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by

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Order $\alpha_s{}^2$ Two-Jet Cross Section in e^+e^- Annihilation

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Abstract: We report results for $e^+e^- \to$ two jets up to order $\alpha_s{}^2$ in the quark-gluon coupling using a jet resolution criterion depending on the jet mass.

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Over the past few years much experimental /1/ and theoretical /2/ work has been done on the role that $O(\alpha_s{}^2)$ corrections play in the description of e^+e^- annihilation into hadrons. Next-to-leading order calculations exist for the total cross section /3/, the thrust and Sterman-Weinberg three-jet distributions /4/, shape parameter distributions /5/ and the energy-energy correlation function /6/. $O(\alpha_s{}^2)$ terms contribute to four parton final states from tree diagrams, to three parton final states through one loop, and to two parton final states through two loops.

The contribution of higher order terms to two parton final states to the annihilation cross section is infrared and collinear divergent. These virtual infrared and collinear divergencies are cancelled by divergencies that arise from soft and collinear partons in contributions from three and four particles in the final state . Such contributions are obtained by integrating the three and four parton terms over small phase space regions which include the infrared singularities. For defining this region one needs parameters to describe the boundary of the region inside which two partons are considered to be irresolvable, i.e. being one jet. Therefore any higher order cross section for the production of a fixed number of jets depends on these boundary parameters in a characteristic manner which reflects the underlying dynamics, i.e. QCD.

In the case of two jets these are the only variables on which the cross section depends. How this comes about was first demonstrated by Sterman and Weinberg /7/ who calculated the two-jet cross section in order α_s with (ε, δ)-cuts. With their method two partons were considered irresolvable if either parton has energy less than $\varepsilon\sqrt{q^2}/2$ or the angle between the two partons is less than δ. The second method for defining irresolvable partons is based on an invariant mass constraint. The two partons i and j are said to be irresolvable if their invariant mass squared $(p_i+p_j)^2$ is less than yq^2. So far the two-jet cross section has been calculated only up to $O(\alpha_s)$, either with the Sterman-Weinberg cuts (ε, δ) /7/ or with the invariant mass cut y /2/. Knowledge of the two-jet cross section up to $O(\alpha_s{}^2)$ is useful for several reasons. First it can be used to determine the coupling constant α_s or the scale parameter Λ by comparing the cut dependence with experimental two-jet cross sections obtained from a cluster analysis /8/ of e^+e^- annihilation data. Second it can be used to check the integrated three-jet and four-jet cross sections in order to see whether they sum up with the two-jet cross section to the well-known $O(\alpha_s{}^2)$ correction of σ_{tot} /3/. Third it might help to resolve some problems encountered in the energy-energy correlation function in the nearly back-to-back limit /9/.

In this letter we report the α_s^2 correction to the two-jet cross section $\sigma_{2\text{-jet}}(y)$ for the invariant mass constraint yq^2. Due to the limited space we will not present any details of the calculation which will be given in a long write-up /10/.

The corresponding 2-, 3- and 4-parton diagrams are shown in fig. 1a,b,c. Individually, the loop-corrected two-parton and three-parton diagrams are infrared and collinear singular. These singularities are supposed to cancel if the 3-parton and 4-parton contributions are integrated over the two-jet region with one or two of the emitted gluons (or quarks) being soft and/or collinear.

The caclulation of $\sigma_{2\text{-jet}}(y)$ proceeds as follows. All the diagrams are calculated in n dimensions. The infrared and ultraviolet singularities then appear as poles in $\varepsilon = (4-n)/2$. We do the renormalization in the $\overline{\text{MS}}$ scheme. All calculations are performed in the Feynman gauge of massles QCD and in the one-photon approximation. Also all correlations with the incoming beam have been integrated out.

In intermediate stages of the calculation the expressions are quite lengthy. The final result, however, has a rather compact form. In particular the sum of the two-parton diagrams with two loops (fig. 1a) has the following form:

$$\sigma_{2\text{-jet}}(q\bar{q}) = \frac{\sigma^{(2)}}{1-\varepsilon}\left\{1-\varepsilon + C_F\,\frac{\alpha_s(\mu^2)}{2\pi}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} A_1 + C_F\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left(\frac{4\pi\mu^2}{q^2}\right)^{2\varepsilon} A_2\right\} \quad (1)$$

where

$$A_1 = \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(-\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} + 6\zeta_2 - 5 + 3\zeta_2\varepsilon - 8\varepsilon\right) \quad (2)$$

$$A_2 = \frac{\Gamma^3(1-\varepsilon)\Gamma(1+2\varepsilon)}{\Gamma(1-3\varepsilon)}\left\{C_F\left[\frac{2}{\varepsilon^4} + \frac{4}{\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{29}{2} - 12\zeta_2\right) + \frac{1}{\varepsilon}\left(\frac{281}{8} - 21\zeta_2 + 6\zeta_3\right) + \frac{1413}{16} - \frac{151}{2}\zeta_2 + 42\zeta_3 + 67\zeta_4\right]\right.$$
$$+ N_C\left[-\frac{11}{12\varepsilon^3} - \frac{1}{\varepsilon^2}\left(\frac{133}{36} - \frac{\zeta_2}{2}\right) - \frac{1}{\varepsilon}\left(\frac{3133}{216} - \frac{26}{3}\zeta_2 - \frac{13}{2}\zeta_3\right) - \frac{4025}{81} + \frac{629}{18}\zeta_2 + \frac{29}{3}\zeta_3 - \frac{11}{4}\zeta_4\right]$$
$$\left. + T_R\left[\frac{1}{3\varepsilon^3} + \frac{11}{9\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{269}{54} - \frac{10}{3}\zeta_2\right) + \frac{5423}{324} - \frac{110}{9}\zeta_2 + \frac{2}{3}\zeta_3\right]\right\} \quad (3)$$

In eq. (3) we have written the contributions of the three SU(3) colour factors separately, where $C_F = 4/3$, $N_C = 3$ and $T_R = N_f/2$. $\sigma^{(2)}$ is the Born cross section for $e^+e^- \to q\bar{q}$ in n dimensions:

$$\sigma^{(2)} = \left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon}\frac{\Gamma(2-\varepsilon)}{\Gamma(2-2\varepsilon)}\,\sigma_0$$
$$\sigma_0 = \frac{4\pi\alpha^2}{3q^2}\,N_C\sum_f Q_f^2 \quad (4)$$

The quarks have charge Q_f and are massless. μ makes the quark-gluon coupling dimensionless. We notice in eq. (3) that the infrared divergencies produce poles in ε up to ε^{-4}. The C_F-term is more singular than the N_C- and the T_R-term. The ζ_n are the normal values of the zeta function, $\zeta_n = \sum_{k=1}^{\infty} k^{-n}$. During completing the calculations that gave us the results in eq. (3) we received the work of Gonsalves /11/ who also calculated the two-loop diagrams. Unfortunately we do not agree with his final result, only the T_R-term agrees. We notice that the most singular terms which are in the C_F-term can be summed into an exponential form. This has been shown recently up to $O(\alpha_s^3)$ also in ref. 12. The next-to-leading terms proportional to $(\frac{11}{6}N_C - \frac{2}{3}T_R)$ can be absorbed into α_s.

In the expression (1) the quark-gluon coupling must be renormalized. This amounts to replacing $\alpha_s(\mu^2)$ in (1) by the renormalized coupling in the $\overline{MS}$-scheme, i.e.

$$\alpha_s(\mu^2) \rightarrow \alpha_s(\mu^2)\left(1 + \left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)\left(-\frac{1}{\varepsilon} + \gamma - \ln 4\pi\right)\frac{\alpha_s(\mu^2)}{2\pi}\right) \tag{5}$$

This is equivalent to adding the following counter term to (1):

$$\sigma^{ct}_{2\text{-}jet}(q\bar{q}) = +\frac{\sigma^{(2)}}{1-\varepsilon} C_F \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \left(\frac{4\pi\mu^2}{q^2}\right)^{2\varepsilon}\left(\frac{11}{6}N_C - \frac{2}{3}T_R\right)\left(-\frac{1}{\varepsilon} + \gamma\right) A_1 \tag{6}$$

The one-loop contributions to $e^+e^- \rightarrow q\bar{q}g$ have been calculated by several groups /4, 5/ in connection with the $O(\alpha_s^2)$ corrections to differential three-jet cross sections. For this case the loop integrals were needed only up to constant terms whereas for our purpose we need to know them including terms proportional to ε and ε^2. The exact basic loop integrals without ε expansion have been given in our earlier work /13/, so that only the trace calculations had to be repeated. The integration over the infrared/collinear singular regions $y_{13} \leqslant y$ and $y_{23} \leqslant y$, where $y_{ij} = 2p_ip_j/q^2$ and p_1, p_2 and p_3 denote q, $\bar{q}$ and g momentum respectively, are done as in lowest order /2/. The final result is:

$$\sigma_{2\text{-}jet}(q\bar{q}g) = \frac{\sigma^{(2)}}{1-\varepsilon}\left\{ C_F \frac{\alpha_s(\mu^2)}{2\pi}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} B_1 + C_F\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left(\frac{4\pi\mu^2}{q^2}\right)^{2\varepsilon} B_2 \right\} \tag{7}$$

where

$$B_1 = \frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\left\{\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} + 4 - 4\zeta_2 - 3\ln y - 2\ln^2 y\right\} \tag{8}$$

$$\begin{aligned} B_2 = \frac{\Gamma(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\Big\{ & C_F\Big[-\frac{4}{\varepsilon^2} - \frac{8}{\varepsilon^3} - \frac{1}{\varepsilon^2}\left(\frac{59}{2} - 24\zeta_2\right) \\ & -\frac{1}{\varepsilon}\left(\frac{147}{2} - 30\zeta_2 - 36\zeta_3\right) + \frac{6}{\varepsilon^2}\ln y + \frac{1}{\varepsilon}\left(22 - 16\zeta_2\right)\ln y \\ & + \frac{4}{\varepsilon^2}\ln^2 y - \frac{1}{\varepsilon}\ln^2 y - \frac{4}{\varepsilon}\ln^3 y - \frac{598}{3} + 128\zeta_2 - 35\zeta_4 \\ & + \left(65 - 20\zeta_2 - 56\zeta_3\right)\ln y - \frac{7}{2}\ln^2 y - \ln^3 y + \frac{7}{3}\ln^4 y\Big] \\ & + N_C\Big[-\frac{1}{2\varepsilon^4} - \frac{1}{\varepsilon^3} - \frac{1}{\varepsilon^2}\left(\frac{5}{2} - \zeta_2\right) - \frac{1}{\varepsilon}\left(6 - 5\zeta_2 + 8\zeta_3\right) \\ & + \frac{3}{\varepsilon^2}\ln y + \frac{5}{\varepsilon}\ln y + \frac{2}{\varepsilon^2}\ln^2 y - \frac{5}{\varepsilon}\ln^2 y - \frac{4}{\varepsilon}\ln^3 y \\ & - \frac{55}{12} + 10\zeta_2 + 11\zeta_3 - 21\zeta_4 + \left(12 - 12\zeta_2 + 16\zeta_3\right)\ln y \\ & - \left(5 + 4\zeta_2\right)\ln^2 y + 6\ln^3 y + \frac{14}{3}\ln^4 y\Big]\Big\} \end{aligned} \tag{9}$$

In (9) there is no contribution proportional to T_R. Such a term appears through the $q\bar{q}g$ counterterm which is

$$\begin{aligned} \sigma^{ct}_{2\text{-}jet}(q\bar{q}g) = & \frac{\sigma^{(2)}}{1-\varepsilon}\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left(\frac{4\pi\mu^2}{q^2}\right)^{2\varepsilon} C_F\left(\frac{11}{6}N_C - \frac{2}{3}T_R\right) \\ & \left(-\frac{1}{\varepsilon} + \gamma\right)\frac{1}{\Gamma(1-\varepsilon)}\Big\{\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} + 4 - 4\zeta_2 - 3\ln y - 2\ln^2 y \\ & + \varepsilon\left(7 - 8\zeta_3 + \zeta_2 + (4\zeta_2 - 4)\ln y + \frac{7}{2}\ln^2 y + 2\ln^3 y\right)\Big\} \end{aligned} \tag{10}$$

The formulas (8), (9) and (10) are only approximate in the sense that we neglected terms O(y). For B_1 and (10) it is rather simple to evaluate them exactly, but for B_2 it would amount to additional effort which is left for later work. As long as y is not too large the terms O(y) in the second order term B_2 should not matter. We have calculated all non-leading O(y) terms for B_1. But they are not given here.

The integration of the four-parton contribution $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}q\bar{q}$ over the two-jet region is rather involved. We have two kinematic configurations depending on whether two partons are collinear with respect to the two other partons, for example the region $y_{13} \leq y$ and $y_{24} \leq y$ or whether two partons are collinear with one parton as $y_{134} \leq y$ for instance (the momenta of the final state are $q(p_1) + \bar{q}(p_2) + g(p_3) + g(p_4)$ and analogous for the $q\bar{q}q\bar{q}$ final state). It is also essential to take the exact boundaries due to the four-parton kinematics into account and to evaluate additional contributions which gave non-leading O(y) terms in the 3-jet calculation /4/. The details will be presented in our longer paper /10/. The result for the real contributions to $\sigma_{2\text{-jet}}$ is:

$$\sigma_{2\text{-jet}}(q\bar{q}gg + q\bar{q}q\bar{q}) = \frac{\sigma^{(2)}}{1-\varepsilon}\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left(\frac{4\pi\mu^2}{q^2}\right)^{2\varepsilon} C_F \frac{1}{\Gamma^2(1-\varepsilon)}$$

$$\Big\{ C_F\Big[\frac{2}{\varepsilon^4} + \frac{4}{\varepsilon^3} + \frac{1}{\varepsilon^2}(15 - 12\zeta_2) + \frac{1}{\varepsilon}\left(\frac{307}{8} - 9\zeta_2 - 30\zeta_3\right)$$

$$+ \frac{5335}{48} - \frac{261}{4}\zeta_2 - 7\zeta_3 - 10\zeta_4 + \Big(-\frac{6}{\varepsilon^2} - \frac{1}{\varepsilon}(22 - 16\zeta_2)$$

$$-\frac{251}{4} + 17\zeta_2 + 44\zeta_3\Big)\ln y + \left(-\frac{4}{\varepsilon^2} + \frac{1}{\varepsilon} + 10 - 6\zeta_2\right)\ln^2 y$$

$$+ \left(\frac{4}{\varepsilon} + 7\right)\ln^3 y - \frac{1}{3}\ln^4 y\Big] + N_C\Big[\frac{1}{2\varepsilon^4} + \frac{23}{12\varepsilon^3}$$

$$+ \frac{1}{\varepsilon^2}\left(\frac{223}{36} - \frac{3}{2}\zeta_2\right) + \frac{1}{\varepsilon}\left(\frac{4033}{216} - 10\zeta_2 + \frac{1}{2}\zeta_3\right) + \frac{434561}{7776}$$

$$-27\zeta_2 - \frac{625}{12}\zeta_3 + \frac{717}{16}\zeta_4 + \left(-\frac{3}{\varepsilon^2} - \frac{21}{2\varepsilon} + \frac{58}{3}\zeta_2 - 10\zeta_3 - \frac{403}{12}\right)\ln y$$

$$+ \left(-\frac{2}{\varepsilon^2} + \frac{4}{3\varepsilon} + \frac{121}{18} + 6\zeta_2\right)\ln^2 y + \left(\frac{4}{\varepsilon} + \frac{4}{3}\right)\ln^3 y - \frac{14}{3}\ln^4 y\Big] \qquad (11)$$

$$+ T_R\Big[-\frac{1}{3\varepsilon^2} - \frac{11}{9\varepsilon^2} - \frac{1}{\varepsilon}\left(\frac{233}{54} - 2\zeta_2\right) - \frac{18092}{1296} + \frac{16}{3}\zeta_2 + \frac{16}{3}\zeta_3$$

$$+ \left(\frac{23}{3} - \frac{8}{3}\zeta_2 + \frac{2}{\varepsilon}\right)\ln y + \left(\frac{4}{3\varepsilon} - \frac{10}{9}\right)\ln^2 y - \frac{8}{3}\ln^3 y\Big]\Big\}$$

The physical 2-jet cross section is the sum of the contributions (1), (6), (7), (10) and (11). In this sum all ε poles cancel and the limit $\varepsilon \to 0$ can be taken. This produces our final result:

$$\sigma_{2\text{-jet}}(y) = \sigma_0\Big\{1 + \frac{\alpha_s(q^2)}{2\pi}C_F\left(-2\ln^2 y - 3\ln y + 2\zeta_2 - 1\right)$$

$$+ \left(\frac{\alpha_s(q^2)}{2\pi}\right)^2 C_F\left[C_F Z_C + N_C Z_N + T_R Z_T\right]\Big\} \qquad (12)$$

where

$$Z_C = 2\ln^4 y + 6\ln^3 y + \left(\frac{13}{2} - 6\zeta_2\right)\ln^2 y$$

$$+ \left(\frac{9}{4} - 3\zeta_2 - 12\zeta_3\right)\ln y + \frac{1}{8} - \frac{51}{4}\zeta_2 + 11\zeta_3 + 4\zeta_4 \qquad (13)$$

$$Z_N = \frac{11}{3}\ln^3 y + \left(2\zeta_2 - \frac{169}{36}\right)\ln^2 y + \left(6\zeta_3 - \frac{57}{4}\right)\ln y$$

$$+ \frac{31}{9} + \frac{32}{3}\zeta_2 - 13\zeta_3 + \frac{45}{2}\zeta_4 \qquad (14)$$

$$Z_T = -\frac{4}{3}\ln^3 y + \frac{11}{9}\ln^2 y + 5\ln y + \frac{19}{9} - \frac{38}{9}\zeta_2 \qquad (15)$$

The formula (12) is correct only up to terms O(y). Terms proportional to $\ln q^2/\mu^2$ have been absorbed into the running coupling constant $\alpha_s(q^2)$. The leading term in the $O(\alpha_s^2)$ correction is in Z_C and is proportional to $\ln^4 y$. The y-dependence of $\sigma_{2\text{-jet}}(y)$ is plotted in fig. 2 for $\alpha_s = 0.12$ and $N_f = 5$, together with the lowest order formula $O(\alpha_s)$, which includes all O(y) terms. For $y = 0.05$ the two-jet rate $\sigma_{2\text{-jet}}/\sigma_0$ is 0.77 with $O(\alpha_s^2)$ terms included and 0.81 in $O(\alpha_s)$. This is a reduction of 5 % compared to the $O(\alpha_s)$ result. Of course, for smaller boundary values the reduction is increased.

We see that the leading terms in Z_N and Z_T can also be absorbed into the coupling constant so that $\alpha_s(q^2) \to \alpha_s(yq^2)$. The first two leading terms in Z_C are such that they are obtained by exponentiating the first two terms of the $O(\alpha_s)$ result. This is in agreement with earlier leading-logarithm calculations for the off-shell quark form factor /14/ and the two-jet cross section for massless /15/ and massive quarks /16/. The exponentiated formula should be applicable for much smaller cut values down to $y \simeq 0.01$. The exponentiated form is plotted in fig. 2. For $y \gtrsim 0.02$ the exponentiated form and the original formula (12) differ only very little. One should notice, however, that the curves in fig. 2 show $\sigma_{2-jet}(y)/\sigma_o$. It must be divided by σ_{tot}/σ_o if one wants to compare to experimental two-jet multiplicity data.

The two-jet cross section (12) or its exponentiated form present a novel possibility to test the structure of higher order QCD matrix elements. This cross section is a complicated superposition of two-loop, one-loop and real contributions to two, three and four parton final states. After having done a cluster analysis of the hadronic final states it should be possible to obtain two-jet multiplicities in a range of cuts between $y = 0.01$ and $y = 0.1$. This way, besides testing the general structure of (12) (or its exponentiated form) one can obtain $\alpha_s(Q^2)$ in a range of Q^2 between, say 20 GeV^2 and 200 GeV^2 (if data at the highest PETRA energies are used) similar to the range of space-like Q^2 tested in deep inelastic lepton-nucleon scattering. This is complementary to the $\alpha_s(Q^2)$ deduced from 3-jet differential distributions, where the effective Q^2 could be somewhat larger.

Unfortunately, so far, a thorough cluster analysis of high energy e^+e^- annihilation data does not exist. The only data available is the two-jet rate at an average CM energy of 34 GeV reported by the JADE Collaboration /17/. In their analysis the two-jet rate is 0.70 (no error given) for $y = 0.04$. Our formula (12) yields $\sigma_{2-jet}/\sigma_{tot} = 0.69$ for $\alpha_s((34)^2) = 0.12$ whereas the exponentiated formula yields 0.66 for $\alpha_s((34)^2) = 0.12$ and $O(\alpha_s)$ yields 0.73. These numbers for $\alpha_s = 0.12$ give also an indication what effect the exponentiation and the change of scale in α_s as compared to eq. (12) has. The α_s values obtained this way are somewhat smaller than other values obtained from e^+e^- data. But we have to keep in mind that the JADE two-jet multiplicity has not been corrected for leakage effects caused by fragmentation.

Actually the two-jet rate as a function of y is rather sensitively dependent on α_s (or $\Lambda_{\overline{MS}}$). This is shown in fig. 3, where we have plotted $\sigma_{2-jet}/\sigma_{tot}$ as a function of y using the exponentiated form for various coupling constants $\alpha_s = 0.12, 0.14, 0.16, 0.18$ at $q^2 = 34$ GeV which corresponds to $\Lambda_{\overline{MS}} = 86, 215, 420, 710$ MeV. We see that a measurement of the two-jet rate with an error less than 10% would determine $\Lambda_{\overline{MS}}$ quite accurately. At this point we must remember that our formula (12) is accurate only up to terms of order y. The calculation of the neglected terms is under way. At the moment we only know the correction for the T_R term in eq. (12). For example, for $y = 0.05$ the T_R-term is increased by 10% if non-leading terms are included. We expect similar changes for the C_F and the N_C-term. Since the $O(\alpha_s^2)$ correction is only 5% in total for $y = 0.05$ the $O(y)$ terms should have only a very small effect. We repeat that these $O(y)$ terms were fully taken into account in the $O(\alpha_s)$ contribution for the curves in fig. 3. Needless to say that our formula can also be applied directly to $Z \to$ two jets and to the $O(\alpha^2)$ corrections for $Z \to e^+e^-$ by replacing $C_F = 1$, $N_C = 0$, $T_R = N_f$ and $\alpha_s \to \alpha$.

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Figure Captions:

Fig. 1 a): Diagrams with $q\bar{q}$ in the final state to order α_s^2 interfering with lowest order diagram.

b): Diagrams with $q\bar{q}g$ in the final state interfering with lowest order diagram for $q\bar{q}g$.

c): Diagrams with four partons ($q\bar{q}gg$ and $q\bar{q}q\bar{q}$) in the final state.

Fig. 2: Two-jet cross section $\sigma_{2-jet}(y)$ as a function of y in units of σ_o for $\alpha_s = 0.12$. $O(\alpha_s)$ is exact lowest order result including all non-leading terms in y. $O(\alpha_s^2)$ is according to eq. (12) and "exponentiated" is the curve as described in the text.

Fig. 3: $\sigma_{2-jet}(y)/\sigma_{tot}$ for various couplings $\alpha_s = 0.12$, 0.14, 0.16 and 0.18 in the exponentiated form as a function of y.

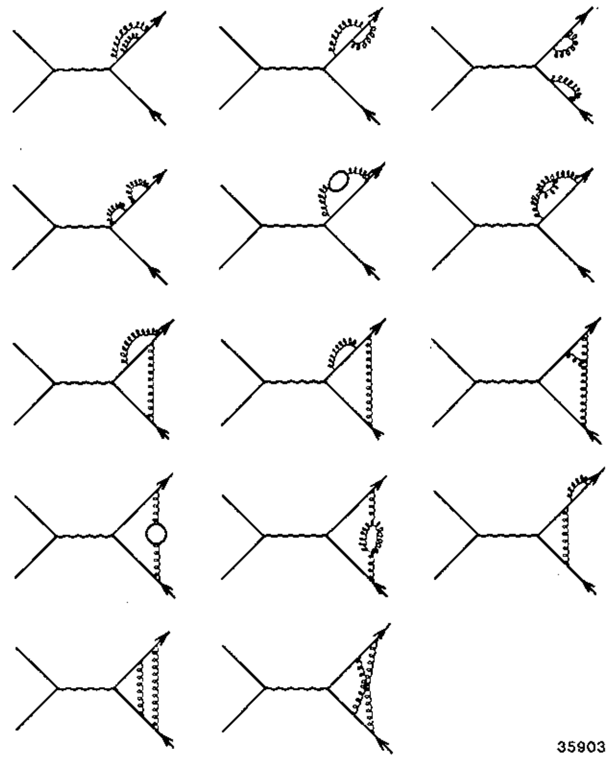


Fig. 1a

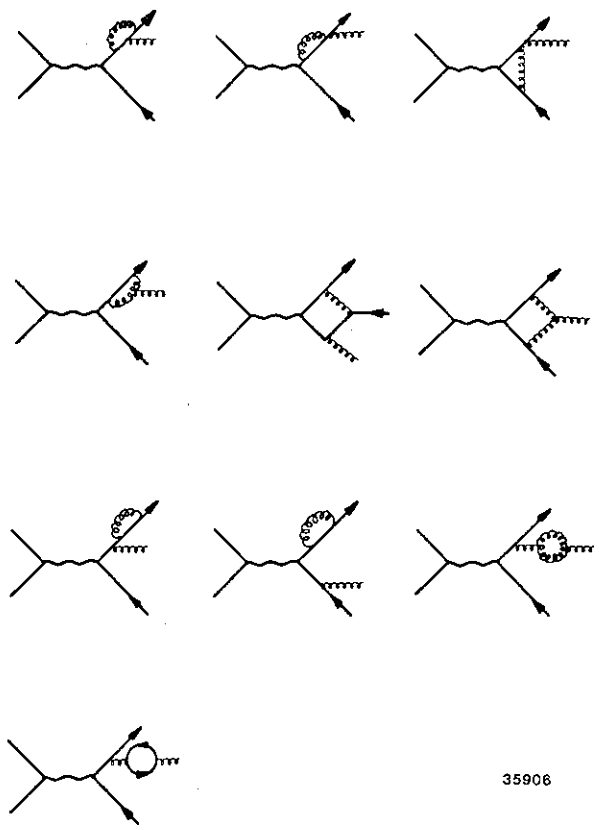


Fig. 1b

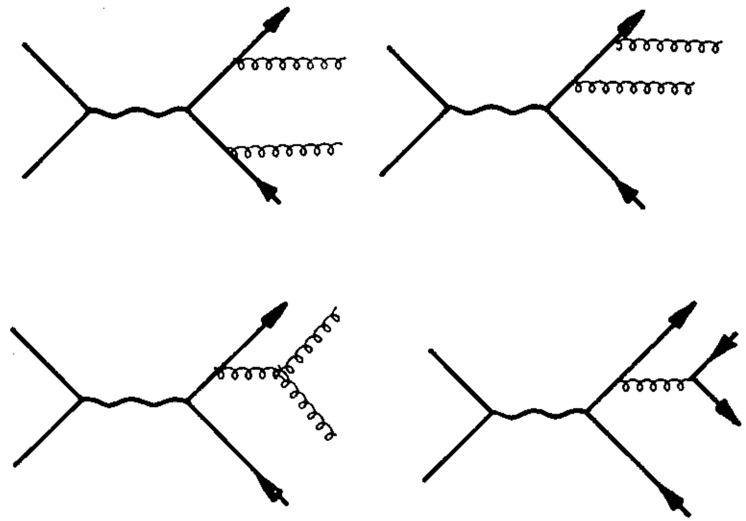
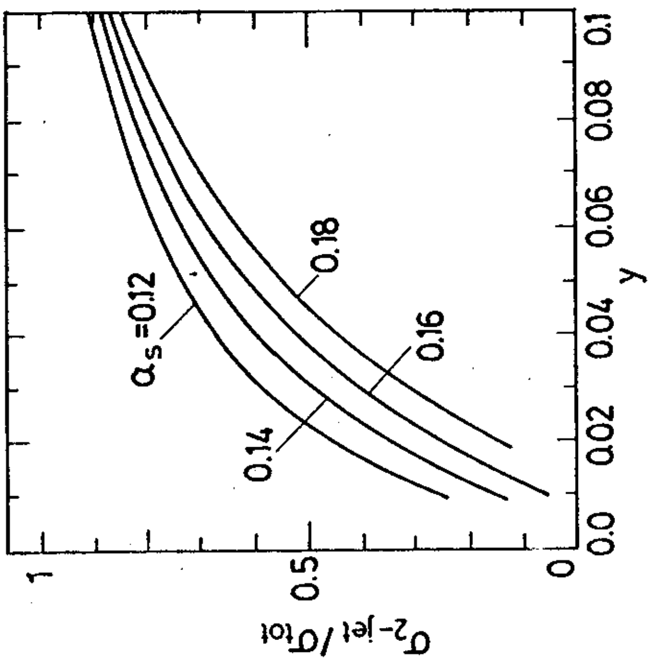
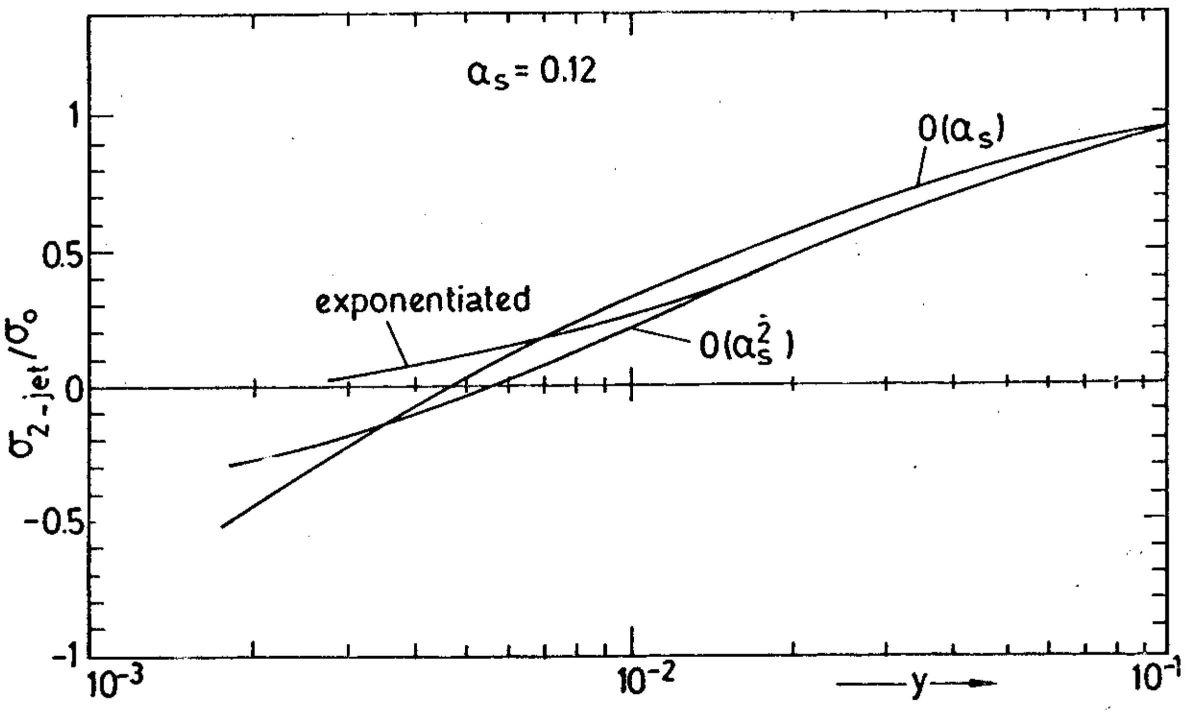


Fig. 1c



αs=0.12
0.14
0.16
0.18
y
0.0
0.02
0.04
0.06
0.08
0.1
0
0.5
1
σ2-jet /σtot

Fig. 3



αs = 0.12
O(αs)
exponentiated
O(αs2)
σ2-jet/σo
1
0.5
0
-0.5
-1
10-3
10-2
10-1
y

Fig. 2