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Nucleon form factors, generalized parton distributions and quark angular momentum

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Abstract

We extract the individual contributions from u and d quarks to the Dirac and Pauli form factors of the proton, after a critical examination of the available measurements of electromagnetic nucleon form factors. From this data we determine generalized parton distributions for valence quarks, assuming a particular form for their functional dependence. The result allows us to study various aspects of nucleon structure in the valence region. In particular, we evaluate Ji's sum rule and estimate the total angular momentum carried by valence quarks at the scale $\mu = 2\,\text{GeV}$ to be $J_v^u = 0.230^{+0.009}_{-0.024}$ and $J_v^d = -0.004^{+0.010}_{-0.016}$.

Contents

1 Introduction

Together with parton distributions, electromagnetic form factors are among the most important quantities that provide information about the internal structure of the nucleon. Their experimental determination has entered the realm of precision physics. Generalized parton distributions (GPDs) combine and enlarge the different types of information contained in ordinary parton densities (PDFs) and form factors, but they remain much less well known experimentally. After pioneering measurements at DESY and Jefferson Lab, the upcoming energy upgrade at Jefferson Lab will significantly advance the determination of GPDs in the valence quark region, whereas measurements at COMPASS will explore the region of sea quarks and gluons with momentum fractions between 10^{-2} and 10^{-1}. For reviews of the many facets of GPDs we refer to [1, 2, 3, 4].

GPDs can be extracted from hard exclusive processes like deeply virtual Compton scattering and meson production. In complement, one can constrain the GPDs for valence quarks indirectly via the sum rules that connect them with electromagnetic form factors. This requires an ansatz for the functional form of the GPDs and in this sense is intrinsically model dependent, but on the other hand it can reach values of the invariant momentum transfer t much larger than what can conceivably be measured in hard exclusive scattering. We performed such an indirect determination some time ago [5]. Since then, there have been significant improvements in the experimental determination of the electromagnetic form factors, and we find it timely to investigate how this progress impacts on the extraction of GPDs and of important quantities such as the total angular momentum carried by quarks in the proton. This is the purpose of the present work.

In section 2 we briefly recall some essentials about form factors and GPDs and introduce our notation. A critical discussion of the form factor data used in our analysis is given in section 3, where we also provide a simple and precise parameterization of the selected data. In section 4 we estimate the contribution of strange quarks to the form factors. Section 5 describes how we combine the experimental results on proton and neutron form factors in order to extract Dirac and Pauli form factors for individual quark flavors, which are most closely connected with GPDs. Our fit of the GPDs to the form factor data, including a number of variants that allow us to investigate systematic uncertainties, is described in sections 6 and 7. In particular, we evaluate Ji's sum rule and thus obtain an estimate for the total angular momentum carried by u and by d quarks minus the corresponding contribution from antiquarks. Using our extracted GPDs, we explore in section 8 a number of further connections, namely the axial form factor, wide-angle Compton scattering, chromodynamic lensing and GPDs at nonzero skewness. We summarize our findings in section 9 and list various numerical results in two appendices.

2 Basics and notation

To begin with, let us recall some basics about the electromagnetic form factors of the nucleon. Experimental results are typically expressed in terms of the Sachs form factors $G_M^p(t)$, $G_E^p(t)$ and $G_M^n(t)$, $G_E^n(t)$, where t is the squared momentum transfer to the proton. Several measurements determine the ratio of electric and magnetic form factors, which is commonly written as

$$R^i(t) = \mu_i G_E^i(t)/G_M^i(t) \tag{1}$$

with $i = p, n$ for the proton and the neutron. The magnetic moments μ_p and μ_n normalize this ratio to unity at $t = 0$.

For convenience the magnetic form factors are often divided by the conventional dipole form

$$G_{\text{dipole}}^i(t) = \frac{\mu_i}{[1 - t/(0.71\,\text{GeV}^2)]^2} \tag{2}$$

with $i = p, n$. Plotting this ratio allows one to discern details in the data over a wide range of t, since the ratios $G_M^i/G_{\text{dipole}}^i$ show only a mild variation, unlike the form factors themselves.

The Dirac and Pauli form factors, F_1^i and F_2^i, are related to the Sachs form factors by

$$G_M^i = F_1^i + F_2^i\,, \qquad G_E^i = F_1^i + \frac{t}{4m^2}\,F_2^i\,, \tag{3}$$

where m is the nucleon mass and again $i = p, n$. One can further decompose

$$F_i^p = e_u F_i^u + e_d F_i^d + e_s F_i^s \,, \\ F_i^n = e_u F_i^d + e_d F_i^u + e_s F_i^s \,, \qquad (4)$$

where F_i^q denotes the contribution from quark flavor q to the form factor F_i^p of the proton. Here $i = 1, 2$ and e_q is the electric charge of the quark in units of the positron charge. It is instructive to rewrite (4) as

$$2F_i^p + F_i^n = F_i^u - F_i^s \,, \\ 2F_i^n + F_i^p = F_i^d - F_i^s \,. \qquad (5)$$

To the extent that the strangeness contributions F_1^s and F_2^s can be neglected, one can hence reconstruct the form factors for u and d quarks from the electromagnetic form factors alone. We will return to the issue of strangeness form factors in section 4. For brevity we will refer to the set of F_i^q as "flavor form factors" in this work. We will also use self-explaining abbreviations

$$F_i^{u-s} = F_i^u - F_i^s \,, \qquad F_i^{u+d} = F_i^u + F_i^d \qquad (6)$$

etc. for linear combinations of these form factors.

The flavor form factors can be written in terms of GPDs at zero skewness. For each quark flavor we have the sum rules

$$F_1^q(t) = \int_0^1 \mathrm{d}x \, H_v^q(x,t) \,, \\ F_2^q(t) = \int_0^1 \mathrm{d}x \, E_v^q(x,t) \qquad (7)$$

with

$$H_v^q(x,t) = H^q(x,0,t) + H^q(-x,0,t) \,, \\ E_v^q(x,t) = E^q(x,0,t) + E^q(-x,0,t) \,, \qquad (8)$$

where $H^q(x,\xi,t)$ and $E^q(x,\xi,t)$ denote the proton GPDs for unpolarized quarks of flavor q in the standard notation [2]. The combinations (8) correspond to the difference of quarks and antiquarks (as it must be for the electromagnetic form factors) and in this sense can be called "valence GPDs". For positive x one recovers the usual quark and antiquark densities as $H^q(x,0,0) = q(x)$ and $H^q(-x,0,0) = -\bar{q}(x)$.

We will also need the combination

$$\widetilde{H}_v^q(x,t) = \widetilde{H}^q(x,0,t) - \widetilde{H}^q(-x,0,t) \qquad (9)$$

for the difference of longitudinally polarized quarks and antiquarks, as well as the antiquark distributions

$$H^{\bar{q}}(x,t) = -H^q(-x,0,t) \,, \\ E^{\bar{q}}(x,t) = -E^q(-x,0,t) \,, \\ \widetilde{H}^{\bar{q}}(x,t) = \widetilde{H}^q(-x,0,t) \,. \qquad (10)$$

With these definitions, the isovector axial form factor of the nucleon can be written as

$$F_A(t) = \int_0^1 \mathrm{d}x \left[\widetilde{H}_v^u(x,t) - \widetilde{H}_v^d(x,t) \right] \\ + 2 \int_0^1 \mathrm{d}x \left[\widetilde{H}^{\bar{u}}(x,t) - \widetilde{H}^{\bar{d}}(x,t) \right] . \qquad (11)$$

The sea quark contribution does not drop out in this sum rule since the axial form factor has positive charge parity and thus corresponds to the sum and not the difference of quark and antiquark contributions. The value of F_A at $t = 0$, the axial charge, is well known from β-decay experiments.

3 Data selection

The determination of the electromagnetic nucleon form factors has not only a long history but remains at the forefront of experimental research. With quoted uncertainties typically in the percent region, the consistency between different measurements and the control of the theory underlying them have become nontrivial issues, as we shall see. In this section we discuss the selection of data used in our subsequent analysis and point out open problems and discrepancies between data sets. Earlier overviews and discussions of form factor data can be found in [6, 8, 7], and for G_E^n also in [27].

A synopsis of the default data set that we use in later sections is given in table 1. Several of these data do not have separated statistical and systematic errors. To have a uniform treatment, we add those errors in quadrature for the data sets where they are available.