DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 85-038
February 1985

INFLUENCE OF THE TRANSVERSE BEAM SIZES ON THE $ep \to ep\gamma$ CROSS SECTION AT HERA

by

G.L. Kotkin, S.I. Polityko, V.G. Serbo

Novosibirsk State University, Novosibirsk

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 85-038
February 1985

ISSN 0418-9833

INFLUENCE OF THE TRANSVERSE BEAM SIZES ON THE $ep \to ep\gamma$ CROSS SECTION AT HERA

G.L.Kotkin, S.I.Polityko and V.G.Serbo
Novosibirsk State University, Novosibirsk, 630090, USSR

ABSTRACT: Luminosity measurement on the HERA accelerator is planned to be carried out by means of the $ep \to ep\gamma$ process. We note that in this process impact parameters occur which are larger than the transverse beam sizes. This leads to a decrease in the number of the observed photons as compared with standard calculations. Our calculations show that the difference is greater than 10% at the photon energy $E_\gamma < 0.04\, E_e$.

1. The accelerator HERA with the colliding ep -beams (which is under construction now) will have electron energy E_e = 30 GeV, proton energy E_p = 820 GeV and luminosity $L = 0.6 \cdot 10^{32}$ cm^{-2}s^{-1} [1]. One of the main parameters of the accelerator, the luminosity, is planned to be measured by detecting the photons of the $ep \to ep\gamma$ process [2]. The standard quantum electrodynamic calculation [3] of this process corresponding to the diagram of fig. 1 is well known (see eq. (1)).

However, for this accelerator the observed cross section will be smaller than that in eq. (1). For example, the deviation from the standard formula (1) is $\gtrsim$ 10% for the photon energy $E_\gamma \lesssim E_e/25$;

certainly, such a difference should be taken into account at the luminosity measurement. It is not difficult to understand this phenomenon. The process under consideration can be represented as the Compton scattering of the equivalent photons produced by a proton on an electron. Equivalent photons with the energy ω form a "disk" whose radius is $\rho \sim E_p/m_p\omega$ where m_p is proton mass. For backward Compton scattering the energy of the observed photon E_γ corresponds to an equivalent photon energy $\omega \sim (m_e/E_e)^2 E_\gamma$ which is ten orders of magnitude smaller than E_γ . That is why the quantity ρ becomes macroscopically large: $\rho \gtrsim$ 0.1 cm at $E_\gamma \lesssim$ 0.1 E_e . The standard cross section (1) corresponds to the interaction of the photons of the "disk" with the infinite electron flow. Meanwhile, the electron beam at HERA has finite transverse size $a \sim 10^{-3}$ cm. Therefore, not all equivalent photons collide with electrons. That leads to the decrease of the number of the observed photons. Below, one calculates the correction to the cross section due to these "missing" equivalent photons.

The discussed effect was found for the first time in the $e^+e^- \to e^+e^-\gamma$ reaction at the VEPP-4 accelerator in Novosibirsk, its magnitude reaches 40% [4]. Corresponding calculations were performed in refs.[5]. In these papers one considered head-on collision only. On the HERA accelerator, however, one plans to collide electron and proton beams at the angle $2\psi = 2\cdot 10^{-2}$ rad. General formulae (which are suitable for this case as well) were obtained in our paper [6]. Below we use formulae (15), (24), (30), (33) from this paper for the considered case of the intersecting ep beams.

2. The well-known cross section of the $ep \to ep\gamma$ process (see fig. 1) has the form [3]:

$$\frac{d\sigma}{dy} = \frac{16\alpha^3}{3m_e^2}\,\frac{1}{y}\left(1-y+\frac{3}{4}y^2\right)\left[\ln\frac{4E_eE_p(1-y)}{m_em_p\,y}-\frac{1}{2}\right],\quad y=\frac{E_\gamma}{E_e} \qquad (1)$$

(The main contribution in this cross section is given by the region of the 4-momentum transfer range $-q^2 \lesssim m_e^2$, therefore, one can calculate this cross section with high accuracy by means of quantum electrodynamics only, neglecting proton structure).

The observed cross section is equal to

$$d\sigma_{obs} = d\sigma - d\sigma_{cor} \qquad (2)$$

where the correction $d\sigma_{cor}$ is expressed in terms of the Compton cross section $d\sigma_C$ and the number of the "missing" equivalent photons dn:

$$d\sigma_{cor} = d\sigma_C(\omega)\cdot dn(\omega); \qquad (3)$$

$$\frac{d\sigma_C}{dy} = \frac{2\pi\alpha^2}{m_e^2\,x}\left[\frac{1}{1-y}+1-y-\frac{4y}{x(1-y)}+\frac{4y^2}{x^2(1-y)^2}\right],\quad x=\frac{4\omega E_e}{m_e^2} \geqslant \frac{y}{1-y}; \qquad (4)$$

$$dn(\omega) = \frac{\alpha}{\pi}\,\frac{d\omega}{\omega}\,G(\omega). \qquad (5)$$

The function G is defined by the density of the electron $n_e(\vec{r},t)$ and proton $n_p(\vec{r},t)$ beams. We choose z-axis along the axis of the proton beam and x-axis - in the plane of the intersecting beams. In this case

$$G = \int \frac{d^2\vec{\rho}}{2\pi\rho}\,\frac{\partial g(\rho,\varphi)}{\partial\rho}\,u^2\left[K_0(u)K_2(u)-K_1^2(u)\right] \qquad (6)$$

where $\vec{\rho}=(\rho_x,\rho_y,0)=\rho(\cos\varphi,\sin\varphi,0)$, $u=\rho\,m_p\omega/E_p$, $K_n(u)$ is the Macdonald function and

$$g(\vec{\rho}) = 1-\left[\int n_p(\vec{r},t)\,n_e(\vec{r}+\vec{\rho},t)\,d^3\vec{r}\,dt\Big/\int n_p(\vec{r},t)\,n_e(\vec{r},t)\,d^3\vec{r}\,dt\right]. \qquad (7)$$

3. It is usually assumed that the beam density has a Gaussian distribution. In this case we have the following densities for the beams intersecting at a small angle 2ψ

$$n_p(\vec{r},t) = \frac{N_p}{(2\pi)^{3/2}\sigma_{px}\sigma_{py}\sigma_{pz}}\exp\left\{-\frac{x^2}{2\sigma_{px}^2}-\frac{y^2}{2\sigma_{py}^2}-\frac{(z-t)^2}{2\sigma_{pz}^2}\right\},$$

$$n_e(\vec{r},t) = \frac{N_e}{(2\pi)^{3/2}\sigma_{ex}\sigma_{ey}\sigma_{ez}}\exp\left\{-\frac{x'^2}{2\sigma_{ex}^2}-\frac{y^2}{2\sigma_{ey}^2}-\frac{(z'+t)^2}{2\sigma_{ez}^2}\right\}, \qquad (8)$$

$$x' = x-2\psi z,\quad z' = z+2\psi x.$$

The beam parameters are given in table 1 taken from ref. [1]. Substituting eq. (8) into eq. (7) and neglecting

Table 1

	σ_x, mm	σ_y, mm	σ_z, mm
p	0.12	0.027	95
e	0.22	0.013	9.3

the terms of order of $(\sigma_{ex}/\sigma_{ez})^2 \sim 5\cdot 10^{-4}$ one obtains

$$g(\vec{\rho}) = 1-\exp\left\{-\frac{\rho_x^2}{2a_x^2}-\frac{\rho_y^2}{2a_y^2}\right\};\quad a_x^2=\sigma_{ex}^2+\sigma_{px}^2+\psi^2(\sigma_{ez}^2+\sigma_{pz}^2); \qquad (9)$$

$$a_y^2 = \sigma_{ey}^2+\sigma_{py}^2.$$

One can see from eq. (9) that further calculations for the intersecting beams differ from those for the head-on collisions by the a_x value only.

Substituting eq. (8) into eq. (6) one finds the function

$G(\omega)$, see fig. 2. The asymptotic behaviour of this function at $E_p/m_p\omega a_y \gg 1$ has the form

$$G^{as}(\omega) = 2\ln\frac{E_p\,(a_x+a_y)}{\sqrt{2}\,m_p\omega\,a_x a_y} - 1{,}577 \tag{10}$$

The function $G(\omega)$ goes very rapidly to zero at $\omega \to \infty$, therefore, integration over ω in the correction $d\sigma_{cor}$ (3) can be expanded up to infinity.

The dependence of relative value of the correction $(d\sigma_{cor}/dy)/(d\sigma/dy)$ on the relative photon energy $y = E_\gamma/E_e$ is shown in fig. 3. The correction increases rapidly with y decrease. At $y \ll 1$ one can obtain, using eq. (10), such an expression

$$\frac{d\sigma^{as}_{cor}}{dy} = \frac{16\alpha^3}{3m_e^2}\left[\left(1-y+\frac{4}{3}y^2\right)\ln\frac{2\sqrt{2}\,E_pE_e(1-y)(a_x+a_y)}{m_e^2\,m_p\,y\,a_x a_y} - \right. \tag{11}$$

$$\left. -\frac{13}{12}(1-y) - \frac{3}{2}y^2\right].$$

The dependence $d\sigma_{cor}$ on ψ is weak under the conditions considered.

4. Let us discuss briefly the results obtained. The luminosity L of the accelerator is defined by the relation $d\dot{N} = L\,d\sigma$ where $\dot{N}$ is the number of observed events per time unit for a process with a cross section σ. Let us consider a process with the following properties: 1) it is sufficiently exactly calculable; 2) it has not too small a cross section; 3) it is convenient for recording. Then its measurement allows one to determine the luminosity; in this case no detailed information on the distribution of the beams density in the interaction range, which is hard to measure, is required. The $ep \to ep\gamma$ process

has a large cross section and is convenient for recording. However, the use of this process for the luminosity determination is connected with a number of difficulties, and it should be taken into account in planning future experiments on HERA. Using this process one should determine the luminosity by the relation

$$d\dot{N} = L\,d\sigma_{obs} \tag{12}$$

where cross section $d\sigma_{obs}$ (2) itself depends on the beams density. Fig. 3 shows that the role of the considered effects can be decreased if the energy E_γ of the observed photons is increased.

All these circumstances lead to the limitations in the applicability of the $ep \to ep\gamma$ process for the luminosity measurements. Therefore, it may be useful to attract attention to the process $ep \to epe^+e^-$ which was proposed for the luminosity measurement in ref. [7]. The cross section of this process is large enough ($\approx$ 6 mbn) and it practically does not depend on the beam sizes.

We are grateful to I.F.Ginzburg and P.Söding for useful discussions, and to K.Steffen for having sent us the HERA materials.

REFERENCES

1. Preprint DESY HERA 81/10, 1981.
2. Preprint DESY HERA 80/01, 1980.
3. Berestetsky V.B., Liphshits E.M., Pitaevsky L.P. Quantum electrodynamics. Moscow, "Nauka", 1980.
4. Blinov A.E., Bondar A.E., Eidelman Yu.I. et al. Phys.Lett. 1982, 113B, 423; Tikhonov Yu.A. Candidate thesis, Novosibirsk, 1982.

5. Bajer V.N., Katkov V.M., Strakhovenko V.M. Yadernaya Fizika, 1982, 36, 163; Burov A.I., Derbenev Ya.S. Preprint INP SO AN SSSR, 81-64, Novosibirsk, 1981.

6. Kotkin G.L., Polityko S.I., Serbo V.G. To be published in Yad.Fiz. 42 (1985) N° 7.

7. V.M.Budnev, I.F.Ginzburg, G.V.Meledin, V.G.Serbo. Nucl.Phys. 1973, B63, 519.

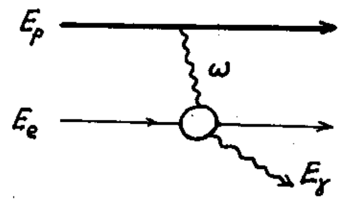

Fig. 1.

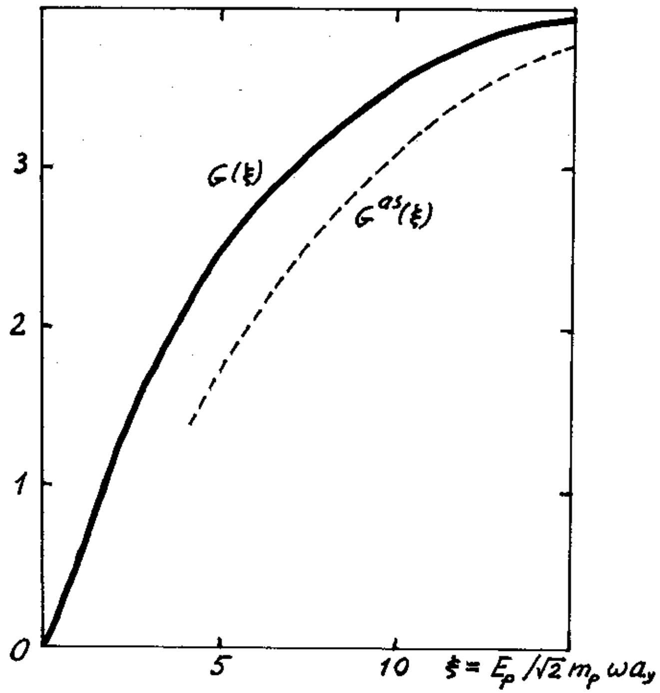

Fig. 2.

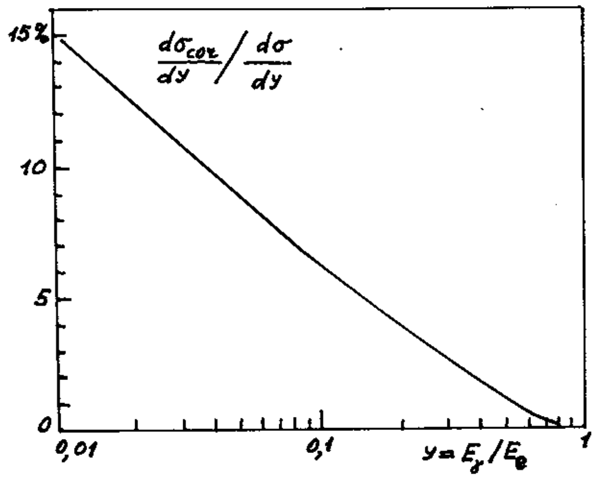

Fig. 3.

Relative value of the correction to the $ep \to ep\gamma$ cross section in dependence on the observed photon energy E_γ