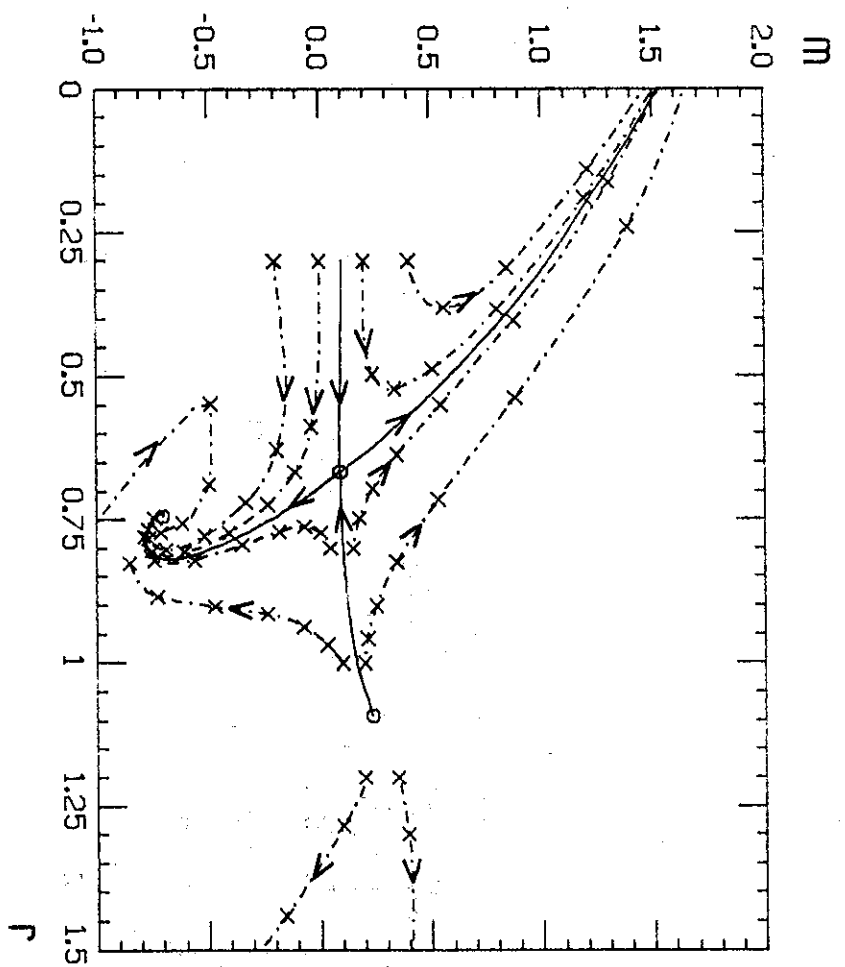

Figure 1c

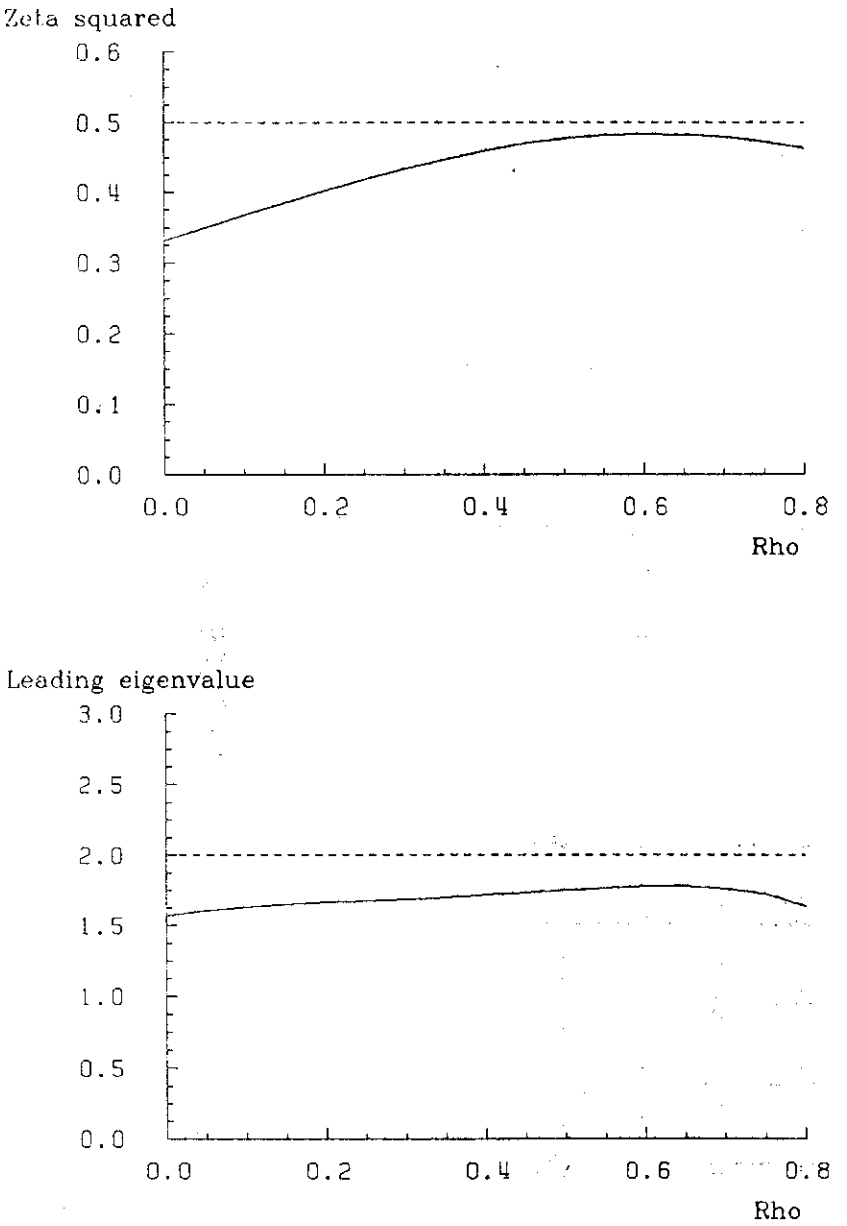

Figure 2