DESY 13-050
IPMU 13-0091
May 2013

# The Gravitational Wave Spectrum from Cosmological $B-L$ Breaking

W. Buchmüller[1], V. Domcke[1], K. Kamada[1], K. Schmitz[2]

[1] *Deutsches Elektronen-Synchrotron DESY, 22607 Hamburg, Germany*
[2] *Kavli IPMU (WPI), University of Tokyo, Kashiwa 277-8583, Japan*

**Abstract**

Cosmological $B-L$ breaking is a natural and testable mechanism to generate the initial conditions of the hot early universe. If $B-L$ is broken at the grand unification scale, the false vacuum phase drives hybrid inflation, ending in tachyonic preheating. The decays of heavy $B-L$ Higgs bosons and heavy neutrinos generate entropy, baryon asymmetry and dark matter and also control the reheating temperature. The different phases in the transition from inflation to the radiation dominated phase produce a characteristic spectrum of gravitational waves. We calculate the complete gravitational wave spectrum due to inflation, preheating and cosmic strings, which turns out to have several features. The production of gravitational waves from cosmic strings has large uncertainties, with lower and upper bounds provided by Abelian Higgs strings and Nambu-Goto strings, implying $\Omega_{\rm GW}h^2 \sim 10^{-13} - 10^{-8}$, much larger than the spectral amplitude predicted by inflation. Forthcoming gravitational wave detectors such as eLISA, advanced LIGO and BBO/DECIGO will reach the sensitivity needed to test the predictions from cosmological $B-L$ breaking.

# 1 Introduction

Relic gravitational waves (GWs) are a fascinating window to the very early universe [1]. They are generated by quantum fluctuations during inflation [2] as well as in the form of classical radiation from cosmic strings [3]. Another important source is preheating after inflation via resonant decay of an oscillating inflaton field [4] or violent collisions of bubble-like structures [5] in tachyonic preheating [6].

We have recently proposed a detailed picture of pre- and reheating where the initial conditions of the hot early universe are generated by the spontaneous breaking of $B-L$, the difference of baryon and lepton number [7–9]. The false vacuum phase of unbroken $B-L$ symmetry yields hybrid inflation with an energy density set by the scale of grand unification [10, 11]. In the $B-L$ breaking phase transition ending inflation most of the vacuum energy density is rapidly transferred to non-relativistic $B-L$ Higgs bosons, a sizable fraction also into cosmic strings. The decays of heavy Higgs bosons and heavy Majorana neutrinos generate entropy and baryon asymmetry via thermal and nonthermal leptogenesis [12, 13]. The temperature evolution during reheating is controlled by the interplay between the $B-L$ Higgs and the neutrino sector. The origin of dark matter are thermally produced gravitinos [14].

In this paper we compute the GW spectrum predicted by cosmological $B-L$ breaking. It receives contributions from all the possible sources mentioned above: inflation, cosmic strings and preheating. Much work has already been done on the stochastic gravitational background from inflation (see, e.g. Refs. [15, 16]). We are particularly interested in features of the GW spectrum caused by the change of the equation of state in the cosmological evolution. This has previously been studied in Ref. [17] with the goal of determining the reheating temperature of the early universe. Our results are consistent with those of Ref. [17]. The main difference is that we can resort to a time-resolved description of the entire reheating process, studied in Ref. [9]. This allows us to gain a better understanding of the connection between features in the GW spectrum and the evolution of the temperature of the thermal bath, pinpointing to which model parameters certain features in the spectrum are related.

A very interesting but also rather uncertain source of GWs are cosmic strings [18]. In the $B-L$ breaking phase transition local cosmic strings are formed. The initial state of such a network can be simulated numerically, and recently the amplitude of the scale-invariant spectrum of GWs produced during the radiation dominated epoch has been determined [19]. Based on these results we obtain the GW spectrum for our model, thereby extending the analysis to GWs produced during reheating and during matter domination. For Abelian Higgs (AH) strings it is usually assumed that strings lose their energy mostly via radiation of massive particles. In this case we find a GW spectrum which has a very similar shape to that generated by inflation, but is amplified by several orders of magnitude. This result opens up the possibility to measure features in the GW spectrum related to the temperature evolution during reheating. Alternatively, one also considers the possibility that, beyond a certain length, strings can be described as Nambu-Goto (NG) strings, which lose their energy by radiating GWs, see e.g. Refs. [21–23]. We shall also study the implications of NG strings for the GW spectrum and compare the results with those obtained for AH strings.

Tachyonic preheating leads to GWs with a spectrum peaked at very high frequencies. For certain parameter regimes of hybrid inflation the spectrum has been determined numerically [24, 25]. We shall base our discussion on analytical estimates for the peak frequencies, which we apply to our model.

Measuring the GW spectrum thus provides a unique possibility to test different aspects of a phase transition in the early universe. Forthcoming space- and ground-based interferometers such as advanced LIGO [20], BBO/DECIGO [26,27] and eLISA [28] will reach the sensitivity necessary to probe this scenario. At the same time, millisecond pulsar timing experiments already now put stringent bounds on NG cosmic strings [29] and future experiments such as SKA [30] will further increase this sensitivity. It will however remain a challenge to disentangle the GW spectrum from a phase transition in the early universe from other sources of GWs, due to both astrophysical processes and subsequent cosmological phase transitions, see e.g. Refs. [31, 32].

The paper is organized as follows. In Sec. 2 we recall some basic formulas for the production of GWs and the transfer function which are needed in the subsequent chapters. The main ingredients of our model for pre- and reheating are described in Sec. 3. Secs. 4 and 5 deal with the production of GWs during inflation and preheating, and in Secs. 6 and 7 GWs from cosmic strings are discussed, for the case of AH strings and NG strings, respectively. Sec. 8 focuses on probing the reheating temperature by measuring a feature in the GW spectrum. Constraints from the cosmic microwave background and observational prospects are the topic of Sec. 9, and we conclude in Sec. 10. Three appendices deal with the scale factor and temperature evolution during reheating as well as the analytical calculation of the GW background from NG strings.

## 2 Cosmic Gravitational Wave Background

In this section we recall some basic formulas which we shall need in our calculation of the various contributions to the GW background. GWs are tensor perturbations of the homogeneous background metric. In a flat Friedmann Robertson Walker (FRW) background, these perturbations can be parametrized as [31]

$$ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \,. \tag{1}$$

Here $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, $a$ is the scale factor and $x^\mu$ are conformal coordinates, with $x^i$ denoting the comoving spatial coordinates and $\tau = x^0$ the conformal time. These are related to the physical coordinates and the cosmic time as $\boldsymbol{x}_{\rm phys} = a(\tau)\,\boldsymbol{x}$ and $dt = a(\tau)\,d\tau$, respectively[1].

Introducing

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\rho_\rho \,, \tag{2}$$

the linearized Einstein equations describing the generation and propagation of GWs read

$$\bar{h}''_{\mu\nu}(\boldsymbol{x},\tau) + 2\frac{a'}{a}\bar{h}'_{\mu\nu}(\boldsymbol{x},\tau) - \nabla^2_{\boldsymbol{x}}\bar{h}_{\mu\nu}(\boldsymbol{x},\tau) = 16\pi G T_{\mu\nu}(\boldsymbol{x},\tau)\,, \tag{3}$$

with a prime denoting the derivative with respect to conformal time; $G$ is Newton's constant and $T_{\mu\nu}$ is the anisotropic part of the stress energy tensor of the source. The total stress energy tensor is the sum of $T_{\mu\nu}$ and an isotropic part which determines the background metric. Outside the source, we can choose the transverse traceless (TT) gauge for the GW, i.e. $h^{0\mu} = 0$, $h^i_i = 0$, $\partial^j h_{ij} = 0$, which implies $\bar{h}_{\mu\nu} = h_{\mu\nu}$. The mode equation which describes the generation and propagation of these degrees of freedom (DOFs) can be obtained by using an appropriate projection operator [31] on the Fourier transform[2] of Eq. (3),

$$\tilde{h}''_{ij}(\boldsymbol{k},\tau) + \left(k^2 - \frac{a''}{a}\right)\tilde{h}_{ij}(\boldsymbol{k},\tau) = 16\pi G a \Pi_{ij}(\boldsymbol{k},\tau)\,, \tag{4}$$

where $\tilde{h}_{ij} = a h_{ij}$, $\Pi_{ij}$ denotes the Fourier transform of the TT part of the anisotropic stress tensor $T_{\mu\nu}$, $k = |\boldsymbol{k}|$, and $\boldsymbol{k}$ is the comoving wavenumber, related to the physical wave number through $\boldsymbol{k}_{\rm phys} = \boldsymbol{k}/a$.

[1] Here and in the following, Greek letters denote Lorentz indices, $\mu,\nu = 0,1,2,3$, whereas Latin letters refer to the spatial indices, $i,j = 1,2,3$, with bold letters indicating 3-vectors.

[2] Our convention for the Fourier transformation is $h_{ij}(\boldsymbol{x},\tau) = \int \frac{d^3k}{(2\pi)^3}\; h_{ij}(\boldsymbol{k},\tau)\exp(i\boldsymbol{k}\boldsymbol{x})$.

A useful plane wave expansion of GWs is given by

$$h_{ij}\left(\boldsymbol{x},\tau\right) = \sum_{A=+,\times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2\hat{\boldsymbol{k}}\, h_A\left(\boldsymbol{k}\right) e_{ij}^{A}\left(\hat{k}\right) T_k(\tau) e^{-ik\left(\tau - \hat{\boldsymbol{k}}\boldsymbol{x}\right)} . \tag{5}$$

Here, $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$, $A = +, \times$ labels the two possible polarization states of a GW in the TT gauge and $e_{ij}^{+,\times}$ denote the two corresponding polarization tensors satisfying the normalization condition $e_{ij}^{A} e^{ij\,B} = 2\delta^{AB}$. $h_A(\boldsymbol{k})$ are the coefficients of the expansion and the red-shift due to the expansion of the universe is captured in the so-called 'transfer function' $T_k(\tau)$.

An analytical expression for $T_k$ can be obtained by studying the homogeneous, i.e. source-free, version of Eq. (4). Using the Friedmann equations, we find $a''/a \sim a^2H^2$. The mode equation describes two distinct regimes. On sub-horizon scales, $k \gg aH$, we can neglect the $a''/a$ term. The solution is thus simply $\tilde{h}_{ij} \sim \cos(\omega\tau)$ and hence $h_{ij} \sim \cos(\omega\tau)/a$, i.e. the modes decay as $1/a$ inside the horizon. On the other hand, on super-horizon scales, $k \ll aH$, we can neglect the $k^2$ term. This yields $2a'h'_{ij} + ah''_{ij} = 0$, with the solution

$$h_{ij}(\tau) = A + B \int^{\tau} \frac{d\tau'}{a^2(\tau')} , \tag{6}$$

where $A$ and $B$ are constants of integration. This solution is a constant plus a decaying mode which can be neglected. Hence on super-horizon scales the amplitude of the mode remains constant, the mode is 'frozen'.

With this, we identify the transfer function $T_k$ capturing the effects due to the expansion of the universe as

$$T_k(\tau_*, \tau) = \frac{h_{ij}^{E}(\boldsymbol{k}, \tau)}{h_{ij}^{E}(\boldsymbol{k}, \tau_*)} . \tag{7}$$

with $h_{ij}^{E}(\boldsymbol{k}, \tau)$ denoting the envelope of the oscillating function $h_{ij}(\boldsymbol{k}, \tau)$. For modes present on super-horizon scales, i.e. GWs produced by inflation, the reference time $\tau_*$ can be equally replaced by any $\tau < \tau_k$, where $\tau_k$ denotes the time when a given mode with wavenumber $k$ enters the horizon, $k = a(\tau_k)H(\tau_k)$. To good approximation, the transfer function can then be estimated as (see e.g. [16])

$$T_k(\tau_*, \tau) \approx \frac{a(\tau_*)}{a(\tau)} \quad \text{with } \tau_* = \begin{cases} \tau_i \text{ for sub-horizon sources} \\ \tau_k \text{ for super-horizon sources} \end{cases} , \tag{8}$$

with $\tau_i$ marking the time when the GW is generated. Here in the latter case, we assume the amplitude to be constant until $\tau = \tau_k$ and then to drop as $1/a$ immediately afterwards. The actual solution to the mode equation yields corrections to both of these assumptions, however as a numerical check reveals the effects compensate each other so that Eq. (8) reproduces the full result very well. We will quantify this statement at the end of Sec. 4 after discussing the transfer function in more detail. For super-horizon sources we will in the following use the more compact notation $T_k(\tau) = T_k(\tau_k, \tau)$.

The GW background is a superposition of GWs propagating with all frequencies in all directions. An important observable characterizing the GW background is the ensemble average of the energy density [31], which is expected to be isotropic,

$$\rho_{\mathrm{GW}}(\tau) = \frac{1}{32\pi G}\left\langle \dot{h}_{ij}\left(\boldsymbol{x},\tau\right)\dot{h}^{ij}\left(\boldsymbol{x},\tau\right)\right\rangle = \int_{-\infty}^{\infty} d\ln k\, \frac{\partial \rho_{\mathrm{GW}}(k,\tau)}{\partial \ln k}\,, \tag{9}$$

with the angular brackets denoting the ensemble average and the dot referring to the derivative with respect to cosmic time. Alternatively, one also uses the ratio of the differential energy density to the critical density $\rho_c = 3H^2/(8\pi G)$,

$$\Omega_{\mathrm{GW}}(k,\tau) = \frac{1}{\rho_c}\frac{\partial \rho_{\mathrm{GW}}(k,\tau)}{\partial \ln k}\,, \tag{10}$$

where $H$ denotes the Hubble parameter. In the model considered in this paper, the energy density receives contributions of quantum as well as of classical origin,

$$\rho_{\mathrm{GW}}(\tau) = \rho_{\mathrm{GW}}^{\mathrm{qu}}(\tau) + \rho_{\mathrm{GW}}^{\mathrm{cl}}(\tau)\,. \tag{11}$$

The quantum part is due to inflation and therefore stochastic, whereas the classical part is determined by the contributions to the stress energy tensor from cosmic strings and from tachyonic preheating,

$$\rho_{\mathrm{GW}}^{\mathrm{cl}}(\tau) = \rho_{\mathrm{GW}}^{\mathrm{CS}} + \rho_{\mathrm{GW}}^{\mathrm{TP}}(\tau)\,. \tag{12}$$

For a stochastic GW background the Fourier modes $h_A\left(\boldsymbol{k}\right)$ in Eq. (5) are random variables and their ensemble average is determined by a time-independent spectral density $S_h(k)$ [31],

$$\left\langle h_A\left(\boldsymbol{k}\right)h_B^*\left(\boldsymbol{k}'\right)\right\rangle = 2\pi\delta\left(k-k'\right)\frac{1}{4\pi}\delta^{(2)}\big(\hat{\boldsymbol{k}}-\hat{\boldsymbol{k}}'\big)\delta_{AB}\frac{1}{2}S_h(k)\,. \tag{13}$$

This relation reflects the fact that different modes are uncorrelated and that the background is isotropic. On sub-horizon scales, $k \gg aH$, Eqs. (5), (8) and (13) yield

$$\left\langle h_{ij}\left(\boldsymbol{x},\tau\right)h^{ij}\left(\boldsymbol{x},\tau\right)\right\rangle = \frac{1}{\pi}\int_{-\infty}^{\infty} dk\; S_h(k)\,\frac{a^2(\tau_*)}{a^2(\tau)}\,, \tag{14}$$

and

$$\left\langle \dot{h}_{ij}\left(\boldsymbol{x},\tau\right)\dot{h}^{ij}\left(\boldsymbol{x},\tau\right)\right\rangle = \frac{1}{\pi a^2(\tau)}\int_{-\infty}^{\infty} dk\, k^2\, S_h(k)\,\frac{a^2(\tau_*)}{a^2(\tau)}\,. \tag{15}$$

Comparing this with Eq. (9) yields the differential energy density

$$\frac{\partial\rho_{\rm GW}\left(k,\tau\right)}{\partial\ln k} = \frac{a^2(\tau_*)}{16\pi^2 G a^4(\tau)} k^3 S_h(k)\,. \tag{16}$$

The classical contribution to the GW energy density is obtained by integrating Eq. (4) from the initial time $\tau_i$ of GW production until today,

$$h_{ij}(\boldsymbol{k},\tau) = 16\pi G\frac{1}{a(\tau)}\int_{\tau_i}^{\tau} d\tau' a(\tau')\mathcal{G}(k,\tau,\tau')\Pi_{ij}(\boldsymbol{k},\tau')\,, \tag{17}$$

where $\mathcal{G}(k,\tau,\tau')$ is the retarded Green's function of the differential operator on the left-hand side of Eq. (4). For sub-horizon modes, i.e. $k\tau \gg 1$, one has $\mathcal{G}(k,\tau,\tau') = \sin(k(\tau-\tau'))/k$. It is now straightforward to evaluate the ensemble average $\langle \dot{h}^2\rangle$. Assuming translational invariance and isotropy of the source,

$$\left\langle \Pi_{ij}(\boldsymbol{k},\tau)\Pi^{ij}(\boldsymbol{k}',\tau')\right\rangle = (2\pi)^3\Pi^2(k,\tau,\tau')\delta(\boldsymbol{k}+\boldsymbol{k}')\ , \tag{18}$$

the resulting differential energy density simplifies to

$$\frac{\partial\rho_{\rm GW}\left(k,\tau\right)}{\partial\ln k} = \frac{2G}{\pi}\frac{k^3}{a^4(\tau)}\int_{\tau_i}^{\tau} d\tau_1\int_{\tau_i}^{\tau} d\tau_2 a(\tau_1)a(\tau_2)\cos(k(\tau_1-\tau_2))\Pi^2(k,\tau_1,\tau_2)\ , \tag{19}$$

Here, in order to perform the ensemble average, we have also averaged the integrand over a period $\Delta\tau = 2\pi/k$, assuming ergodicity.

# 3 Cosmological $B{-}L$ Breaking

The main goal of this paper is to derive the full spectrum of GWs whose origin is either directly or indirectly related to the $B{-}L$ phase transition. In the next chapters, we are going to discuss in turn all of the relevant sources for GWs. For now, let us first review how spontaneous $B{-}L$ breaking at the end of hybrid inflation can be embedded into supersymmetric theories.

Cosmological $B{-}L$ breaking is implemented by the superpotential

$$W_{B-L} = \frac{\sqrt{\lambda}}{2}\Phi\left(v_{B-L}^2 - 2S_1S_2\right)\ , \tag{20}$$

where the chiral superfields $\Phi$, $S_1$ and $S_2$ represent standard model gauge singlets carrying $B-L$ charges 0, $-2$ and $+2$, respectively. The radial component $\varphi$ of the complex scalar $\phi = \varphi/\sqrt{2}e^{i\theta} \subset \Phi$ is identified as the inflaton. Similarly, the Higgs multiplet $S$ breaking $B-L$ at the scale $v_{B-L}$ is contained in the fields $S_{1,2} = S/\sqrt{2}e^{\pm i\Lambda}$. The actual scalar $B-L$ breaking Higgs boson $\sigma$ corresponds in particular to the real part of the complex scalar $s \subset S$. The parameter $\lambda$ is a dimensionless coupling constant.

Assuming a canonical Kähler potential for $\phi$, the tree-level scalar potential induced by $W_{B-L}$ is exactly flat in the direction of the inflaton $\varphi$. For $\varphi$ larger than some critical value, $\varphi > \varphi_c = v_{B-L}$, the complex scalars in $S_1$ and $S_2$ are stabilized at their origin, $S_{1,2} = 0$, such that $B-L$ is unbroken. In this phase of unbroken $B-L$, the energy density of the vacuum is non-zero, $V_0 = \frac{1}{4}\lambda v_{B-L}^4$, corresponding to an explicit breaking of supersymmetry and entailing a stage of hybrid inflation. The supersymmetric vacuum is stabilized by radiative corrections at the one-loop level, forcing $\varphi$ to slowly roll down to $\varphi = 0$. Once $\varphi$ passes below $\varphi_c$, the $B-L$ Higgs boson becomes tachyonically unstable, i.e. it acquires a negative mass squared. This triggers the sudden end of inflation and the spontaneous breaking of $B-L$. In the true groundstate, we eventually have $\varphi = 0$ and $S_{1,2} = v_{B-L}/\sqrt{2}$.

The slow-roll parameters $\epsilon$ and $\eta$ of hybrid inflation as well as the amplitude $\Delta_s^2$ of the scalar metric perturbations can be readily expressed in terms of $\lambda$ and $v_{B-L}$,

$$\epsilon \approx \frac{\lambda}{16\pi^2}\left|\eta\right| , \qquad \eta \approx -\frac{\lambda\, M_{\mathrm{Pl}}^2}{32\pi^3\, \varphi_*^2} \approx -\frac{1}{2N_e^*} , \tag{21}$$

$$\Delta_s^2(k_*) = \frac{H_{\mathrm{inf}}^2}{8\pi^2 \epsilon M_{\mathrm{Pl}}^2} \approx \frac{64\pi^2}{3} N_e^* \left(\frac{v_{B-L}}{M_{\mathrm{Pl}}}\right)^4 . \tag{22}$$

Here, $M_{\mathrm{Pl}} = (8\pi G)^{-\frac{1}{2}} = 2.44\times 10^{18}$ GeV is the reduced Planck mass, $k_* = 0.002\ \mathrm{Mpc}^{-1}$ is the chosen pivot scale, which is probed by observations of the CMB, and $N_e^* \simeq 50$ denotes the number of e-folds before the end of inflation, at field value $\varphi_*$ when the pivot scale leaves the Hubble horizon. In Eq. (22) we have used the slow-roll relation $3M_{\mathrm{Pl}}^2 H_{\mathrm{inf}}^2 = V_0$, where $H_{\mathrm{inf}}$ denotes the Hubble parameter during inflation. The value of $\Delta_s^2$ measured by the PLANCK satellite, $\Delta_s^2 \simeq 2.18\times 10^{-9}$ [33], fixes $v_{B-L}$ to a value close to the GUT scale. More precisely, in a detailed study of hybrid inflation that also takes into account the production of cosmic strings as well as non-canonical contributions to the Kähler potential, the authors of Ref. [34] find consistency among all relevant observations for $v_{B-L}$ values ranging between $3 \times 10^{15}$ GeV and $7 \times 10^{15}$ GeV and couplings in the range $10^{-4} \lesssim \sqrt{\lambda} \lesssim 10^{-1}$. Taking into account the recent PLANCK data [33], the upper bound on $v_{B-L}$ comes down to $v_{B-L} \lesssim 6 \times 10^{15}$ GeV. For definiteness, we shall work with $v_{B-L} = 5 \times 10^{15}$ GeV in the following.

Let us now turn to tensor perturbations. As evident from Eq. (21), $\epsilon$ is suppressed by a loop factor as compared to $\eta$. This results in a very small tensor-to-scalar ratio $r$ and hence a very small amplitude $\Delta_t^2$ of the tensor metric perturbations,

$$\Delta_t^2 = \frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2} = \frac{\lambda}{6\pi^2}\left(\frac{v_{B-L}}{M_{\rm Pl}}\right)^4 = r\,\Delta_s^2 \simeq r \times 2.18 \times 10^{-9}\,, \tag{23}$$

$$r = 16\,\epsilon \simeq 1.0 \times 10^{-7}\left(\frac{\lambda}{10^{-4}}\right)\left(\frac{50}{N_e^*}\right)\,. \tag{24}$$

According to the consistency relation $n_t = -r/8$, we then immediately conclude that our inflationary model always predicts a negligibly small tensor spectral index $n_t$. In the calculation of the GW spectrum, we can therefore neglect any variation of the Hubble scale during inflation.

The spontaneous breaking of $B{-}L$ at the end of hybrid inflation is accompanied by two important non-perturbative processes. The first is tachyonic preheating which denotes the transfer of the initial vacuum energy density $V_0$ into a gas of non-relativistic $B{-}L$ Higgs bosons $\sigma$ along with the non-adiabatic production of all particle species coupled to the $B{-}L$ Higgs field.[3] The second process is the production of topological defects in the form of cosmic strings. They are characterized by their energy density per unit length $\mu$, which, in the Abelian Higgs model, is given by [36]

$$\mu = 2\pi v_{B-L}^2\, B\left(\frac{m_S}{m_Z}\right)\,, \qquad \text{with } B(\beta) = 2.4\left(\ln\frac{2}{\beta}\right)^{-1} \text{ for } \beta < 10^{-2}\,, \tag{25}$$

where $m_S$ and $m_Z$ denote the masses of the $B{-}L$ Higgs and gauge bosons, respectively. Preheating as well as cosmic strings act as sources of GWs and we will discuss their respective contributions to the GW spectrum in Secs. 5, 6 and 7.

The $B{-}L$ breaking sector couples to the supersymmetric standard model (supplemented by three generations of right-handed neutrinos) via Yukawa terms in the superpotential,

$$W \supset W_{\rm MSSM} + W_n\,, \quad W_n = h_{ij}^\nu \mathbf{5}_i^* n_j^c H_u + \frac{1}{2} h_i^n n_i^c n_i^c S\,, \tag{26}$$

where $n_i^c$ denote the superfields containing the charge conjugates of the right-handed neutrinos, the matrices $h_{ij}^n$ and $h_{ij}^\nu$ encompass Yukawa coupling constants, and $W_{\rm MSSM}$ is

[3] An interesting question in this context, which requires further investigation, concerns the effect of the inflaton on tachyonic preheating, see e.g. Ref. [35].

the superpotential of the minimal supersymmetric standard model (MSSM). We assume that the flavour structure of our superpotential derives from a $U(1)$ flavour symmetry of the Froggatt-Nielsen type that commutes with $SU(5)$, cf. Ref. [37]. This is the reason why we arrange all the superfields of our model in $SU(5)$ multiplets. In particular, we have $\mathbf{5}_i^* = (d_i^c,\, \ell_i)$, $i = 1, 2, 3$. Furthermore, we also assume that the colour triplet partners of the electroweak Higgs doublets $H_u$ and $H_d$ have been projected out. During the $B{-}L$ and the electroweak phase transitions, the fields $S$ and $H_{u,d}$ acquire vacuum expectation values $v_{B-L}$ and $v_{u,d}$, respectively. After electroweak symmetry breaking, the superpotential $W_n$ hence turns into the usual seesaw superpotential featuring a neutrino Dirac as well as a neutrino Majorana mass term, thereby providing us with a natural explanation for the smallness of the standard model neutrino masses.

After preheating most of the total energy density is stored in non-relativistic $\sigma$ particles. These then slowly decay into all three generations of heavy Majorana neutrinos $N_i$ and sneutrinos $\tilde{N}_i$ via the second operator in the superpotential $W_n$. Subsequently, the heavy (s)neutrinos decay in turn into the lepton-Higgs pairs of the MSSM via the first operator in $W_n$. The (s)neutrino decay products thermalize immediately, thereby giving rise to a hot thermal bath of MSSM radiation. This chain of decay and thermalization processes represents the actual reheating phase of the early universe. As explained in more detail in Refs. [8,9], it is accompanied by the generation of a primordial lepton asymmetry in the decay of the heavy (s)neutrinos as well as the production of a thermal abundance of gravitinos. Electroweak sphaleron processes convert the lepton asymmetry into the baryon asymmetry of the universe. Our scenario of cosmological $B{-}L$ breaking hence naturally accommodates baryogenesis via leptogenesis. Moreover, given an appropriate superparticle mass spectrum, the thermally produced gravitinos either account themselves for the relic density of dark matter or they generate the dark matter abundance in the form of MSSM neutralinos in their decays.

The two main quantities controlling the time evolution of the reheating process are $\Gamma_S^0$ and $\Gamma_{N_1}^0$, i.e. the vacuum decay rate of the $B{-}L$ Higgs bosons and its superpartners as well as the vacuum decay rate of the heavy (s)neutrinos of the first generation,[4]

$$\Gamma_S^0 = \frac{m_S}{32\pi}\left(\frac{M_1}{v_{B-L}}\right)^2\left[1-\left(\frac{2M_1}{m_S}\right)^2\right]^{1/2}, \quad \Gamma_{N_1}^0 = \frac{\widetilde{m}_1}{4\pi}\left(\frac{M_1}{v_u}\right)^2, \tag{27}$$

[4]For simplicity, we shall assume that the decay of the $B{-}L$ Higgs multiplet into the two heavier (s)neutrino generations is kinematically forbidden (cf. also Ref. [9]).

with $\widetilde{m}_1$ denoting the effective neutrino mass of the first generation,

$$\widetilde{m}_1 = \left[(h^\nu)^\dagger \, h^\nu\right]_{11} \frac{v_u^2}{M_1} \,. \tag{28}$$

According to the Froggatt-Nielsen flavour model that our earlier study in Ref. [9] is based on, the Higgs and (s)neutrino masses, $m_S$ and $M_1$, are expected to differ by some power of the Froggatt-Nielsen hierarchy parameter $\eta \simeq 1/\sqrt{300}$. Just as in our previous work, we shall thus assume for definiteness that $m_S = M_1/\eta^2$. This reduces the number of free and independent parameters to two, namely the two neutrino masses $M_1$ and $\widetilde{m}_1$ which then end up being in one-to-one correspondence to the two decay rates $\Gamma_S^0$ and $\Gamma_{N_1}^0$. A further important quantity, which can be determined as a function of $\Gamma_S^0$ and $\Gamma_{N_1}^0$, is the effective (s)neutrino decay rate $\Gamma_{N_1}^S$,

$$\Gamma_{N_1}^S(a) = \gamma^{-1}(a)\, \Gamma_{N_1}^0 \,, \quad \gamma^{-1}(a) = \left\langle \frac{M_1}{E_{N_1}} \right\rangle_a^{(S)} \,, \tag{29}$$

which accounts the for the fact that the (s)neutrinos which are produced with very high momenta $p_{N_1} \gg M_1$ in the decay of the $B\!-\!L$ Higgs particles remain relativistic up to their decay. Correspondingly, the factor $\gamma^{-1}$ multiplying $\Gamma_{N_1}^0$ in Eq. (29) denotes the time-dependent inverse Lorentz factor for the heavy (s)neutrinos averaged over the entire (s)neutrino phase space (cf. Ref. [8] for an explicit computation of $\gamma^{-1}$).

In order to obtain a detailed and time-resolved picture of the reheating process, one needs to solve the set of Boltzmann equations describing the evolution of all relevant particle species. Such a study has been performed in Ref. [9]. For completeness, we now recall some of the results of our earlier work (cf. Fig. 1, upper panel). A remarkable feature of reheating after the $B\!-\!L$ phase transition is an approximate plateau in the radiation temperature around the time when the heavy (s)neutrinos decay (cf. Fig. 1, lower panel). This constancy of the temperature over some extended period of time is a direct consequence of a temporary balance between entropy production and cosmic expansion. The temperature at which the plateau is located represents the characteristic temperature scale for leptogenesis as well as for the thermal production of gravitinos. It is typically larger by some $\mathcal{O}(1)$ factor than the actual reheating temperature $T_{\mathrm{RH}}$, which is reached towards the end of reheating when half of the total energy has been converted into relativistic particles, cf. Sec. 8.