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LONGITUDINAL AND TRANSVERSE SINGLE BUNCH INSTABILITIES INDUCED BY ORBIT DEPENDENT HIGHER ORDER MODE LOSSES

by

R. D. Kohaupt

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NOTKESTRASSE 85 · 2 HAMBURG 52

Numerical estimates

The maximum possible growth (damping) rate is given by (6) or (19)

$$|\delta| = f_o \frac{Z_{\|\perp} J_B}{E/e} |D_o X_{oo}| \qquad (56)$$

For PETRA we use the following parameters:

D_o = 30 cm, X_{oo} = 5 mm, E = 7 GeV, J = 5 mA, f_o = 130 kHz and
$Z_{\|\perp} = 1.3 \cdot 10^8\ \Omega/cm^2$ at σ_s = 1 cm for 56 seven-cell and 56 fifteen-cell cavities [3].

From equ. (55) we obtain

$$\frac{1}{|\delta|} \approx 5.4 \text{ msec}$$

In the transverse direction this effect could not occur in PETRA, since the transverse feedback system cures any instability of such strength.

In the longutudinal direction, however, the instability could have occured. Sometimes we observe longitudinal instabilities in PETRA which can be cured by artificial bunch lengthening. These instabilities were interpreted as induced by parasitic cavity modes balanced by Landau-damping.

Experimental study is necessary to find out whether longitudinal instabilities in PETRA can be identified as effects described in this note.

On could think of applying this effect to stabilize longitudinal multi-bunch instabilities in a high single bunch current machine by providing a sufficient amplitude $X_{co}\ D_o < 0$ in the cavity and applying only a transverse feedback system*).

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*) In this note only coherent oscillations were considered. However, also higher order internal longitudinal and higher head-tail modes can become unstable. Work along this line in in progress.

Acknowledgements

The author is grateful to H. Mais for many helpful discussions.

References

1) T. Weiland, Nuclear Instrumentation and Methods 216 (1983), p. 31-34.

2) A. Piwinski and A. Wrulich, DESY Report 76/07 (1976).

3) T. Weiland, private communication.

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Abstract

In the presence of a transverse cavity impedance there is a contribution to the higher order mode losses depending on the beam position within the cavity. This part of the higher order mode losses causes longitudinal and transverse single bunch instabilities depending on the product of dispersion and closed orbit deviation in the rf section.

Introduction

The discrepancy between observed (longitudinal and) transverse single bunch instabilities and the theoretical predictions led to an intensive search for "missing impedances". It was suspected, however, that part of these discrepances might be due to effects that had not yet been considered.

In this note an example of such an effect, not considered before, is studied.

That part of the higher order mode losses which depends on the closed orbit induces longitudinal and transverse single bunch instabilities dependent on closed orbit deviation and the dispersion in the cavities.

The mechanism

In the presence of a transverse impedance $Z_\perp$ there is also a longitudinal impedance $Z_{|\perp}$ which causes a contribution to the higher order mode losses of a single bunch depending on the off-axis coordinate X_D in the cavity[1]:

$$\frac{\delta E}{E} = - \frac{Z_{|\perp} \cdot J_B}{E/e} X_D^2 \tag{1}$$

$\frac{\delta E}{E}$ $\hat{=}$ relative energy loss

E $\hat{=}$ energy

$Z_{|\perp}$ $\hat{=}$ "long.-transv." impedance with dimension Ω/m^2

J_B $\hat{=}$ bunch current.

Since the total deviation X_D from the cavity axis can be split into a "closed orbit" deviation X_{co} and a varying deviation X, we find a contribution to $\frac{\delta E}{E}$ proportional to X *):

$$\frac{\delta E}{E} \longrightarrow -2 \frac{Z_{|\perp} \cdot J_B}{E/e} X_{co} \cdot X \tag{2}$$

This part is the source of longitudinal and transverse instabilities.

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*) the non-linear term does not contribute to instability.

Differential equation approach

a) longitudinal

We assume the synchrotron frequency f_s $(= \frac{\Omega_s}{2\pi})$ to be slow with respect to the fractional part of betatron oscillations (far from satellite resonances). Then the linearized equation for coherent synchrotron oscillations can be written as

$$\frac{\Delta E}{E} = A\,\varphi + \frac{L}{T}\cdot X$$

$$\varphi = -B\cdot\frac{\Delta E}{E} \qquad (3)$$

$$X = D_x\cdot\frac{\Delta E}{E}$$

$$L = -2\,\frac{Z_{|\perp}\cdot J}{E/e}\,X_{co} \quad , \quad A\cdot B = \Omega_s^2 \qquad (4)$$

with $T \hat{=}$ revolution time.

From eqs. (3) one derives

$$\varphi + \Omega_s^2\varphi - L\,D_x\varphi = 0 \quad , \qquad (5)$$

an equation containing damping or antidamping depending on the sign of $L\,D_x$. The complexe frequency shift $\Delta\omega_s$ is simply given by

$$\Delta\omega_s = -\,i\,\frac{L D_x}{2}\,f_o \qquad (6)$$

keeping only linear terms in L.

$f_o \hat{=}$ revolution frequency.

b) transverse

We start with the coordinate of transverse deviation from the closed orbit X and define according to Courant-Snyder

$$\xi = \frac{X}{\sqrt{\beta_x}} \qquad (7)$$

$\beta_x \hat{=}$ amplitude function.

In order to keep the model simple we assume $D_x/\sqrt{\beta_x}$ to be constant as a function of longitudinal coordinate s. Then, we can write a differential equation for the coordinate ξ of coherent betatron motion

$$\overset{\circ\circ}{\xi} + \omega_\beta^2\,\xi = \omega_\beta^2\cdot\frac{D_x}{\sqrt{\beta_x}}\cdot\frac{\delta E}{E} \qquad (8)$$

with $\overset{\circ}{} = \frac{d}{d\tau}$, where τ is the "quasi time" defined by

$$\tau = \frac{\varphi_x}{\omega_\beta} \qquad (9)$$

$\omega_\beta \hat{=}$ circular betatron frequency
$\varphi_x \hat{=}$ phase advance.

From the definition (9) one finds

$$\frac{d\tau}{dt} = \frac{1}{\overline{\beta}} \quad ; \quad \overline{\beta} = \beta_x/\frac{R}{Q_x} \qquad (10)$$

$R \hat{=}$ machine radius
$Q_x \hat{=}$ betatron tune.

The energy loss in a cavity is given by

$$\frac{d}{ds}\,\frac{\delta E}{E} = L\sqrt{\beta_x}\,\xi\cdot\delta_p(s - s_o) \qquad (11)$$

$\delta_p(s-s_o)$ = periodic δ-function (periodic in the circumference $2\pi R$)
$s_o \hat{=}$ cavity coordinate.

According to (9) and (10) one obtains from (11)

$$\overset{\circ}{\frac{\delta E}{E}} = L\sqrt{\beta_x}\cdot\xi\cdot c\cdot\overline{\beta}\cdot\delta_p(s - s_o) \qquad (12)$$

$c \hat{=}$ velocity of light.

Differentiating equ. (8) with respect to τ and using (12) yields

$$\overset{\circ\circ\circ}{\xi} + \omega_\beta^2\,\overset{\circ}{\xi} = \omega_\beta^2\,L\,D_x\cdot\xi\cdot c\cdot\overline{\beta}\cdot\delta_p(s - s_o) \qquad (13)$$

In order to solve equ. (13) we make use of the "averaging process" keeping only the lowest Fourier component in the δ-function, i.e. we make the replacement

$$\delta(S[\tau] - \tilde{C}_0) \dashrightarrow \frac{1}{T} \int_0^T d\tau \;\; \delta_p(S[\tau] - S_0) \tag{14}$$

$T \hat{=}$ revolution time

Applying again (10) this is equivalent to

$$\delta_p(S[\tau] - S_0) \dashrightarrow \frac{1}{T}\, \frac{1}{\beta c} \tag{15}$$

and (13) yields

$$\overset{\circ\circ\circ}{\xi} + \omega_\beta^2 \cdot \mathring{\xi} = L\, D_x\, \omega_\beta^2 \cdot \xi \tag{16}$$

For vanishing perturbation ($L = 0$) equ. (8) has two solutions

$$\omega_\pm^\circ = \pm\,\omega_\beta \tag{17}$$

Putting

$$\begin{aligned} \xi &= \hat{\xi}\, e^{i\omega_\pm \tau} \\ \omega_\pm &= \omega_\pm^\circ + \Delta\omega_\pm \end{aligned} \tag{18}$$

and keeping only linear terms in L we obtain from (16)

$$\Delta\omega_+ = \Delta\omega_- = i\, \frac{L D_x}{2}\, f_0 \tag{19}$$

So we conclude from (6) and (19) that synchrotron or betatron oscillations of a single bunch can become unstable depending on the product of LD_x.

The calculations done so far are based on the averaging process (14) and on the separate treatment of synchrotron and betatron oscillations.

In the next section we will study the effect of cavity localization and will combine synchrotron and betatron motion.

Transfer matrix approach

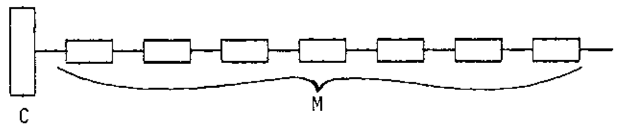

Figure 1

In figure 1 we consider a simple model of a machine built up of a single (localized) cavity and a sequence of magnets. We define the coordinate vector $\vec{y}$

$$\vec{y} = \begin{pmatrix} x \\ x' \\ \eta \\ \sigma \end{pmatrix} \tag{20}$$

where x is the total orbit deviation with respect to the closed orbit, η is the relative energy deviation and σ is the orbit length with respect to the equilibrium particle trajectory.

Assuming constant dispersion and amplitude function in the cavity, the transfer matrix M can be written as

$$M = \begin{pmatrix} \cos\mu & \beta_0 \sin\mu & (1-\cos\mu) D_0 & 0 \\ -\frac{1}{\beta_0}\sin\mu & \cos\mu & \frac{1}{\beta_0}\sin\mu\, D_0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\beta_0}\sin\mu\, D_0 & (1-\cos\mu) D_0 & -(\alpha + \frac{D_0^2}{\beta_0}\sin\mu) & 1 \end{pmatrix} \tag{21}$$

$\mu \;\hat{=}$ phase advance of the machine
$\beta_0 \;\hat{=}$ amplitude function in the cavity
$D_0 \;\hat{=}$ dispersion in the cavity
$\alpha \;\hat{=}$ compaction factor

Writing down (21) use has been made of the relation

$$\sigma_\beta = \frac{1}{\beta_0}\left(D_0 \sin\mu \cdot X_\beta + D_0\, \beta_0 (1-\cos\mu)\, X_\beta'\right) \tag{22}$$

where σ_β denotes the change of trajectory length due to pure betatron oscillations X_β, X_β'. This relation was derived under general conditions in ref. (2).

Since we have introduced the total deviation X instead of X_β the matrix M is symplectic.

The transfer matrix for the cavity "section" can be written as

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ L & 0 & 1 & A \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

Here A together with α is related to the phase advance μ_S of the synchrotron oscillation, as we will see later. The matrix C is only symplectic for L = 0. This is not surprising, since L leads to damping (antidamping). The total transfer matrix then reads

$$S = M\,C = \begin{pmatrix} [\cos\mu + LD_0(1-\cos\mu)] & \beta_0 \sin\mu & (1-\cos\mu)D_0 & A D_0(1-\cos\mu) \\ -\frac{1}{\beta_0}\sin\mu(1-LD_0) & \cos\mu & \frac{1}{\beta_0}\sin\mu\, D_0 & \frac{AD_0}{\beta_0}\sin\mu \\ L & 0 & 1 & A \\ [\frac{1}{\beta_0}D_0\sin\mu + LD_0(1-\cos\mu)] & \mu D_0(1-\cos\mu) & -(\alpha + \frac{D_0^2}{\beta_0}\sin\mu) & [1-A(\alpha+\frac{D_0^2}{\beta_0}\sin\mu)] \end{pmatrix} \tag{24}$$

In order to study damping (antidamping), we solve

$$R(\lambda) = |S - \underline{1}\cdot\lambda| = 0 \tag{25}$$

with $$\lambda = e^{2\pi i Q} \tag{26}$$

and $$\lambda_0 = e^{2\pi i Q_0} \tag{27}$$

where λ_0 is an unperturbed (L = 0) solution of (25). Defining

$$\lambda = \lambda_0 + \Delta\lambda \tag{28}$$

we find $$\Delta\lambda \approx 2\pi i \lambda_0 \cdot \Delta Q$$

or $$\Delta\omega = -\frac{\Delta\lambda}{\lambda_0} f_0 \tag{29}$$

For L = 0, equ. (25) has four solutions

$$Q_{0\beta\pm} = \pm\mu \qquad Q_{0\beta\pm} = \pm\mu_S \tag{30}$$

as expected.

Before we solve (25) for $L \neq 0$ we restrict ourselves to a working point which stays "far away" from a first order satellite resonance. In that case, the oscillating orbit length σ (synchrotron frequency) does not follow the betatron oscillation (fractional part of betatron frequency). According to this the last row of (24) will be replaced by

$$(0 \quad 0 \quad -\alpha \quad 1-\alpha A) \tag{31}$$

The first and second rows remain uncharged, since the total X of course follows the (slowly) oscillating η via the dispersion. Instead of $R(\lambda)$ we therefore study $R_{offres}(\lambda)$ where S is modified by (31). Equations (24), (25) and the replacement (31) yields $\Delta_S^2 = \alpha A$,

$$R_{off\,res}(\lambda) =$$

$$[1-\lambda_S^2-\lambda](1-\lambda)+\Delta_S^2][(\cos\mu-\lambda)^2+\sin^2\mu] - [1-\Delta_S^2-\lambda]\lambda LD_0[(1-\cos\mu)(\cos\mu-\lambda)-\sin^2\mu] \tag{32a}$$

With help of $\cos\mu_S$ and $\sin\mu_S$ equ. (32) can be written as

$$R_{off\,res}(\lambda) =$$

$$[(\cos\mu_S-\lambda)^2+\sin^2\mu_S][(\cos\mu-\lambda)^2+\sin^2\mu] - [2\cos\mu_S-1-\lambda]\lambda LD_0[(1-\cos\mu)(\cos\mu-\lambda)-\sin^2\mu]\cdot \tag{32b}$$

Putting

$$\lambda = \lambda_{0\beta} + \Delta\lambda \tag{33}$$

and retaining only terms linear in $\Delta\lambda$ and L we obtain from equ. (32a)

$$\Delta\lambda = -\frac{1}{2K_\beta} LD_0 \cdot \lambda_{0\beta} \tag{34}$$

with $$K_\beta = 1 + \frac{\Delta_S^2}{(1-\Delta_S^2-\lambda_{0\beta})(1-\lambda_{0\beta})}$$

For $\Delta_s \ll 1$, equ. (34) is equivalent to equ. (19)

Putting

$$\lambda = \lambda_{os} + \Delta\lambda \tag{35}$$

we obtain from (32b)

$$\Delta\lambda = \frac{1}{2K_s} LD_0 \cdot \lambda_{os} \tag{36}$$

with $\quad \frac{1}{K_s} = (1 + i \cdot \frac{\cos\mu_s - 1}{\sin\mu_s}) \frac{[(\lambda_{os} - \cos\mu)(1 - \cos\mu) + \sin^2\mu]}{(\lambda_{os} - \cos\mu)^2 + \sin^2\mu}$

For $\Delta_s^2 \ll 1$, equ. (36) is equivalent to equ. (6)

Thus, for "weak" longitudinal focussing the "differential equation approach" is a sufficient approximation if the working point is far from the 1st order satellite resonance.

Adiabatic approximation approach

If the growth or damping described in the forgoing sections is sufficiently slow as compared to the corresponding oscillation, there is a third method to drive the growth (damping) rates, making use of the "adiabatic" change of the Hamiltonian function.

a) longitudinal

The Hamiltonian H of the synchrotron motion is given by

$$H = \frac{\eta^2}{2} + \Omega_s^2 \cdot \frac{\varphi^2}{2} \tag{37}$$

The variation of η, φ due to the unperturbed motion does not change the Hamiltonian. The variation

$$\frac{\delta\eta}{\delta t} = f_o L D_x \eta \tag{38}$$

due to the perturbation, however, causes a variation of the Hamiltonian

$$\frac{\delta H}{\delta t} = \dot{H} = f_o L D_x \eta^2 \tag{39}$$

Putting

$$\eta = \hat{\eta} \cos(\Omega_s t + \Psi) \tag{40}$$

yields

$$\dot{H} = f_o L D \hat{\eta}^2 \cos^2(\Omega_s t + \Psi) \tag{41}$$

Since

$$\hat{\eta}^2 = 2H \tag{42}$$

one obtains from (41)

$$\dot{H} = 2 f_o L D_x H \cos^2(\Omega_s t + \Psi) \tag{43}$$

Averaging equ. (43) over a synchrotron period leads to

$$\frac{1}{T_s} \int_{(period)} dt\, \dot{H} = 2 L D_x f_o H \frac{1}{T_s} \int dt \cos^2(\Omega_s t + \Psi) \tag{44}$$

Here use has been made of the fact that the variation of H due to the perturbation is slow as compared to the synchrotron oscillation, so that H can be taken out of the integral on the r.h.s. of (44).

Applying the "adiabatic approximation" also on the r.h.s. of (44), this relation can be written as

$$\frac{1}{T_S} \int_{(\text{period})} dt\ \dot{H} \approx \dot{H} = f_0 L D_X H \tag{45}$$

or

$$H = H_0\, e^{f_0 L D_X t}$$

Because of $\hat{\eta} \sim \sqrt{H}$ we finally arrive at

$$\hat{\eta} \approx e^{\frac{1}{2} f_0 L D_X t}$$

so that the growth (damping) rate becomes

$$\delta_S = \frac{1}{2} f_0 L D_X \tag{46}$$

in agreement with (6).

b) <u>transverse</u>

For the transverse case we write down the emittance in terms of betatron parameters in the cavity

$$\varepsilon = \gamma_0 X^2 + 2\alpha_0 XX' + \beta_0' X'^2 \tag{47}$$

with $\alpha_0 = -\frac{1}{2}\beta'$ (set to zero in the cavity)

$$\gamma_0 = \frac{1+\alpha_0^2}{\beta_0}$$

The emittance is a constant for "free" betatron motion. The change δX of X due to the energy loss $\frac{\delta E}{E}$ in the cavity (for $D_X' = 0$ in the cavity) is given by

$$\delta X = -D_X \frac{\delta E}{E} = -L D_X X \tag{48}$$

For one cavity only, this the change "per turn":

$$-LD_X \cdot X = \frac{\delta X}{\delta n_{rev}} \tag{49}$$

The emittance changes according to

$$\frac{\delta\varepsilon}{\delta n_{rev}} = -\frac{2}{\beta_0} L D_X \cdot X^2 \tag{50}$$

Putting

$$x = \sqrt{\varepsilon}\sqrt{\beta_0} \cos\varphi_X \tag{51}$$

yields

$$\frac{\delta\varepsilon}{\delta n_{rev}} = -2\varepsilon L D_X \cos^2\varphi_X \tag{52}$$

Averaging equ. (52) over the betatron periode and using the adiobatic approximation, we obtain

$$\frac{1}{2\pi}\int \frac{\delta\varepsilon}{\delta n_{rev}} d\varphi_X \approx \frac{\delta\varepsilon}{\delta n_{rev}} = -L D_X \varepsilon \tag{53}$$

and therefore

$$\varepsilon = \varepsilon_0 \cdot e^{-L D_X n_{rev}} \tag{54}$$

or

$$x \sim e^{-\frac{1}{2} L D_X n_{rev}} \tag{55}$$

In agreement with (19).

Numerical estimates

The maximum possible growth (damping) rate is given by (6) or (19)

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For PETRA we use the following parameters:

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