DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 85-027
March 1985

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

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KADANOFF-WILSON TRANSFORMATIONS IN QUANTUM GAUGE THEORIES

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Abstract: A transparent condition restricting KW transformations of correlation functions is used to establish the behavior of QFT functions. The arising proportionality factors are calculated for large classes of transformations and correlations. Gauge-invariant block transformations for matter fields are introduced, too. KW invariant functions are constructed which are suited for the QFT limit and allow firm conclusions about the parameters of the theory.

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The fact that masses in nonperturbative quantum field theory (QFT) arise from a singularity in the parameter space of the regularized theory suggests the treatment by Kadanoff-Wilson (KW) transformations[1]. Because the definition of a QFT as a limit of a regularized theory basically involves particular correlation functions, the transformation properties of these functions are important. This so far has not received appropriate attention.

Since gauge theories are the ones describing particle physics, their consideration is of prime interest. The lattice regularization[2] to be used in the nonperturbative case has the additional advantage of allowing a strictly gauge-invariant formulation. Gauge invariant KW transformations of the blocking type have first been proposed by Swendsen[3].

In the present letter a transparent condition expressing the restrictions for transformations of correlation functions is given and used to investigate the behavior of the particular functions which enter the definition of QFT. The arising proportionality factors turn out to be straightforwardly calculable, which is done for the main types of correlations and a large class of transformations. In this context gauge-invariant transformations are introduced for matter fields, too. On the basis of these results next KW invariant functions are constructed which at the same time are suited for the QFT limit. Their properties allow, after specifying a fixed point structure, to derive firm conclusions about nature and handling of the parameters of the theory.

The general form of the transformations to be used is

$$e^{S(\tilde{U},g,\lambda)} = \int_U T_{\lambda\lambda_0}(\tilde{U},U)\, e^{S(U,g,\lambda_0)} . \qquad (1)$$

The given action is denoted by $S(U,g,1)$ and the one after a transformation reducing the degrees of freedom by a factor λ^d by $S(U,g,\lambda)$. U stands for all occurring fields and $\int_U$ for the integrations over them. $g = (g_1, \ldots, g_\ell)$ are the parameters prescribed for $\lambda = 1$. In order to have

$$\int_{\tilde{U}} e^{S(\tilde{U},g,\lambda)}\, \sigma(\tilde{U},\lambda) = \int_U e^{S(U,g,\lambda_0)}\, \sigma(U,\lambda_0) \qquad (2)$$

according to (1) the condition

$$\int_{\tilde U} \sigma(\tilde U,\lambda)\, \mathcal{T}_{\lambda\lambda_0}(\tilde U, U) = \sigma(U,\lambda_0) \tag{3}$$

must be satisfied. For $\sigma(\tilde U,\lambda)=\sigma(U,\lambda_0)=1$ (3) corresponds to the usual requirement of keeping the partition function fixed. For general σ (3) means a severe restriction on the transformations of correlation functions. In order that (2) is defined at all, one must choose $\sigma(U,\lambda_0)$ out of the image set of (3).

The lattice correlation functions are of form $G(n,g)$ with $n = (n_1, \ldots, n_r)$, where n_ϱ are integer variables. The relation of the n_ϱ to physical lengths x_ϱ is $n_\varrho(\nu) = \mathrm{int}\,(\nu x_\varrho/b)$ where b is the length unit and ν the continuous numbering of the sequence which is to define the QFT limit for $\nu \to \infty$. To be able to transform $G(n(\nu),g) = \int_U e^{S(U,g,1)} \sigma_G(n(\nu),U) \big/ \int_U e^{S(U,g,1)}$ one has to find a way for satisfying (3). In the case of a decimation transformation (for which in Fig. 1 only the straight connection contributes) and $n(\nu)/\lambda$ integer it is straightforward to use $\sigma(\tilde U,\lambda) = Z\sigma_G(n(\nu)/\lambda,\tilde U)$ (the proportionality factor Z is discussed below) which gives $\sigma(U,1)=\sigma_G(n(\nu),U)$. If $n(\nu)/\lambda$ is not integer the obvious modification is to put $\sigma(\tilde U,\lambda)=Z\sigma_G(n(\nu/\lambda),\tilde U)$ which leads to $\sigma(U,1)\cong\sigma_G(n(\nu),U)$, where the equivalence sign $\cong$ denotes asymptotic equality for large ν. For a general block transformation, again starting from $\sigma(\tilde U,\lambda)=Z\sigma_G(n(\nu/\lambda),\tilde U)$, $\sigma(U,1)$ obtained from (3) in general consists of many contributions, the locations in which are distributed around those of the decimation case. Since the respective fluctuations are governed by λ (which is finite) while the mean extensions depend on ν (which is very large) one arrives at $\sigma(U,1)\cong\sigma_G(n(\nu),U)$ as before. Thus one obtains

$$G(n(\nu),g)\cong Z\int_{\tilde U} e^{S(\tilde U,g,\lambda)}\,\sigma_G\!\left(n\!\left(\tfrac{\nu}{\lambda}\right),\tilde U\right)\Big/\int_{\tilde U} e^{S(\tilde U,g,\lambda)} \tag{4}$$

for the transformed functions.

To calculate the Z factor for gauge fields along a loop, the class of gauge-invariant KW transformations shown for one step in Fig. 1 is used, which have p_i contributions of weight β_i and thin by a factor r^d per step. After s steps one then has $\lambda = r^s$. In the calculation of $\sigma(U,1)$ according to (3), for $\nu \gg \lambda$ after each step the blocking fluctuations can be smoothed to the decimation mean such that a sequence of lengths $L, rL, \ldots, r^s L$ in lattice units occurs. By the j-th step from one contribution of length $r^{j-1}L$ then contributions are generated the weighted number of which can be summed by the multinomial theorem to give $\gamma^{r^{j-1}L}$

where $\gamma = \sum_i \beta_i p_i$. Thus starting from $\sigma(\tilde U,\lambda) = Z_{\mathcal{L}}\,\sigma_G(n(\nu/\lambda),\tilde U)$ one arrives at $\sigma(U,1) \cong Z_{\mathcal{L}}\,\gamma^{(1+r+\ldots+r^{s-1})L}\,\sigma_G(n(\nu),U)$ from which one gets

$$Z_{\mathcal{L}} = e^{-c(\lambda-1)L}, \tag{5}$$

where $c=(\ln\gamma)/(r-1)$ and $L = \mathrm{int}\left(\frac{\nu}{\lambda}\,\frac{\mathcal{L}}{b}\right)$, $\mathcal{L}$ being the physical perimeter.

As an example of the inclusion of matter fields the factor Z_P occurring for the gauge-invariant combination $\bar\psi_{n'} U(P_{n'n})\psi_n$ is calculated. For the gauge fields along the path $P_{n'n}$ again the transformations of Fig. 1 are used, while for the matter fields the gauge-invariant class shown in Fig. 2 is introduced, which has p_i^ψ contributions of weight β_i^ψ. Similarly as before one now obtains

$$Z_P = \lambda^{-2\delta}\, e^{-c(\lambda-1)L} \tag{6}$$

where $\delta = \ln\gamma^\psi/\ln r$ and $\gamma^\psi = \sum_i \beta_i^\psi p_i^\psi$, with $\mathcal{L}$ now being the path length.

Choosing $\gamma \neq 1$ and $\gamma^\psi \neq 1$, which makes (5) and (6) nontrivial, corresponds to the introduction of the factor the tuning of which is subtle in linear blocking[4] and for which there may be more freedom in the nonlinear case[5]. Also parallels to wave-function renormalization of perturbative QFT are obvious. In particular, (5) is close to the factor obtained there[6] for smooth loops with d = 4 and Pauli-Villars type regularization. The corner divergences there, however, have no counterpart here.

In order that the transformed form (4) becomes attractive for QFT one has to get rid of the Z factors without loosing KW invariance and without giving up the possibility to vary γ and γ^ψ. This is achieved by forming combinations (in particular ratios) of the functions of type (4) such that all Z factors cancel out, which gives functions of the form

$$P(n(\nu),g) \cong \mathcal{P}\left(n\left(\tfrac{\nu}{\lambda}\right), g, \lambda\right). \tag{7}$$

Since the Z factors can be calculated as indicated, the remarkable result at this point is that all the functions of type (7) can readily be constructed.

It is to be noted here that Creutz ratios[7], formed to escape perimeter divergences, according to (5) are under the combinations admitted in (7). Furthermore, all ratios from which in perturbative QFT wave-function renormalization cancels out, are candidates. The class of this type which is of the form $\Gamma(x_1,\dots,x_n)/(\Gamma(y_1,z_1)\dots\Gamma(y_m,z_m))^{1/2}$ actually gives all S-matrix elements, physical masses and physical coupling strengths. In the present nonperturbative and gauge-invariant framework the combinations of direct physical interest are a subclass of (7).

For the QFT limit an appropriate fixed point of the KW transformations is needed. To describe it a representation by auxiliary parameters c_i (e. g. coefficients of some expansion of S or of e^S, or functions of such coefficients) is used for which one has $S(U,g,\lambda) = \hat{S}(U,c(g,\lambda))$, where $c = (c_1, c_2, \dots)$ and where $c(g,1)$ is prescribed. One then transforms to relevant, irrelevant and marginal parameters, which gives $S(U,g,\lambda) = \bar{S}(U,\zeta(g,\lambda))$ with $\zeta = (\zeta_1, \zeta_2, \dots)$. Marginal is understood here as substantially marginal[8], i. e. beyond the linear approximation (the occurrence of marginal parameters obviously implies that one has a manifold[9] of fixed points rather than a single point). It will be sufficient to have this representation in a domain of which the considered fixed point is an interior point.

In terms of the parameters related to the fixed point, (7) gets the form

$$P(n(\nu), g) \cong \bar{P}(n(\tfrac{\nu}{\lambda}), \zeta(g,\lambda)). \tag{8}$$

With $c(g,1)$ also the ζ_i are prescribed for $\lambda = 1$. In the following it is convenient to use transforms $\bar{g}_i$ of the g_j such that $\zeta_i(\bar{g}_i,\lambda)$ is fixed by $\bar{g}_i$ at $\lambda = 1$.

In order that a reasonable QFT limit exists, the ν dependence entering (8) via n(apart from that occurring in ratios $n_{\varrho'}/n_{\varrho}$) must be compensated by letting g depend on ν. This means for a change from ν to $\alpha\nu$ to require that $P(n(\alpha\nu), g(\alpha\nu)) \cong P(n(\nu), g(\nu))$ from which by using (8) and its λ independence it follows that

$$\zeta(g(\alpha\nu), \alpha\lambda) \cong \zeta(g(\nu), \lambda). \tag{9}$$

Thus the remarkable fact is obtained that the compensation condition can be separated from the particular functions P.

Since the marginal parameters are independent of λ, they allow to satisfy (9) with $\bar{g}_i$ independent of ν, by which the marginal $\zeta_i(\bar{g}_i)$ themselves become constants of the theory.

In the case of relevant parameters, to be in accordance with (9), for increasing ν one must shift $\zeta_i(\bar{g}_i(\nu),1)$ towards the fixed-point value ζ_i^*. The change of $\bar{g}_i$ with ν dictated by (9) then corresponds to what is conventionally expressed by a β function. One should, however, be aware that the functions of QFT and those related to KW transformations differ by definition, as has been emphasized by careful authors[10] some time ago. Here, due to the separation of (9) from the P, it follows that the bare β function of QFT and the KW function coincide for large ν.

For irrelevant parameters, in order to satisfy (9), when ν is increased $\zeta_i(\bar{g}_i(\nu),1)$ must be shifted away from the fixed-point value and thus outside the domain under consideration (e.g. to infinity or to another fixed point) unless it is set to its fixed-point value ζ_i^* from the beginning. Therefore, in order to be able to perform the limit in the considered domain one must put $\zeta_i(\bar{g}_i,1) = \zeta_i^*$, i. e. go to the "renormalized" manifold (RM). That the "renormalized" trajectory is related to the definition of QFT has been noticed a long time ago[11]. Here, on the basis of (7), it has become possible to derive the precise facts.

Using (8) the basic quantities of QFT now get the form

$$Q = \lim_{\nu\to\infty} \bar{P}(n(\tfrac{\nu}{\lambda}), \zeta^R(g(\nu),\lambda), \zeta^C) \tag{10}$$

independent of λ, with relevant parameters ζ_i^R satisfying (9) and marginal ones ζ_i^C being constants. Since (9) only determines the change for large ν, it is not suited for fixing a constant. For this purpose one has to go back to the P and to require definite limiting values, which to each ζ_i^R associates a constant R_i of the theory with the meaning of an independent mass scale. In detail, for fixing ℓ constants R_i and ζ_i^C one has to select ℓ quantities $\bar{P}_i$ and to prescribe physical values Q_i to them[12], which (provided the $\bar{P}_i$ have suitable invertibility properties) allows to solve the system of equivalence relations $\bar{P}_i \cong Q_i$ with $i = 1,\dots,\ell$. This determines $g(\nu)$ up to asymptotic equivalences which is sufficient for the calculation of physical quantities from (10).

The fact that it is a necessity to go to the RM affects some popular views. Instead of the usually assumed universality class of actions there is rather an equivalence class of pairs each of which consists of a KW transformation and of an associated RM action. To go to another pair means a change of representation. The chance for improvement[13] then is that in selected ones of these representations the approach to the limit for some of the $\mathcal{P}$ is faster[14].

Acknowledgements

I wish to thank F. Gutbrod, R.D. Peccei and the DESY Theory Group for the kind hospitality extended to me. I am, in particular, grateful to H. Joos for conversations.

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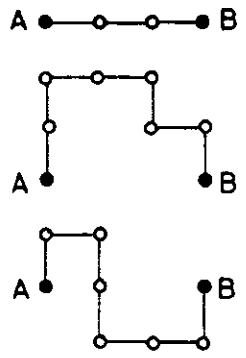
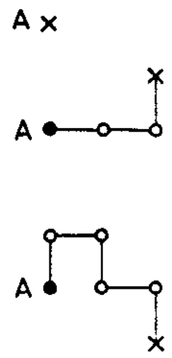
Figure Captions

Fig. 1 Typical contributions to block link from A to B. Links between dots correspond to gauge fields on finer lattice.

Fig. 2 Typical blocking contributions for matter field at point A. Cross denotes matter field and links between dots gauge fields on finer lattice.



Fig·1



Fig·2