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5D Maximally Supersymmetric Yang-Mills in 4D Superspace: Applications

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Abstract

We reformulate 5D maximally supersymmetric Yang-Mills in 4D Superspace, for a manifold with boundaries. We emphasise certain features and conventions necessary to allow for supersymmetric model building applications. Finally we apply the holographic interpretation of a slice of AdS and show how to generate Dirac soft masses between external source fields, as well as kinetic mixing, as a boundary effective action.

1 Introduction

Convincing evidence has been shown that maximal super Yang-Mills in 5D is a good effective description of the 6D $(2,0)$ M5-brane CFT on an S^1, for example see [1, 2]. This suggestion is interesting from the perspective of beyond standard model building. Whilst we usually think of quantum field theories as effective field theories, with some perhaps unmentioned cutoff, extra dimensional models suffer more than most from their lack of UV completion as they appear, at least from power counting, to be non renormalisable. If 5D MSYM is indeed UV finite, after accounting for both the perturbative and non-perturbative spectrum, then it may be possible to build 5D models which have a self contained UV completion.

Precedents for this sort of model building are well known: The Horava-Witten construction in eleven dimensional supergravity [3] motivated studies of 5D global super Yang-Mills with boundaries [4]. Later Randall-Sundrum [5] models motivated warped or "a slice of" AdS_5 models. AdS/QCD constructions [6, 7] have also flourished as they capture the relevant local quantum field theory degrees of freedom of string

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theory, in five dimensions. Already in [1, 2] a T^2 compactification of the $(2, 0)$ theory was used to consider four dimensional physics. As the Torus is the product of two circles $T^2 = S^1 \times S^1$ with Radii R_5 and R_4, naturally the 5D MSYM action is the limit that R_5 is small relative to R_4. We would like to explore the resulting action where R_4 is an orbifold $S^1/\mathbb{Z}_2$. In 4d superspace notation this leads to a vector superfield and chiral adjoint with positive parity (V, H) and two chiral superfields with negative parity (Φ, H^c). In some sense this model is already in use for model building: whenever one wishes for a positive parity vector V and H for instance to generate Dirac soft masses between the fermions contained in those multiplets as in [8, 9], it may be natural to consider MSYM in 5D and not simply SYM in 5D. Furthermore in the examples we choose despite Φ and H^c having negative parity, they actually play a signification role, due to the equations of motion between the left and right handed fermions of each multiplet. Additional chiral adjoints such as (H_{adj}) of $SU(3)_c$ are indeed well motivated extensions which have been shown to be able to increase the Higgs mass substantially [10]. Indeed theories with more supersymmetry may fair better than models with less, as discussed in [11]. Brane constructions, such as those found in this paper, offer the possibility that matter multiplets appear in $\mathcal{N} = 1$ multiplets and gauge fields in $\mathcal{N} = 2$ multiplets, something that arises quite naturally in supersoft supersymmetry breaking.

In this paper we do not concern ourselves with the question of UV finiteness, instead wish to study this setup from the perspective of model building. A natural first step to make use of the 5D MSYM action for model building purposes is to reduce this action to 4D superspace, in two component spinor notation. Conventions are important here: we have chosen the gamma matrices of the various dimensions to be built from natural tensor products of the four dimensional Weyl representation. In this paper we will compactify on an interval $S^1/\mathbb{Z}_2$ the x_4 direction, preserving x_5 as the direction of M-theory, and hence hopefully preserving some of the UV-finiteness of this theory. In particular the choice of an interval instead of a circle allows for the possibility to break some supersymmetry, leaving a theory with only 8 supercharges from the four dimensional perspective. In addition the orbifold fixed points allows one to introduce boundary localised matter, which may break the supersymmetry down further, hopefully in which the final supersymmetry (last 4 real supercharges) can be broken dynamically.

The current paper establishes the procedure for reducing MSYM in 5D starting from 11D spinors, to four dimensional $\mathcal{N} = 1$ superspace notation, with two component spinors. The reduction of $\mathcal{N} = 1$, 5D SYM has been carried out in [4, 12, 13]. The procedure, starting from $\mathcal{N} = 1$, 7d SYM, may be found in [14]. Recent papers on supersymmetry breaking and model building using the 5D SYM action may be found in [15–19]. It is also interesting to consider their deconstruction [8, 20–23].

This structure of the paper is as follows: In section 2 we outline the maximal

super Yang-Mills action in five dimensions written using eleven dimensional spinors. In section 3 we construct the same action using symplectic Majorana spinors, which are the natural spinors of five dimensions. In section 4 we start from section 3 and take an orbifold of the fifth direction x_4, constructing the resulting lagrangian and notation for the theory in superspace. In section 5 we demonstrate an application of this setup by generating an effective action for Dirac soft masses between boundary external sources. We conclude in section 6. A very convenient set of conventions may be found in appendix A, which also briefly comments on this construction's descent from $\mathcal{N} = 1$ super Yang-Mills in seven dimensions.

2 5D maximal super Yang-Mills

In this section we outline the maximal super Yang-Mills action in five dimensions written using eleven dimensional spinors living in the spacetime of M-theory.

The 5D maximal super Yang-Mills action [2] is given by

$$S_{MSYM} = \int d^5x \mathcal{L}_{MSYM} \tag{2.1}$$

where the lagrangian $\mathcal{L}_{MSYM}$ is given by

$$\mathrm{tr}\left(-\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}D_M X^I D^M X^I + \frac{i}{2}\bar{\Psi}\Gamma^M D_M \Psi + \frac{ig_5}{2}\bar{\Psi}\Gamma^5\Gamma^I[X^I,\Psi] + \frac{g_5^2}{4}\sum_{I,J}[X^I,X^J]^2\right) \tag{2.2}$$

The indices μ, ν are 4D, $\mu = 0,1,2,3$. M, N are 5D indices with metric $\eta_{MN} = \mathrm{diag}(-1,1,1,1,1)$. A, B are the 11D indices. $I, J = 6,7,8,9,10$. The spinors are real with 32 components. $\bar{\Psi} = \Psi^\dagger\Gamma^0$. We have chosen to define $D_M = \partial_M + ig_5 A_M$, which rescales the $1/g_5^2$ inside the field strength tensor. In this non canonical normalisation[1], the mass dimensions of $[D_M, A_M, X^I, \Psi]$ are $[1, 3/2, 3/2, 2]$. Similarly $[\delta_\epsilon, \epsilon, \theta, Q]$ has $[0, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}]$. The complete conventions may be found in appendix A.

The supersymmetry transformations are given by

$$\delta_\epsilon A_M = i\bar{\epsilon}\Gamma_M\Gamma_5\Psi \tag{2.3}$$

$$\delta_\epsilon X^I = i\bar{\epsilon}\Gamma^I\Psi \tag{2.4}$$

$$\delta_\epsilon \Psi = \frac{1}{2}F_{MN}\Gamma^{MN}\Gamma_5\epsilon + D_M X^I\Gamma^M\Gamma^I\epsilon - \frac{ig_5}{2}[X^I,X^J]\Gamma^{IJ}\Gamma^5\epsilon \tag{2.5}$$

where ϵ is a supersymmetry transformation parameter. Useful identities are

$$\Gamma^{MN} = \frac{1}{2}[\Gamma^M,\Gamma^N] \tag{2.6}$$

[1] This form of the action is more suited to perturbation theory in g_5, the canonical form being most suited to finding solitonic solutions.

$$D_M X^I = \partial_M X^I - ig_5[A_M, X^I] \tag{2.7}$$

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig_5[A_N, A_M]. \tag{2.8}$$

This theory has 16 supercharges,

$$\Psi = \Psi_+ + \Psi_- \tag{2.9}$$

$$\Psi_\pm = \pm\Gamma^5\Psi_\pm \tag{2.10}$$

$$\epsilon_\pm = \mp\Gamma^5\epsilon_\pm. \tag{2.11}$$

For more details see [14]. The action is written in terms of full 11D M-theory spinors, which are unconstrained real 32-component spinors, despite only the projection of the spinor with respect to Γ_5 actually living on the M5-brane.

2.1 Important boundary terms

We will wish to study the inclusion of supersymmetric matter fields in both the orbifold and boundary perspectives. In the orbifold picture one can neglect total derivatives in satisfying the supersymmetry transformations. The are however two types of boundary conditions that must be satisfied for delta-function localised matter: primary boundary conditions make the bulk and boundary supersymmetric and secondary boundary conditions also make the boundary conditions themselves supersymmetric. These are naturally included in the superspace formulation, but must be introduced to the component action to complete supersymmetry. These terms are of particular interest as, in the boundary picture, they are the boundary terms for this action (analogous to the Gibbons-Hawking-York boundary terms of gravity as discussed in [24, 25]). For an example of completing actions with the correct supersymmetric boundary term see [26] and also for double field theory [27].

These terms play a significant role in AdS systems, where this action determines the boundary to boundary two point functions using the operator field correspondence, for a given bulk action with boundary sources [19, 28].

On a manifold with a boundary at $x_4 = 0$ one must in addition include the boundary terms (in canonical normalisation)

$$\frac{1}{g_5^2}\int_{\partial\mathcal{M}} d^4x \left(\eta^{4M}\eta^{PQ}F_{MP}A_Q - \eta^{4M}(D_M X_I)X^I + \frac{1}{2}\bar{\Psi}\Psi\right) \tag{2.12}$$

for full closure of supersymmetry of the bulk action. Strictly speaking it is actually the second two terms that are the additional boundary terms, the first term being already contained in the Yang-Mills action, but we include them together because of their equivalent roles in the boundary action. As we shall see later, the superfield notation automatically includes these boundary terms and are related to the boundary conditions of the bulk fields.

3 As symplectic Majorana spinors

In this section we define the maximal super Yang-Mills action using symplectic Majorana spinors, which are the natural objects of five dimensions. This action has and $SO(5)_R$ symmetry and no additional auxiliary fields. This action may be compared with that of $\mathcal{N}=1$ SYM [12], in which the action has an $SU(2)_R$ symmetry and a triplet of auxiliary fields X^a, under the $SU(2)_R$.

The lagrangian is given by

$$\begin{aligned}\mathcal{L}_{MSYM} = & -\frac{1}{4}F_{MN}F^{MN} - \frac{1}{2}D_M X_I D^M X^I + \frac{i}{2}\bar{\psi}^I \Gamma^M D_M \psi_I \\ & + \frac{g_5^2}{4}\sum_{a,b}[X^a, X^b]^2 + \frac{g_5}{2}\bar{\psi}^I (G^a)_I{}^J [X_a, \psi_J]\end{aligned} \tag{3.1}$$

where the trace, tr, is implicit. The I, J label the full $SP(2)$ R-symmetry, which decomposes into two copies of the usual $SU(2)$ R-symmetry for 5D symplectic Majorana spinors, labelled by i, j. Incidentally, $\bar{\epsilon}_I = (\epsilon_I)^\dagger \Gamma^0$.

We define a basis of fermions

$$\psi_I = \begin{pmatrix} \Psi^1 \\ \Psi^2 \\ \Omega^1 \\ \Omega^2 \end{pmatrix}, \qquad \left(\bar{\psi}_I\right)^T = \begin{pmatrix} \bar{\Psi}_1 \\ \bar{\Psi}_2 \\ \bar{\Omega}_1 \\ \bar{\Omega}_2 \end{pmatrix}, \qquad \epsilon_I = \begin{pmatrix} \epsilon^1 \\ \epsilon^2 \\ \xi^1 \\ \xi^2 \end{pmatrix}, \qquad (\bar{\epsilon}_I)^T = \begin{pmatrix} \bar{\epsilon}_1 \\ \bar{\epsilon}_2 \\ \bar{\xi}_1 \\ \bar{\xi}_2 \end{pmatrix}. \tag{3.2}$$

The supersymmetry transformations are given by

$$\delta_\epsilon X^a = i\bar{\epsilon}^I (G^a)_I{}^J \psi_J \tag{3.3a}$$

$$\delta_\epsilon A_M = i\bar{\epsilon}_i \gamma_M \Psi^i + i\bar{\xi}_i \gamma_M \Omega^i \tag{3.3b}$$

$$\delta\psi_I = F_{MN}\Gamma^{MN}\epsilon_I + \not{D} X^a \, (G^a)_I{}^J \epsilon_J + \frac{g_5}{2}[X_a, X_b](G^{ab})_I{}^J \epsilon_J \tag{3.3c}$$

The super algebra may be written as

$$\{Q^i_A, \bar{Q}^j_B\} = 2\Gamma^M P_M \delta^{ij} \delta_{AB} \tag{3.4}$$

$A, B = 1, 2$ where the Q's labelled $A = 1$ couple to ϵ^i and those labelled $A = 2$ to ξ^i. We label

$$X^a = X^6, \ldots, X^{10} \tag{3.5}$$

while the matrices G^a are explained below in section 3.2. We will not need the full R-symmetry in the following discussions and therefore we singled out half of the fermions in writing (3.2).

We also briefly comment that we could have written the action using unconstrained Dirac spinors (of five dimensions). The resulting action would contain

$$\mathcal{L} \supset \frac{i}{2}\bar{\lambda}\Gamma^M D_M \lambda + \frac{i}{2}\bar{\chi}\Gamma^M D_M \chi \tag{3.6}$$

where $\lambda = \Psi^1$ and $\chi = \Omega^1$ as in Eqn. (4.1). Although in this case the R-symmetry would not have been so manifest.

3.1 The boundary term

The 5D maximal super Yang-Mills boundary term with symplectic Majorana spinors is

$$\frac{1}{g_5^2}\int_{\partial\mathcal{M}} d^4x \left(G^{4M}G^{PQ}F_{MP}A_Q - \frac{1}{2}G^{4M}(D_M X_a)X^a + \frac{1}{4}\bar{\psi}_I\psi^I \right). \tag{3.7}$$

It is straightforward to determine from this the boundary terms in 2 component spinor notation.

3.2 The $SO(5)$ R-symmetry

The 5D maximally supersymmetric Yang-Mills theory has $SO(5) \cong Sp(2)$ R-symmetry. The two pairs of symplectic-Majorana fermions transform as **4** of $Sp(2)$ whereas the five scalars transform as **5** of $SO(5)$. These two representations can be related by using five-dimensional, Euclidean gamma matrices

$$(G^a)_I{}^J = \left\{ \begin{pmatrix} 0 & -i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i\sigma^1 \\ +i\sigma^1 & 0 \end{pmatrix}, \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix} \right\}. \tag{3.8}$$

They satisfy

$$\{G^a, \bar{G}^b\} = -2\delta^{ab}, \qquad a, b = 6, \ldots, 10. \tag{3.9}$$

The index a relates to the scalar components X^a for which $X^{10} \equiv \Sigma$.

4 In terms of 4D superspace

We now wish to rewrite the maximal super Yang-Mills description in terms of four dimensional superfields. For dimensional reduction to four dimensional superspace, it is more natural to first formulate a description using 5D symplectic Majorana spinors, which each decompose into two 4D Weyl spinors. This will also make the R-symmetry more manifest. The spinors of Eqn. (3.2) are given by

$$\Psi^1 = \begin{pmatrix} \lambda_{L\alpha} \\ \bar{\lambda}_R^{\dot{\alpha}} \end{pmatrix}, \quad \Omega^1 = \begin{pmatrix} \chi_{L\alpha} \\ \bar{\chi}_R^{\dot{\alpha}} \end{pmatrix}, \quad \Psi^2 = \begin{pmatrix} \lambda_{R\alpha} \\ -\bar{\lambda}_L^{\dot{\alpha}} \end{pmatrix}, \quad \Omega^2 = \begin{pmatrix} \chi_{R\alpha} \\ -\bar{\chi}_L^{\dot{\alpha}} \end{pmatrix}. \tag{4.1}$$

The reality condition defines the barred fermions by

$$\Psi^i = \epsilon^{ij}C_5\bar{\Psi}_j^T \quad \text{and} \quad \Psi^i = \epsilon^{ij}B_5\Psi_j^*. \tag{4.2}$$

where the $SU(2)_R$ symmetry indices are raised and lowered with

$$\epsilon^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{4.3}$$

The supersymmetry transformation parameters ϵ^i, ξ^i are defined similarly:

$$\epsilon^1 = \begin{pmatrix} \epsilon_{L\alpha} \\ \bar{\epsilon}_R^{\dot{\alpha}} \end{pmatrix}, \quad \xi^1 = \begin{pmatrix} \xi_{L\alpha} \\ \bar{\xi}_R^{\dot{\alpha}} \end{pmatrix}, \quad \epsilon^2 = \begin{pmatrix} \epsilon_{R\alpha} \\ -\bar{\epsilon}_L^{\dot{\alpha}} \end{pmatrix}, \quad \xi^2 = \begin{pmatrix} \xi_{R\alpha} \\ -\bar{\xi}_L^{\alpha} \end{pmatrix}. \tag{4.4}$$

For these spinors $\bar{\epsilon}_1 = (\epsilon^1)^\dagger\gamma^0 = (\epsilon_R^\alpha, \bar{\epsilon}_{L,\dot{\alpha}})$ and $\bar{\epsilon}_2 = (-\epsilon_L^\alpha, \bar{\epsilon}_{R,\dot{\alpha}})$. It will be useful to label $\Sigma \equiv X^{10}$.

We are actually interested to explore manifolds with boundaries, as they have the most useful practical applications. The presence of constant boundaries preserves only half the supersymmetry of the bulk system. As the commutator of a supersymmetry transformation generates a translation, we may define the translation parameter a^M in terms of the supersymmetry transformation parameters,

$$a^M = 2i(\bar{\epsilon}_I \Gamma^M \eta^I). \tag{4.5}$$

Allowing only $a^5 = 0$ to break translation invariance, fixes a relation between the supersymmetry transformations that

$$2i(\bar{\epsilon}_1\gamma^5\eta^1 - \bar{\epsilon}_2\gamma^5\eta^2 - \bar{\xi}_1\gamma^5\tilde{\eta}^1 + \bar{\xi}_2\gamma^5\tilde{\eta}^2) = 0 \tag{4.6}$$

$$(\epsilon_R\eta_L - \bar{\epsilon}_L\bar{\eta}_R - \epsilon_L\eta_R + \bar{\epsilon}_R\bar{\eta}_L) - (\xi_R\tilde{\eta}_L - \bar{\xi}_L\bar{\tilde{\eta}}_R - \xi_L\tilde{\eta}_R + \bar{\xi}_R\bar{\tilde{\eta}}_L) = 0 \tag{4.7}$$

This means we can either make the ϵ's be related to the ξ's i.e. $\epsilon^i = \beta\xi^i$, or we can make $\epsilon_L = \beta\epsilon_R$ and $\xi_L = \beta\xi_R$. In the first case the ϵ's would be preserved and we could set $\xi_i = 0$. To solve the coupled fermion equations of motion, it turns out to be more practical to use $\epsilon_R = \xi_R = 0$. This second case is actually more familiar as it allows for parity $(+,-)$ to be related to handedness (L,R).

Setting $\epsilon_R = \xi_R = 0$, preserves only ϵ_L and ξ_L or 8 real supercharges of $\mathcal{N} = 2$ supersymmetry. We may also temporarily set $\xi_L = 0$ to obtain $\mathcal{N} = 1$ multiplets. The positive parity fields fill a vector multiplet V and a chiral multiplet H: the field content of the preserved $\mathcal{N} = 2$ SYM. The negative parity fields fill two chiral multiplets Φ and H^c in the adjoint which amount to an $\mathcal{N} = 2$ Hypermultiplet. This matter content also has a natural descent from applying a quiver to 4D $\mathcal{N} = 4$ super Yang-Mills [29, 30].

4.1 The Lagrangian

In order to write the action for the 5D maximally supersymmetric Yang-Mills theory in the 4D superspace, we need to collect the field content in $\mathcal{N} = 1$ multiplets. In other words, we need to show that, after specialising to a $\mathcal{N} = 1$ subset of the full supersymmetry, the fields, or certain linear combinations of them, transform as components of three chiral superfields and a vector superfield.

The terms in the Lagrangian for the Vector superfield and Chiral field Φ is

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}\int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} + \frac{1}{2g_5^2}\int d^4\theta\ \mathrm{tr}Z^2 \tag{4.8}$$

Field	V	Φ	H	H^c
Parity	$+$	$-$	$+$	$-$

Table 1: *The parity assignments of the bulk fields*

with

$$Z = e^{-2g_5V}\left(\partial_4 e^{2g_5V} + ig_5\bar{\Phi}e^{2g_5V} - ig_5 e^{2g_5V}\Phi\right). \tag{4.9}$$

The field strength tensor W^α is a left handed chiral Superfield defined by $W^\alpha = -\frac{1}{4}\bar{D}^2 D^\alpha V$. The lagrangian for the additional adjoint fields are given by

$$\mathcal{L} = \int d^4\theta \mathrm{tr}\left(e^{-g_5V}H^\dagger e^{g_5V}H + e^{-g_5V}(H^c)^\dagger e^{g_5V}H^c\right)$$

$$+ \frac{1}{4}\int d^2\theta \ \mathrm{tr}\left(H^c\partial_4 H + g_5\Phi[H,H^c]\right) + c.c. \tag{4.10}$$

The gauge transformations are given by

$$e^{-V} \to e^{-\Lambda}e^{-V}e^{-\Lambda^\dagger} \quad , \quad H^\dagger \to e^{\Lambda^\dagger}H^\dagger e^{-\Lambda^\dagger}. \tag{4.11}$$

The vector superfield in Wess-Zumino gauge is

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda}_L - i\bar{\theta}^2\theta\lambda_L + \frac{1}{2}\theta^2\bar{\theta}^2(D) \tag{4.12}$$

where $D = D_4\Sigma = D_4X^{10}$. The adjoint chiral Superfield is

$$\Phi = (\Sigma + iA_4) + \sqrt{2}\theta(-i\sqrt{2}\lambda_R) + \theta^2(F_\Phi) \tag{4.13}$$

These field assignments within the multiplet are determined by the preserved supersymmetry transformations, below. We wish to choose the boundary conditions following table 1, and so have chosen to preserve ϵ_L and ξ_L and set $\epsilon_R = \xi_R = 0$ and then $P(\partial_5) = -1$ as normal. The resulting 4D $\mathcal{N} = 2$ vector multiplet is the combination $V + H$ instead of $V + \Phi$.

It is also instructive to see that if we choose to preserve only ϵ_L and set $\epsilon_R = 0$ we may then break the symmetry down to $\mathcal{N} = 1$ SYM and three adjoint chiral superfields. In the orbifold direction x_4 there is still a residual gauge transformation and we can construct a super gauge covariant derivative operator in this direction

$$\nabla_4 \equiv \partial_4 + g_5\Phi \ \text{ where } \ \nabla_4(\cdot) \equiv \partial_4(\cdot) + g_5\Phi(\cdot) - g_5(\cdot)\Phi, \tag{4.14}$$

when acting on chiral objects and

$$\nabla_4(\cdot) \equiv \partial_4(\cdot) - g_5\Phi^\dagger(\cdot) - g_5(\cdot)\Phi, \tag{4.15}$$

when acting on real linear objects e.g. $(\cdot) = e^{2V}$. The chiral super field strength tensor is

$$W_\alpha = -i\lambda_L + \sqrt{2}\theta_\beta(\delta^\beta_\alpha D - (\sigma^{\mu\nu})^\beta_\alpha F_{\mu\nu}) + \theta^2\sigma^\mu\partial_\mu\bar{\lambda}_L \tag{4.16}$$

and the additional adjoint fields are

$$H = (X^8 + iX^9) + \sqrt{2}\theta(-i\sqrt{2}\chi_L) + \theta^2(F_H) \tag{4.17}$$

$$H^c = (X^6 + iX^7) + \sqrt{2}\theta(\sqrt{2}\chi_R) + \theta^2(F_{H^c}). \tag{4.18}$$

The F-terms of the chiral fields are given by

$$F^\dagger_\Phi = -\frac{1}{2}g_5[(X^6 + iX^7), (X^8 + iX^9)] \tag{4.19}$$

$$F^\dagger_H = -\frac{1}{2}\left[\partial_4(X^6 + iX^7) + g_5[(\Sigma + iA_4), (X^6 + iX^7)]\right] \tag{4.20}$$

$$F^\dagger_{H^c} = -\frac{1}{2}\left[\partial_4(X^8 + iX^9) + g_5[(\Sigma + iA_4), (X^8 + iX^9)]\right]. \tag{4.21}$$

If one chooses $\xi_L = \xi_R = \bar{\xi}_L = \bar{\xi}_R = \epsilon_R = \bar{\epsilon}_R = 0$ and considers only the supersymmetry transformations parameterised by ϵ_L and $\bar{\epsilon}_L$, in other words only $\mathcal{N} = 1$ in four dimensions, the general transformations (3.3) for the scalars reduces to

$\mathcal{N} = 2$	$\mathcal{N} = 1$	
$\delta X^6 = \chi_R\epsilon_L + \bar{\chi}_R\bar{\epsilon}_L - \bar{\lambda}_R\bar{\xi}_L - \lambda_R\xi_L$	$\delta_{\epsilon_L} X^6 = \chi_R\epsilon_L + \bar{\chi}_R\bar{\epsilon}_L$	(4.22a)
$\delta X^7 = i(\bar{\chi}_R\bar{\epsilon}_L - \chi_R\epsilon_L + \bar{\lambda}_R\bar{\xi}_L - \lambda_R\xi_L)$	$\delta_{\epsilon_L} X^7 = -i\chi_R\epsilon_L + i\bar{\chi}_R\bar{\epsilon}_L$	(4.22b)
$\delta X^8 = i(\bar{\chi}_L\bar{\epsilon}_L - \chi_L\epsilon_L - \bar{\lambda}_L\bar{\xi}_L + \lambda_L\xi_L)$	$\delta_{\epsilon_L} X^8 = -i\chi_L\epsilon_L + i\bar{\chi}_L\bar{\epsilon}_L$	(4.22c)
$\delta X^9 = -\bar{\chi}_L\bar{\epsilon}_L - \chi_L\epsilon_L + \bar{\lambda}_L\bar{\xi}_L + \lambda_L\xi_L$	$\delta_{\epsilon_L} X^9 = -\chi_L\epsilon_L - \bar{\chi}_L\bar{\epsilon}_L$	(4.22d)
$\delta\Sigma = i(\bar{\epsilon}_L\bar{\lambda}_R - \epsilon_L\lambda_R - \bar{\chi}_R\bar{\xi}_L + \chi_R\xi_L),$	$\delta_{\epsilon_L}\Sigma = -i\lambda_R\epsilon_L + i\bar{\lambda}_R\bar{\epsilon}_L.$	(4.22e)

This gives some natural combinations under $\mathcal{N} = 2$

$$\delta_\epsilon(X^6 + iX^7) = 2(\chi_L\epsilon_L - \bar{\lambda}_R\bar{\xi}_L) \tag{4.23}$$

$$\delta_\epsilon(X^8 + iX^9) = 2i(\lambda_L\xi_L - \chi_L\epsilon_L) \tag{4.24}$$

The gauge field transforms as

$\mathcal{N} = 2$	$\mathcal{N} = 1$	
$\delta_{\epsilon_L} A_\mu = i(\epsilon_L\sigma^\mu\bar{\lambda}_L + \bar{\epsilon}_L\bar{\sigma}^\mu\lambda_L + \bar{\xi}_L\bar{\sigma}^\mu\chi_L + \xi_L\sigma^\mu\bar{\chi}_L)$	$\delta_{\epsilon_L} A_\mu = i\epsilon_L\sigma_\mu\bar{\lambda}_L + i\bar{\epsilon}_L\bar{\sigma}_\mu\lambda_L$	(4.25a)
$\delta_\epsilon A_4 = -(\bar{\xi}_L\bar{\chi}_R + \xi_L\chi_R) - (\bar{\epsilon}_L\bar{\lambda}_R + \epsilon_L\lambda_R)$	$\delta_{\epsilon_L} A_4 = -(\epsilon_L\lambda_R + \bar{\epsilon}_L\bar{\lambda}_R)$	(4.25b)

The fermions transform under $\mathcal{N}=1$ as

$$\delta_{\epsilon_L}\lambda_L = F_{\mu\nu}\sigma^{\mu\nu}\epsilon_L - iD_4\Sigma\epsilon_L + 2[X^6, X^7]\epsilon_L + 2[X^8, X^9]\epsilon_L \tag{4.26a}$$

$$\delta_{\epsilon_L}\lambda_R = -i\sigma^\mu F_{\mu 4}\bar{\epsilon}_L - D_\mu\Sigma\sigma^\mu\bar{\epsilon}_L + 2[X^6 - iX^7, X^8 - iX^9]\epsilon_L \tag{4.26b}$$

$$\delta_{\epsilon_L}\chi_L = -\sigma^\mu D_\mu X^8\bar{\epsilon}_L - i\sigma^\mu D_\mu X^9\bar{\epsilon}_L - 2[X^6 - iX^7, \Sigma]\epsilon_L + D_4(X^6 - iX^7)]\epsilon_L \tag{4.26c}$$

$$\delta_{\epsilon_L}\chi_R = i\sigma^\mu D_\mu X^6\bar{\epsilon}_L - \sigma^\mu D_\mu X^7\bar{\epsilon}_L - 2i[X^8 - iX^9, \Sigma]\epsilon_L + iD_4(X^8 - iX^9)]\epsilon_L. \tag{4.26d}$$

Under $\mathcal{N}=2$ the fermions transform as

$$\begin{aligned}\delta_\epsilon\lambda_L = {} & +F_{\mu\nu}\sigma^{\mu\nu}\epsilon_L - iD_4\Sigma\epsilon_L - 2[X^6, X^7]\epsilon_L + 2[X^8, X^9]\epsilon_L + \\ & \xi_L(D_4X^6 - iD_4X^7) + \sigma^\mu\bar{\xi}_L D_\mu X^8 + i\sigma^\mu\bar{\xi}_L D_\mu X^9 + 2\xi_L[X^6, \Sigma] - 2i\xi_L[X^7, \Sigma]\end{aligned} \tag{4.27a}$$

$$\begin{aligned}\delta_\epsilon\lambda_R = {} & i\sigma^\mu F_{4\mu}\bar{\epsilon}_L - \sigma^\mu D_\mu\Sigma\bar{\epsilon}_L + D_\mu\Sigma\left(\sigma^\mu\bar{\epsilon}_L\right) - \frac{1}{2}[X^6 - iX^7, X^8 + iX^9]\epsilon_L \\ & +\xi_L D_4X^8 + \sigma^\mu\bar{\xi}_L D_\mu X^6 - i\sigma^\mu\bar{\xi}_l D_\mu X^7 - i\xi_L D_5X^9 - 2\xi_L[X^8 + iX^9, \Sigma]\end{aligned} \tag{4.27b}$$

$$\begin{aligned}\delta_\epsilon\chi_L = {} & D_5(X^6 - iX^7)\epsilon_L - \frac{1}{2}[X^6 - iX^7, \Sigma]\epsilon_L - \frac{i}{2}[\Sigma, X^8 - iX^9]\epsilon_L \\ & i\xi_L D_4\Sigma + \xi_L(F_{\mu\nu}\sigma^{\mu\nu} - 2[X^6, X^7] + 2[X^8, X^9])\end{aligned} \tag{4.27c}$$

$$\begin{aligned}\delta_\epsilon\chi_R = {} & iD_\mu(X^6 + iX^7)\sigma^\mu\bar{\epsilon}_L - \frac{i}{2}[\Sigma, X^8 + iX^9]\epsilon_L + D_5(X^6 + iX^7)\epsilon_R \\ & - iF_{\mu 4}\sigma^\mu\bar{\xi}_L + \sigma^\mu\bar{\xi}_L D_\mu X^6 + 2\xi_L([X^6, X^8 - iX^9] + [X^7, X^8 + iX^9]).\end{aligned} \tag{4.27d}$$

This completes our analysis of the orbifolded MSYM theory reduced to $\mathcal{N}=2$ in four dimensional superspace. Additional conventions may be found in the appendix. The primary purpose of this detailed exposition of the orbifolded MSYM action was that it may have future model building applications. So next we change tone slightly and demonstrate an application of this setup.

5 The boundary effective action

In this section we explore applications of maximal super Yang Mills on a five dimensional orbifold. There are likely to be many uses to the construction of the MSYSM with an orbifold, but we wish to discuss one particular example that makes application also of the boundary terms. In addition all fields V, H and Φ, H^c, with both positive and negative parity play an important role. The application is to supersymmetry breaking in gauge mediation.

In this section we will imagine that the sector that breaks supersymmetry is a strongly coupled system and admits something analogous to an AdS dual. In addition, for gauge mediated supersymmetry breaking, we expect that this strongly coupled

sector has some subset that is charged under the standard model gauge groups. As envisaged in [16, 17, 19], we may wish to imagine a situation in which there is a weakly gauged global symmetry $SU(N_F)$ of the strongly coupled system, which will be identified with the standard model gauge groups (or some GUT embedding). We will model this system with a bulk slice of AdS_5 with extra dimension ranging $L_0 < z < L_1$. In those papers $\mathcal{N} = 1$ super Yang-Mills is considered for the bulk fields. Here we extend this to maximal super Yang-Mills and focus here only on new results not contained in [19]. The metric is given by

$$ds^2 = a^2(z)(\eta^{\mu\nu} dx_\mu dx_\nu + dz^2) \quad \text{where} \quad a^2(z) = \left(\frac{R}{z}\right)^2. \tag{5.1}$$

The action of this paper may be extended to warped or AdS space following [31–34]. In particular, consistency with AdS space means that the action contains

$$\mathcal{L}_{AdS} \supset \left(m_\psi \bar{\psi}^I \psi_I + \frac{1}{2} m^2_{X^I} X^I X_I \right). \tag{5.2}$$

These mass terms have

$$m_\psi R = c \quad \text{with} \quad \Delta = \frac{3}{2} + |c + \frac{1}{2}| \tag{5.3}$$

Δ is the scaling dimension and c is a real number which controls the localisation of the field profiles in the z direction ($c = 1/2$ is flat). For the scalars one finds

$$m^2_{X^I} R = c^2 + c - \frac{15}{4} \tag{5.4}$$

where $c \to -c$ is also possible [33], although $\Delta = 2$.

The boundary terms of the MSYM theory will now play an important role for us in generating a boundary effective action. The boundary values of the bulk gauge fields are the sources and we wish to compute the tree level effective action, essentially the correlators of operators that couple to these sources:

$$\langle O(p) O(-p) \rangle = \mathrm{Lim}_{L_0 \to 0} (p^2 \Pi(p^2) + \text{UV counter terms}). \tag{5.5}$$

p, q are four dimensional momentum. For this model, the sources are

$$A^0_\mu(x), \lambda^0_{\alpha,L}(x), D^0(x), \chi^0_{\alpha,L}(x), \phi^0_6(x), \phi^0_7(x), \phi^0_8(x), \phi^0_9(x), \tag{5.6}$$

where x is the four dimensional position and the scalar sources are given by

$$\phi^0_{6,7} = \partial_z X^{6,7}|_{z=L_0} \quad D^0 = \partial_z X^{10}|_{z=L_0} \quad \text{and} \quad \phi^0_{8,9}| = X^{8,9}|_{z=L_0}. \tag{5.7}$$

The other fermions λ_R, χ_R are free to vary [19], it is just the sources that are fixed. A^0, D^0, λ^0_L and χ^0_L have even parity. These source fields are sources for (non) CFT operators. As well as the identification of operators and fields found in [19], the additional scalars and fermions are identified as in table 2, where in this instance the L

4D: operator		Field	Δ	m^2
$\mathcal{O}^I(x)$	$\rightarrow$	$X^I(x,z)$	2	-4
$\mathcal{O}^\alpha(x)_L$	$\rightarrow$	$\chi^\alpha(x,z)_L$	5/2 or 3/2	1/2

Table 2: *Operators corresponding to the bulk fields of the model.*

on the fermion labels parity under γ^4 Eqn. (A.14) (and should not be confused with a flavour symmetry label). The non vanishing boundary terms at $z = L_0$ are

$$\frac{1}{g_5^2}\int \frac{d^4p}{(2\pi)^4}\left[\frac{a(z)}{2}\left(\eta^{\mu\nu}A_\mu(p,z)\partial_z A_v(-p,z) - 2\eta^{\mu\nu}A_\mu(p,z)\partial_\nu A_5(-p,z)\right)\right] +$$

$$\frac{1}{g_5^2}\int \frac{d^4p}{(2\pi)^4}\left[ia^3(z)(\partial_z X_I)X^I + a^4(z)\left(\lambda_L\lambda_R + \chi_L\chi_R + \bar{\lambda}_L\bar{\lambda}_R + \bar{\chi}_L\bar{\chi}_R\right)\right]. \qquad (5.8)$$

with $X^I, I = 6,7,8,9,10$ and $X^{10} = \Sigma$. One may compute the effective actions for these fields, taking into account the bulk to boundary field profiles and canonical normalisation,

$$\lambda^0 \rightarrow \lambda^0 a^{3/2}(L_0) \quad , \quad \phi^0 \rightarrow \phi^0 a(L_0) \quad , \quad A^0 \rightarrow A^0. \qquad (5.9)$$

In particular the bulk fermion field λ_L is coupled to λ_R

$$\sigma^\mu_{\alpha\dot{\alpha}} p_\mu \bar{\lambda}^0_R = p\frac{Q_-(q,L_0)}{Q_+(q,L_0)}\lambda^0_L \qquad (5.10)$$

where $Q_\pm(q,z)$ are bulk profile functions that are solutions to the bulk fermion equations of motion. The bulk fields may be decomposed in terms of a source and a bulk profile

$$(\{\lambda_L,\chi_L\},\{\lambda_R,\chi_R\})(q,z) = \frac{1}{Q_\pm(q,L_0)}(\{\lambda^0_L,\chi^0_L\}_+,\{\lambda^0_R,\chi^0_R\}_-)Q_\pm(q,z), \qquad (5.11)$$

suitably normalised by the boundary value of the profile function. A particular solution is

$$Q_+(q,z) = z^{5/2}\left[J_\alpha(qz)Y_\beta(qL_1) - J_\beta(qL_1)Y_\alpha(pz)\right], \qquad (5.12)$$

with $\alpha = c + 1/2$ and $\beta = \alpha - 1$. When this is carried out for all the bulk fields one obtains a boundary effective action

$$\int\frac{d^4q}{(2\pi)^4}\left[\Pi_1(q^2)F_0^{\mu\nu}F_{0,\mu\nu} - i\Pi^{\lambda_L}_{1/2}(q^2)\lambda^0_L\sigma^\mu\partial_\mu\bar{\lambda}^0_L - i\Pi^{\chi_L}_{1/2}(q^2)\chi^0_L\sigma^\mu\partial_\mu\bar{\chi}^0_L + \frac{1}{2}\Pi_0(q^2)D_0^2\right]$$

$$+\int\frac{d^4q}{(2\pi)^4}\left[\Pi(q^2)_{\phi_6}(\phi^0_6)^2 + \Pi(q^2)_{\phi_7}(\phi^0_7)^2 + \Pi(q^2)_{\phi_8}(\phi^0_8)^2 + \Pi(q^2)_{\phi_9}(\phi^0_9)^2\right]. \qquad (5.13)$$

One should interpret $(\phi^0)^2$ in the above to mean $\phi^0(q)\phi^0(-q)$ etc. This effective action could not be generated without the additional fields such as λ_R, χ_R contained in the

MSYSM action. This is the boundary action for a four dimensional $\mathcal{N} = 2$ SYM $SU(N_F)$ flavour symmetry, in a large N_c strongly coupled system.

The actual form of the correlators turn out to be similar to those found in [19]. As we have not broken supersymmetry yet, these boundary to boundary correlators of the tree level effective action should cancel, in sets for instance

$$\left[\Pi_0(q^2) - 4\Pi^{\lambda_L}_{1/2}(q^2) + 3\Pi_1(q^2)\right] \equiv 0. \tag{5.14}$$

But now there is also

$$\Pi^{\chi_L}_{1/2}(q^2) \quad \text{and} \quad \Pi_0(q^2)_{\phi_I} \tag{5.15}$$

The general form is

$$\Pi^{\chi_L}_{1/2}(q^2) = \frac{a(z)}{q g_5^2}\left(\frac{Q_-(q,z)}{Q_+(q,L_0)}\right)_{z=L_0}. \tag{5.16}$$

where the tree level matching condition is given by

$$\frac{R}{g_5^2} = \frac{N_c}{12\pi^2}. \tag{5.17}$$

It is also interesting to consider that the source-field construction on the UV boundary may be written in superspace,

$$a^3(L_0)\int d^4x \int d^2\theta \left[\Phi^0\Phi(L_0) + H^c(L_0)H^0 + H(L_0)H^{c,0}\right] \tag{5.18}$$

which also incorporate F^0 term sources, for instance

$$\int d^2\theta \left[H^c H^0\right] = F_{\mathcal{O}}(\phi_8^0 + i\phi_9^0) + \mathcal{O}^\alpha \chi^0_{L,\alpha} + \mathcal{O}F_H^0 \tag{5.19}$$

although for our use we find it easier to work with components.

5.1 Dirac masses from a strongly coupled system

These results have some interesting applications. Not only does it extend the work of [7,35] in exploring how one may use 5D supersymmetric models to study 4D strongly coupled systems, importantly this setup will allow for Dirac soft masses [8, 36–39] to arise in AdS_5 between the external source fields λ_L^0 and χ_L^0 (contained in V and H_{adj}). Extending the current correlator programme of [38], we take an IR localised superpotential

$$W = Y_X H_{adj}\mathcal{J}_2 \tag{5.20}$$

with $\mathcal{J}_2(y) = J_2 + \sqrt{2}\vartheta j_2 + \vartheta^2 F_2$, a chiral superfield made of operators that may live in the bulk or IR boundary. Applying [19], a Dirac soft mass can be interpreted as additional terms in a supersymmetry breaking effective action on the UV boundary, as well as interesting kinetic mixing terms:

$$S_{eff} \supset \int \frac{d^4p}{(2\pi)^4}\left[g_{SM}\tilde{Y}_X M \tilde{H}_{1/2}(p^2)\lambda_L^0\chi_L^0 - i g_{SM}\tilde{Y}_X \tilde{G}_{1/2}(p^2)\chi_L^0\sigma_\mu\partial^\mu\bar{\lambda}_L^0\right] \tag{5.21}$$

$$+\int \frac{d^4p}{(2\pi)^4}\left[-\frac{\tilde{Y}_X^2}{2}M\tilde{I}_{1/2}(p^2)\chi_L^0\chi_L^0-\frac{i}{2}\tilde{Y}_X^2\tilde{E}_{1/2}(p^2)\chi_L^0\sigma^\mu\partial_\mu\bar{\chi}_L^0\right]+c.c. \tag{5.22}$$

The fields have been canonically normalised. We expect that these soft masses are taken in the limit $p^2 \to 0$, such that any dressing functions $\Lambda(p^2)$ (which may incidentally be aborbed into the definition of the Yukawa $\tilde{Y}_X$), associated with the intermediate states, will not suppress this result: $\Lambda(0) = 1$ typically. In summary, this effective action is found by integrating out the bulk states and generate effective Dirac masses between external source fields of fermions $\lambda_L^0$and χ_L^0 .

5.2 Cross sections for Dirac soft masses

In [40] cross sections of visible to hidden sector matter were considered for a straightforward messenger model. These techniques may be applied to the correlators associated with Dirac soft masses, and moreover, if the hidden sector is strongly coupled these correlators may develop certain form factors as documented in [19, 41]. For these reasons we explore the cross-sections for the Dirac soft mass correlators. In general these cross sections will be valid for both $\sigma(\lambda_L \to \text{hidden})$ or $\sigma(\chi_L \to \text{hidden})$.

If a hidden U(1) gauge field develops a vev $W'_\alpha = \theta_\alpha D$ the messenger fields $Q, \tilde{Q}$ with opposite charges under the U(1) are split $M^2 \pm D$ with a current [38]

$$\mathcal{J}_2 = Q\tilde{Q} \tag{5.23}$$

coupled to an adjoint chiral superfield

$$W \supset Y_X H_{adj}\mathcal{J}_2 \tag{5.24}$$

then one obtains a Dirac soft mass between λ_L and χ_L

$$m_D = gY_X M\tilde{H}_{1/2}(0). \tag{5.25}$$

As the function $\tilde{H}_{1/2}(p^2/M^2)$ is structurally the same as $M\tilde{B}_{1/2}$ we may compute the cross section to be

$$\sigma_D = \frac{(4\pi)\alpha_X}{m_0 s}\text{Im}\left[iM\tilde{H}_{1/2}(s)\right] = \frac{(4\pi)\alpha_X}{m_0 s}\frac{1}{2i}\text{Disc}\left[iM\tilde{H}_{1/2}(s)\right] \tag{5.26}$$

where we have defined $\alpha_X = g_{SM}Y_X/4\pi$ and

$$\text{Disc}\left[iM\tilde{H}_{1/2}(s)\right] = \frac{m_0}{4\pi s}\lambda^{1/2}(s, m_0^2, m_+^2)\theta(s-(m_0+m_+)^2) - (m_+ \to m_-) \tag{5.27}$$

In the result above we have used some notation. We have introduce the 'triangle function':

$$\lambda(s, m_1^2, m_2^2) = 4s|\mathbf{p}|^2 = (s^2 + m_1^4 + m_2^4) - 2sm_1^2 - 2sm_2^2 - 2m_1^2m_2^2 \tag{5.28}$$

where s is the centre of mass energy squared Mandelstam variable $s = (p_1 + p_2)^2$. Multiplying by

$$\theta(s - (m_1 + m_2)^2) \tag{5.29}$$

which is equivalent to

$$\lambda^{1/2}(s, m_1^2, m_2^2) = 2\sqrt{s}|\vec{p^{\,}}| = \sqrt{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]}\theta(s - (m_1 + m_2)^2). \tag{5.30}$$

For equal mass scattering it has a very simple form

$$\lambda^{1/2}(s, m, m) = [s(s - 4m^2)]^{1/2} = s\sqrt{1 - \frac{4m^2}{s}} \tag{5.31}$$

and for unequal masses with $m_0 = M$, $\quad m_\pm^2 = M^2 \pm F$ (or similarly, $m_\pm^2 = M^2 \pm D$) one gets

$$\lambda^{1/2}(s, m_0, m_+) = \sqrt{s^2 + F^2 - 2s(m_0^2 + m_+^2)} \tag{5.32}$$

$$\lambda^{1/2}(s, m_0, m_+) = \sqrt{s^2 + F^2 - 2s(m_0^2 + m_-^2)} \tag{5.33}$$

$$\lambda^{1/2}(s, m_-, m_+) = \sqrt{s^2 - 4m_0^2 s + 4F^2}. \tag{5.34}$$

Conversely, for a model with an R-symmetric F term [38], the Dirac mass is given by

$$\tilde{H}_{1/2}(p^2/M^2) = \frac{1}{\sqrt{2}} M \cos(\xi/v) \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q+p)^2 + M^2} \tag{5.35}$$

$$\times \left(\frac{1}{q^2 + m_+^2} + \frac{1}{q^2 + m_-^2} - \frac{2}{q^2 + M^2} \right).$$

This will give a cross section

$$\text{Disc}\left[iM\tilde{H}_{1/2}(s) \right] = \tag{5.36}$$

$$\frac{\cos(\xi/v)}{\sqrt{2}} \frac{M}{4\pi s} \left[\lambda^{1/2}(s, m_0^2, m_+^2)\theta + \lambda^{1/2}(s, m_0^2, m_-^2)\theta - 2\lambda^{1/2}(s, m_0^2, m_0^2)\theta \right].$$

These cross sections may be dressed by the appropriate form factor squared $|F(s)|^2$

$$F(s) = \frac{Q_-(s, L_1)}{Q_+(s, L_0)} = -g_5 \sum_{n=1} \frac{F_n \psi_n(L_1)}{s - m_n^2} \tag{5.37}$$

associated with the bulk to boundary propagator between the UV boundary and the IR brane. We hope that these ideas have aided in extending the Dirac soft mass programme [8, 38] also to certain strongly coupled systems.

6 Discussion and conclusions

In this paper we have determined the superspace formulation of maximal super Yang-Mills in five dimensions, on a manifold with boundaries. This type of setup perhaps offers a new resolution of the issue of non renormalisability of five dimensional model building [1, 2]. In addition it allows for a positive parity vector V and chiral field H_{adj}, consistent with supersymmetry and dimensional reduction, which may allow for Dirac soft masses in a natural manner.

In addition we have looked at the boundary terms that result from closure of supersymmetry of the action. This motivated us to discuss various examples where the boundary action, and necessity of descending from the full MSYM, allows one to compute the complete boundary to boundary correlators and Dirac soft masses between external sources, in a slice of AdS_5.

In particular, by applying the holographic interpretation of this setup, we have shown that some fermionic source fields λ_L^0 and χ_L^0 which couple to some (non) CFT operators $\mathcal{O}_{L,R}$, may develop Dirac soft masses after the bulk action is completely integrated out. This setup can be straightforwardly extended to more general or more complicated AdS models. It is natural to also write this action in a fully warped superspace following [32–34]. It could also be possible to construct an entirely four dimensional quiver Deconstruction of this setup, which is likely to be more palatable to some. Finally it might be interesting to see M-theory play a more prominent role in phenomenology.

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A Conventions and spinors

In this appendix we outline the conventions used. The indices μ, ν are 4D , $\mu = 0, 1, 2, 3$. M, N are 5D indices with metric $\eta_{MN} = \text{diag}(-1, 1, 1, 1, 1)$. A, B are the 11D indices. $I, J = 6, 7, 8, 9, 10$. In particular the $\Gamma^{\hat{M}\hat{N}}$ multiply 7d spinors, the γ^{MN} multiply symplectic Majorana spinors and the σ^{MN} multiply 2-component spinors.

We define the 11d Gamma matrices to satisfy the Clifford algebra

$$\{\Gamma^A, \Gamma^B\} = -2\eta^{AB}\mathbb{1} \tag{A.1}$$

$$(\Gamma^A)^T = C_{11}\Gamma^A C_{11}^{-1} \tag{A.2}$$

$$(\Gamma^A)^* = B_{11}\Gamma^A B_{11}^{-1} \tag{A.3}$$

$$B_{11} = C_{11}\Gamma^0. \tag{A.4}$$

To dimensionally reduce to four dimensions we choose the explicit representation

$$\Gamma^M = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \gamma^M \tag{A.5}$$

$$\Gamma^6 = \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \otimes \mathbb{1}_4 \tag{A.6}$$

$$\Gamma^7 = \sigma_3 \otimes \sigma_3 \otimes \sigma_2 \otimes \mathbb{1}_4 \tag{A.7}$$

$$\Gamma^8 = \sigma_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_4 \tag{A.8}$$

$$\Gamma^9 = -\sigma_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_4 \tag{A.9}$$

$$\Gamma^{10} = \sigma_3 \otimes \sigma_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_4 \tag{A.10}$$

$$\Gamma^5 = \sigma_3 \otimes \sigma_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_4. \tag{A.11}$$

Γ^5 should satisfy:

$$\Gamma^5 = i\Gamma^0\Gamma^1...\Gamma^9\Gamma^{10}. \tag{A.12}$$

In two component spinor notation

$$\gamma^M = \left(\begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot{\alpha}} \\ \bar{\sigma}^{\mu\dot{\alpha}\alpha} & 0 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \right), \quad \text{and} \quad C_5 = \begin{pmatrix} -\epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \tag{A.13}$$

where $\sigma^\mu_{\alpha\dot{\alpha}} = (1, \vec{\sigma})$ and $\bar{\sigma}^{\mu\dot{\alpha}\alpha} = (1, -\vec{\sigma})$. $\alpha, \dot{\alpha}$ are spinor indices of $\mathrm{SL}(2, C)$. The $\gamma^4_{5d} = -i\gamma^5_{4d}$ where explicitly

$$\gamma^5_{4d} = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix}. \tag{A.14}$$

This may also be written as $\gamma^5_{4d} = i\gamma^0\gamma^1\gamma^2\gamma^4\gamma^5$. C_5 is the 5d charge conjugation matrix such that

$$C_5\gamma^M C_5^{-1} = \left(\gamma^M\right)^T. \tag{A.15}$$

We may also define the complex conjugation matrix

$$B_5 = C_5\gamma^0 = \begin{pmatrix} 0 & -\epsilon^{\alpha\beta} \\ \epsilon_{\dot{\alpha}\dot{\beta}} & 0 \end{pmatrix}. \tag{A.16}$$

We include also the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.17}$$

such that one may verify the tensor products.

A.1 Seven dimensions

For obtaining the 5d Lagrangian from seven-dimensional $\mathcal{N}=1$ supersymmetric Yang-Mills theory [14], the following set of gamma matrices is particularly convenient:

$$\Gamma^{\hat{M}} = \left\{\sigma^3 \otimes \gamma^M,\, \sigma^1 \otimes \mathbb{1}_4,\, \sigma^2 \otimes \mathbb{1}_4\right\}. \tag{A.18}$$

Note that the five dimensional matrices are embedded in the first five components of $\Gamma^{\hat{M}}$. Moreover, Γ^5 and Γ^6 act as the identity operator to the two halves of the eight-component spinors of seven dimensions.

Symplectic Majorana spinors in seven dimensions are defined as follows:

$$\psi_I = \epsilon_{IJ} C_7 \left(\bar{\psi}^T\right)^J, \quad \psi_I = \epsilon_{IJ} B_7 \left(\psi^*\right)^J, \tag{A.19}$$

with the seven-dimensional charge conjugation matrix being

$$C_7 = i\Gamma^{\hat{0}}\Gamma^{\hat{2}}\Gamma^{\hat{4}}\Gamma^{\hat{5}} = \sigma^2 \otimes \mathbb{1}_2 \otimes \sigma^2 = -i\sigma^2 \otimes C_5 \tag{A.20}$$

and complex conjugation is defined as

$$B_7 = C_7\Gamma^{\hat{0}}. \tag{A.21}$$

In this basis, a pair of seven-dimensional symplectic Majorana spinors satisfying (A.19) have the form

$$\psi_1 = \begin{pmatrix} \Omega^1 \\ \Psi^1 \end{pmatrix}, \qquad \psi_2 = \begin{pmatrix} \Psi^2 \\ \Omega^2 \end{pmatrix}, \qquad \epsilon_1 = \begin{pmatrix} \xi^1 \\ \epsilon^1 \end{pmatrix}, \qquad \epsilon_2 = \begin{pmatrix} \epsilon^2 \\ \xi^2 \end{pmatrix}, \tag{A.22}$$

where (Ω^1, Ω^2) and (Ψ^1, Ψ^2) are pairs of four component spinors, separately satisfying the five-dimensional symplectic Majorana condition:

$$\Omega^i = \epsilon^{ij} C \bar{\Omega}^T_j, \qquad \Psi^i = \epsilon^{ij} C \bar{\Psi}^T_j. \tag{A.23}$$

The 7d supersymmetry parameters may be written

$$(\bar{\eta}^1)^T = \begin{pmatrix} \bar{\xi}_1 \\ -\bar{\epsilon}_1 \end{pmatrix}, \qquad (\bar{\eta}^2)^T = \begin{pmatrix} \bar{\epsilon}_2 \\ -\bar{\xi}_2 \end{pmatrix}. \tag{A.24}$$

It is useful to identify the degrees of freedom of the 7d and 5d theories

$$(B_1 + iB_2) = (X_8 + iX_9) \;\text{ and }\; B_3 = X_{10} = \Sigma, \tag{A.25}$$

where Σ is a real adjoint scalar matching the notation of $\mathcal{N}=1$ 5d SYM. The seven dimensional SYM boundary action (of x_4) is given by

$$\frac{1}{g_5^2}\int d^4x \left(G^{4M}G^{PQ}F_{MP}A_Q - \frac{1}{2}G^{4M}(D_M B_i)B^i + \frac{1}{4}\bar{\psi}^I\psi_I\right) \tag{A.26}$$

which are naturally contained in the superfield, but not the component action of [14].

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