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A CONFINEMENT CRITERION FOR QCD WITH DYNAMICAL QUARKS

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A b s t r a c t

We propose a criterion that tests the existence of isolated quarks in QCD.

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A longstanding open problem in QCD is the problem of quark confinement. In the limit of infinitely heavy (static) quarks several confinement criteria have been proposed, the best known being that of Wilson [1]. All these criteria fail however in the presence of light (dynamical) quarks since vacuum fluctuations of quark-antiquark pairs screen the confining forces.

In experiments quark confinement manifests itself in the fragmentation of sufficiently far separated quark-antiquark pairs into ordinary hadrons. On the other hand, if free quarks are to exist it should be possible to separate a quark-antiquark pair. The free quark is then obtained by sending the antiquark to infinity. One is therefore led to investigate theoretically the sequence of "dipole" states

$$\Phi_{\underline{x}\underline{y}} = \sum_{i,\alpha,\beta} \psi_{i,\alpha}(\underline{x})\, \overline{\psi}_{i,\beta}(\underline{y})\; U_{\alpha\beta}(\mathcal{C}_{\underline{x}\underline{y}})\,\Omega \tag{1}$$

where $\psi_{i\alpha}(\underline{x})$ are the quark fields at the point $\underline{x}$ of space with color index α and Dirac and flavor index i, $U(\mathcal{C}_{\underline{x}\underline{y}})$ is the path ordered integral of the gauge field over the path $\mathcal{C}_{\underline{x}\underline{y}}$

$$U(\mathcal{C}_{\underline{x}\underline{y}}) = P\exp\Big\{i\int_{\mathcal{C}_{\underline{x}\underline{y}}} A\Big\} \tag{2}$$

and Ω is the vacuum.

The energy of the state $\Phi_{\underline{x}\underline{y}}$ diverges as $|\underline{x}-\underline{y}|$ tends to infinity. In refs.[3]-[5] we discussed a possible way out of this difficulty. Denote by $U^{(n)}(\mathcal{C}_{\underline{x}\underline{y}})$ the result of translating $U(\mathcal{C}_{\underline{x}\underline{y}})$ by n steps into euclidean time:

$$U^{(n)}(\mathcal{C}_{\underline{x}\underline{y}}) = T^{n}\, U(\mathcal{C}_{\underline{x}\underline{y}})\, T^{-n}. \tag{3}$$

Here T is the transfer matrix. The energy of the sequence of states

$$\Phi^{(n)}_{\underline{x}\underline{y}} = \sum_{i,\alpha,\beta} \psi_{i,\alpha}(\underline{x})\, \overline{\psi}_{i,\beta}(\underline{y})\; U^{(n)}_{\alpha\beta}(\mathcal{C}_{\underline{x}\underline{y}})\,\Omega \tag{4}$$

stays bounded as $\underline{y}$ goes to infinity if we take n proportional to $|\underline{x}-\underline{y}|$ (this is a consequence of the perimeter law for the Wilson loop which is always true if the gauge fields are coupled to matter fields carrying the fundamental charge - see e.g. [2]). For the sake of simplicity we shall take $\underline{x}-\underline{y}$ along a lattice axis and choose $n=\frac{1}{2}|\underline{x}-\underline{y}|$. The Ansatz (4) mimics the attempt to separate a quark-antiquark pair with a given energy.

If quark fragmentation occurs as $\underline{y}\to\infty$, the transition probability of $\Phi^{(n)}_{\underline{x}\underline{y}}$ into hadronic states (including the vacuum) should go to one. In particular, since all hadronic states are local excitations of the vacuum, one expects

$$\lim_{|\underline{x}-\underline{y}|\to\infty} \frac{|(\Omega,\Phi^{(n)}_{\underline{x}\underline{y}})|^2}{\|\Phi_{\underline{x}\underline{y}}\|^2} = \text{const} \neq 0 \qquad (5)$$

If on the other hand the limit (5) is zero, this is an indication that the sequence of dipole states becomes orthogonal to all hadronic states and therefore approximates an isolated quark.

In order to compute the ratio of eq.(5) we express the matrix elements in terms of euclidean expectation values of gauge invariant strings and loops. Denote by $\mathcal{C}_{xy}$ a rectangular path in euclidean space-time with endpoints $\underline{x}$ and $\underline{y}$ in the time-zero-hyperplane (fig.1).

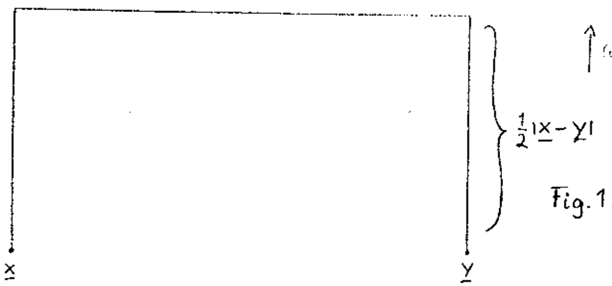

Fig.1: The path $\mathcal{C}_{xy}$

The numerator of (5) is given in graphical terms by

$$\Big|\sum_i \Big\langle \overset{\mathcal{C}_{xy}}{\ulcorner\!\!\urcorner}_{\;i\;\;i} \Big\rangle\Big|^2 \qquad (6)$$

and the denominator is given by

$$\sum_{i,j} \Big\langle \underset{\Theta\mathcal{C}_{xy}}{\overset{\mathcal{C}_{xy}}{{}_{i}^{i}\Box_{j}^{i}}} \Big\rangle \qquad (7)$$

where Θ denotes the reflection through the time-zero-hyperplane.

In the continuum theory the denominator of eq.(5) has to be regularized because of the singularity of products of quark fields at coinciding points. A simple way of doing this is to replace it by the expectation value of the euclidean Wilson loop $\mathcal{C}_{xy}\circ\Theta\mathcal{C}_{xy}$. Thus the parameter to be tested can be represented in graphical terms by [3]

$$\varrho = \lim_{|x-y|\to\infty} \frac{\Big|\sum_i \Big\langle \ulcorner\!\!\urcorner_{\;i\;\;i} \Big\rangle\Big|^2}{\langle \Box \rangle} \qquad (8)$$

A regularization of the corners of the path $\mathcal{C}_{xy}$ is probably not necessary since the associated divergences in the numerator and denominator should cancel.

We would like to point out that our parameter tests in some sense the origin of the perimeter behaviour of the Wilson loop in the presence of dynamical quarks. If this behaviour is caused by fluctuations of the matter fields, then both the numerator and the denominator in eq.(8) should decrease exponentially with the same rate. As a result $\varrho\neq 0$ which means confinement. If however the perimeter behaviour is partially due to fluctuations of the gauge field, the denominator is expected to decay more slowly. Then ϱ vanishes, which indicates deconfinement.

The picture described above has emerged from a general analysis of the particle structure in gauge theories [4], and has been confirmed both analytically [5] and numerically [6] on the example of the Z_2 Higgs theory. In the Z_2 model there exists both a confining and a deconfining phase, and ϱ shows the expected behaviour.

As an aside we mention that our parameter looks similar to a parameter proposed by Bricmont and Fröhlich [7]. However, the information contained in the latter is different: the B.F. parameter is not sensitive to the presence of free quarks.; it rather tests the existence of bound states of a dynamical and a static quark [5]. Since the existence of bound states does not imply the absence of isolated quarks in the scattering states, the B.F. criterion is not directly relevant to the confinement problem.

For the application of our confinement criterion it is important to control the behaviour of the parameter ϱ under renormalization group transformations. Let $\varrho=\varrho(g)$ denote the dependence of ϱ on the coupling constant g in massless QCD on a lattice. By imposing a renormalization condition the lattice spacing a is fixed as a function of g. The l.h.s. of (8) should behave like a gauge invariant two-point-function $W_2(r,g)$ of the fields ψ and $\bar{\psi}$ (r is the distance between $\underline{x}$ and $\underline{y}$ on the lattice) at a physical distance ra. The asymptotic behaviour of $W_2(r,g)$ for small distances ra can be determined from perturbation theory. The continuum value ϱ_c of our order parameter is obtained by taking the limit

$$\rho_c = \lim_{g \to 0} \frac{\rho(g)}{W_2(r,g)} \Big|_{ra = \text{const.}} \qquad (9)$$

The parameter ρ_c could be determined in a Monte-Carlo simulation by computing $\rho(g)$ and using perturbative results for the renormalization group equation and for $W_2(r,g)$. At a later stage the accuracy of the simulation could be improved to determine the scaling relations and the short distance behaviour of $W_2(r,g)$ directly from the simulation.

The parameter ρ_c should be directly related to measured quantities in the quark fragmentation process. It would be very interesting to isolate such quantities, but this requires a more detailed study of the fragmentation process on the lattice.

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