



85-6-261

DEUTSCHES

DEUTSCHES ELEKTRONEN-SYNCHROTRON **DESY**

DESY 85-028
April 1985

ON THE EXISTENCE OF THE REAL TIME EVOLUTION IN
EUCLIDEAN LATTICE GAUGE THEORIES

by

K. Fredenhagen

II. Institut für Theoretische Physik, Universität Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY behält sich alle Rechte für den Fall der Schutzrechtserteilung und für die wirtschaftliche Verwertung der in diesem Bericht enthaltenen Informationen vor.

DESY reserves all rights for commercial use of information included in this report, especially in case of filing application for or grant of patents.

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX ,
send them to the following address (if possible by air mail) :

DESY
Bibliothek
Notkestrasse 85
2 Hamburg 52
Germany

On the Existence of the Real Time Evolution

in

Euclidean Lattice Gauge Theories

Klaus Fredenhagen (*)

II. Institut für Theoretische Physik

Universität Hamburg

Abstract: A simple argument is given which excludes the occurrence of zero eigenvalues of the transfer matrix in euclidean lattice gauge theories.

(*) Heisenberg fellow

1. Introduction

If the transfer matrix T of a classical statistical mechanics is positive and has no zero eigenvalue, one can associate to this theory a quantum theory where the Hamiltonian H is defined by $H = -\ln T$. The positivity of T is implied by the reflection positivity of the statistical mechanics; euclidean lattice gauge theories have this property for the standard actions /1, 2/. The absence of zero eigenvalues of T in the thermodynamic limit, however, is an apparently hard problem which remained unsolved over several years /3/.

In striking contrast to the difficulties which complicate a direct proof, the solution becomes simple if one formulates the problem in terms of the algebra of observables of the quantum theory. This is possible for finite gauge groups and has first been carried out in /4/. The argument is the following one.

The local transfer matrices T_{Δ} implement (non *-) automorphisms of the algebra of local observables \mathcal{A} ,

$$\alpha_i^{\Delta}(A) = T_{\Delta} A T_{\Delta}^{-1} \quad (1.1)$$

In a theory with finite range interactions these automorphisms converge to a (non *-) automorphism α_i of \mathcal{A} which may be interpreted as time translation by one unit in imaginary direction.

A ground state ω_0 is defined to be a state with

$$\begin{aligned} (i) \quad & \omega_0 \alpha_i = \omega_0 \\ (ii) \quad & 0 \leq \omega_0(A^* \alpha_i(A)) \leq \omega_0(A^* A) \quad , A \in \mathcal{A} \end{aligned} \quad (1.2)$$

Limits of local ground states fulfil these conditions. In the GNS-representation π of \mathcal{A} with cyclic vector Ω inducing ω_0 one can define the transfer matrix T by

$$T \pi(A) \Omega = \pi \alpha_i(A) \Omega \quad , A \in \mathcal{A} \quad (1.3)$$

From (1.2), T is a positive contraction with $T \Omega = \Omega$. Since α_i is an automorphism of \mathcal{A} and Ω is cyclic for \mathcal{A} , the range of T is dense in the

representation space \mathcal{H}_π , hence T cannot have a zero eigenvalue. The inverse of T is densely defined and is given by the formula

$$T^{-1} \pi(A) \Omega = \pi \alpha_i^{-1}(A) \Omega, \quad A \in \mathcal{A} \quad (1.4)$$

The inverse of α_i , i.e. the translation in negative euclidean time, has no simple counterpart in the statistical mechanics. This might explain why this argument has been overlooked before (as far as I know).

In the case of a continuous gauge group essentially the same argument applies, but the unboundedness of the inverses of the local transfer matrices causes some problems. I treat this case in Section 2. In Section 3 the equivalence of the algebraic approach with the statistical mechanical approach is shown. In particular it is proven that as a consequence of the DLR equations for the Gibbs state of the statistical mechanics the ground state ω_0 of the quantum system is locally normal.

2. The Inverse of the Transfer Matrix

Consider the hypercubic lattice \mathbb{Z}^d , $d \geq 2$ and a compact group G, the gauge group. For each box Δ in \mathbb{Z}^d , $B(\Delta)$ denotes the set of positively oriented bonds and $P(\Delta)$ the set of positively oriented plaquettes in Δ . The set of local configurations is

$$G_\Delta = \prod_{b \in B(\Delta)} G_b \quad (2.1)$$

where for each $b \in B(\Delta)$ G_b is a copy of G, and

$$\mathcal{H}_\Delta = \mathcal{L}^2(G_\Delta), \quad (2.2)$$

the space of square integrable functions on G_Δ with respect to normalized Haar measure, is the Hilbert space of local state vectors. The algebra of local observables in Δ is defined to be the algebra of all bounded operators in \mathcal{H}_Δ :

$$\mathcal{A}(\Delta) = \mathcal{B}(\Delta) \quad (2.3)$$

If $\Delta_1 < \Delta_2$, $\mathcal{A}(\Delta_1)$ can be identified with a subalgebra of $\mathcal{A}(\Delta_2)$. Using these identifications one can introduce the algebra of all local observables by

$$\mathcal{A} = \bigcup_{\Delta} \mathcal{A}(\Delta) \quad (2.4)$$

The dynamics of the system is characterized by local transfer matrices $T_\Delta \in \mathcal{A}(\Delta)$ which stem from an euclidean action. As operator on $\mathcal{L}^2(G_\Delta)$, T_Δ has the following integral kernel

$$T_\Delta(g, g') = \exp \left\{ \frac{1}{2} \sum_{p \in P(\Delta)} [\chi(\partial g(p)) + \chi(\partial g'(p))] + \sum_{b \in B(\Delta)} \chi(g(b)g'(b)^{-1}) \right\} \quad (2.5)$$

where $\partial g(p)$ denotes an element in the conjugacy class of the product of $g(b)$ for the bonds b in the boundary of p and where χ is a real continuous invariant function on G of positive type.

T_Δ is a positive operator of Hilbert-Schmidt class. It has no zero eigenvalue if and only if in the expansion of e^χ as a linear combination of simple characters each character occurs with a positive weight. This condition is satisfied e.g. if χ is a positive multiple of the character of a selfconjugate faithful representation of G /5/.

We now want to prove that under this condition also the global transfer matrix has no zero eigenvalue. For this purpose we try to define the imaginary time translation α_i . Since T_Δ^{-1} is not bounded in general we have to find a suitable domain of definition for α_i .

First we introduce the (non *-) automorphism of \mathcal{A} implemented by the multiplication part of T_Δ :

$$\beta(A) = \lim_{\Delta \uparrow \mathbb{Z}^d} e^{\frac{1}{2} \sum_{p \in P(\Delta)} \chi(\partial g(p))} A e^{-\frac{1}{2} \sum_{p \in P(\Delta)} \chi(\partial g(p))} \quad (2.6)$$

Due to the compactness of the gauge group and the continuity of χ , $\beta(A)$ exists for all $A \in \mathcal{A}$.

The convolution part of $T_{\underline{\Lambda}}$ is also invertible, but the inverse is not bounded in general. Let

$$e^{\chi} = \sum_{\xi} c_{\xi} \chi_{\xi} \quad (2.7)$$

be the expansion of e^{χ} as a linear combination of simple characters. Let $P_{\xi}(b)$ denote the operator of convolution by $\chi_{\xi}(g(b))$ on $\mathcal{X}_{\underline{\Lambda}}$ for $b \in B(\underline{\Lambda})$. Then the convolution part $C_{\underline{\Lambda}}$ of $T_{\underline{\Lambda}}$ is

$$C_{\underline{\Lambda}} = \prod_{b \in B(\underline{\Lambda})} \left(\sum_{\xi} c_{\xi} P_{\xi}(b) \right) \quad (2.8)$$

with the (in general) unbounded inverse

$$C_{\underline{\Lambda}}^{-1} = \prod_{b \in B(\underline{\Lambda})} \left(\sum_{\xi} c_{\xi}^{-1} P_{\xi}(b) \right) \quad (2.9)$$

Let

$$\alpha_f(\underline{\Lambda}) = \bigcup_P P \alpha(\underline{\Lambda}) P \quad (2.10)$$

where P runs over the set of projections on the finite dimensional subrepresentations of the left regular representation of $G_{\underline{\Lambda}}$ on $\mathcal{X}_{\underline{\Lambda}}$, and let

$$\alpha_f = \bigcup_{\underline{\Lambda}} \alpha_f(\underline{\Lambda}) \quad (2.11)$$

For each bond b we define an automorphism γ_b on α_f by

$$\gamma_b(A) = \sum_{\xi, \xi'} c_{\xi} c_{\xi'}^{-1} P_{\xi}(b) A P_{\xi'}(b) \quad (2.12)$$

If $A \in \alpha_f(\underline{\Lambda})$ and $b \notin B(\underline{\Lambda})$, γ_b acts trivially on A . Therefore the limit

$$\gamma(A) = \lim_{\underline{\Lambda} \nearrow \mathbb{Z}^d} \prod_{b \in B(\underline{\Lambda})} \gamma_b(A) \quad (2.13)$$

exists for all $A \in \alpha_f$, and γ is a (non *-) automorphism of α_f .

Now we can define the imaginary time translation α_i for all $A \in \mathcal{D}(\alpha_i) = \beta^{-1}(\alpha_f)$ by

$$\alpha_i(A) = \beta_T \beta(A) \quad (2.14)$$

Then for each $A \in \mathcal{D}(\alpha_i)$ and for $\underline{\Lambda}$ sufficiently large we have the relation

$$\alpha_i(A) T_{\underline{\Lambda}} = T_{\underline{\Lambda}} A \quad (2.15)$$

and $\alpha_i(A)$ is uniquely fixed by (2.15).

$\mathcal{D}(\alpha_i)$ is a subalgebra of α with the following density property: for each $\underline{\Lambda}$ there exists some $\underline{\Lambda}_1 \supset \underline{\Lambda}$ such that $\alpha(\underline{\Lambda})$ is contained in the w^* -closure of $\mathcal{D}(\alpha_i) \cap \alpha(\underline{\Lambda}_1)$, considered as a subalgebra of the von Neumann algebra $\alpha(\underline{\Lambda}_1)$.

The inverse α_{-i} of α_i has domain $\mathcal{D}(\alpha_{-i}) = \beta(\alpha_f)$ and is defined by

$$\alpha_{-i}(A) = \beta^{-1} \gamma^{-1} \beta^{-1}(A) \quad (2.16)$$

$\mathcal{D}(\alpha_{-i})$ has the same density property as $\mathcal{D}(\alpha_i)$.

A ground state of α with respect to α_i is defined to be a locally normal^(*) state ω_0 on α with

$$\begin{aligned} (i) \quad & \omega_0 \alpha_i(A) = \omega_0(A) \\ (ii) \quad & 0 \leq \omega_0(A^* \alpha_i(A)) \leq \omega_0(A^* A) \end{aligned} \quad (2.17)$$

for all $A \in \mathcal{D}(\alpha_i)$. Let ω_0 be a ground state and $(\pi, \mathcal{H}, \Omega)$ the corresponding GNS-representation which is characterized by the relation

$$(\Omega, \pi(A) \Omega) = \omega_0(A), \quad A \in \alpha. \quad (2.18)$$

(*) We recall that a state ω on α is locally normal if for every $\underline{\Lambda}$ there exists some density matrix $\rho_{\underline{\Lambda}}$ in $\mathcal{X}_{\underline{\Lambda}}$ with $\text{Tr} \rho_{\underline{\Lambda}} A = \omega(A)$ for all $A \in \alpha(\underline{\Lambda})$.

Then the transfer matrix T is defined as an operator in \mathcal{H} by

$$T \pi(A) \Omega = \pi \alpha_i(A) \Omega, \quad A \in \mathcal{D}(\alpha_i). \quad (2.19)$$

The density property of $\mathcal{D}(\alpha_i)$ together with the local normality of ω_0 implies that $\mathcal{D}(\alpha_i)\Omega$ is dense in \mathcal{H} , hence T is densely defined. From (2.15) T is a positive contraction.

That T cannot have an eigenvalue zero is now obvious from the fact that its inverse,

$$T^{-1} \pi(A) \Omega = \pi \alpha_i(A) \Omega, \quad A \in \mathcal{D}(\alpha_i), \quad (2.20)$$

is densely defined.

3. Equivalence with the Statistical Mechanical Approach

The euclidean lattice gauge theory corresponding to the quantum system which was introduced in Section 2 is defined on the hypercubic lattice \mathbb{Z}^{d+1} with action $S = \sum_p \chi(\partial g(p))$. Euclidean fields are continuous functions f of finitely many bond variables g(b). Let $\langle \cdot \rangle$ denote a Gibbs state of the system. This means that $\langle \cdot \rangle$ satisfies the DLR equations /6, 7/

$$\langle f \rangle = \langle m_L(f) \rangle \quad (3.1)$$

for all euclidean fields f and all finite sets of bonds L. Here $m_L(f)$ denotes the function

$$m_L(f)(g) = Z_L(g)^{-1} \int \prod_{b \in L} dg'(b) f(g') e^{\sum_{p \in \partial^* L} \chi(\partial g'(p))} \Big|_{g'(b) = g(b), b \notin L} \quad (3.2)$$

where $\partial^* L$ is the set of plaquettes which contain bonds of L in their boundary and where $Z_L(g)$ is fixed by the condition

$$m_L(1)(g) = 1 \quad (3.3)$$

If f depends only on variables g(b) with $b \in L$ then

$$\sup_g |m_L(f)(g)| \leq \text{const} \int dg |f(g)|, \quad (3.4)$$

hence $\langle \cdot \rangle$ can be uniquely extended to \mathcal{L}^1 -functions of finitely many variables. Note that due to (3.1) $\langle \cdot \rangle$ is automatically invariant under gauge transformations (Absence of spontaneous breakdown of gauge symmetry (cf. /8/)).

A construction of a Hilbert space and a positive transfer matrix is possible if the Gibbs state $\langle \cdot \rangle$ is translation invariant in the o-direction and fulfils reflection positivity at the hyperplanes $x^0 = 0$ and $x^0 = \frac{1}{2}$ in the temporal gauge (see e.g. /9/). This condition can be described in the following way.

Let \mathcal{F} denote the set of euclidean fields which are invariant under gauge transformations at points not contained in the $(x^0 = 0)$ -hyperplane. These functions are fixed by their restrictions to the set \mathcal{G}_t of configurations g satisfying the temporal gauge condition $g(b) = 1$ for all bonds b pointing in the o-direction. In the following we shall identify the functions in \mathcal{F} with their restriction to \mathcal{G}_t . By τ_n we denote translation in \mathcal{F} by n steps in o-direction,

$$(\tau_n f)(g) = f(g_n), \quad g \in \mathcal{G}_t \quad (3.5)$$

with $g_n(b) = g(b + n(1, \underline{0}))$, and by Θ the operation

$$(\Theta f)(g) = \overline{f(g^{\mathcal{D}})}, \quad g \in \mathcal{G}_t \quad (3.6)$$

with $g^{\mathcal{D}}(b) = g(\mathcal{D}(b))$, where \mathcal{D} denotes the reflection through the $(x^0 = 0)$ -hyperplane.

Let $\mathcal{F}_+ \subset \mathcal{F}$ be the set of fields which depend only on bond variables g(b) with b in the half space $x^0 \geq 0$. Then the condition of reflection positivity at hyperplanes $(x^0 = 0)$ and $(x^0 = \frac{1}{2})$, respectively, reads

$$\begin{aligned} (i) \quad & \langle (\Theta f) f \rangle \geq 0 \\ (ii) \quad & \langle (\Theta f) \tau_1 f \rangle \geq 0 \end{aligned} \quad (3.7)$$

for all $f \in \mathcal{F}_+$.

We recall that the physical Hilbert space $\hat{\mathcal{K}}$ is the completion of \mathcal{F}_+/N , equipped with the scalar product $(h+N, f+N) = \langle (\Theta h) f \rangle$ where $N = \{f \in \mathcal{F}_+, \langle (\Theta f) f \rangle = 0\}$ and that the transfer matrix \hat{T} is the positive contraction in $\hat{\mathcal{K}}$ with

$$\hat{T}(f+N) = \tau_1 f + N \quad (3.8)$$

(See e.g. /3/).

We now want to show that $\langle \cdot \rangle$ defines a ground state ω_0 of the quantum system (\mathcal{A}, α_i) of Section 2 such that there is a unitary operator U from the GNS-Hilbert space \mathcal{H} onto $\hat{\mathcal{K}}$ such that

$$\begin{aligned} (i) \quad UT &= \hat{T}U \\ (ii) \quad U\pi(f) &= f^{(0)}U \end{aligned} \quad (3.9)$$

for all multiplication operators f where $f^{(0)}(g) = f(g^{(0)})$, $g^{(i)}$, $i \in \mathbb{Z}$, denotes the restriction of g onto the set of bonds in the $(x^0 = i)$ -hyperplane, identified with \mathbb{Z}^d , and $f^{(0)}$ acts by multiplication on \mathcal{F}_+/N (note that $f^{(0)}N \subset N$)

Let $A \in \mathcal{A}(\Delta)$. We associate to A two functions $E_{\pm}(A) \in \mathcal{F}$:

$$\begin{aligned} E_+(A)(g) &= \lim_{\Delta_1 \nearrow \mathbb{Z}^d} (AT_{\Delta_1})(g^{(0)}, g^{(1)}) / T_{\Delta_1}(g^{(0)}, g^{(1)}) \\ E_-(A)(g) &= \lim_{\Delta_1 \nearrow \mathbb{Z}^d} (T_{\Delta_1}A)(g^{(-1)}, g^{(0)}) / T_{\Delta_1}(g^{(-1)}, g^{(0)}) \end{aligned} \quad (3.10)$$

Due to the finite range of interactions the limits exist and are reached already for a finite Δ_1 . Since T_{Δ_1} is a Hilbert-Schmidt operator in \mathcal{H}_{Δ_1} with continuous integral kernel, $E_+(A)$ and $E_-(A)$ are \mathcal{L}^2 -functions. For a multiplication operator f one has the relations

$$\begin{aligned} E_+(fA) &= f^{(0)}E_+(A) \\ E_-(Af) &= E_-(A)f^{(0)} \end{aligned} \quad (3.11)$$

We now set

$$\omega_0(A) = \langle E_+(A) \rangle \quad (3.12)$$

ω_0 is a locally normal linear functional on \mathcal{A} . For the investigation of its properties we use the following lemma.

Lemma: Let $A, B \in \mathcal{A}(\Delta)$, and let L be the set of bonds in $\{0\} \times \Delta$

Then

- (i) $\Theta E_+(A) = E_-(A^*)$
- (ii) $m_L(E_+(A)) = m_L(E_-(A))$
- (iii) $m_L(E_+(AB)) = m_L(E_-(A)E_+(B))$
- (iv) $E_+(\alpha_i(A)) = \tau_i E_-(A)$

Proof: Let $A(g_0, g'_0)$ and $B(g_0, g'_0)$ denote the integral kernels of A and B , respectively, considered as operators on \mathcal{H}_{Δ} . We use the abbreviations

$$\begin{aligned} M(g_0) &= \exp \left\{ \frac{1}{2} \sum_{p \in \partial^+ B(\Delta)} \chi(\partial g_0(p)) \right\} \\ N(g_0, g'_0) &= \exp \left\{ \sum_{b \in B(\Delta)} \chi(g_0(b) g'_0(b)^{-1}) \right\} \end{aligned}$$

Then for $g \in \mathcal{E}_t$

$$\begin{aligned} E_+(A)(g) &= \int \prod_{b \in B(\Delta)} \alpha_{g_0}(b) A(g_0, g'_0) \cdot \frac{M(g_0)}{M(g^{(0)})} \cdot \frac{N(g_0, g^{(1)})}{N(g^{(0)}, g^{(1)})} \Big|_{g_0(b) = g^{(0)}(b), b \in B(\Delta)} \\ E_-(A)(g) &= \int \prod_{b \in B(\Delta)} \alpha_{g_0}(b) A(g^{(-1)}, g_0) \cdot \frac{M(g_0)}{M(g^{(0)})} \cdot \frac{N(g^{(-1)}, g_0)}{N(g^{(-1)}, g_0)} \Big|_{g_0(b) = g^{(0)}(b), b \in B(\Delta)} \end{aligned}$$

Now (i) follows from the fact that the integral kernel of A^* is $A^*(g_0, g'_0) = \overline{A(g'_0, g_0)}$. (ii) is a special case of (iii) ($B = 1$).

(iii) The mean m_L has the form

$$m_L(f)(g) = \int \prod_{b \in L} \alpha_{g'}(b) f(g') M(g^{(0)})^2 N(g^{(-1)}, g^{(0)}) N(g^{(0)}, g^{(1)}) \Big|_{g'(b) = g(b), b \in L}$$

Inserting for $f E_+(AB)$ and $E_-(A)E_+(B)$, respectively, gives (iii).

(iv) follows from (3.10) and (2.15):

$$\begin{aligned} E_+(\alpha_i(A))(g) &= (\alpha_i(A)T_{\Delta_1})(g^{(0)}, g^{(1)}) / T_{\Delta_1}(g^{(0)}, g^{(1)}) \\ &= (T_{\Delta_1} A)(g^{(0)}, g^{(1)}) / T_{\Delta_1}(g^{(0)}, g^{(1)}) \\ &= \tau_1 E_-(A) \end{aligned}$$

q.e.d.

Theorem: ω_0 is a ground state of \mathcal{A} with respect to α_i .

Proof: Using (3.1), (3.7), (3.12) and the Lemma, we find

$$\omega_0(A^*A) = \langle E_+(A^*A) \rangle = \langle E_-(A^*)E_+(A) \rangle = \langle (\Theta E_+(A))E_+(A) \rangle \geq 0$$

hence ω_0 is a state. For $A \in \mathcal{D}(\alpha_i)$,

$$\omega_0 \alpha_i(A) = \langle E_+(\alpha_i(A)) \rangle = \langle \tau_1 E_-(A) \rangle = \langle E_-(A) \rangle = \langle E_+(A) \rangle = \omega_0(A),$$

so ω_0 is invariant under α_i . Furthermore

$$\begin{aligned} \omega_0(A^* \alpha_i(A)) &= \langle E_+(A^* \alpha_i(A)) \rangle = \langle E_-(A^*) E_+(\alpha_i(A)) \rangle \\ &= \langle (\Theta E_+(A)) \tau_1 E_-(A) \rangle = \langle (\Theta E_+(A)) \tau_1 E_+(A) \rangle = \langle (E_+(A) + \mathcal{N}) \hat{T}(E_+(A) + \mathcal{N}) \rangle \end{aligned}$$

where the 4th equality sign follows from the Lemma together with (3.1) for the set $L_1 = \{1\} \times B(\Delta)$ since $m_{L_1}(fh) = f m_{L_1}(h)$ if f does not depend on variables $g(b)$ with $b \in L_1$. Since T is a positive contraction ω_0 is a ground state.

q.e.d.

We now have the means to prove (3.9). Let

$$U \pi(A) \Omega = E_+(A) + \mathcal{N} \quad (3.13)$$

We have

$$\begin{aligned} \|U \pi(A) \Omega\|^2 &= \|E_+(A) + \mathcal{N}\|^2 = \langle (\Theta E_+(A))E_+(A) \rangle = \langle E_+(A^*A) \rangle = \omega_0(A^*A) \\ &= \|\pi(A) \Omega\|^2, \end{aligned}$$

so U is an isometry. Moreover, for a multiplication operator f from (3.11)

$$\begin{aligned} U \pi(f) \pi(A) \Omega &= U \pi(fA) \Omega = E_+(fA) + \mathcal{N} = f^{(0)} E_+(A) + \mathcal{N} \\ &= f^{(0)} U \pi(A) \Omega, \end{aligned}$$

hence we have (3.9) (ii). Finally, for $A \in \mathcal{D}(\alpha_i)$

$$\begin{aligned} U T \pi(A) \Omega &= U \pi \alpha_i(A) \Omega = E_+(\alpha_i(A)) + \mathcal{N} = \tau_1 E_-(A) + \mathcal{N} \\ &= \tau_1 m_L(E_-(A)) + \mathcal{N} = \tau_1 m_L(E_+(A)) + \mathcal{N} = \tau_1 E_+(A) + \mathcal{N} = \hat{T}(E_+(A) + \mathcal{N}) \\ &= \hat{T} U \pi(A) \Omega, \end{aligned}$$

thus (3.9) (i) follows from the density of $\pi(\mathcal{D}(\alpha_i))\Omega$ in \mathcal{H} . It remains to show that $\hat{\mathcal{H}} = U\mathcal{H}$. But $U\mathcal{H}$ is invariant under \hat{T} and under multiplication with time zero fields and contains $(1 + \mathcal{N})$, thus $U\mathcal{H} = \hat{\mathcal{H}}$.

4. Concluding Remarks

In this paper I treated only the case of a pure gauge theory. It is easy to see that the same methods work in the case where a bounded Higgs field is coupled to the gauge theory. For unbounded Higgs fields and for Fermi fields technical complications may occur which have not been investigated so far.

The absence of zero eigenvalues of the transfer matrix is necessary for the existence of a time evolution of the time zero fields. It does not imply, however, that there is any trace of Einstein causality for this evolution. This is connected with the open question whether the norm closure or at least the weak closure of the quantum algebra of observables is invariant under time evolution. A clarification of these points would be very important for a direct physical interpretation of lattice gauge theories before the continuum limit has been performed.

Acknowledgements: It is a pleasure for me to thank Mihail Marcu, Ricardo Schor and Erhard Seiler for stimulating discussions.

References

- /1/ Lüscher, M.: Construction of a selfadjoint, strictly positive transfer matrix for Euclidean lattice gauge theories, Commun. Math. Phys. 54, 283 (1977)
- /2/ Osterwalder, K., Seiler, E.: Gauge field theories on a lattice, Ann. Phys. 110, 440 (1978)
- /3/ Glimm, J., Jaffe, A.: Quantum Physics. A functional point of view. Springer Verlag, New York-Heidelberg-Berlin 1981
- /4/ Fredenhagen, K., Marcu, M.: Charged states in \mathbb{Z}_2 gauge theories, Commun. Math. Phys. 92, 81-119 (1983)
- /5/ Borgs, C., Seiler, E.: Lattice Yang-Mills theory at nonzero temperature and the confinement problem, Commun. Math. Phys. 91, 329-380 (1983)
- /6/ Dobrushin, R.L.: Description of a random field by means of its conditional probabilities and conditions of its regularity. Theory Probability Appl. 13, 197-224 (1968)
- /7/ Lanford III, O.E., Ruelle, D.: Observables at infinity and states with short range correlations in statistical mechanics, Commun. Math. Phys. 13, 194-215 (1969)
- /8/ Elitzur, S.: Impossibility of spontaneously breaking local symmetries, Phys. Rev. D12, 3978-3981 (1975)
- /9/ Seiler, E.: Gauge theories as a problem of constructive quantum field theory and statistical mechanics. Lect. Notes in Physics, Vol. 159. Berlin-Heidelberg-New York, Springer 1982