Modulated Electron Bunch with Amplitude Front Tilt in an Undulator

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Abstract

In a previous paper we discussed the physics of a microbunched electron beam kicked by the dipole field of a corrector magnet by describing the kinematics of coherent undulator radiation after the kick. We demonstrated that the effect of aberration of light supplies the basis for understanding phenomena like the deflection of coherent undulator radiation by a dipole magnet. We illustrated this fact by examining the operation of an XFEL under the steady state assumption, that is a harmonic time dependence. We argued that in this particular case the microbunch front tilt has no objective meaning; in other words, there is no experiment that can discriminate whether an electron beam is endowed with a microbunch front tilt of not. In this paper we extend our considerations to time-dependent phenomena related with a finite electron bunch duration, or SASE mode of operation. We focus our attention on the spatiotemporal distortions of an X-ray pulse. Spatiotemporal coupling arises naturally in coherent undulator radiation behind the kick, because the deflection process involves the introduction of a tilt of the bunch profile. This tilt of the bunch profile leads to radiation pulse front tilt, which is equivalent to angular dispersion of the output radiation. We remark that our exact results can potentially be useful to developers of new generation XFEL codes for cross-checking their results.

1 Introduction

A well-known result of classical particle tracking states that after an electron beam is kicked there is a change in the trajectory of the electron beam, while the orientation of the microbunching front remains as before, see Fig 1. In other words the kick results in a difference between directions of the electron motion and of the normal to microbunching front [1, 2]. In XFEL simulations it is generally accepted that coherent radiation from the undulator placed after the kicker is emitted along the normal to the microbunching front [1].

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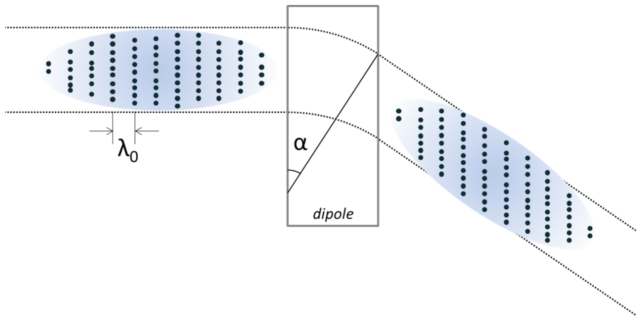

Fig. 1. Illustration of the problem as arises according to classical particle tracking when a microbunched electron beam is deflected by a dipole magnet. After passing the dipole, the microbunching is preserved, but only along its original direction.

The experiment [3] showed the surprising effect that coherent undulator radiation can be kicked by an angle of about five times the rms radiation divergence practically without suppression. The maximal kick angle was limited only by the photon beamline aperture.

In a previous paper [4] we presented a kinematical description the experiment [3] and demonstrated that the effect of aberration of light supplies the basis for an understanding of a phenomena like deflection of coherent undulator radiation by a dipole magnet. In order to focus on the ideal mechanism of microbunch tilt influence, in [4] we restricted our attention to the steady-state theory using the approximation of a continuous electron beam. However, in practical situations the electron beam has a finite pulse duration, and a question arises concerning the region of applicability of results in [4]. The present study answers such question, extending the analysis of the tilt influence by taking into account time-dependent effects.

2 Spatiotemporal transformation of X-ray FEL pulses by a kicker

Let us first consider a rectangular electron bunch of duration T. The transverse distribution of the electron bunch has a Gaussian shape with standard deviation σ_x. We make the assumption that the spatial profile of the bunching factor is close to that of the electron beam and that the modulation of the electron bunch is fully longitudinally and transversely coherent. Self-seeding schemes have been experimentally studied to reduce the bandwidth of XFEL pulses up to the Fourier limit [5, 6]. If the bunch is sufficiently long and the stretch in bunch duration due to the kick is much less that the bunch length, $vT \gg \alpha\sigma_x$, one can neglect edge effects and use the steady state approach. Using the plots presented in this paper one can give a quantitative answer to the question about the region of applicability of the steady state model in the case of fully coherent modulation.

Let us now consider the case of an electron bunch with a modulation induced by the SASE process in the undulator upstream of the dipole kick. As a consequence of the start-up from shot noise, the longitudinal coherence of the bunch modulation is rather poor. Here we are interested in the impact of the distortion of the microbunching front of the kicked bunch on the performance of the SASE X-ray beam splitting setup [3], where spatiotemporal distortions affect the degree of transverse coherence value. When describing the physical principles, it is always important to find an analytical description. In fact, finding analytical solutions is always fruitful for understanding XFEL physics and for testing numerical simulation codes. From this point of view, a SASE XFEL is a rather complicated object. It is important to find a model that provides the possibility of an analytical description without loss of essential information about the features of the random process. One can satisfy this condition by modeling the electron beam by means of a long rectangular bunch and by using a stationary random process for modeling radiation pulses. An analysis of the SASE X-ray pulse quality is given in terms of analytical results.

There is an urgent need to develop electron beam instrumentation allowing for a full measurement of the microbunching structure, including the spatiotemporal coupling of the modulation within the bunch. In order to characterize such modulated electron bunches, a conventional undulator module does not have enough resolution, and special diagnostic techniques are needed. A method for obtaining a full characterization of the bunch modulation could be based on the generation of an exact radiation replica of the electron bunch modulation. In our case, the synthesizer would simply consist of a short, few period $(5 - 10)$ radiator undulator installed after the kicker. Since the undulator is short, longitudinal dynamics in the undulator is purely governed by single-particle effects, where results do not depends on the presence of other particles.

In the following we assume that we actually have such a setup so that, after the kicker, the beam enters the short radiator undulator, which is resonant at the beam modulation frequency. Since the beam has a large bunching component, coherent emission is copiously produced: in this paper we study the characteristics of this pulse of radiation. The energy of the output radiation pulse can be estimated to be in the μJ-level. A realistic measurement of the microbunching heavily relies on the assumption that the beam density modulation does not appreciably change as the beam propagates through the radiator undulator. This approximation means that only the contributions to the radiation field arising from the initial density modulation are taken into account, and not those arising from the induced bunching. The effect of betatron motion on the preservation of micro bunching should be accounted for. In fact, the finite angular divergence of the electron beam, linked with the betatron motion, yields a spread of the longitudinal velocity

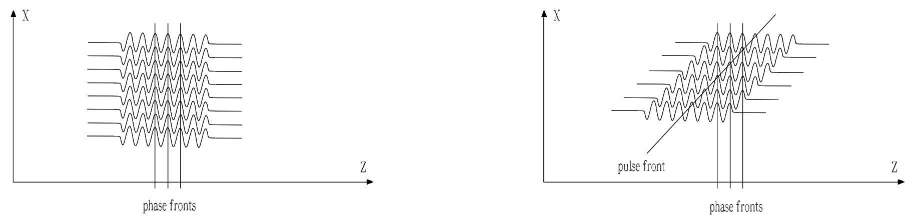

Fig. 2. Schematic representation of the electric field profile of an undistorted pulse beam (left) and of a beam with pulse front tilt (right). The z axis is along the beam propagation direction.

of the electrons, leading to microbunching suppression. This factor can be easily estimated and one can conclude that due to the short length of the radiator undulator it does not constitute a serious problem.

The signal produced by the radiator undulator is therefore expected to be an electric field pulse with amplitude $E(x, y, t)$. If no further longitudinal beam dynamics effects are present in the radiator undulator, this field is directly proportional to the microbunching amplitude $M(x, y, t)$.

X-ray FEL electric field pulses are usually represented with sufficient accuracy as the product of factors separately depending on space and time. However, when the manipulation of the microbunched electron beam requires the introduction of a kick, such assumption fails. The direction of the energy flow is always orthogonal to the surface of constant phase, that is to the wavefronts of the corresponding propagating wave. If one deals with coherent undulator radiation from the kicked microbunched beam, one has to consider, in addition, planes of constant intensity, that is pulse fronts. Fig 2 shows a schematic representation of the electric field profile of an undistorted pulse and one with a pulse-front tilt. Let us suppose that the X-ray pulse propagates in the $x - z$ plane, and that the pulse front tilt is also in this plane. We start by writing the field of an undisturbed pulse as $E(x, t) = a(t)b(x)\exp(-i\omega_0 t)$, where ω_0 is the pulse carrier frequency, which is linked to wave vector k by $k = \omega_0/c$. A distortion of the pulse front does not affect propagation, because the phase fronts remain unaffected. However for most applications, it is desirable that this fronts be parallel to the phase fronts, and therefore orthogonal to the propagation direction. A pulse-front tilt can indeed be introduced in the beam due to the propagation of an electron bunch through a kicker, which is our case of interest. The electric field of a pulse including tilt distortion can be expressed in the space-time domain $[x, t]$ as $E = a(t - px)b(x)\exp(-i\omega_0 t)$, while the Fourier transform from the $[x, t]$ domain to the $[k_x, \omega]$ domain can be expressed as $\hat{E}(k_x + p\Delta\omega, \Delta\omega)$, which is the Fourier component of the electric field of a pulse with angular dispersion and $\Delta\omega = \omega - \omega_0$. The tilt angle α is given by

$\tan\alpha = cp$ and $p = dk_x/d\omega = k(d\theta/d\omega)$ [1] . One concludes that the pulse-front tilt is invariably accompanied by angular dispersion. Any kick upstream of the radiator undulator, which introduces a bunch profile tilt, also introduces a pulse-front tilt and the output radiation is accompanied by angular dispersion.

3 Situation treatable analytically

We first consider the case of SASE radiation. We perform an analysis on the $x - z$ plane specified by the vertical and longitudinal directions in Fig. 2. Upon definition of

$$x_1 = \bar{x} + \frac{\Delta x}{2}$$
$$x_2 = \bar{x} - \frac{\Delta x}{2} \ . \qquad (1)$$

we can specify in full generality the spatial correlation function of the electric field between points x_1 and x_2 at time t as

$$\Gamma_t(\bar{x}, \Delta x) = \left\langle E\left(t, \bar{x} + \frac{\Delta x}{2}\right) E^*\left(t, \bar{x} - \frac{\Delta x}{2}\right)\right\rangle \ , \qquad (2)$$

where $\langle ... \rangle$ indicates an average over an ensemble of shots and the subscript t is a reminder of the dependence of the correlation function on time. As discussed before, when studying this time correlation function, we assume that the statistical process is stationary. Though this model is very idealized, making use of it is justified by the fact that in many real cases the radiation pulse is much longer that the coherence length. In other words, partial

[1] Let us give an elementary derivation of this expression, linking the angular dispersion and the pulse front tilt. This derivation is possible for ultrashort X-ray pulse having large transverse size. In this case the short pulse consists of monochromatic plane wave components with different frequencies. The surface of constant phase (the phase front) is determined by $\phi = -\omega t + k_x x + k_z z = \mathrm{const}$. This equation describes a line in the $x - z$ plane with slope $\tan\theta = -k_x/k_z$. The pulse front is the surface where intensity is at a maximum. The condition for this is that the plane wave components with different frequencies have the same phase that is $d\phi/d\omega = -t + dk_x/d\omega + dk_z/d\omega = 0$. This equation describes a line with slope $\tan\alpha = -dk_x/dk_z = \tan\theta + [k_z/(\cos\theta)^2]d\theta/dk_z$. We choose the coordinates system so that $\theta = 0$ for the mean ω and the mean k_z values. In this case, the slope of the phase fronts is zero. Because of this, the slope of the pulse fronts becomes $\tan\alpha = kd\theta/dk$ [7].

coherence is an essential feature of SASE XFEL radiation in the time domain. Under the assumption of ergodicity we can substitute the ensemble average in Eq. (2) with a temporal average, thus obtaining

$$\Gamma(\bar{x}, \Delta x) = \int_{-\infty}^{\infty} dt\, E\left(t, \bar{x} + \frac{\Delta x}{2}\right) E^*\left(t, \bar{x} - \frac{\Delta x}{2}\right) , \tag{3}$$

where we dropped the temporal dependence of Γ. Next we introduce a Fourier representation of the field according to

$$E(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\Delta\omega\, \bar{E}(\Delta\omega, x) \exp(-i\omega t) . \tag{4}$$

Using this representation and the relation

$$\delta(\omega_2 - \omega_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt\ \exp\left[i(\omega_2 - \omega_1)t\right] \tag{5}$$

we obtain from Eq. (3)

$$\Gamma(\bar{x}, \Delta x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\Delta\omega\, \bar{E}\left(\Delta\omega, \bar{x} + \frac{\Delta x}{2}\right) \bar{E}^*\left(\Delta\omega, \bar{x} - \frac{\Delta x}{2}\right) . \tag{6}$$

In the frequency domain, the beam modulation envelope before the kick can be modeled as

$$\bar{M}(\Delta\omega, x) = f(\Delta\omega) g(x) , \tag{7}$$

This means that we are assuming, in first approximation, full transverse coherence of the SASE modulation upstream the kicker. Following [4], the envelope of the modulation after the kick is

$$\bar{M}(\Delta\omega, x) = f(\Delta\omega) g(x) \exp\left(\frac{i\Delta\omega\alpha}{v} x\right) . \tag{8}$$

The diagnostic undulator behind the kicker generates an exact radiation replica of the electron beam modulation and substituting Eq. (8) into Eq. (6) we obtain

$$\Gamma(\bar{x},\Delta x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\Delta\omega \; \left|f(\Delta\omega)\right|^2 \exp\left(\frac{i\Delta\omega\alpha}{v}\Delta x\right) g\left(\bar{x}+\frac{\Delta x}{2}\right) g^*\left(\bar{x}-\frac{\Delta x}{2}\right) . \quad (9)$$

We further model the radiation spectrum and the transverse dependence of the pulse as smooth Gaussian functions of rms size σ_ω and σ_x respectively

$$\left|f(\Delta\omega)\right|^2 = A\exp\left[-\frac{\Delta\omega^2}{2\sigma_\omega^2}\right] , \quad (10)$$

and

$$g(x) = B\exp\left(-\frac{x^2}{2\sigma_x^2}\right) . \quad (11)$$

Substitution in Eq. (9) finally gives

$$\Gamma(\bar{x},\Delta x) = \frac{AB}{\sqrt{2\pi}}\sigma_\omega \exp\left[-\frac{\sigma_\omega^2\alpha^2(\Delta x)^2}{2v^2}\right]\exp\left[-\frac{\bar{x}^2+(\Delta x)^2/4}{\sigma_x^2}\right] , \quad (12)$$

which can be written equivalently as

$$\Gamma(\bar{x},\Delta x) = \frac{AB}{\sqrt{2\pi}}\sigma_\omega \exp\left[-\left(\frac{\sigma_\omega^2\alpha^2}{2v^2}+\frac{1}{2\sigma_x^2}\right)(x_1^2+x_2^2)+\frac{\sigma_\omega^2\alpha^2}{2v^2}x_1x_2\right] . \quad (13)$$

We now introduce, as a figure of merit for $\Gamma(\bar{x},\Delta x)$, the following definition of degree of transverse coherence

$$\xi = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx_1 dx_2 \left|\Gamma(x_1,x_2)\right|^2}{\left|\int_{-\infty}^{\infty} dx\Gamma(x,x)\right|^2} . \quad (14)$$

Using Eq. (13) we obtain

$$\xi = \frac{1}{\sqrt{1+\hat{\alpha}^2}} \quad (15)$$

where

$$\hat{\alpha} = \left(\frac{\sqrt{2}\sigma_\omega\sigma_x}{v}\right)\alpha \quad (16)$$

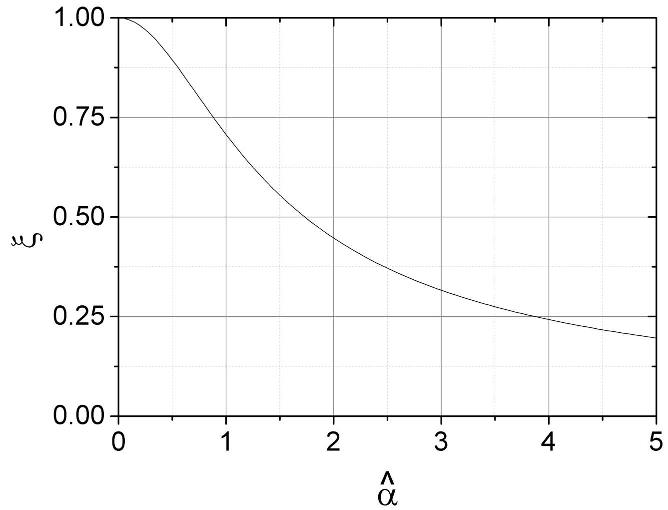

Fig. 3. Dependence of the degree of coherence ξ, on the normalized kick angle $\tilde{\alpha}$.

A plot of the output from the above result for ξ is given in Fig 3. It has a very simple functional form. It is clear from this that the effect of the kick is only significant when $\hat{\alpha}^2 \gtrsim 1$.

We give an example of the use of this formula, involving the typical parameters of a soft X-ray FEL. In the framework of the accepted model the input parameter is the standard deviation of the SASE radiation spectrum, σ_ω. We expect that the typical FWHM of the SASE radiation spectrum envelope at saturation, $\Delta\omega_{\mathrm{FWHM}}$, is of the order of $2\rho_{\mathrm{1D}}\omega_0$, where ρ_{1D} is the 1D FEL parameter [8] and ω_0 is the resonance frequency. Therefore, setting $\sigma_\omega \simeq \rho_{\mathrm{1D}}\omega_0$ gives a good estimate of the rms deviation. Another input parameter is the rms deviation of the transverse Gaussian distribution of the radiation field amplitude, σ_x. Within our model, the transverse distribution of the field amplitude at the radiator undulator exit is close to that of the electron beam. For a typical soft X-ray case with $\lambda \simeq 1$nm, $\rho_{\mathrm{1D}} \simeq 0.002$, and $\sigma_x \simeq 20\mu$m, the SASE radiation can be kicked by an angle of order of a mrad without suppression of transverse coherence.

4 Figure of merit measuring the spatiotemporal coupling

A useful figure of merit measuring the spatiotemporal coupling can be found in [9]. Considering the angular dispersion this parameter can be written as

$$\rho = \int dk_x d(\Delta\omega)\, I(k_x, \Delta\omega) \frac{k_x \Delta\omega}{< (\delta k_x)^2 >^{1/2} < (\delta\omega)^2 >^{1/2}} \,, \tag{17}$$

where

$$< (\delta k_x)^2 >= \int dk_x d(\Delta\omega)\ I(k_x, \Delta\omega) k_x^2 ,$$
$$< (\delta\omega)^2 >= \int dk_x d(\Delta\omega)\ I(k_x, \Delta\omega)\omega^2 ,$$
$$I(k_x, \Delta\omega) = |E(k_x, \Delta\omega)|^2 \tag{18}$$

The range of ρ is in $[-1, 1]$ and readily indicates the severity of these distortions. Thus, in order to estimate the pulse front tilt distortion we can simply calculate the pulse front tilt parameter ρ. The case $\rho = 0$ corresponds to a pulse free of spatiotemporal distortions, while $0 < |\rho| < 1$ indicates that some distortions are present. The ρ parameter is very sensitive to a small amount of dispersion: a value of $\rho \simeq 0.1$ corresponds to a pulse stretched by only about 10%. This correlation coefficient is ideal for controlling a setup that should approach the Fourier limit. However, there is a little change in ρ in the region of strong distortions [9].

We start by writing the electron bunch modulation immediately before the kicker as $M(x, t) = A(t)B(x) \exp(-i\omega_0 t)$. In this section we study the case when the bunch modulation is fully coherent in space and time. We further model the spectrum and the transverse dependence of the bunch modulation as smooth Gaussian functions of rms size σ_ω and σ_x respectively. This model has proven to be very fruitful, and gives the possibility of obtaining simple analytical expressions for the main characteristics of the radiation from an undulator behind the kicker.

Using Eq. (8) we can write the electric field of the radiation pulse from the short diagnostic undulator that we supposed placed after the kick. In the spatial-frequency and temporal-frequency domain one has

$$\hat{E}_1(\Delta\omega, k_x) = f(\Delta\omega) h\left(k_x + \frac{\Delta\omega\alpha}{v}\right) . \tag{19}$$

The spectrum distribution of the radiation pulse with Gaussian profile is given by

$$\left|f(\Delta\omega)\right|^2 = A \exp\left[-\frac{\Delta\omega^2}{2\sigma_\omega^2}\right] . \tag{20}$$

Moreover,

$$h(k_x) = \sqrt{2\pi} B \sigma_x \exp\left(-\frac{1}{2} k_x^2 \sigma_x^2\right) . \tag{21}$$

With the help of Eq. (18) we can write

$$< (\delta k_x)^2 >= 2\pi B^2 \sigma_x^2 A \int_{-\infty}^{\infty} d(\Delta\omega) \int_{-\infty}^{\infty} dk_x k_x^2 \exp\left[-\frac{(\Delta\omega)^2 \alpha^2 \sigma_x^2}{v^2}\right] \times \exp\left[-k_x^2 \sigma_x^2 - \frac{2 k_x \sigma_x^2 \alpha (\Delta\omega)}{v}\right] \exp\left[-\frac{\Delta\omega^2}{2\sigma_\omega^2}\right] \tag{22}$$

and

$$< (\delta\omega)^2 >= 2\pi B^2 \sigma_x^2 A \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk_x (\Delta\omega)^2 \exp\left[-\frac{(\Delta\omega)^2 \alpha^2 \sigma_x^2}{v^2}\right] \times \exp\left[-k_x^2 \sigma_x^2 - \frac{2 k_x \sigma_x^2 \alpha\ \Delta\omega}{v}\right] \exp\left[-\frac{\Delta\omega^2}{2\sigma_\omega^2}\right] . \tag{23}$$

Explicit calculations give

$$< (\delta k_x)^2 >= 2^{1/2} \pi^2 A B^2 \sigma_\omega \left[\frac{v^2 + 2\alpha^2 \sigma_x^2 \sigma_\omega^2}{v^2 \sigma_x}\right] \tag{24}$$

and

$$< (\delta\omega)^2 >= 2^{3/2} \pi^2 A B^2 \sigma_x \sigma_\omega^3 . \tag{25}$$

In a similar fashion one obtains

$$2\pi B^2 \sigma_x^2 A \int_{-\infty}^{\infty} d(\Delta\omega) \int_{-\infty}^{\infty} dk_x (\Delta\omega) k_x \exp\left[-\left(k_x + \frac{\Delta\omega\alpha}{v}\right)^2 \sigma_x^2\right] \exp\left[-\frac{\Delta\omega^2}{2\sigma_\omega^2}\right] = -2^{3/2} \pi^2 A B^2 \sigma_x \alpha \frac{\sigma_\omega^3}{v} . \tag{26}$$

Substituting Eq. (26), Eq. (25) and Eq. (24) into Eq. (17) one finally gets

$$\rho = -\left(\frac{\sqrt{2}\alpha\sigma_x\sigma_\omega}{v}\right) \backslash \left(1 + 2\frac{\alpha^2 \sigma_x^2 \sigma_\omega^2}{v^2}\right)^{1/2} , \tag{27}$$

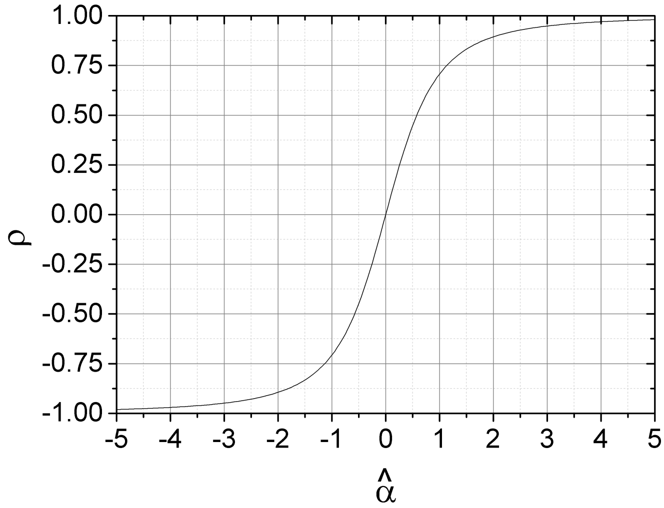

Fig. 4. Dependence of the tilt parameter ρ, on the normalized kick angle $\hat{\alpha}$.

that can also be expressed as

$$\rho = -\frac{\hat{\alpha}}{\sqrt{1 + \hat{\alpha}^2}} , \tag{28}$$

where $\hat{\alpha}$ is the normalized kick angle given by Eq. (16). The behavior of ρ as a function of $\hat{\alpha}$ is shown in Fig. 4. We believe that this parameter can be used as a benchmark enabling the comparison of the performance of X-ray FEL beam splitting setups.

We give an example of the use of this formula, involving parameters typical of a soft X-ray FEL, as in the previous section. One of the input parameters of our example is the standard deviation of the bandwidth limited spectrum of a Gaussian X-ray FEL pulse, σ_ω. Since the temporal and spectral characteristics of the field are related to one another through a Fourier transform, the standard deviation σ_τ of the X-ray pulse intensity distribution in the time domain and σ_ω cannot vary independently of each other: the relation $\sigma_\omega = 0.5/\sigma_\tau$ holds.

In order to understand what performance can be achieved in a practical application, we consider the shortest possible duration of the X-ray pulse $\sigma_\tau \simeq 1$fs and $\sigma_x \simeq 20\mu$m, which are typical values for XFELs. For a kick angle $\alpha = 1$ mrad one obtains the value $\rho \simeq 0.05$. From this result it becomes apparent that such ultrashort X-ray pulse can be kicked by an angle of about a mrad with spatiotemporal distortions that can be acceptable in many situations.

5 Conclusions

A modulated electron bunch is usually represented as the product of an amplitude function and a phase term. In our case of interest the spectral amplitude is centered around a mean frequency ω_0 and has appreciable values only in a narrow frequency interval $\Delta\omega$ such that $\Delta\omega/\omega_0 \ll 1$. Because of this, in the time domain it is convenient to introduce a carrier frequency ω_0 and to write the modulation as product of a slowly varying modulation envelope and a phase term $\exp[i\omega_0(z/v - t)]$. The modulation of the electron bunch due to the XFEL process is usually represented with sufficient accuracy as the product of factors separately depending on space and time. However, when the manipulation of the modulated electron bunch requires the introduction of a kick, such assumption fails. As discussed in [4], the carrier microbunching phase front is readjusted and always orthogonal to the propagation direction. In this article we extended the analysis of the influence of the kick by taking into account time-dependent effects. As discussed here, the kick introduces a tilt of the modulation envelope with respect to the phase front. The delay across the bunch is characterized by a certain tilt angle, which is simply equal to the kick angle α. A method for obtaining a full characterization of the bunch modulation can be based on the generation of an exact radiation replica of the electron bunch modulation in a short undulator installed after the kicker, which functions as a dedicated diagnostics design. If one deals with coherent undulator radiation from a kicked microbunched beam, one has to consider, in addition to an extra constant phase, planes of constant intensity, that is pulse fronts. It follows that any kick upstream of the radiator undulator introduces an X-ray pulse front tilt, and the output radiation is accompanied by angular dispersion. In the SASE regime, spatiotemporal distortion affects the degree of transverse coherence. An analysis of the quality of the SASE X-ray pulse is given in terms of analytical results, which are expected to serve as a primary standard for testing future FEL codes including effects related to the kick.

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