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by

Xiaoyuan Li
*Deutsches Elektronen-Synchrotron DESY, Hamburg*

R. E. Marshak
*Department of Physics*
*Virginia Polytech Institute and State University Blacksburg*

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# Towards a Realistic Composite Model of Quarks and Leptons

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Xiaoyuan Li*+

Theory Group, DESY, Hamburg, West Germany

and

R. E. Marshak

Department of Physics

Virginia Polytechnic Institute and State University

Blacksburg, VA 24061

Abstract

Within the context of the 't Hooft anomaly matching scheme, some guiding principles for model building are discussed with an eye to low energy phenomenology. It is argued that $\Lambda_{ch}$ (chiral symmetry breaking scale of the global color-flavor group $G_{CF}$) ~ $\Lambda_{MC}$ (metacolor scale) and $\Lambda_{g_{CF}}$ (unification scale of the gauge subgroup of $G_{CF}$) $\lesssim \Lambda_{ch}$. As illustrations of the method, two composite models are suggested that can give rise to three or four generations of ordinary quarks and leptons without exotic fermions.

*Alexander von Humboldt fellow
+Permanent address: Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735 Beijing, People's Republic of China



## §1. Introduction

A composite (preon) model of leptons and quarks has been proposed as one way to understand the existence of at least three generations of leptons and quarks (the so-called "generation problem") [1]. A central issue in preon models is to explain why the masses of the composite leptons and quarks can be so much smaller than the confining scale $\Lambda_{MC}$ (the metacolor scale) [2]. At least two possibilities have been suggested:

a. Unbroken chiral symmetries of the preons can protect the composite fermions from acquiring masses of the order of $\Lambda_{MC}$, and the currents of the unbroken chiral symmetries satisfy the 't Hooft anomaly matching condition [3].
b. Composite fermions can also be massless in the supersymmetric theory if they are the supersymmetric partners of the Goldstone bosons associated with spontaneously broken flavor symmetries [4].

Although large efforts have been made to construct a realistic composite model of leptons and quarks along these lines [5], no such model has emerged.

In this paper, we return to the preon model without supersymmetry and formulate some guiding principles that appear neccessary to achieve a realistic composite model of leptons and quarks. By realistic, we mean that the preon model must do the following:

a. reproduce the observed three generations of ordinary leptons and quarks;
b. protect the leptons and quarks from acquiring masses of the order of $\Lambda_{MC}$;
c. remove most of the exotic fermions usually contained in the composite representations;
d. break the color-flavor gauge symmetry at the right mass scale ("gauge hierarchy problem");
e. generate the observed masses of leptons and quarks at a suitably low mass scale;
f. manage any excess Goldstone bosons associated with the spontaneously broken global color-flavor symmetry.

This paper is organized as follows. In §2, we begin with some simple chiral preon models which give a solution to the 't Hooft anomaly matching condition with respect to the entire global color-flavor chiral symmetry group $G_{CF}$ of the preon Lagrangian. It will be seen, however, that there seems to be no straightforward way to find a potentially realistic model among these index solutions. Something must be missing. This exercise leads us to impose a series of constraints on preon model building on the basis of low energy phenomenology. In §3, the relation between the global chiral symmetry breaking scale $\Lambda_{ch}$ and the unification scale $\Lambda_{g_{CF}}$ of the gauge subgroup of the global chiral symmetry $G_{CF}$ is discussed. It turns our that we must have $\Lambda_{g_{CF}} < \Lambda_{ch}$. Thus, the gauge subgroup $g_{CF}$ of the global chiral group $G_{CF}$ can only be identified as the observed low energy color-flavor gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, or as a fairly modest extension (in mass scale) such as $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, etc. It is unlikely that $g_{CF}$ can be identified as a grand unification group.



In §4, the transformation properties of the preons under a given color-flavor gauge group are further determined by the additional constraints of freedom from anomalies, vector-like electric charge and color charges, etc. The next section (§5) is devoted to the exotic fermion problem. It is proposed that if the global chiral group of $G_{CF}$ of the preon lagrangian is broken down maximally to an unbroken chiral subgroup $H_{CF}$ which minimally contains $g_{CF}$ as a subgroup, then the 't Hooft anomaly matching condition can be used to remove exotic fermions. In §6, we apply this principle to the preon model in §2 and we identify one model that can reproduce four generations of massless leptons and quarks (on the metacolor scale) but no exotic fermions. Another model with three generations of massless leptons and quarks but no exotic fermions is also obtained. Finally, §7 contains some concluding comments.

## §2. Some simple chiral preon models obeying 't Hooft anomaly matching

Let us begin with a preon model in which the metacolor interaction is described by the exceptional group $E_6$, and the left-handed preons are in the fundamental representation $\underline{27}$ of $E_6$ with multiplicity n (see Table 1).

Table 1. $(E_6)_{HC}$ x SU(n) Model

| preons | representation of $(E_6)_{MC}$ | multiplicity | global chiral symmetry SU(n) | |
|---|---|---|---|---|
| p | 27 | n | $\square$ | |
| composites | representation of $(E_6)_{MC}$ | | global chiral symmetry | 't Hooft index |
| ppp | 1 | | $\square\square\square$ | $\ell_+$ |
| ppp | 1 | | $\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$ | $\ell_o$ |
| ppp | 1 | | $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ | $\ell_-$ |

This model possesses the following features:

i) the metacolor dynamics of the preons is chiral,

ii) there is no metacolor anomaly;

iii) the metacolor interaction is asymptotically free if $n < 22$;

iv) there are metacolor singlet three-preon composite fermions;

v) there is global chiral symmetry $G_{CF} = SU(n)$ supposing that the color-flavor gauge interactions can be turned off (we will come back to this point).

A particularly interesting point is that the 't Hooft anomaly matching condition can be satisfied. In fact, from Table 1, the 't Hooft anomaly matching equation can be written as



$$\ell_+ A(\square\square\square) + \ell_o A(\begin{smallmatrix}\square\square\\\square\end{smallmatrix}) + \ell_- A(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}) = 27 \tag{1}$$

or

$$\frac{(n+3)(n+6)}{2}\ell_+ + (n^2-9)\ell_o + \frac{(n-3)(n-6)}{2}\ell_- = 27 \tag{2}$$

An n-independent solution can be found with 't Hooft indices:

$$\begin{cases} \ell_+ = \ell_- = 1 \\ \ell_o = -1 \end{cases} \tag{3}$$

This means that the global chiral symmetry SU(n) of the preon Lagrangian can operate as an unbroken chiral symmetry, and it can protect one <u>left-handed</u> composite fermion multiplet in each of the two representations $\square\square\square$ and $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ of SU(n) and one <u>right-handed</u> composite fermion multiplet in the representation $\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$ of SU(n).

For the above solution to describe reality, more discussion of the global chiral symmetry group SU(n) is needed. SU(n) can not be gauged without introducing spectator fermions since the preons are in the fundamental representation of SU(n) which is not free of SU(n) anomaly. Thus only a subgroup $g_{CF}$ of SU(n) can be gauged such that the preon representation is free of $g_{CF}$ anomaly. This can be done in several different ways. The simplest way is to assume that the preons transform according to one single complex representation of $g_{CF}$. Then $g_{CF}$ must be an anomaly-free group. From the metacolor asymptotic freedom condition, $n < 22$, and we immediately have $n = 16$ and the subgroup SO(10) of SU(16) is gauged such that preons transform according to the $\overline{16}$ spinor representation of SO(10) [6]. The resulting massless composite fermion spectra can be obtained by using the branching theorem for SU(16) → SO(10). Then the index solution $\ell_{\pm} = 1$ and $\ell_{o} = -1$ of the 't Hooft anomaly matching condition implies that the resulting massless composite fermions are the following:

left-handed composite fermion multiplets:

1 in $\underline{144}$ representation

1 in $\underline{560}$ representation

1 in $\underline{672}$ representation

and right-handed composite fermion multiplets:

1 in $\underline{16}$ representation

1 in $\underline{144}$ representation

1 in $\underline{1200}$ representation

There is only one family but many exotics at the SO(10) level. One may argue that we can have more families when SO(10) breaks down to $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$ since the $\underline{144}$, $\underline{560}$ and $\underline{1200}$ representations of SO(10) all contain the combined representation $(4,2,1) + (\bar{4},1,2)$ of $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$. However, Georgi's survival principle [7] tells us that the left and right composite fermions in the $(4,2,1) + (\bar{4},1,2)$ representation will be paired to give a mass of the order of the unification scale of $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$. We again have only one family of ordinary leptons and quarks but many exotic fermions.

The same unfavorable situation occurs in the model $SU(N)_{MC} \times SU(N+4) \times U(1)$, with two complex representations of the metacolor group, and even in the model $SU(N)_{MC} \times SU(N+5) \times U(1) \times U'(1)$ (see



Tables 2 and 3), with three complex representations of the metacolor group, [8].

Table 2. $SU(N)_{MC} \times SU(N+4) \times U(1)$ Model

| preons | representation of $SU(N)_{MC}$ | multiplicity | global chiral symmetry SU(N+4) | U(1) | |
|---|---|---|---|---|---|
| $p_1$ | □ | N+4 | □ | N+2 | |
| $p_2$ | □□ | 1 | 1 | −(N+4) | |
| **composites** | **representation of $SU(N)_{MC}$** | | **global chiral symmetry SU(N+4)** | **U(1)** | **Solution to 't Hooft anomaly matching eqn.** |
| $p_1p_1p_2$ | 1 | | □□ | N | $\ell_+=0$ |
| $p_1p_1p_2$ | 1 | | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | N | $\ell_-=1$ |

Table 3. $SU(N)_{MC} \times SU(N+5) \times U(1) \times U'(1)$ Model

| preons | representation of $SU(N)_{MC}$ | multiplicity | global chiral symmetry SU(N+5) | U(1) | U'(1) | |
|---|---|---|---|---|---|---|
| $p_1$ | □ | 1 | 1 | 1−N | −(N+5) | |
| $p_2$ | □ | N+5 | □ | −1 | 2N+5 | |
| $p_3$ | □□ | 1 | 1 | 2 | −2(N+5) | |
| **composites** | **representation of $SU(N)_{MC}$** | | **global chiral symmetry SU(N+5)** | **U(1)** | **U'(1)** | **Solution to 't Hooft anomaly matching eqn.** |
| $p_2p_2p_3$ | 1 | | □□ | 0 | 2N | $\ell_+=0$ |
| $p_2p_2p_3$ | 1 | | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 0 | 2N | $\ell_-=1$ |
| $p_1{}^*p_1{}^*p_3$ | 1 | | 1 | 2N | 0 | m=0 |
| $p_2{}^*p_3{}^*p_1$ | 1 | | □ | −N | −N | n=1 |

In these models we can take the global chiral symmetry $G_{CF}$ = SU(15) x U(1) or SU(15) x U(1) x U(1)', and gauge its subgroup $g_{CF}$ = SU(5) such that the preons transform according to 5 + $\overline{10}$ representations of SU(5):

$$\square \rightarrow 5 + \overline{10} \qquad (4)$$

then

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} \rightarrow \bar{5} + 10 + 45 + \overline{45} \qquad (5)$$

$$\boxed{\square} \rightarrow \bar{5} + 10 \qquad (6)$$

Taking Georgi's survival hypothesis into account, the index solutions to the 't Hooft anomaly matching condition imply that there is only one family of massless leptons and quarks in the model $SU(11)_{MC}$ x SU(15) x U(1), and just two families in the $SU(10)_{MC}$ x SU(15) x U(1) x U(1)' model.

These simple examples show that there seem to be no straightforward way to identify a potentially realistic example among the index solutions to the 't Hooft anomaly matching condition although there exist possible metacolor groups and metacolor representations (preons) that provide solutions to the 't Hooft condition [8]. We must therefore change our strategy. For this purpose, we note that the only gauge group for which there is strong experimental evidence is the standard group $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ and that, indeed, there is contrary evidence for SU(5) grand unification. We therefore try to construct a realistic composite model by deriving the constraints from low energy



phenomenology and then see in which way these constraints can be satisfied.

§3. Phenomenological constraints on the preon model

Since the gauge subgroup $g_{CF}$ of the global chiral symmetry $G_{CF}$ and its representations must correspond to experiment, we consider them first. Obviously, they should contain the phenomenologically interesting color-flavor gauge group; the possible candidates for $g_{CF}$ are:

$$\begin{array}{l} SU(3)_C \times SU(2)_L \times U(1)_Y \\ SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ SU(4)_{L+R} \times SU(2)_L \times SU(2)_R \\ SU(5) \\ SO(10), \text{ etc..} \end{array} \qquad (7)$$

The exceptional group $E_6$ can not be a candidate for $g_{CF}$ since the global chiral group $G_{CF}$ is unitary. At first glance, it seems that all the above candidates can be used to construct a realistic preon model. However, this is not true if we take into account the constraint due to the chiral symmetry breaking scale $\Lambda_{ch}$.

Suppose that the global chiral group $G_{CF}$ is spontaneously broken down to an unbroken subgroup $H_{CF}$, and a subgroup $g_{CF}$ of $G_{CF}$ is gauged. The geometic relations among $G_{CF}$, $H_{CF}$ and $g_{CF}$ are sketched in Figs. 1, 2, and 3.

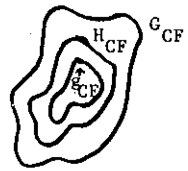


Fig. 1

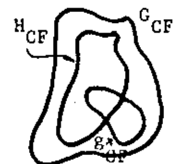


Fig. 2

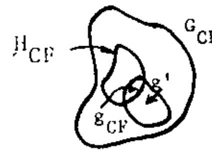


Fig. 3

In case i), the color-flavor group $g_{CF}$ is a subgroup of unbroken chiral symmetry $H_{CF}$. Thus, $g_{CF}$ is exact and unbroken at $\Lambda_{ch}$. In case ii), $g_{CF}$ is spontaneously broken down to a subgroup which is determined by vacuum alignment. The breaking scale is $\Lambda_{ch}$. In case iii), $g_{CF}$ is a subgroup of $H_{CF}$; However, an extension $g_{CF} \times g'$ of $g_{CF}$ is gauged. $g_{CF}$ may be broken due to misalignment of the vacuum state at a lower scale than $\Lambda_{ch}$[1,9]. In any case, the breaking scale of $g_{CF}$ should be equal to or less than the chiral symmetry breaking scale $\Lambda_{ch}$:

$$\Lambda_{g_{CF}} \lesssim \Lambda_{ch} \tag{8}$$

Thus, $g_{CF}$ could be identified as a grand unification group only if the chiral symmetry breaking scale $\Lambda_{ch}$ exceeds the grand unification scale $\Lambda_{GUT}$. However, the composite model of leptons and quarks becomes really interesting physics only if:

$$\Lambda_{MC} \ll \Lambda_{GUT} \tag{9}$$

It is reasonable to assume that $\Lambda_{ch}$ should not be vastly different from $\Lambda_{MC}$, although we have no knowledge about the exact relation between these two mass scales:

$$\Lambda_{MC} \sim \Lambda_{ch} \tag{10}$$



Hence we conclude that the gauge subgroup $g_{CF}$ of the global chiral symmetry group $G_{CF}$ should not be a grand unification group, but rather a color-flavor gauge group with a (partial) unification scale, say, in the neighborhood of 100 TeV [10]:

$$\begin{aligned} &SU(3)_C \times SU(2)_L \times U(1)_Y \\ &SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &SU(4)_{L+R} \times SU(2)_L \times SU(2)_R \quad (\text{if } g_L > g_R) \end{aligned} \tag{11}$$

where $g_L$ ($g_R$) are the gauge coupling constants to the gauge bosons $W_L$ ($W_R$). From this point of view, the approach in the last section i.e. to identify the gauge subgroup $g_{CF}$ of $G_{CF}$ as the grand unification group SO(10) or SU(5) can only make sense if the chiral symmetry breaking scale $\Lambda_{ch}$ is vastly different from the metacolor confinement scale $\Lambda_{MC}$. This is possible but unnatural.

## §4. Preon models with low energy gauge subgroups of $G_{CF}$

Let us now discuss the transformation properties of preons under a given color-flavor gauge subgroup $g_{CF}$. Obviously, there should be preons which transform non-trivially under each factor of $g_{CF}$. Furthermore, the resulting preon model must be free of $g_{CF}$ anomalies and have vector-like electric charge and color SU(3) charges. These constraints, in fact, can be used to determine the color-flavor number n of preons and transformation properties of preons under $g_{CF}$.

One "naive" way is to take the color-flavor number of preons n=16 or n=15 and let the preons have exactly the same quantum numbers as those of one generation of leptons and quarks, as we did in §2. In this direct identification of preons, the preon representation is also chiral with respect to $g_{CF}$. However, in this case, in order to reproduce the three generations of leptons and quarks, we may need more (even much more) than three generations of preons with exactly the same quantum numbers as those of one generation of ordinary leptons and quarks. We have no explanation for the origin of the color-flavor quantum numbers. This is not very attractive.

Is there any way to reduce the color-flavor number of preons? There is, if we let one left-handed preon carry just one color or one flavor. In addition, we must introduce a "mirror" preon to make the theory free of $g_{CF}$ anomaly and to have vector-like electric charge and SU(3) color charges at the preon level. The preon contents for the "low energy" color-flavor gauge groups are summarized in Table 4.

Table 4. Preon contents

| color-flavor gauge group | transformation properties of preons | number of color-flavor |
|---|---|---|
| $SU(3)_C \times SU(2)_L \times U(1)_Y$ | $(3,1)_a + (\bar{3},1)_b + (1,2)_c + (1,1)_d + (1,1)_e$ | 10 |
| $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ | $(3,1,1)_a + (\bar{3},1,1)_b + (1,2,1)_c + (1,1,2)_d + (1,1,1)_e + (1,1,1)_f$ | 12 |
| $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$ | $(4,1,1) + (\bar{4},1,1) + (1,2,1) + (1,1,2)$ | 12 |



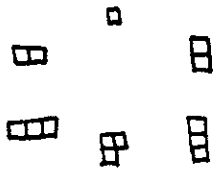
In Table 4, the $U(1)_Y$ quantum numbers are determined by the vector-like condition. In such a physical identification the preon representation may not be color-flavor chiral. However, since the representation of quarks and leptons must be chiral with respect to the color-flavor group $g_{CF}$, we must ask whether a non-chiral preon representation can lead to a color-flavor chiral theory at the composite fermion level? Firstly, the preon representation is chiral with respect to the metacolor group; thus a metacolor and color-flavor gauge invariant bare mass term for preons could not be developed even if the preon contents are not chiral with respect to the color-flavor gauge group. Secondly, a composite fermion can be identified either as a quark-lepton or as a mirror fermion depending on the sign of the 't Hooft index associated with this composite fermion. Therefore, whether the resulting composite fermion spectra are color-flavor chiral can be determined only after solving the 't Hooft anomaly matching condition. As we shall see, a solution without mirror fermions can be found.

After the determination of the color-flavor gauge group and the preon transformation property, one important consistency check must be made. That is, at the preon level, $g_{CF}$ must be asymptotically free. In fact, one key assumption in the preon theory is that all forces, except the strong metacolor forces can be neglected, at the metacolor confinement scale $\Lambda_{MC}$. This is why the fundamental preon Lagrangian possesses the underlying global chiral symmetry $G_{CF}$. Consequently, the gauge bosons can only be weakly coupled to the currents of the chiral symmetry $G_{CF}$. If $g_{CF}$

is not asymptotically free at the preon level, it violates this basic assumption. It turns out that the asymptotic freedom condition for $g_{CF}$ at the preon level places a very strong constraint on the dimension of the preon metacolor representation. As one example, it is easy to show that in the $(E_6)_{MC} \times SU(n)$ model described in §2, if we take n=16 and gauge the subgroup SO(10) of SU(16) such that the preons transform according to the 16-dim spinor representation, there is no asymptotic freedom for the SO(10) color-flavor gauge group [2].

## §5. Elimination of exotic fermions

Now we turn to the composite fermion spectra. They are determined by the preon model construction and can be classified according to the representations of the global chiral symmetry $G_{CF}$. As we have shown in §2, the composites in general can be in the representations of SU(n)



(12)

and their complex conjugates since the composite fermions are metacolor singlets made out of three fermionic preons. Their transformation properties under $g_{CF}$ can be determined by considering the branching theorem of a given representation of $G_{CF}$ under $G_{CF} \to g_{CF}$. Evidently, the resulting composite fermion spectra should contain the ordinary leptons and quarks. However, for a given model, not all preon physical identification described



in the last section can result in composite fermion spectra which contain quarks and leptons. For example, if preons transform as a 16-dim spinor representation of $g_{CF} \equiv SO(10)$, then the model with composite fermions in the representations  and  of $G_{CF} \equiv SU(16)$ will not give rise to the observed leptons and quarks. Hence, only certain combinations of the metacolor group, preon contents, gauge color-flavor group and preon physical identification can lead to ordinary leptons and quarks. This is certainly an important constraint on preon model building.

A common feature of most preon models is that the resulting composite fermion spectra contain a large number of exotic fermions. The reason is simple. The composite fermions are classified according to the representations of $G_{CF}$; however, only a subgroup of $G_{CF}$ is gauged and the leptons and quarks are to be identifed only in the irreducible representations of the subgroup. Some exotics may be welcome and could serve as the signature of compositeness in future accelerators. However, too many exotic fermions are certainly not an attractive feature. To get rid of most exotic fermions becomes a difficult problem in composite models. The survival hypothesis, effective Yukawa coupling to "preon condensates", etc. are often used to do this job.

What we would like to suggest in this paper is a straightforward procedure to eliminate exotic fermions. We propose to exploit the 't Hooft anomaly matching condition for this purpose as follows:

a. Find an unbroken subgroup $H_{CF}$ of the global chiral symmetry group $G_{CF}$ which minimally contains the color-flavor gauge group $g_{CF}$ as a subgroup; the preon representation should not be free of $H_{CF}$ anomaly;

b. Work out the classification of the fermion spectra under $H_{CF}$, and define a 't Hooft index associated with each representation of $H_{CF}$;

c. Write down the 't Hooft anomaly matching condition for this unbroken $H_{CF}$ and look for solutions such that all (or most) of the exotic indices (these are associated with those representations of $H_{CF}$ which do not give any leptons and quarks after $g_{CF}$ is gauged) are zero and the only non-vanishing indices are those associated with the representations of $H_{CF}$ that yield leptons and quarks after $g_{CF}$ is gauged.

A few remarks. a) Here the unbroken chiral symmetry $H_{CF}$ is essentially selected on the basis of phenomenological requirements (let us recall that the 't Hooft anomaly matching condition is a necessary - not sufficient -condition for an unbroken chiral symmetry to be maintained). In particular, $H_{CF}$ is not the entire global chiral symmetry $G_{CF}$ of the preon Lagrangian. $G_{CF}$ is assumed to be spontaneously broken down to $H_{CF}$ by preon condensates even though the 't Hooft anomaly matching condition is satisfied. In this sense, the choice of the unbroken chiral symmetry can only be considered a working hypothesis. The method works because at present there is no completely reliable procedure



for dealing with very strong gauge couplings and we do not know any sufficient condition for a given chiral symmetry to remain unbroken. We also do not know how a given chiral symmetry breaks and which subgroup remains unbroken; this is especially true for chiral gauge theories. b) It is essential that the preon representations should not be free of $H_{CF}$ anomaly. This makes the solution to the 't Hooft anomaly matching condition non-trivial. In particular, $H_{CF}$ can not be identical with the color-flavor gauge group $g_{CF}$, since the preon representations must be free of $g_{CF}$ anomaly. c) The solution we are looking for may not be a unique solution to the 't Hooft anomaly matching condition. It is possible that other solutions will be found. However, if the solution with no exotic indices does exist, then we have at least provided a scenario in which exotic fermions are removed naturally.

## §6. <u>Two realistic preon models</u>

Let us now summarize the salient features of our program to find a realistic composite model of leptons and quarks:

a. Identify the gauge part of the global chiral symmetry $G_{CF}$ as one of the "low-energy" color-flavor gauge groups in order to make the composite model physically interesting.

b. Select one of the preon physical identifications in §4 and reduce the number of color-flavors to less than 15 or 16. The global chiral symmetry $G_{CF}$ of the preon system can be determined in accordance with this minimal color-flavor number n; for example, $G_{CF} = SU(n)$, $SU(n) \times U(1)$ and $SU(n) \times U(1) \times U(1)'$, depending on the model in which there are one, two and three different kinds of preons respectively (only one of them has multiplicity $n > 1$).

c. Determine the unbroken chiral symmetry $H_{CF}$ which minimally contains the "low-energy" color-flavor group as a subgroup,

d. Find a metacolor group and metacolor representation (preons) which satisfy the condition for the metacolor group (discussed in §2) and for the color-flavor part (discussed in §3-5).

e. Look for the solution to the 't Hooft anomaly matching condition with no exotic indices.

In order to illustrate this program, let us return to the model $SU(N)_{MC} \times SU(N+4) \times U(1)$ considered in §2. We now choose the color-flavor gauge group $g_{CF}$ as $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$. The minimal color-flavor number n is now taken to be 12, and the preon identifications are shown in Table 4. The unbroken chiral symmetry $H_{CF}$, which contains $g_{CF}$ minimally, is then:

$$H_{CF} = SU(4)_L \times SU(4)_R \times SU(2)_L \times SU(2)_R$$

The preon contents and the composite fermion spectra under this $H_{CF}$ are summarized in Table 5.



Table 5.

Preon contents and the composite fermion spectra under $H_{CF}$

| preons | | SU(12) | Q | $SU(4)_L$ | $SU(4)_R$ | $SU(2)_L$ | $SU(2)_R$ | 't Hooft indices |
|---|---|---|---|---|---|---|---|---|
| $p_1$ | c | $\square$ | -5 | $\square$ | 1 | 1 | 1 | |
| | $\bar{c}$ | | | 1 | $\square$ | 1 | 1 | |
| | $W_L$ | | | 1 | 1 | $\square$ | 1 | |
| | $W_R$ | | | 1 | 1 | 1 | $\square$ | |
| $p_2$ | | 1 | 6 | 1 | 1 | 1 | 1 | |
| **composite fermions** | | **SU(12)** | **Q** | **$SU(4)_L$** | **$SU(4)_R$** | **$SU(2)_L$** | **$SU(2)_R$** | **'t Hooft indices** |
| $ccp_2$ | | $\square\square$ | (-4) | $\square\square$ | 1 | 1 | 1 | $s_1$ |
| $W_LW_Lp_2$ | | | | 1 | 1 | $\square\square$ | 1 | $s_2$ |
| $\bar{c}\bar{c}p_2$ | | | | 1 | $\square\square$ | 1 | 1 | $s_3$ |
| $W_RW_Rp_2$ | | | | 1 | 1 | 1 | $\square\square$ | $s_4$ |
| $c\bar{c}p_2$ | | | | $\square$ | $\square$ | 1 | 1 | $t_1$ |
| $W_RW_Lp_2$ | | | | 1 | 1 | $\square$ | $\square$ | $t_2$ |
| $W_Lcp_2$ | | | | $\square$ | 1 | $\square$ | 1 | $\ell_1$ |
| $W_Rcp_2$ | | | | $\square$ | 1 | 1 | $\square$ | $m_1$ |
| $\bar{c}W_Lp_2$ | | | | 1 | $\square$ | $\square$ | 1 | $m_2$ |
| $\bar{c}W_Rp_2$ | | | | 1 | $\square$ | 1 | $\square$ | $\ell_2$ |
| $ccp_2$ | | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | (-4) | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 1 | 1 | 1 | $a_1$ |
| $W_LW_Lp_2$ | | | | 1 | 1 | 1 | 1 | $a_2$ |
| $\bar{c}\bar{c}p_2$ | | | | 1 | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 1 | 1 | $a_3$ |
| $W_RW_Rp_2$ | | | | 1 | 1 | 1 | 1 | $a_4$ |
| $c\bar{c}p_2$ | | | | $\square$ | $\square$ | 1 | 1 | $t_1$ |
| $W_RW_Lp_2$ | | | | 1 | 1 | $\square$ | $\square$ | $t_2$ |
| $W_Lcp_2$ | | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | (-4) | $\square$ | 1 | $\square$ | 1 | $\ell_1$ |
| $W_Rcp_2$ | | | | $\square$ | 1 | 1 | $\square$ | $m_1$ |
| $\bar{c}W_Lp_2$ | | | | 1 | $\square$ | $\square$ | 1 | $m_2$ |
| $\bar{c}W_Rp_2$ | | | | 1 | $\square$ | 1 | $\square$ | $\ell_2$ |

From Table 5, it is easy to see that the indices associated with the repressentations which will give us leptons and quarks after gauging $SU(4)_{L+R} \times SU(2)_L \times SU(2)_R$ or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are $\ell_1$, $\ell_2$, $m_1$, $m_2$. Since all other indices belong to exotic fermions, it is sufficient to set all exotic indices equal to zero. Then the 't Hooft anomaly matching condition for $H_{CF}$ can be written as

$$2(\ell_1 + m_1) = D(r) \qquad (13)$$

$$-2(\ell_2 + m_2) = -D(r) \qquad (14)$$

where D(r) is the dimension of the metacolor representation of preons which transforms non-trivially under $SU(4)_L \times SU(4)_R$. It is important to realize that the index solution corresponds to leptons and quarks only if $\ell_1$, $\ell_2 \geq 0$ and $m_1$, $m_2 \leq 0$; otherwise, they correspond to mirror fermions. Therefore, the left-right symmetric condition and the absence of mirror fermions imply:

$$\ell \equiv \ell_1 = \ell_2 > 0 \qquad (15)$$

$$m \equiv m_1 = m_2 = -|m| < 0. \qquad (16)$$

From Eqs. (13)-(16), we obtain the solution:

$$m = -|m| \qquad \ell = D(r)/2 + |m| \qquad (17)$$

Consequently, the number of families is:



$$\ell + |m| = D(r)/2 + |m| \qquad (18)$$

Furthermore, if asymptotic freedom of $g_{CF} = SU(4)_C \times SU(2)_L \times SU(2)_R$ is also required at the composite fermion level, it is found that

$$m = 0 \qquad (19)$$

Thus, we finally have D(r)/2 generations of massless leptons and quarks but no exotic fermions. In the present model, D(r) = 8, and we obtain four generations of leptons and quarks. Another possible model is the group $SU(6)_{MC} \times SU(12) \times Q_1 \times Q_2$ where we obtain three generations of massless leptons and quarks (see Table 6).

Table 6.

Preon model with three families of leptons and quarks

| preons | $SU(6)_{MC}$ | SU(12) | $Q_1$ | $Q_2$ |
|---|---|---|---|---|
| $P_1$ | $\square$ | $\square$ | 8 | 4 |
| $P_2$ | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 1 | -16 | 4 |
| $P_3$ | $\square\square$ | 1 | -4 | -8 |
| composites | $SU(6)_{MC}$ | SU(12) | $Q_1$ | $Q_2$ |
| $P_1P_1P_2$ | 1 | $\square\square$ | 0 | 12 |
| $P_1P_1F_2$ | 1 | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 0 | 12 |
| $P_1P_1P_3$ | 1 | $\square\square$ | 12 | 0 |
| $P_1P_1P_3$ | 1 | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | 12 | 0 |

## §7. Conclusion

In this paper, we have shown that if the global chiral symmetry group $G_{CF}$ of the preon Lagrangian is maximally spontaneously broken down to an unbroken chiral subgroup $H_{CF}$ which maximally contains a "low-energy" color-flavor gauge subgroup $g_{CF}$, then the 't Hooft anomaly matching condition can be used to find a solution with no exotic fermions. In particular, if the SU(12) x U(1) group in the $SU(8)_{MC}$ x SU(12) x U(1) model (or SU(12) x U(1) x U(1)' in the model $SU(6)_{MC}$ x SU(12) x U(1) x U(1)') is spontaneously broken down to the unbroken chiral subgroup $SU(4)_L$ x $SU(4)_R$ x $SU(2)_L$ x $SU(2)_R$, which subsequently breaks down to the subgroup $SU(4)_{L+R}$ x $SU(2)_L$ x $SU(2)_R$ or $SU(3)_C$ x $SU(2)_L$ x $SU(2)_R$ x $U(1)_{B-L}$ , then an index solution to the 't Hooft anomaly matching condition with four or three generations of massless leptons and quarks without exotic fermions can be found. Another advantage is that the Georgi-Kaplan mechanism [9] can be easily implemented in this scheme to break the electroweak symmetry further and to generate the masses of leptons and quarks at a later stage. Thus, the program suggested may be capable of leading to a realistic composite model of leptons and quarks[11].

We have chosen the unbroken chiral subgroup $H_{CF}$ to be $SU(4)_L$ x $SU(4)_R$ x $SU(2)_L$ x $SU(2)_R$ because the 't Hooft anomaly matching equation can be solved easily. However, there are other possible solutions. One interesting possibility might be to work with $SU(3)_L$ x $SU(3)_R$ x $SU(2)_L$ x $U(1)_Y$ unbroken chiral symmetry and gauge $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$; then the 't Hooft anomaly matching condition would become more restrictive and if there is a solution with no exotic index, the result may be more interesting.



In our approach, the number of families is determined by solving the 't Hooft anomaly matching condition. This has the consequence that the number of families is related to the dimension of the metacolor representation of preons which transforms non-trivially under $H_{CF}$. At the composite level, we automatically have global family symmetry. More work must be done to explain the observed differences among the three different generations.

Finally, we emphasize that chiral symmetry breaking play a most important role in the present approach. Any further progress in understanding the chiral symmetry property of chiral gauge theory would be welcome.

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11. We should point out the connection of our work to that of I. Bars.
    Bars ( Nucl. Phys. B208, 77 (1982) and Phys. Lett. B114, 118 (1982) ) searched for solutions to the 't Hooft anomaly matching condition for $G_{CF}$ that also satisfied the decoupling condition. He introduced the family group at the preon level whereas, in our approach, the family number is related to the 't Hooft index. In the Bars model, preons have exactly the same color-flavor quantum numbers as those of one generation of quarks and leptons. Instead, we looked for a model in which the number of color-flavors can be reduced in order to give some explanation of the origin of color-flavor. Finally, in Bars approach, $G_{CP}$ is assumed not to be spontaneously broken and additional technicolor interactions or Higgs must be introduced to break electroweak symmetry. On the other hand, one of our models, $SU(6)_{MC} \times SU(12) \times U(1) \times U(1)$, falls outside the Bars classification because it does not even satisfy the 't Hooft anomaly matching condition for $G_{CF}$.