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Finite-width effects in unstable-particle production at hadron colliders

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Abstract

We present a general formalism for the calculation of finite-width contributions to the differential production cross sections of unstable particles at hadron colliders. In this formalism, which employs an effective-theory description of unstable-particle production and decay, the matrix element computation is organized as a gauge-invariant expansion in powers of Γ_X/m_X, with Γ_X and m_X the width and mass of the unstable particle. This framework allows for a systematic inclusion of off-shell and non-factorizable effects whilst at the same time keeping the computational effort minimal compared to a full calculation in the complex-mass scheme. As a proof-of-concept example, we give results for an NLO calculation of top-antitop production in the $q\bar{q}$ partonic channel. As already found in a similar calculation of single-top production, the finite-width effects are small for the total cross section, as expected from the naïve counting $\sim \Gamma_t/m_t \sim 1\%$. However, they can be sizeable, in excess of 10%, close to edges of certain kinematical distributions. The dependence of the results on the mass renormalization scheme, and its implication for a precise extraction of the top-quark mass, is also discussed.

1 Introduction

The performance of the Large Hadron Collider (LHC) so far has been extremely successful, with about 5fb^{-1} of integrated luminosity collected in the 2011 run and more than $20\,\text{fb}^{-1}$ in 2012. This has led to an unprecedented accuracy in the measurements of many Standard Model (SM) cross sections and distributions, both in the electroweak and strong sectors, and also to the discovery of a new particle consistent with a SM Higgs boson. With the high-energy run which will follow the 2013 shutdown, experimental errors are bound to reduce even further, leading to higher-precision measurements and to the possible observation of new-physics effects beyond the SM. This high experimental precision clearly motivates similarly accurate theoretical predictions for cross sections and kinematical distributions, so that on the one hand the clean extraction of signals from the data is possible and, on the other hand, contributions to the backgrounds to processes of interest can be accurately constrained.

Most of the phenomenologically interesting processes at the LHC, such as W and Z boson production, top-quark production and Higgs production, not to mention beyond-the-Standard-Model (BSM) processes, like supersymmetric (SUSY) particle production, involve massive unstable particles. These particles are not asymptotic states and show up in detectors as energetic jets and leptons, often being accompanied by large transverse missing energy $\not\!\!E_T$. The necessity of precise theoretical predictions therefore raises the question of how to correctly treat the decay of the intermediate unstable particle to the physical final states. For observables which are inclusive in the final states originating from the unstable-particle decay, it is often sufficient to treat the massive particles as stable, ignoring their decay. The error associated with this approximation is formally of order Γ_X/m_X, where Γ_X and m_X are the width and mass of the unstable particle. For the aforementioned processes this corresponds to less than a few percent, i.e. typically smaller than the experimental errors. Clearly, while the stable approximation is appropriate for the total cross section, it cannot be used to predict arbitrary kinematical observables. Moreover, the error associated with this approximation could actually be numerically sizeable for new, yet undiscovered wide resonances, for example strongly-decaying SUSY particles.

A step forward towards a realistic description of production and decay of an unstable particle X is the narrow-width approximation (NWA) which is a framework commonly used in the context of high-energy calculations for hadron colliders. In the NWA the particle is produced and allowed to decay to the physical final states while remaining on shell. At next-to-leading order (NLO), radiative corrections are given by factorizable virtual and real contributions to the on-shell production and decay subprocesses. While technically only slightly more involved than the stable-top approximation, the NWA preserves spin correlations between the production and decay subprocesses, and allows for realistic kinematical cuts on the momenta of the physical final states (i.e. leptons and jets).

The NWA includes neither off-shell effects related to the virtuality of the intermediate unstable-particle propagator, nor non-factorizable corrections linking the production and decay subprocesses. Sub-resonant or non-resonant contributions, which correspond to diagrams with the correct physical final state but which involve fewer or no intermediate unstable-particle propagators, are also neglected. As in the stable approximation, these finite-width effects are expected to be small, of order Γ_X/m_X, for inclusive-enough observables. This is due to large cancellations between virtual and real non-factorizable corrections and also because of the suppression of non-resonant contributions. However, for arbitrary kinematical distributions and in particular, close to certain kinematical thresholds where the cancellations mentioned above are less effective, finite-width effects can be large. Strikingly, in Refs. [1–3] it was pointed out that the naïve expectation of the error associated with the NWA can be underestimated by an order of magnitude for BSM processes where the mass of daughter particles approaches the mass of the parent particle X. This is relevant for searches of SUSY in decay cascades, where one often observes some degree of mass degeneracy between particles in different steps of the cascade. Recently, non-negligible off-shell effects were observed even in Higgs production and decay to massive vector bosons [4], due to interferences between resonant and non-resonant contributions. Thus, it is clear that an approach that goes beyond the NWA and which includes at least the dominant finite-width effects is desirable.

A possible solution to the issue of finite-width effects is clearly the calculation of the full, gauge-invariant set of diagrams corresponding to a given physical final state. This approach includes the coherent sum of resonant and non-resonant contributions, treats the intermediate resonant particles as fully off-shell and contains both factorizable and non-factorizable corrections at NLO. Self-energy contributions can be resummed in the unstable-particle propagator in a consistent gauge-invariant way using, for example, the complex-mass scheme [5, 6]. Examples applying the complex-mass scheme to production of unstable particles at NLO include the calculations of four-fermion production at an e^+e^- collider [5], Higgs decay to vector-boson pairs [7] and two recent independent calculations of off-shell effects in $t\bar{t}$ production [8–10]. While the complex-mass scheme approach is completely general and very flexible, allowing the calculation of arbitrary kinematical distributions, the full NLO computation is technically challenging, requiring both the calculation of a much larger set of diagrams than for the corresponding on-shell process and special techniques to handle 5- or 6-point functions with complex masses.

An alternative approach to the full NLO calculation was presented in Ref. [11] and applied to processes of t-channel and s-channel single-top production [11,12]. The approach of Ref. [11] is the generalization of the effective field theory (EFT) description of *resonant*-particle production of Ref. [13], which was employed in the calculation of inclusive W-pair production at an e^+e^- collider [14,15]. The EFT calculation results in a systematic, gauge-invariant expansion of the matrix elements in powers of Γ_X/m_X, in a way which can be considered a generalization of the pole approximation [16, 17]. Compared to the full NLO calculation in the complex-mass scheme, the effective-theory approach has the advantage of identifying the terms that are relevant to achieving a given target accuracy prior to the actual calculation. This greatly reduces the complexity of the computation while at the same time allows for the inclusion of the leading off-shell and non-factorizable effects in a completely differential manner. For single-top production finite-width effects were found to be small for inclusive-enough observables, although they can be large close to the kinematical edges of some distributions. This general picture is consistent with the results found by the full NLO calculations of top-pair production [18].

In this paper we give a second example of the application of the EFT formalism of Ref. [11] and calculate the cross section for the pair-production process $q\bar{q} \to t\bar{t} \to W^+W^-b\bar{b}$. The top-pair production process has been studied extensively over the years in the stable-top and narrow-width approximations and, more recently, using the complex-mass scheme (a detailed list of references is given in Section 3). It thus represents a perfect proof-of-concept calculation by which to test the validity of the EFT formalism, extend it to more than one unstable particle and to compare it to different available approaches. The paper is organized as follows: in Section 2 we review the effective-theory formalism and introduce a treatment of real corrections which differs slightly from the one used in Ref. [11]. The calculation of the LO and NLO relevant amplitudes for the specific example of $t\bar{t}$ production is described in Section 3. In Section 4 we present results for several distributions and assess the effect of finite-width contributions by comparing the effective-theory predictions with results obtained in the NWA. In that same section we also discuss the effects of using different mass-renormalization schemes (more precisely the pole and PS schemes) on the cross section and distributions, and their possible implications for a precise extraction of the top-quark mass from data. Finally, our conclusions are given in Section 5.

2 Effective-theory description of unstable-particle production

The effective-theory framework for the description of unstable-particle production used in this work was first formulated for the total cross section in Ref. [13] and applied to the case of inclusive W^+W^- production at an e^+e^- collider in Refs. [14, 15]. The formalism was later extended to the more general case of differential cross sections and applied to single-top production at hadron colliders [11, 12]. In this section we review the main features of this approach, referring the reader to the aforementioned references for further details.

Unstable-particle effective theory is built upon the hierarchy of two scales, namely, the typical virtuality of the resonant unstable particle X, which is set by its decay width, $p_X^2 - m_X^2 \sim m_X\Gamma_X$, and the particle mass m_X. This hierarchy is encoded in the ratio $\Gamma_X/m_X \ll 1$. The latter is treated as a small parameter, δ, in line with the strong and electroweak coupling constants α_s, and α_{ew} and allows for a systematic expansion of the full matrix elements. These different expansion parameters are related by the counting scheme [1]

$$\frac{\Gamma_X}{m_X} \sim \alpha_s^2 \sim \alpha_{ew} \,. \tag{1}$$

In the following we will generically refer to any of the above parameters as δ. The expansion in δ is implemented at the Lagrangian level, replacing the (B)SM fields and interactions

[1] Note that in (1) we assume that the unstable-particle decay proceeds via electroweak decay channels, $\Gamma_X \propto \alpha_{ew}$. For resonances decaying via strong interactions the counting scheme is $\Gamma_X \sim \alpha_s \sim \sqrt{\alpha_{ew}}$.

with effective fields and vertices. The effective fields are associated with different momentum regions defined according to the scaling of their momenta with respect to the parameter δ and encode the physics at the two very different scales that characterize the production and decay process[2]. For the problem at hand these momentum regions are a *hard* region ($q_0 \sim |\vec{q}| \sim m_X$), a *soft* region ($q_0 \sim |\vec{q}| \sim m_X\delta$) and *collinear* regions ($n_i \cdot q \sim m_X\delta$, $\bar{n}_i \cdot q \sim m_X$, $q_\perp \sim m_X\sqrt{\delta}$). Here n_i, $\bar{n}_i$ are light-like vectors associated with the momenta of the external massless particles, $n_i = (1, \vec{p}_i/|\vec{p}_i|)$, $\bar{n}_i = (1, -\vec{p}_i/|\vec{p}_i|)$, and $q_\perp$ is the remaining, perpendicular component of the momentum q.

In the effective theory only low-virtuality modes with $p^2 \lesssim m_X^2\delta$ are kept as dynamical degrees of freedom, and are described by effective fields in the Lagrangian. These include, in particular, a field Φ_X to describe the resonant unstable particle X. The finite-width of the particle is resummed into the leading EFT kinetic term in a generalisation of the heavy-quark effective theory (HQET) Lagrangian in the case of a non-vanishing width [13],

$$\mathcal{L}^{(0)}_{\mathrm{EFT},kin} = 2\hat{m}_X \Phi_x^\dagger \left(iv \cdot \partial - \frac{\Omega_X}{2} \right) \Phi_X \,, \tag{2}$$

where $\hat{m}_X v$, with $v^2 = 1$, represents the large, on-shell component of the resonant-particle momentum, and $\hat{m}_X$ is the renormalized mass in a generic renormalization scheme. The coefficient Ω_X is related to the complex pole $\mu_X^2 \equiv m_X^2 - i m_X \Gamma_X$ of the full unstable-particle propagator,

$$\Omega_X = \frac{\mu_X^2 - \hat{m}_X^2}{\hat{m}_X} \,. \tag{3}$$

In the pole scheme, $\hat{m}_X = m_X$, Ω_X has the simple form $\Omega_X = -i\Gamma_X$. Additional terms in the EFT Lagrangian are given by bilinear terms for soft and collinear fields, power-suppressed corrections to (2) and terms describing the interaction of Φ_X and collinear fields with soft fields. Hard modes are not explicitly part of the effective Lagrangian and their contribution is encoded in *matching coefficients* multiplying effective interaction vertices. These can be schematically parameterized as

$$\mathcal{C}_{i,P}(\mu_X)\mathcal{F}^i_P(\Phi^\dagger_X, \phi_c, \phi_s, \partial_\mu), \quad \mathcal{C}_{j,D}(\mu_X)\mathcal{F}^j_D(\Phi_X, \phi_c, \phi_s, \partial_\mu), \quad \mathcal{C}_{k,NR}(\mu_X)\mathcal{F}^k_{NR}(\phi_c, \phi_s, \partial_\mu) \,, \tag{4}$$

where $\mathcal{F}^i_P$, $\mathcal{F}^j_D$ and $\mathcal{F}^k_{NR}$ denote functions of fields and derivatives and the indices i, j, k label different Lorentz structures. $\phi_{c,s}$ generically represent collinear and soft fields and $\mathcal{C}_{i,P}$ and $\mathcal{C}_{j,D}$ are the hard matching coefficients of the production and decay effective vertices, which are computed from on-shell SM amplitudes. In this context "on-shell" has to be understood as $p_X^2 = \hat{m}_X^2 + \hat{m}_X\Omega_X = \mu_X^2$, meaning that the effective couplings in the Lagrangian are in general complex. This is a feature that the EFT framework shares with the complex-mass scheme. The interaction terms $\mathcal{C}_{k,NR}\mathcal{F}^k_{NR}$ encode the contribution of non-resonant configurations which also contribute to the cross section starting from a certain order in δ. Note that in order to describe pair-production of unstable particles,

[2]Here and in the following we assume that the invariants $s_{ij} = 2p_i \cdot p_j$ constructed from the external momenta are of the same order of m_X^2, and we treat them as a single scale.