DESY 15-157
MS-TP-15-13

Next-to-next-to-leading order contributions to inclusive jet production in deep-inelastic scattering and determination of α_s

Thomas Biekötter[a], Michael Klasen[a],* and Gustav Kramer[b]
[a] *Institut für Theoretische Physik, Westfälische Wilhelms-Universität Münster, Wilhelm-Klemm-Straße 9, D-48149 Münster, Germany*
[b] *II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany*
(Dated: September 4, 2015)

We present the first calculation of inclusive jet production in deep-inelastic scattering with approximate next-to-next-to-leading order (aNNLO) contributions, obtained from a unified threshold resummation formalism. The leading coefficients are computed analytically. We show that the aNNLO contributions reduce the theoretical prediction for jet production in deep-inelastic scattering, improve the description of the final HERA data in particular at high photon virtuality Q^2 and increase the central fit value of the strong coupling constant.

INTRODUCTION

The HERA collider, which operated at DESY from 1992 to 2007, has produced many important physics results, first of all perhaps the most precise determinations to date of the quark and gluon densities in the proton from single experiments (H1, ZEUS) [1, 2] and their combined data sets [3]. These data, taken in deep-inelastic electron-proton scattering, are complemented by a wealth of data from photoproduction at low virtuality Q^2 of the exchanged photon, in particular on jet production [4], giving access also to the distributions of partons in the photon [5] and to measurements of the strong coupling constant [6].

Using the data set of the HERA-II phase of the HERA collider from 2003-2007 with an integrated luminosity of 351 pb^{-1}, the H1 collaboration have recently published final measurements of inclusive jet, dijet and three-jet production in deep-inelastic scattering (DIS) [7, 8] and used them to determine the strong coupling constant (at the mass M_Z of the Z-boson) to be

$$\alpha_s(M_Z) = 0.1185 \pm 0.0016(\text{exp.}) \pm 0.0040\,(\text{th.}), \quad (1)$$

taking into account absolute double-differential inclusive jet, dijet and three-jet cross section data as functions of Q^2 and the jet transverse momentum p_T. A more precise value was obtained from normalized jet cross sections, yielding

$$\alpha_s(M_Z) = 0.1165 \pm 0.0008(\text{exp.}) \pm 0.0038\,(\text{th.})\,. \quad (2)$$

Unsatisfactorily, only the value obtained by unrenormalized results is in agreement with the current world average of $\alpha_s(M_Z) = 0.1185 \pm 0.0006$ [9]. The latter uses only observables that are known to next-to-next-to-leading order (NNLO) of perturbative QCD, while the analysis of the H1 collaboration was done in next-to-leading order (NLO) accuracy. The lack of knowledge of higher-order contributions becomes manifest in a bigger theoretical uncertainty due to scale variation in Eq. (1). The absolute double-differential cross section measurement of inclusive jets alone led H1 to a value of the strong coupling constant of

$$\alpha_s(M_Z) = 0.1174 \pm 0.0022(\text{exp.}) \pm 0.0050\,(\text{th.}). \quad (3)$$

In this Letter, we compute the inclusive jet production DIS cross section for the first time including NNLO contributions, obtained from a unified threshold resummation formalism [10], and extract an approximate NNLO (aNNLO) value for the strong coupling constant. Our calculations are based on previous work on inclusive jet production in deep-inelastic scattering up to NLO [11]. They reduce, as we will see, the theoretical prediction and increase the central fit value of the strong coupling constant, improving the description of the final HERA data in particular at high photon virtuality Q^2.

NNLO CONTRIBUTIONS TO JET PRODUCTION IN DIS

The QCD factorization theorem allows to write the differential cross section for inclusive jet production in neutral-current DIS with high momentum transfer $Q^2 = -q^2$ as a convolution of the partonic cross section $\mathrm{d}\sigma_{\gamma a}$ with the parton densities in the proton $f_{a/P}$ and the flux of photons in electrons $f_{\gamma/e}$ as

$$\mathrm{d}\sigma = \sum_a \int \mathrm{d}y\, f_{\gamma/e}(y) \times \int \mathrm{d}x_P\, f_{a/P}(x_P, \mu_F) \mathrm{d}\sigma_{\gamma a}(\alpha_s, \mu_R, \mu_F)\,, \quad (4)$$

where we define $y = (p \cdot q)/(p \cdot k)$ with p and k the momenta of the incoming proton and electron, respectively, and q the momentum of the exchanged photon. In deep-inelastic scattering the highly off-shell photon has no time to decay, so resolved photon contributions can safely be neglected.

From a unified threshold resummation formalism a master formula can be obtained that permits to compute soft and virtual corrections to arbitrary partonic hard scattering cross sections [10]. At NLO it reads

$$\mathrm{d}\sigma_{ab} = \mathrm{d}\sigma_{ab}^{B}\frac{\alpha_s(\mu)}{\pi}\left[c_3 D_1(z) + c_2 D_0(z) + c_1\delta(1-z)\right] , \tag{5}$$

where for just one color-charged parton in the initial state we only need the formula for simple color flow. The functions

$$D_l(z) \ = \ \left[\frac{\ln^l(1-z)}{1-z}\right]_+ \tag{6}$$

with decreasing l are the leading and subleading logarithms at partonic threshold ($z \to 1$) in pair-invariant-mass kinematics. The NNLO master formula is given in the reference cited above, as are the general formulæ for the coefficients c_i.

We state here the coefficients for the two partonic processes that contribute to jet production in DIS. For $\gamma^* q \to qg$, where γ^* represents the off-shell photon, g a gluon and q a quark or an anti-quark, we find

$$c_3 = C_F - N_C\,, \tag{7}$$

$$\begin{aligned} c_2 = & 2C_F \ln\left(\frac{-u}{M^2}\right) + N_C \ln\left(\frac{t}{u}\right) \\ & - C_F \ln\left(\frac{\mu_F^2}{M^2}\right) - \frac{3}{4}C_F - \frac{\beta_0}{4} \end{aligned} \tag{8}$$

and $c_1 = c_1^\mu + T_1$ with

$$c_1^\mu = -\frac{3}{4}C_F \ln\left(\frac{\mu_F^2}{M^2}\right) + \frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{M^2}\right). \tag{9}$$

For the second process $\gamma^* g \to q\bar{q}$ we find

$$c_3 = 2\left(N_C - C_F\right)\,, \tag{10}$$

$$c_2 = N_C \ln\left(\frac{tu}{M^4}\right) - N_C \ln\left(\frac{\mu_F^2}{M^2}\right) - \frac{3}{2}C_F\,, \tag{11}$$

and

$$c_1^\mu = \frac{\beta_0}{4}\left[\ln\left(\frac{\mu_R^2}{M^2}\right) - \ln\left(\frac{\mu_F^2}{M^2}\right)\right]. \tag{12}$$

These coefficients agree with those found in photoproduction for massless jets for the direct part [6]. For massive jets, additional logarithms depending on the jet radius R appear (e.g. in c_2) [12], which are however irrelevant in the case $R = 1$ as in the H1 analysis considered here [7]. The calculation is analogous to the case of single-jet production in hadron-hadron collisions [13]. The above coefficients further depend on the QCD color factors $C_F = 4/3$ and $N_C = 3$, on the one-loop β-function $\beta_0 = (11N_C - 2n_f)/3$ with n_f the number of active quark flavors, the Mandelstam variables t and u, the renormalization and factorization scales μ_R and μ_F and the fixed large invariant scale M^2, that in DIS is equal to Q^2, whereas in photoproduction it was the Mandelstam variable s. The part T_1 of c_1 does not contain any dependence either on the renormalization or the factorization scale. It includes the NLO virtual corrections and is not predicted by the threshold resummation formalism. If available, it can be read off from a full NLO calculation. For the case of transverse photon polarization it could be found in reference [11]. For longitudinal polarization we took the formula directly from the source code of the corresponding program JetViP [14, 15], that calculates inclusive jet production in DIS to NLO accuracy. Some two-loop quantities appearing in the NNLO master formula were not given explicitly in [10] and could not been found in the respective literature. As in our previous work on jet photoproduction [6], they were therefore neglected.

COMPARISONS TO H1 DATA

The NNLO contributions have been implemented in the code JetViP for inclusive jet and dijet production in DIS, where the convolution over z was already included for NLO initial-state corrections on the proton side. At NLO, we use of course our complete calculation and not only the logarithmically enhanced terms described above. As a numerical check, we have repeated the NLO analysis of inclusive single-differential jet production of the H1 collaboration, performed with NLOJet++ [16] and presented in reference [7, 8], and found excellent agreement, confirming previous successful comparisons of different NLO programs for jet production in DIS [16, 17].

The measurement took place during the HERA-II running period with an integrated luminosity of 351 pb^{-1}. The beam energies were 27.6 GeV for electrons or positrons and 920 GeV for protons, which gives a center-of-mass energy of 319 GeV. The leptonic phase space was given by 150 GeV^2 $< Q^2 <$ 15 000 GeV^2 and $0.2 < y < 0.7$. The jet phase space was restricted to the rapidity interval $-1.0 < \eta_{lab} < 2.5$, where η_{lab} is the pseudorapidity of a jet in the HERA lab frame. The cross section was measured differentially in the jet transverse momentum p_T and the virtuality Q^2. Jets were reconstructed using the k_T-clustering algorithm [18] in the Breit frame, where exlusively electroweak processes can be ruled out by demanding a minimum of jet transverse momentum (here $p_T > 7$ GeV). In inclusive jet production in DIS, an almost identical fit result for $\alpha_s(M_Z)$ was obtained with the anti-k_T algorithm. The jet radius was $R = 1$.

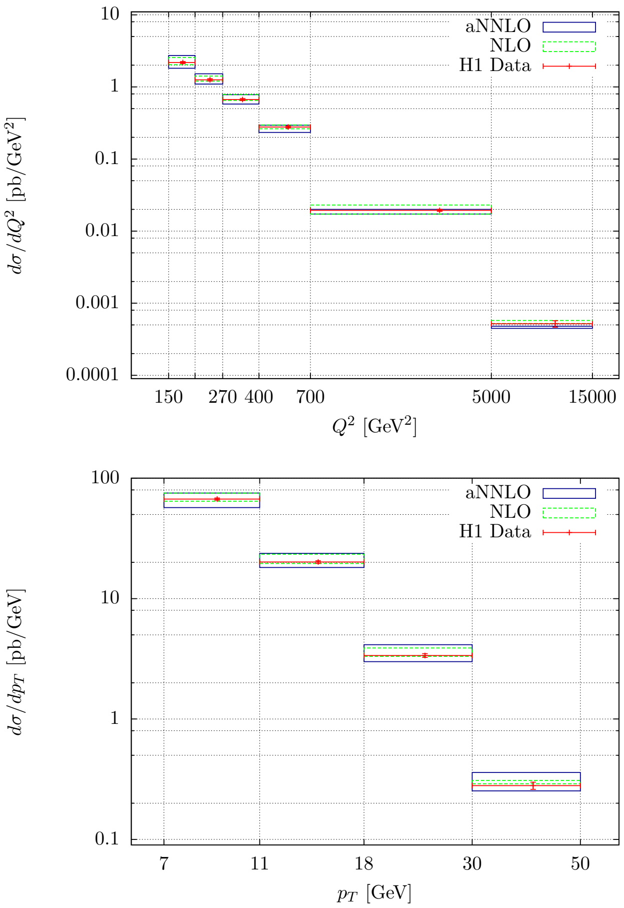

FIG. 1: Single-differential inclusive jet cross sections as a function of photon virtuality Q^2 (top) and jet transverse momentum p_T (bottom) in NLO (green dashed lines) and aNNLO (blue full lines) with the corresponding scale uncertainty bands, obtained by varying μ_R and μ_F simultaneously by a factor of two up and down, compared to the final H1 data (red points, color online).

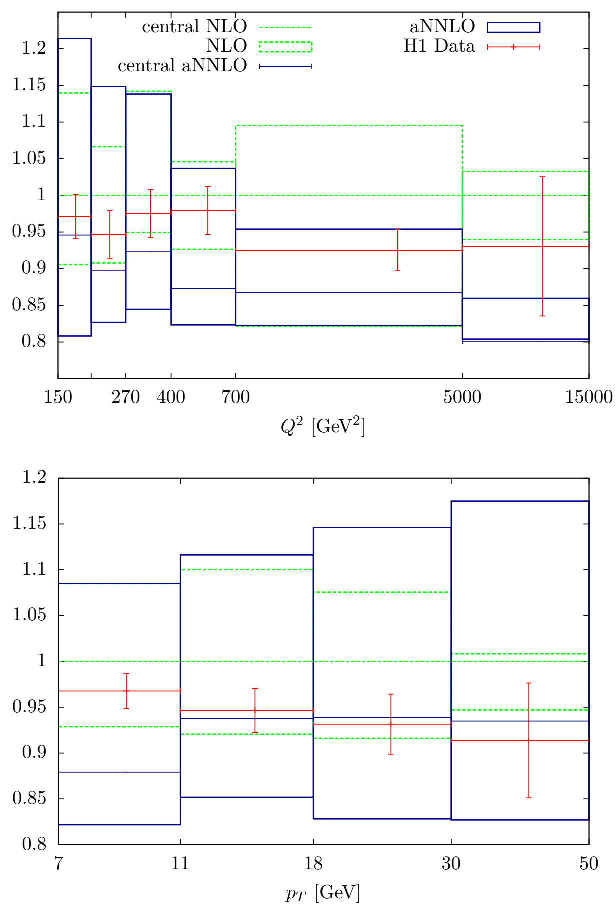

FIG. 2: Same as Fig. 1, but normalized to the central NLO predictions.

The perturbative scales were chosen to be

$$\mu_R^2 = (Q^2 + p_T^2)/2 \quad \text{and} \quad \mu_F^2 = Q^2\,. \tag{13}$$

The perturbative hard-scattering functions were convoluted with the MSTW2008 set of parton distribution functions in the proton with different fixed $\alpha_s(M_Z)$ values [19]. This PDF set especially offers the possibility to determine a best-fit $\alpha_s(M_Z)$. The number of active flavors was $n_f = 5$, since sea contributions of a heavy top quark inside the proton can safely be neglected. Following the H1 analysis, we choose the PDF member with $\alpha_s(M_Z) = 0.118$ in all plots shown in this Letter, which together with the scale choice from Eq. (13) defines our central fit.

In Fig. 1, we compare our NLO (green dashed lines) and aNNLO (blue full lines) results to the experimental data of the H1 collaboration (red points). The uncertainty bands are obtained by varying both scales simultaneously about the central scales up and down by a factor of two as it was also done by the H1 collaboration. We have verified that our NLO results and uncertainty bands agree very well with those of the H1 analysis. Comparing our calculations to the measurements, we see that the data points lie in the error bands of both the NLO and the aNNLO calculations. For the Q^2 distribution (top), the scale uncertainty is of similar size at NLO and aNNLO at low Q^2 and is considerably reduced in aNNLO at higher Q^2 as expected. We checked that this tendency continues and becomes more pronounced for even higher Q^2, where unluckily we have no data to compare with. For the p_T distribution (bottom), the scale uncertainty is not reduced from NLO to aNNLO, since analytically no logarithms of ratios of μ_R or μ_F over p_T appear (see above). While in our previous analysis of jet photoproduction the jet transverse momentum p_T ranged from 17 to 71 GeV [6], its range is restricted in DIS and this analysis to lower values of 7 to 50 GeV, while the photon virtuality reaches values up to (122.5 GeV)^2.

In Fig. 2, we show the same comparison normalized to the central NLO result. We also depict here the central aNNLO result, which is always smaller than the NLO central result by approximately 6%. The central Q^2 distribution and even more the central p_T distribution agree better with the H1 data at aNNLO than at NLO, as they then lie right within the experimental uncertainties. While the central NLO results overestimate the measured cross sections, indicating that the value of the strong coupling constant $\alpha_s(M_Z)$ used at this order is too large, the central aNNLO results underestimate the data and require a slightly larger value of α_s. The data are thus clearly sensitive to the strong coupling constant and can be used for an extraction not only at NLO, but also at aNNLO. However, we do not expect a significant reduction of the theoretical error from the scale uncertainty, in particular from the p_T distribution.

DETERMINATION OF α_S

To determine the strong coupling constant from these comparisons, the theoretical calculations have to be performed with a set of parton densities in the proton obtained from global fits assuming different values of $\alpha_s(M_Z)$. For our analysis at aNNLO, we employ the latest fits of the MSTW group, which have been obtained with NNLO running of the coupling, evolution of the parton densities, deep-inelastic scattering and vector-boson production matrix elements [19]. 22 different MSTW2008 NNLO members were used, which correspond to values of $\alpha_s(M_Z) = 0.107$ to 0.127. To compare the aNNLO best-fit α_s to a corresponding NLO α_s based on our full NLO calculation, we carry out the same approach with the NLO MSTW2008 parton distribution functions. These are available for the range of $\alpha_s(M_Z) = 0.110$ to 0.130.

The strong coupling constant α_s was determined by comparing the theoretical predictions at NLO and aNNLO to the experimental measurements by H1 and then finding the minimum value of the reduced χ^2. We present here our results for the single-differential p_T-distribution, but have verified that fitting the Q^2-distribution yields very similar results. At NLO we find

$$\alpha_s^{\rm NLO}(M_Z) = 0.115 \pm 0.002({\rm exp.}) \pm 0.005({\rm th.}), \quad (14)$$

where the central value is slightly lower than the one obtained by H1 from the unnormalized double-differential inclusive jet cross section (cf. Eq. (3)), but where the total experimental error and the theoretical error obtained from a simultaneous variation of the renormalization and factorization scales agree very well. At aNNLO, α_s gets shifted upwards, since we demonstrated above that the aNNLO contributions reduce the differential cross sections compared to NLO. We find

$$\alpha_s^{\rm aNNLO}(M_Z) = 0.122 \pm 0.002({\rm exp.}) \pm 0.013({\rm th.}), \quad (15)$$

where the central value is now slightly above the world average, the experimental error is of course unchanged, and the theoretical error is slightly larger, reflecting the observation made above that the aNNLO calculation is not yet sufficiently stabilized by threshold logarithms at these values of p_T and Q^2. The numerical situation would only improve at higher values of Q^2, where unfortunately no experimental data are available.

CONCLUSIONS

In conclusion, we have presented here a first calculation of inclusive-jet production in neutral-current deep-inelastic scattering up to NNLO of perturbative QCD. Leading and subleading logarithmic contributions were extracted from a unified threshold resummation formalism for virtual photon-parton scattering processes and shown to agree with those appearing in our full NLO calculations. The aNNLO contributions implemented in our NLO program improve the description of the final H1 data on inclusive-jet production in the Q^2 distribution and even more in the p_T distribution, when the world average value of α_s is used and for central scale choices. The scale uncertainties are reduced only at the highest values of Q^2, where threshold corrections are most important. An aNNLO fit of these data with the MSTW2008 set of parton densities resulted in a new determination of the strong coupling constant at the mass of the Z-boson that increased the central fit value from below to above the current world average, but did not reduce the theoretical error.

We thank W. Vogelsang for useful discussions. This work has been supported by the BMBF Theorie-Verbund "Begleitende theoretische Untersuchungen zu den Experimenten an den Großgeräten der Teilchenphysik."

* michael.klasen@uni-muenster.de

[1] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **21**, 33 (2001).
[2] S. Chekanov *et al.* [ZEUS Collaboration], Eur. Phys. J. C **21**, 443 (2001).
[3] F. D. Aaron *et al.* [H1 and ZEUS Collaboration], JHEP **1001**, 109 (2010).
[4] M. Klasen, G. Kramer and S. G. Salesch, Z. Phys. C **68**, 113 (1995); M. Klasen and G. Kramer, Z. Phys. C **72**, 107 (1996); M. Klasen and G. Kramer, Z. Phys. C **76**, 67 (1997); M. Klasen, T. Kleinwort and G. Kramer, Eur. Phys. J. direct C **1**, 1 (1998); M. Klasen, G. Kramer and B. Pötter, Eur. Phys. J. C **1**, 261 (1998); M. Klasen and G. Kramer, Phys. Rev. D **56**, 2702 (1997); M. Klasen and G. Kramer, Eur. Phys. J. C **71**, 1774 (2011); M. Klasen, Rev. Mod. Phys. **74**, 1221 (2002).
[5] S. Albino, M. Klasen and S. Söldner-Rembold, Phys. Rev. Lett. **89**, 122004 (2002).

[6] M. Klasen, G. Kramer and M. Michael, Phys. Rev. D **89**, 074032 (2014).

[7] V. Andreev *et al.* [H1 Collaboration], Eur. Phys. J. C **75**, 65 (2015).

[8] D. A. Britzger [H1 Collaboration], DESY-THESIS-2013-045 (2013).

[9] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[10] N. Kidonakis, Int. J. Mod. Phys. A **19**, 1793 (2004).

[11] B. Pötter, PhD thesis, DESY, University of Hamburg (1997)

[12] D. de Florian and W. Vogelsang, Phys. Rev. D **76**, 074031 (2007) and M. C. Kumar and S. O. Moch, Phys. Lett. B **730**, 122 (2014).

[13] N. Kidonakis and J. F. Owens, Phys. Rev. D **63**, 054019 (2001).

[14] B. Pötter, Comp. Phys. Commun. **133**, 105 (2000).

[15] M. Klasen, G. Kramer and B. Pötter, Euro. Phys. J. **C1**, 261 (1998).

[16] Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. **87**, 082001 (2001).

[17] B. Pötter, Comput. Phys. Commun. **133**, 105 (2000).

[18] S. Catani, Y. L. Dokshitzer, M. H. Seymour and B. R. Webber, Nucl. Phys. B **406**, 187 (1993); S. D. Ellis and D. E. Soper, Phys. Rev. D **48**, 3160 (1993).

[19] A. Martin, W. Stirling, R. Thorne and G. Watt, Eur. Phys. J. C **64**, 653 (2009).