# Deep inelastic cross-section measurements at large $y$ with the ZEUS detector at HERA 

ZEUS Collaboration


#### Abstract

The reduced cross sections for $e^{+} p$ deep inelastic scattering have been measured with the ZEUS detector at HERA at three different centre-of-mass energies, 318, 251 and 225 GeV . The cross sections, measured double differentially in Bjorken $x$ and the virtuality, $Q^{2}$, were obtained in the region $0.13 \leq y \leq 0.75$, where $y$ denotes the inelasticity and $5 \leq Q^{2} \leq 110 \mathrm{GeV}^{2}$. The proton structure functions $F_{2}$ and $F_{L}$ were extracted from the measured cross sections.


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## 1 Introduction

Deep inelastic scattering (DIS) in $e^{ \pm} p$ collisions is an important tool to investigate the structure of the proton and to test different theoretical approaches to solving quantum chromodynamics (QCD). At low virtuality, $Q^{2}$, the inclusive $e^{ \pm} p$ DIS cross section can be expressed in terms of the two structure functions, $F_{2}$ and $F_{L}$, as

$$
\begin{equation*}
\frac{d^{2} \sigma^{e^{ \pm} p}}{d x d Q^{2}}=\frac{2 \pi \alpha^{2} Y_{+}}{x Q^{4}}\left[F_{2}\left(x, Q^{2}\right)-\frac{y^{2}}{Y_{+}} F_{L}\left(x, Q^{2}\right)\right]=\frac{2 \pi \alpha^{2} Y_{+}}{x Q^{4}} \tilde{\sigma}\left(x, Q^{2}, y\right) \tag{1}
\end{equation*}
$$

where $\alpha$ is the fine structure constant, $x$ is the Bjorken scaling variable, $y$ is the inelasticity and $Y_{+}=1+(1-y)^{2}$. The quantity $\tilde{\sigma}$ is referred to as the reduced cross section. The kinematic variables are related via $Q^{2}=x y s$, where $\sqrt{s}$ is the $e p$ centre-of-mass energy.

The magnitude of $F_{L}$ is proportional to the absorption cross section for longitudinally polarised virtual photons by protons, $F_{L} \propto \sigma_{L}$, while $F_{2}$ includes also the absorption cross section for transversely polarised virtual photons, $F_{2} \propto\left(\sigma_{T}+\sigma_{L}\right)$. At low values of $x$, the ratio $R=F_{L} /\left(F_{2}-F_{L}\right) \approx \sigma_{L} / \sigma_{T}$ gives the relative strengths of the two components. HERA measurements [1] of $\tilde{\sigma}$ provide the strongest constraints on the proton parton distribution functions (PDFs) at low $x$. Within the DGLAP formalism [2], $F_{2}$ at low $x$ is dominated by the quark sea distributions while the scaling violations of $F_{2}$ reflect the gluon distribution.

The value of $F_{L}$ is zero in zeroth-order perturbative QCD. Contributions to $F_{L}$ arise from higher-order processes initiated by quarks and gluons in the proton. At low $Q^{2}$ and $x$, the gluon-initiated processes dominate and $F_{L}$ is directly correlated to the gluon density. The description of the $F_{L}$ contribution is the theoretically most challenging part in the QCD analysis (PDF extraction) of the HERA reduced cross-section data. The various available model calculations [3-9] differ sizeably in their predictions of $F_{L}$, in particular at low $Q^{2}$.

This paper presents new measurements of the reduced $e^{+} p$ cross sections up to large values of $y$ using the ZEUS detector. The data were taken at different centre-of-mass energies, $\sqrt{s}=318,251$ and 225 GeV , allowing for measurements at fixed $\left(x, Q^{2}\right)$ but different values of $y$. This allows $F_{2}$ to be decoupled from $F_{L}$, thus providing direct sensitivity to the gluon density. The variation of $\sqrt{s}$ was achieved by varying the proton beam energy, $E_{p}$, while keeping the electron ${ }^{1}$ beam energy constant, $E_{e}=27.5 \mathrm{GeV}$. The data were collected in 2006 and 2007 with $E_{p}=920$, 575 and 460 GeV , referred to respectively as the high- (HER), medium- (MER) and low-energy-running (LER) samples. The corresponding integrated luminosities of the HER, MER and LER samples are 44.5, 7.1 and $13.9 \mathrm{pb}^{-1}$, respectively. The cross-section measurements at fixed $\left(x, Q^{2}\right)$ and

[^0]different $y$ were used to extract $F_{2}$ and $F_{L}$ separately, as has been previously done in fixedtarget experiments [10-13], and recently by the ZEUS [14] and H1 [15-18] collaborations at HERA. For the results reported in this paper, high-precision data taken with an 820 GeV proton beam [19] were also included in the extraction. This data set, referred to as the ZEUS97 sample, has an integrated luminosity of $30.0 \mathrm{pb}^{-1}$.

The data described here supersede those in the previous publication [14]. The main improvements with respect to the previous analysis are an extension of the kinematic coverage of the measurements to lower values of $Q^{2}$, improved analysis techniques and a better understanding of systematic uncertainties, leading to more accurate and more precise measurements of the reduced cross sections.

## 2 The ZEUS detector

A detailed description of the ZEUS detector can be found elsewhere [20]. A brief outline of the components most relevant for this analysis is given below.

In the kinematic range of the analysis, charged particles were tracked in the central tracking detector (CTD) [21] and the microvertex detector (MVD) [22]. These components operated in a magnetic field of 1.43 T provided by a thin superconducting solenoid. The CTD drift chamber covered the polar-angle region ${ }^{2} 15^{\circ}<\theta<164^{\circ}$. The MVD silicon tracker consisted of a barrel (BMVD) and a forward (FMVD) section. The BMVD provided polarangle coverage for tracks from $30^{\circ}$ to $150^{\circ}$. The FMVD extended the polar-angle coverage in the forward region to $7^{\circ}$. For CTD-MVD tracks that pass through all nine CTD superlayers, the momentum resolution was $\sigma\left(p_{T}\right) / p_{T}=0.0029 p_{T} \oplus 0.0081 \oplus 0.0012 / p_{T}$, with $p_{T}$ in GeV .

The high-resolution uranium-scintillator calorimeter (CAL) [23] consisted of three parts: the forward (FCAL), the barrel (BCAL) and the rear (RCAL) calorimeters. Each part was subdivided transversely into towers and longitudinally into one electromagnetic section (EMC) and either one (in RCAL) or two (in BCAL and FCAL) hadronic sections (HAC). The smallest subdivision of the calorimeter was called a cell. The CAL energy resolutions, as measured under test-beam conditions, were $\sigma(E) / E=0.18 / \sqrt{E}$ for electrons and $\sigma(E) / E=0.35 / \sqrt{E}$ for hadrons, with $E$ in GeV .

The rear hadron-electron separator (RHES) [24] consisted of a layer of approximately $10000\left(3 \times 3 \mathrm{~cm}^{2}\right)$ silicon-pad detectors inserted in the RCAL at a depth of approximately 3 radiation lengths. The small-angle rear tracking detector (SRTD) [25] was attached

[^1]to the front face of the RCAL and consisted of two planes of scintillator strips. The detector covers the total area of $68 \times 68 \mathrm{~cm}^{2}$, with a $20 \times 20 \mathrm{~cm}^{2}$ cutout in the centre for the beam-pipe. The polar-angle coverage is $162^{\circ}<\theta<176^{\circ}$, with full acceptance for $167^{\circ}<\theta<174.5^{\circ}$.

A small tungsten-scintillator calorimeter located approximately 6 m from the interaction point in the rear direction is referred to as the 6 m Tagger [26]. For scattered electrons at small scattering angles in the energy range from 4.1 to 7.4 GeV , the acceptance was very close to unity with very high purity.

The luminosity was measured using the Bethe-Heitler reaction $e p \rightarrow e \gamma p$ by a luminosity detector which consisted of independent lead-scintillator calorimeter [27] and magnetic spectrometer [28] systems. The fractional systematic uncertainty on the measured luminosity was $1.8 \%$ [29], composed of a correlated uncertainty of $1.5 \%$ for the HER, LER and MER data sets and an additional $1 \%$ uncorrelated uncertainty for each data set. The ZEUS97 data set has an independent luminosity uncertainty of $1.5 \%$.

## 3 Monte Carlo samples

The DIS signal processes were simulated using the DJangoh 1.6 Monte Carlo (MC) model [30] with the CTEQ5D [31] parameterisation of the proton PDF. The hadronic final state of the DJangoh MC was simulated using the colour-dipole model of Ariadne 4.12 [32]. Photoproduction events $\left(Q^{2}<1.5 \mathrm{GeV}^{2}\right)$, which are the largest background for the measurement, were simulated using the Pythia 6.221 [33] MC model in conjunction with the GRV-G-96 HO [34] parameterisation of the photon PDF and with the CTEQ5D parameterisation of the proton PDF. Additional background components that were considered were elastic QED Compton (QEDC) scattering and mis-reconstructed low- $Q^{2}$ DIS events, simulated using the Grape-Compton [35] and Djangoh 1.6 MC models, respectively. For all Djangoh- and Pythia-generated samples the Lund string model of Jetset 7.4 [36] was used for the hadronisation. The ZEUS detector response was simulated using a program based on Geant 3.21 [37]. The generated events were passed through the detector simulation, subjected to the same trigger requirements as the data and processed by the same reconstruction programs.

The Djangoh and Pythia samples included a diffractive component and first-order electroweak corrections. The diffractive and non-diffractive components of the DJangoh sample were scaled (reweighted) to improve the description of the $\eta_{\max }$ distribution, where $\eta_{\max }$ is equal to the pseudorapidity of the most forward CAL energy deposit.

The electroweak corrections were simulated using the Heracles 4.6 [30,38] MC model. Their uncertainty was evaluated by comparing the predictions from Heracles to the higher-order predictions from Hector 1.0 [39]. The predictions were found to agree
within $0.5 \%$ and the remaining uncertainty was not included in the systematic uncertainties on the cross sections.

The predicted photoproduction backgrounds were compared with photoproduction data selected using the 6 m -Tagger and a photoproduction-enriched sample, separately for all three data sets, HER, LER and MER [40]. An event weight was determined for this background as a function of the candidate electron energy to bring the MC in agreement with the data. The same weighting function was found to suffice for all data sets. The weight reduced from 1.2 at electron-candidate energy of 6 GeV to 1.0 at electron-candidate energy of 10 GeV . A remaining uncertainty of $\pm 10 \%$ in the photoproduction background was accounted for in the systematic uncertainties.

## 4 Vertex distribution

Events were separately analysed for two different vertex regions: the central-vertex region, $\left|Z_{\mathrm{vtx}}\right|<30 \mathrm{~cm}$, and the shifted-vertex region, $30<Z_{\mathrm{vtx}}<100 \mathrm{~cm}$. The latter region contained fewer events, but the scattered electrons from these events had a greatly increased geometrical acceptance at low $Q^{2}$.

The distribution of event vertices depended on the bunch structure of the proton and electron beams. Primary bunches were separated by 96 ns , whereas secondary electron bunches occurred at 2 ns intervals and secondary proton bunches occurred at 4.8 ns intervals. Assuming that one of the interacting particles was in a primary bunch, nine separate peaks of the vertex distribution along the $Z$ direction, within $\pm 100 \mathrm{~cm}$ of the nominal interaction point, were produced.

A dedicated analysis of the vertex distributions using separate data sets was performed, leading to a parameterisation with 10 Gaussian functions, two Gaussian functions being required to fit the central peak [40]. The obtained efficiency-corrected $Z_{\mathrm{vtx}}$ distributions for the HER, MER and LER samples are shown in Fig. 1.

The vertex distributions of the MC samples were accurately tuned to match the distributions in the data. This was particularly crucial to obtain proper acceptance corrections for the events selected in the two different vertex regions. The underlying vertex distributions were extracted using a number of different data selections, leading to the determination of an increased uncorrelated normalisation uncertainty for the shifted-vertex region of 3.0\%.

## 5 Event reconstruction and selection

The event kinematics were evaluated based on the reconstruction of the scattered electron [41] using

$$
\begin{gather*}
y_{e}=1-\frac{E_{e}^{\prime}}{2 E_{e}}\left(1-\cos \theta_{e}\right),  \tag{2}\\
Q_{e}^{2}=2 E_{e}^{\prime} E_{e}\left(1+\cos \theta_{e}\right), \tag{3}
\end{gather*}
$$

where $\theta_{e}$ and $E_{e}^{\prime}$ are the polar angle and energy of the scattered electron, respectively.
Electrons were identified using a neural network based on the moments of the threedimensional shower profile of clusters found in the CAL [42]. The quantity $E_{e}^{\prime}$ was reconstructed using the CAL and $\theta_{e}$ was determined using the reconstructed interaction vertex and scattered-electron position in the SRTD or, if outside the SRTD acceptance, in the RHES. In less than $2 \%$ of events, $\theta_{e}$ could not be determined in this way; such events were rejected.

The quantity $\delta \equiv \sum_{i}\left(E-p_{Z}\right)_{i}$ was used both in the trigger and in the offline analysis. The sum runs over all CAL energy deposits. Conservation of energy, $E$, and longitudinal momentum, $p_{Z}$, implies that $\delta \approx 2 E_{e}=55 \mathrm{GeV}$. Undetected particles that escape through the forward beam-pipe hole contribute negligibly to $\delta$. Undetected particles that escape through the rear beam-pipe hole, such as the final-state electron in a photoproduction event, cause a substantial reduction in $\delta$. Events not originating from $e p$ collisions often exhibit a very large $\delta$.

A three-level trigger system was used to select events online [20, 43-45]. A dedicated trigger was developed providing high efficiency for high-y events [46]. The trigger required an event to have $\delta>30 \mathrm{GeV}$ and either an electron candidate with $E^{\prime}>4 \mathrm{GeV}$ in the RCAL outside a $30 \times 30 \mathrm{~cm}^{2}$ box centred around the beam-pipe, or a $\delta^{\theta<165^{\circ}}>20 \mathrm{GeV}$, where $\delta^{\theta<165^{\circ}}$ denotes $\delta$ calculated only from the CAL energy deposits at polar angles less than $165^{\circ}$. The electron energy requirement was reduced to $E^{\prime}>2 \mathrm{GeV}$ for the MER and LER data runs.

Events were selected offline if:

- $42<\delta<65 \mathrm{GeV}$;
- the reconstructed interaction vertex fulfilled $\left|Z_{\mathrm{vtx}}\right|<30 \mathrm{~cm}$ for the central- and $30<Z_{\mathrm{vtx}}<100 \mathrm{~cm}$ for the shifted-vertex region;
- the energy of the electron candidate satisfied $E_{e}^{\prime}>6 \mathrm{GeV}$;
- the event topology was not compatible with an elastic QEDC scattering event, which has an event signature of two, and only two, electromagnetic deposits back to back in azimuthal angle;
- the event timing was consistent with that of an $e p$ interaction;
- $y_{e}<0.95$ and $y_{\mathrm{JB}}>0.05$, where $y_{\mathrm{JB}}$ is the Jacquet-Blondel estimator [47] of $y$;
- $p_{T, h} / p_{T, e}>0.3$, where $p_{T, h}$ and $p_{T, e}$ refer to the transverse momentum of the hadronic system and electron candidate, respectively.

The projected path of the electron candidate was required to:

- exit the CTD at a radius $>20 \mathrm{~cm}$ and hence traverse the MVD fiducial volume (typically three active layers) and at least four active layers of the CTD;
- enter the RCAL at a radius $<135 \mathrm{~cm}$, to ensure full energy containment in the RCAL.

Hit information from the MVD and CTD was used to validate electron candidates [46]. The procedure was based on the ratios of the number of observed to the maximum number of possible hits in the MVD and CTD along the track trajectory, denoted $f_{\text {hit }}^{\text {MVD }}$ and $f_{\text {hit }}^{C T D}$. The requirements on the ratios were $f_{\text {hit }}^{\mathrm{MVD}}>0.45$ and $f_{\mathrm{hit}}^{\mathrm{CTD}}>0.6$. This method was used to increase the polar-angle acceptance compared to the regular tracking capability of the MVD+CTD [40]. The efficiency of this selection was studied in data and MC using specially selected data sets. A reweighting of the MC for the central-vertex event sample was found to be necessary for electrons at small scattering angles, with a maximum weight of 1.06 for the smallest angles used in this analysis. The uncertainty on this reweighting procedure was included in the systematic uncertainties. No reweighting was necessary for the shifted-vertex events, as the efficiency of the selection is high and uniform for this sample.

Figures 2 and 3 show the distributions of the variables $E_{e}^{\prime}$ and $\theta_{e}$ for the HER, MER and LER data sets compared to the combined detector-level predictions from the MC models, for the central- and shifted-vertex events. In the region $E_{e}^{\prime}>6 \mathrm{GeV}$, which is relevant for the analysis, the agreement is adequate for the extraction of the cross sections. According to the MC models, the final data samples contained $97 \%$ DIS signal and $3 \%$ background events for both the central- and shifted-vertex data samples in all data sets. The vast majority of the background events were found at low $Q^{2}$ and high $y$ and resulted from photoproduction processes. In the kinematic bin most affected the background fraction was 30\%.

## 6 Cross-section measurements

The reduced cross section at a given $\left(x, Q^{2}\right)$ value was evaluated from the number of events reconstructed in a bin according to

$$
\begin{equation*}
\tilde{\sigma}\left(x, Q^{2}\right)=\frac{N_{\mathrm{data}}-N_{\mathrm{MC}}^{\mathrm{bg}}}{N_{\mathrm{MC}}^{\mathrm{DIS}}} \tilde{\sigma}_{\mathrm{SM}}\left(x, Q^{2}\right) . \tag{4}
\end{equation*}
$$

Here $N_{\text {data }}, N_{\mathrm{MC}}^{\mathrm{bg}}$ and $N_{\mathrm{MC}}^{\mathrm{DIS}}$ denote, respectively, the number of observed events in the data and the expected number of background and signal DIS events. The latter two numbers
were taken from the MC simulations normalised to the data luminosity. The quantity $\tilde{\sigma}_{S M}\left(x, Q^{2}\right)$ denotes the Standard Model electroweak Born-level reduced cross section and was calculated with the DJANGOH simulation with electroweak corrections switched off.

The reduced cross sections were measured according to Eq. (4) for kinematic values in the ranges $0.13 \leq y \leq 0.75$ and $5 \leq Q^{2} \leq 110 \mathrm{GeV}^{2}$. They were measured separately for the central- and shifted-vertex regions, referred to as $\tilde{\sigma}_{\text {cen }}$ and $\tilde{\sigma}_{\text {sh }}$. The following $y$-bin boundaries were used for the measurement: $0.090,0.175,0.265,0.355,0.440,0.520,0.600$, $0.660,0.720,0.780$, together with the following $Q^{2}$-bin boundaries: $4.5,6,8,11,15,20,28$, $38,52,70,95,130 \mathrm{GeV}^{2}$. Figures 4-6 show the obtained cross sections at fixed values of $Q^{2}$ as functions of $y$ for the HER, MER and LER samples, separately for the central- and shifted-vertex regions. Reasonable agreement is observed between them. The predictions based on the HERAPDF1.5 PDF set [48] are also depicted in the Figs. 4-6 and provide a good description of the data. The reduced cross sections are reported in Tables 1-6 for central- and shifted-vertex regions and are used for the extraction of structure functions.

## 7 Systematic uncertainties

A number of possible effects that could affect the cross-section measurements have been investigated. These are classified according to whether they affect measurements in a correlated way, can be treated as uncorrelated, or are negligibly small. The systematic uncertainties are denoted with the symbols $\delta_{\text {source }}$. The numbers in the parentheses give an indication of the typical values observed in the cross-section bins and the maximum observed value. The correlated systematic uncertainties are:

- $\left\{\delta_{\gamma p}\right\}$, the $\pm 10 \%$ normalisation uncertainty on the level of photoproduction background ( $0.5 \%, 3 \%$ );
- $\left\{\delta_{E_{\text {had }}}\right\}$, the $\pm 2 \%$ hadronic-energy-scale uncertainty, evaluated by varying the scale in simulated events $(0.5 \%, 4 \%)$;
- $\left\{\delta_{\text {diff }}\right\}$, the uncertainty on the scale factors applied to the diffractive DJangoh component ( $0.2 \%, 0.5 \%$ );
- $\left\{\delta_{\text {hits }}\right\}$, the electron validation using CTD and MVD information $(1 \%, 2 \%)$.

These $\delta_{\text {source }}$ values are listed in Tables 1-6 for the reduced cross sections at the three different centre-of-mass energies. The reduced cross-section changes are quoted with a sign indicating how they should be handled in a fit; positive quantities indicate an increase in the cross sections, while negative quantities indicate a decrease. The probability distributions for the cross sections are taken to be Gaussian. The quoted numbers in the table correspond to variations of one standard deviation.

A number of systematic tests produced sizeable changes in the extracted cross sections, but should be considered as uncorrelated in fits to the cross sections. These are:

- $\left\{\delta_{E_{e}}\right\}$, the electron-energy-scale uncertainty [14] of $\pm 0.5 \%$ for $E_{e}^{\prime}>20 \mathrm{GeV}$, increasing to $\pm 1.9 \%$ at $E_{e}^{\prime}=6 \mathrm{GeV}$, evaluated by varying the scale in simulated events $(1 \%, 5 \%)$;
- $\left\{\delta_{e I D}\right\}$, the uncertainty of the electron-finding efficiency, evaluated by loosening or tightening the criterion applied to the output of the neural network used to select electron candidates, both for data and MC $(0.2 \%, 5 \%)$;
- $\left\{\delta_{\mathrm{Zvtx}}\right\}$, the uncertainty on the $Z_{\mathrm{vtx}}$ distribution, that was evaluated by varying the event-selection criteria used in the extraction of the distribution ( $0.5 \%, 10 \%$ );
- $\left\{\delta_{d x}, \delta_{d y}\right\}$, the SRTD and HES position uncertainty of $\pm 2 \mathrm{~mm}$ in both the horizontal and vertical directions $(0.5 \%, 4 \%)$.

These uncertainties are combined in quadrature for each bin and quoted under the label $\delta_{\text {unc. }}$. This uncertainty also includes the statistical uncertainty in the MC sample.

Finally, a number of checks were performed with resulting cross-section variations small enough that they are considered negligible. These include:

- the trigger-efficiency uncertainty;
- the uncertainty due to the electroweak corrections. In a separate analysis [49] the differential cross sections for radiating an initial-state photon were also extracted. These results are in excellent agreement with the expectations from Heracles, adding confidence that the radiative effects are well simulated.

In addition to the uncertainties listed above, the normalisation uncertainties for the different data sets are:

- a correlated luminosity uncertainty of $1.5 \%$ for the HER, MER and LER cross sections;
- an additional uncorrelated luminosity and vertex-distribution uncertainty of $1.0 \%$ for the central-vertex HER, MER and LER cross sections and $3.0 \%$ for the shifted-vertex cross sections.

The total systematic uncertainty, $\delta_{\text {sys }}$, in each bin formed by adding the individual uncertainties in quadrature, is also given in Tables 1-6. The total systematic uncertainties in the tables do not include the normalisation uncertainty.

## 8 The ZEUS97 data set

In order to increase the precision of the $F_{L}$ and $F_{2}$ extractions, the reduced NC cross sections measured by ZEUS from the $e^{+} p$ data taken in 1996 and 1997 with the proton beam energy $E_{p}=820 \mathrm{GeV}$ [19] were included in the analysis. The precision of these
data is comparable to the HER data presented in this paper. The binning for the ZEUS97 sample is similar but not identical to that used for the present measurement, so a binning correction was applied to the cross sections from the ZEUS97 sample, using the procedure introduced in an earlier publication [1]. An interpolation of a measurement to the required point on the $\left(x, Q^{2}\right)$ grid is performed by multiplying the measured cross section by the ratio of predicted double-differential cross sections at two different $\left(x, Q^{2}\right)$ points. For the theory calculation, the HERAPDF1.0 NLO prediction [1] was used. Only the points which required less than $2 \%$ adjustments were included in the fits to extract $F_{2}$ and $F_{L}$. The following systematic uncertainties [19] on the cross sections were included in the extraction of $F_{2}$ and $F_{L}$ :

- $\left\{\tilde{\delta}_{\text {eID }}\right\}$, the uncertainty in the positron-finding efficiency;
- $\left\{\tilde{\delta}_{\text {dxdy }}\right\}$, the uncertainty in the positron position;
- $\left\{\tilde{\delta}_{e \theta}\right\}$, the uncertainty in the positron-scattering angle;
- $\left\{\tilde{\delta}_{E_{e}}\right\}$, the uncertainty in the positron-energy scale;
- $\left\{\tilde{\delta}_{\text {hdFC }}\right\},\left\{\tilde{\delta}_{\text {hdBC }}\right\},\left\{\tilde{\delta}_{\text {hdRC }}\right\}$, the uncertainty in the hadronic-energy scales in FCAL, BCAL and RCAL;
- $\left\{\tilde{\delta}_{\text {hdF }}\right\},\left\{\tilde{\delta}_{\text {Bhd }}\right\}$, the uncertainty in the hadronic-energy flow;
- $\left\{\tilde{\delta}_{\gamma p}\right\}$, the uncertainty in the photoproduction background.

Following the prescription in the publication [19], all uncertainties were treated as correlated.

The fractional systematic uncertainties after the cross-section adjustment were kept unchanged. In addition to the uncertainties mentioned above, the ZEUS97 cross sections have a $2 \%$ normalisation uncertainty, which resulted from a $1.5 \%$ luminosity uncertainty, a $1 \%$ trigger uncertainty, a $1 \%$ uncertainty from the vertex distribution and a $0.5 \%$ uncertainty due to radiative corrections.

The reduced cross sections after the adjustment are given double differentially in bins of $x$ and $Q^{2}$ in Table 7.

## 9 Extraction of $\boldsymbol{F}_{2}, \boldsymbol{F}_{L}$ and $\boldsymbol{R}$

### 9.1 Fit method

The values of $F_{L}, F_{2}$ and $R=F_{L} /\left(F_{2}-F_{L}\right)$ were extracted by performing a fit to the reduced cross sections using Eq. (1). The fit was performed with the BAT package [50]. Prior to fitting, the HER, MER and LER cross sections were normalised to ZEUS97 data
at low $y$. This resulted in the following normalisation factors for the central- and shiftedvertex samples: $c_{\text {HER,cen }}=1.025 \pm 0.003 ; c_{\text {MER,cen }}=1.004 \pm 0.007 ; c_{\text {LER,cen }}=0.998 \pm 0.004$; $c_{\mathrm{HER}, \mathrm{sh}}=1.042 \pm 0.012 ; c_{\mathrm{MER}, \mathrm{sh}}=1.056 \pm 0.020 ;$ and $c_{\mathrm{LER}, \mathrm{sh}}=1.039 \pm 0.013$. Within the assigned global normalisation uncertainties of the respective data sets these numbers are consistent with unity. A different cross-section binning was used for the structurefunction extraction in order to cover similar $Q^{2}$ and $x$ ranges in each data set, resulting in $27\left(x, Q^{2}\right)$ bins. After normalisation, the reduced cross sections from the central- and shifted-vertex regions were combined using a weighted average based on their combined statistical and uncorrelated systematic uncertainties. This procedure, and the use of ZEUS97 data, yielded a total of 104 reduced cross sections.

The procedure to extract $F_{L}, F_{2}$ and $R$ is explained in the previous ZEUS publication [14]. Free parameters were introduced in the fits for each of the data sets to allow the normalisation factors to vary according to Gaussian distributions with standard deviations given by the uncertainties quoted above for the HER, MER and LER data sets, respectively. The uncertainties for the central-vertex sample were used for $Q^{2} \geq 17 \mathrm{GeV}^{2}$, where these data dominated the combinations. The uncertainties for the shifted-vertex sample were used at lower $Q^{2}$, where they are more precise. The normalisation of the ZEUS97 data was kept fixed in the fit. The resulting values of $F_{2}$ and $F_{L}$ therefore have an additional uncertainty resulting from the normalisation uncertainty of this data set $(2 \%)$.

Additional nuisance parameters were introduced in the fits to account for the correlated systematic uncertainties.
All systematic uncertainties of the ZEUS97 data set were treated as uncorrelated with those of the HER, MER and LER samples. A global fit was performed to extract $F_{L}$ and $F_{2}$ values from the reduced cross sections. The joint posterior probability distribution for all parameters was then extracted under different sets of conditions as described below.

### 9.2 Fit results

Constrained and unconstrained parameter fits were performed to extract $F_{2}\left(x, Q^{2}\right)$ and $F_{L}\left(x, Q^{2}\right)$. Flat priors were used for the physics parameters within the range allowed. For the constrained fits, $0.8<F_{2}<2$ and $0 \leq F_{L} \leq F_{2}$ were required. Example Rosenbluth plots [51] showing the linear fit to the reduced cross sections together with a $68 \%$ uncertainty band are shown in Fig. 7 for the nine lowest $x$ points for each $Q^{2}$ bin for the unconstrained fits. The resulting $F_{2}$ and $F_{L}$ values for all $\left(x, Q^{2}\right)$ bins are given in Table 8. The results given are for the parameter value at the mode of the marginalised probability distribution [50]. For each parameter, the narrowest $68 \%$ probability interval around the marginalised mode is taken as an uncertainty. These ranges contain the full experimental uncertainty. Relative uncertainties as small as $2 \%$ were achieved for $F_{2}$ and (absolute) uncertainties for $F_{L}$ were in the range $0.1-0.2$. The results for $F_{L}$ found in this
analysis are somewhat lower than those in the previous ZEUS analysis [14] in the region of kinematic overlap. The difference compared to the previous ZEUS analysis is primarily due to improvements in the treatment of the diffractive events in the MC simulation and of the electron validation at small scattering angles.

The results from the unconstrained fit are shown in Fig. 8 together with predictions based on the HERAPDF1.5 expectations. Reasonable agreement between extracted and predicted $F_{2}$ and $F_{L}$ values is observed although with a tendency for the prediction to lie below the extracted $F_{L}$ values.

Further fits to the data were performed to extract $F_{L}\left(Q^{2}\right), R\left(Q^{2}\right)$, and a single overall value of $R$ for the full data set. In each case, the same fitting procedure as described above was used, but with a reduced number of parameters.

To extract $F_{L}\left(Q^{2}\right)$, first $r\left(Q^{2}\right)$ was fitted, where $r=F_{L} / F_{2}$, taking a single value of $r$ for all $x$ points in the same $Q^{2}$ bin. The value of $F_{L}\left(Q^{2}\right)$ was then evaluated as $F_{L}\left(x_{i}, Q^{2}\right)=$ $r\left(Q^{2}\right) F_{2}\left(x_{i}, Q^{2}\right)$, where for each $Q^{2}$ point, $x_{i}$ was chosen such that $Q^{2} / x_{i}$ was constant, which for $\sqrt{s}=225 \mathrm{GeV}$ corresponds to $y=0.71$. Both constrained and unconstrained fits were made. For the constrained $R\left(Q^{2}\right)$ and overall $R$ fits it was required that $R\left(Q^{2}\right) \geq 0$ and $R \geq 0$. The results for $F_{L}\left(Q^{2}\right)$ and $R\left(Q^{2}\right)$ are given in Table 9 . The overall value of $R$ from both the unconstrained and constrained fits is $R=0.105_{-0.037}^{+0.055}$. Figures 9a and 9b show a comparison of $F_{L}\left(Q^{2}\right)$ and $R\left(Q^{2}\right)$ with the H1 data [17] and the NNLO QCD predictions based on the HERAPDF1.5. The H1 measurements generally lie above the ZEUS results. The differences were examined: taking into account the correlations between the ZEUS data points and neglecting the correlations between the H1 data points a $\chi^{2}$ of 12.2 is obtained for 8 degrees of freedom. The predictions based on HERAPDF1.5 are in reasonable agreement with both data sets.

## 10 Summary

The reduced cross sections $\tilde{\sigma}\left(x, Q^{2}\right)$ for $e^{+} p$ neutral current deep inelastic scattering have been measured with the ZEUS detector at HERA, using data collected at $\sqrt{s}=318,251$ and 225 GeV , in the kinematic region $0.13 \leq y \leq 0.75$ and $5 \leq Q^{2} \leq 110 \mathrm{GeV}^{2}$. The extension of the kinematic range in comparison to the previous ZEUS publication [14] was made possible with the use of shifted-vertex data. The new results supersede those in the previous publication. The reduced cross sections were used together with those from previous ZEUS data collected at $\sqrt{s}=300 \mathrm{GeV}$ to extract the proton structure functions $F_{2}$ and $F_{L}$ for 27 values of $x$ and $Q^{2}$. Relative uncertainties as small as $2 \%$ were achieved for $F_{2}$ and (absolute) uncertainties for $F_{L}$ were in the range $0.1-0.2$. In addition, $F_{L}$ and the ratio, $R=F_{2} /\left(F_{2}-F_{L}\right)$, have also been extracted as a function of $Q^{2}$ together with an overall value of $R=0.105_{-0.037}^{+0.055}$. The $F_{L}$ measurements reported here are lower than
but compatible with those in the previous ZEUS and H 1 publications and in reasonable agreement with theoretical predictions for $F_{L}$.

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| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { HER } \end{gathered}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{aligned} & \hline \delta_{\text {sys }} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\gamma p} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{E_{\text {had }}}$ (\%) | $\delta_{\text {diff }}$ <br> (\%) | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0.000110 | 0.63 | 1.810 | 10.05 | 17.30 | 16.70 | 0.00 | -4.09 | $-0.61$ | 1.83 |
| 7 | 0.000100 | 0.69 | 1.470 | 5.39 | 10.06 | 9.63 | $-0.97$ | -1.69 | -0.56 | 2.06 |
| 7 | 0.000092 | 0.75 | 1.257 | 3.45 | 10.20 | 7.36 | -3.14 | -5.86 | $-0.63$ | 2.29 |
| 9 | 0.000185 | 0.48 | 1.372 | 8.67 | 10.84 | 10.62 | -0.57 | -0.70 | -0.10 | 1.99 |
| 9 | 0.000159 | 0.56 | 1.334 | 4.25 | 6.32 | 5.95 | -0.61 | 0.32 | -0.25 | 2.00 |
| 9 | 0.000141 | 0.63 | 1.382 | 2.68 | 5.09 | 4.57 | -0.44 | -1.05 | -0.42 | 1.89 |
| 9 | 0.000129 | 0.69 | 1.434 | 1.98 | 5.14 | 3.84 | -1.23 | -2.52 | -0.51 | 1.90 |
| 9 | 0.000119 | 0.75 | 1.419 | 1.72 | 6.77 | 4.54 | -2.51 | -3.86 | -0.58 | 1.90 |
| 12 | 0.000539 | 0.22 | 1.196 | 10.66 | 13.24 | 13.13 | 0.00 | 0.45 | 0.09 | 1.68 |
| 12 | 0.000383 | 0.31 | 1.124 | 6.16 | 7.98 | 7.76 | 0.00 | -0.13 | 0.04 | 1.86 |
| 12 | 0.000297 | 0.40 | 1.239 | 3.43 | 4.95 | 4.55 | -0.03 | -0.48 | -0.20 | 1.89 |
| 12 | 0.000247 | 0.48 | 1.340 | 2.16 | 4.05 | 3.50 | -0.44 | -0.59 | -0.16 | 1.90 |
| 12 | 0.000212 | 0.56 | 1.279 | 1.55 | 3.27 | 2.47 | -0.81 | -0.61 | -0.28 | 1.86 |
| 12 | 0.000188 | 0.63 | 1.384 | 1.44 | 3.36 | 2.42 | -0.89 | -1.21 | -0.38 | 1.74 |
| 12 | 0.000172 | 0.69 | 1.404 | 1.34 | 3.80 | 2.47 | -1.29 | -1.96 | -0.40 | 1.63 |
| 12 | 0.000158 | 0.75 | 1.416 | 1.45 | 5.86 | 3.53 | -2.58 | -3.56 | -0.52 | 1.54 |
| 17 | 0.001293 | 0.13 | 0.954 | 3.65 | 6.45 | 6.11 | 0.00 | -0.75 | 0.09 | 1.91 |
| 17 | 0.000764 | 0.22 | 1.129 | 2.47 | 4.08 | 3.60 | 0.00 | -0.24 | 0.10 | 1.89 |
| 17 | 0.000542 | 0.31 | 1.272 | 1.68 | 3.10 | 2.49 | -0.03 | -0.30 | -0.10 | 1.83 |
| 17 | 0.000420 | 0.40 | 1.353 | 1.29 | 2.42 | 1.65 | -0.08 | -0.20 | -0.18 | 1.75 |
| 17 | 0.000350 | 0.48 | 1.373 | 1.13 | 2.31 | 1.54 | -0.23 | -0.29 | -0.21 | 1.67 |
| 17 | 0.000300 | 0.56 | 1.389 | 1.03 | 2.28 | 1.51 | -0.42 | -0.55 | -0.24 | 1.55 |
| 17 | 0.000267 | 0.63 | 1.402 | 1.21 | 2.65 | 1.72 | -0.92 | -1.01 | $-0.35$ | 1.44 |
| 17 | 0.000244 | 0.69 | 1.440 | 1.30 | 3.77 | 2.46 | -1.45 | -2.08 | -0.37 | 1.28 |
| 17 | 0.000224 | 0.75 | 1.409 | 1.48 | 5.22 | 3.11 | -2.08 | -3.46 | -0.46 | 1.05 |
| 24 | 0.001826 | 0.13 | 1.048 | 0.98 | 2.88 | 2.16 | 0.00 | -0.72 | 0.08 | 1.76 |
| 24 | 0.001079 | 0.22 | 1.190 | 0.86 | 2.61 | 1.98 | -0.00 | -0.21 | -0.04 | 1.69 |
| 24 | 0.000766 | 0.31 | 1.257 | 0.81 | 1.98 | 1.14 | -0.06 | -0.15 | -0.09 | 1.61 |
| 24 | 0.000593 | 0.40 | 1.360 | 0.81 | 1.77 | 0.91 | -0.09 | -0.19 | -0.14 | 1.50 |
| 24 | 0.000494 | 0.48 | 1.422 | 0.83 | 1.75 | 1.00 | $-0.28$ | -0.27 | -0.16 | 1.38 |
| 24 | 0.000424 | 0.56 | 1.416 | 0.88 | 1.73 | 1.05 | -0.37 | -0.52 | $-0.24$ | 1.20 |
| 24 | 0.000377 | 0.63 | 1.440 | 1.10 | 2.21 | 1.53 | -0.65 | -1.01 | -0.25 | 1.02 |
| 24 | 0.000344 | 0.69 | 1.485 | 1.24 | 3.57 | 2.89 | -1.08 | -1.55 | $-0.28$ | 0.83 |
| 24 | 0.000316 | 0.75 | 1.495 | 1.42 | 3.78 | 2.25 | $-1.80$ | $-2.36$ | $-0.36$ | 0.58 |
| 32 | 0.002434 | 0.13 | 1.013 | 0.64 | 2.36 | 1.69 | -0.00 | -0.62 | 0.06 | 1.52 |
| 32 | 0.001438 | 0.22 | 1.163 | 0.66 | 2.13 | 1.57 | -0.00 | -0.19 | -0.03 | 1.42 |
| 32 | 0.001021 | 0.31 | 1.280 | 0.71 | 1.51 | 0.76 | -0.02 | -0.13 | -0.09 | 1.30 |
| 32 | 0.000791 | 0.40 | 1.358 | 0.79 | 1.61 | 1.07 | -0.09 | -0.26 | -0.13 | 1.16 |
| 32 | 0.000659 | 0.48 | 1.407 | 0.89 | 1.44 | 0.99 | -0.19 | -0.18 | -0.18 | 1.00 |
| 32 | 0.000565 | 0.56 | 1.462 | 1.00 | 1.62 | 1.25 | -0.39 | $-0.47$ | $-0.20$ | 0.81 |
| 32 | 0.000502 | 0.63 | 1.419 | 1.29 | 1.96 | 1.49 | -0.61 | -0.90 | -0.25 | 0.60 |
| 32 | 0.000459 | 0.69 | 1.490 | 1.39 | 2.62 | 1.93 | -0.69 | -1.56 | -0.24 | 0.40 |
| 32 | 0.000422 | 0.75 | 1.453 | 1.67 | 4.03 | 2.97 | -1.90 | -1.92 | -0.29 | 0.21 |

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| ---: | ---: | ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: | ---: |
| 45 | 0.003423 | 0.13 | 0.981 | 0.58 | 1.89 | 1.31 | 0.00 | -0.65 | 0.08 | 1.21 |
| 45 | 0.002023 | 0.22 | 1.128 | 0.64 | 1.92 | 1.57 | -0.01 | -0.15 | -0.02 | 1.08 |
| 45 | 0.001435 | 0.31 | 1.243 | 0.73 | 1.22 | 0.76 | -0.00 | -0.12 | -0.09 | 0.94 |
| 45 | 0.001112 | 0.40 | 1.357 | 0.85 | 1.44 | 1.19 | -0.03 | -0.16 | -0.13 | 0.78 |
| 45 | 0.000927 | 0.48 | 1.372 | 1.00 | 1.40 | 1.24 | -0.10 | -0.14 | -0.18 | 0.62 |
| 45 | 0.000795 | 0.56 | 1.454 | 1.11 | 1.74 | 1.53 | -0.28 | -0.63 | -0.17 | 0.43 |
| 45 | 0.000706 | 0.63 | 1.481 | 1.43 | 2.27 | 2.11 | -0.52 | -0.59 | -0.20 | 0.26 |
| 45 | 0.000645 | 0.69 | 1.431 | 1.62 | 3.07 | 2.69 | -1.03 | -1.06 | -0.23 | -0.13 |
| 45 | 0.000593 | 0.75 | 1.491 | 1.79 | 3.48 | 2.85 | -1.38 | -1.43 | -0.27 | -0.02 |
| 60 | 0.004564 | 0.13 | 0.939 | 0.66 | 1.73 | 1.39 | 0.00 | -0.58 | 0.10 | 0.85 |
| 60 | 0.002697 | 0.22 | 1.108 | 0.74 | 1.72 | 1.56 | -0.00 | -0.13 | 0.02 | 0.73 |
| 60 | 0.001914 | 0.31 | 1.216 | 0.86 | 1.04 | 0.86 | -0.00 | -0.07 | -0.09 | 0.58 |
| 60 | 0.001483 | 0.40 | 1.298 | 1.01 | 1.45 | 1.38 | -0.04 | -0.11 | -0.08 | 0.43 |
| 60 | 0.001236 | 0.48 | 1.379 | 1.17 | 1.32 | 1.26 | -0.09 | -0.21 | -0.17 | 0.28 |
| 60 | 0.001060 | 0.56 | 1.446 | 1.31 | 1.48 | 1.41 | -0.22 | -0.32 | -0.23 | -0.14 |
| 60 | 0.000942 | 0.63 | 1.505 | 1.68 | 2.51 | 2.31 | -0.46 | -0.82 | -0.26 | -0.05 |
| 60 | 0.000860 | 0.69 | 1.543 | 1.82 | 2.53 | 2.20 | -0.44 | -1.15 | -0.25 | -0.01 |
| 60 | 0.000791 | 0.75 | 1.445 | 2.17 | 4.64 | 3.70 | -1.55 | -2.33 | -0.27 | 0.00 |
| 80 | 0.006085 | 0.13 | 0.875 | 0.76 | 1.58 | 1.36 | 0.00 | -0.61 | 0.14 | 0.49 |
| 80 | 0.003596 | 0.22 | 1.044 | 0.85 | 1.53 | 1.47 | 0.00 | -0.13 | 0.03 | 0.38 |
| 80 | 0.002552 | 0.31 | 1.185 | 0.98 | 1.02 | 0.98 | -0.01 | 0.04 | -0.04 | 0.26 |
| 80 | 0.001978 | 0.40 | 1.257 | 1.17 | 1.39 | 1.38 | -0.05 | 0.06 | -0.07 | 0.15 |
| 80 | 0.001648 | 0.48 | 1.336 | 1.35 | 1.36 | 1.34 | -0.07 | 0.16 | -0.12 | -0.06 |
| 80 | 0.001413 | 0.56 | 1.414 | 1.50 | 1.92 | 1.83 | -0.24 | -0.52 | -0.16 | -0.01 |
| 80 | 0.001256 | 0.63 | 1.477 | 1.91 | 2.19 | 2.13 | -0.32 | -0.36 | -0.23 | 0.00 |
| 80 | 0.001147 | 0.69 | 1.413 | 2.19 | 2.74 | 2.55 | -0.65 | -0.73 | -0.16 | 0.00 |
| 80 | 0.001055 | 0.75 | 1.397 | 2.55 | 6.08 | 5.67 | -1.41 | -1.67 | -0.20 | 0.00 |
| 110 | 0.008367 | 0.13 | 0.834 | 0.88 | 1.50 | 1.36 | 0.00 | -0.57 | 0.19 | 0.19 |
| 110 | 0.004944 | 0.22 | 1.019 | 0.99 | 1.76 | 1.75 | -0.00 | -0.12 | 0.03 | -0.12 |
| 110 | 0.003509 | 0.31 | 1.117 | 1.17 | 1.15 | 1.14 | -0.01 | -0.10 | -0.03 | -0.05 |
| 110 | 0.002719 | 0.40 | 1.226 | 1.36 | 1.46 | 1.45 | -0.03 | -0.14 | -0.08 | -0.01 |
| 110 | 0.002266 | 0.48 | 1.265 | 1.61 | 2.22 | 2.20 | -0.11 | -0.24 | -0.12 | 0.00 |
| 110 | 0.001942 | 0.56 | 1.386 | 1.81 | 2.21 | 2.13 | -0.38 | -0.43 | -0.12 | 0.00 |
| 110 | 0.001727 | 0.63 | 1.402 | 2.25 | 2.65 | 2.63 | -0.12 | -0.26 | -0.11 | 0.00 |
| 110 | 0.001576 | 0.69 | 1.447 | 2.57 | 3.19 | 3.08 | -0.46 | -0.66 | -0.13 | 0.00 |
| 110 | 0.001450 | 0.75 | 1.358 | 4.14 | 6.90 | 6.74 | -1.16 | -0.88 | -0.25 | 0.00 |

Table 1: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=318 G e V$ for the central-vertex region. The first three columns contain the bin centres in $Q^{2}, x$, and $y$, the next four contain the measured cross section, the statistical uncertainty, total systematic uncertainty and the uncorrelated uncertainty, respectively. The final four list the bin-to-bin correlated uncertainties from each systematic source, $\delta_{\gamma p}, \delta_{E_{\text {had }}}, \delta_{\text {diff }}$, $\delta_{\text {hits }}$. The normalisation uncertainty (see Section 7) is not included.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { HER } \end{gathered}$ | $\delta_{\text {stat }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\text {sys }} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\gamma p} \\ & (\%) \\ & \hline \end{aligned}$ | $\begin{array}{r} \hline \delta_{E_{\mathrm{had}}} \\ (\%) \\ \hline \end{array}$ | $\delta_{\text {diff }}$ <br> (\%) | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 0.000088 | 0.56 | 1.134 | 10.30 | 11.78 | 11.68 | -0.55 | -1.06 | -0.13 | 0.97 |
| 5 | 0.000078 | 0.63 | 1.503 | 7.34 | 9.60 | 9.50 | -0.40 | 0.73 | -0.22 | 1.08 |
| 5 | 0.000072 | 0.69 | 1.096 | 8.96 | 13.47 | 12.82 | -1.74 | -3.50 | -0.41 | 1.26 |
| 7 | 0.000173 | 0.40 | 1.185 | 8.02 | 9.73 | 9.67 | -0.34 | 0.44 | 0.01 | 0.87 |
| 7 | 0.000144 | 0.48 | 1.101 | 6.07 | 6.50 | 6.39 | -0.19 | -0.53 | -0.04 | 1.03 |
| 7 | 0.000124 | 0.56 | 1.210 | 5.40 | 7.79 | 7.68 | -0.32 | 0.72 | -0.23 | 1.06 |
| 7 | 0.000110 | 0.63 | 1.598 | 5.73 | 7.54 | 7.45 | -0.10 | 0.58 | -0.22 | 1.02 |
| 7 | 0.000100 | 0.69 | 1.117 | 7.69 | 16.42 | 16.17 | -2.42 | -0.74 | -0.39 | 1.26 |
| 7 | 0.000092 | 0.75 | 1.546 | 7.29 | 12.57 | 11.88 | -1.51 | -3.63 | $-0.30$ | 1.16 |
| 9 | 0.000685 | 0.13 | 0.870 | 7.37 | 8.00 | 7.95 | 0.00 | -0.35 | 0.02 | 0.80 |
| 9 | 0.000405 | 0.22 | 1.060 | 4.99 | 6.17 | 6.09 | 0.00 | -0.44 | -0.16 | 0.90 |
| 9 | 0.000287 | 0.31 | 1.157 | 4.19 | 5.37 | 5.27 | 0.00 | 0.07 | 0.08 | 1.00 |
| 9 | 0.000222 | 0.40 | 1.274 | 4.09 | 5.00 | 4.89 | 0.00 | 0.18 | -0.14 | 1.01 |
| 9 | 0.000185 | 0.48 | 1.261 | 4.29 | 5.28 | 5.15 | -0.19 | -0.58 | -0.06 | 1.02 |
| 9 | 0.000159 | 0.56 | 1.199 | 4.82 | 8.03 | 7.93 | -0.62 | 0.34 | -0.05 | 1.08 |
| 9 | 0.000141 | 0.63 | 1.383 | 5.50 | 7.31 | 7.10 | -0.45 | -1.29 | -0.08 | 1.05 |
| 9 | 0.000129 | 0.69 | 1.385 | 6.08 | 8.80 | 8.63 | -1.08 | -0.64 | -0.39 | 1.11 |
| 9 | 0.000119 | 0.75 | 1.293 | 7.54 | 12.59 | 12.09 | -1.45 | -2.98 | -0.21 | 1.16 |
| 12 | 0.000913 | 0.13 | 0.972 | 3.14 | 4.15 | 3.94 | 0.00 | -0.88 | -0.02 | 1.00 |
| 12 | 0.000539 | 0.22 | 1.179 | 3.02 | 4.02 | 3.89 | 0.00 | -0.14 | 0.04 | 1.00 |
| 12 | 0.000383 | 0.31 | 1.192 | 3.43 | 3.61 | 3.45 | -0.02 | -0.27 | 0.04 | 1.01 |
| 12 | 0.000297 | 0.40 | 1.247 | 3.86 | 4.36 | 4.22 | -0.25 | -0.20 | -0.16 | 1.03 |
| 12 | 0.000247 | 0.48 | 1.325 | 4.21 | 4.81 | 4.70 | -0.05 | 0.12 | -0.07 | 1.01 |
| 12 | 0.000212 | 0.56 | 1.424 | 4.39 | 5.23 | 5.10 | -0.16 | -0.48 | -0.12 | 1.02 |
| 12 | 0.000188 | 0.63 | 1.288 | 5.89 | 10.16 | 10.06 | -0.69 | 0.61 | -0.11 | 1.07 |
| 12 | 0.000172 | 0.69 | 1.335 | 6.86 | 9.47 | 9.24 | -1.45 | -0.91 | -0.12 | 1.16 |
| 12 | 0.000158 | 0.75 | 1.562 | 7.61 | 13.34 | 11.59 | -1.74 | $-6.25$ | -0.49 | 1.17 |
| 17 | 0.001293 | 0.13 | 0.946 | 2.90 | 3.60 | 3.39 | 0.00 | -0.68 | 0.05 | 1.00 |
| 17 | 0.000764 | 0.22 | 1.199 | 3.08 | 4.84 | 4.72 | 0.00 | -0.35 | 0.08 | 1.00 |
| 17 | 0.000542 | 0.31 | 1.258 | 3.48 | 3.82 | 3.68 | -0.02 | -0.04 | -0.17 | 1.01 |
| 17 | 0.000420 | 0.40 | 1.419 | 3.88 | 4.80 | 4.68 | 0.00 | 0.34 | -0.14 | 1.01 |
| 17 | 0.000350 | 0.48 | 1.473 | 4.34 | 6.02 | 5.92 | -0.11 | 0.35 | 0.02 | 1.02 |
| 17 | 0.000300 | 0.56 | 1.381 | 5.20 | 6.47 | 6.33 | $-0.67$ | -0.28 | -0.18 | 1.09 |
| 17 | 0.000267 | 0.63 | 1.200 | 7.41 | 14.06 | 13.90 | -1.72 | -0.27 | -0.22 | 1.17 |
| 17 | 0.000244 | 0.69 | 1.398 | 7.22 | 9.61 | 9.52 | -0.34 | 0.66 | -0.40 | 1.03 |
| 17 | 0.000224 | 0.75 | 1.536 | 8.98 | 21.29 | 21.11 | $-2.27$ | -0.78 | -0.09 | 1.25 |
| 24 | 0.001826 | 0.13 | 0.968 | 2.71 | 3.75 | 3.55 | 0.00 | -0.67 | 0.12 | 1.00 |
| 24 | 0.001079 | 0.22 | 1.169 | 2.89 | 3.22 | 3.05 | 0.00 | -0.12 | 0.02 | 1.01 |
| 24 | 0.000766 | 0.31 | 1.264 | 3.40 | 4.37 | 4.25 | 0.00 | 0.14 | -0.01 | 1.00 |
| 24 | 0.000593 | 0.40 | 1.379 | 3.97 | 4.97 | 4.83 | -0.16 | 0.46 | -0.30 | 1.02 |
| 24 | 0.000494 | 0.48 | 1.413 | 4.62 | 5.22 | 5.11 | 0.00 | 0.26 | 0.01 | 1.01 |
| 24 | 0.000424 | 0.56 | 1.327 | 5.38 | 6.21 | 6.11 | -0.08 | -0.45 | -0.25 | 1.02 |
| 24 | 0.000377 | 0.63 | 1.472 | 6.73 | 7.21 | 7.13 | -0.28 | 0.15 | -0.03 | 1.04 |
| 24 | 0.000344 | 0.69 | 1.385 | 8.27 | 12.20 | 11.63 | -1.47 | -3.17 | -0.12 | 1.16 |
| 24 | 0.000316 | 0.75 | 1.843 | 7.69 | 12.39 | 12.31 | $-0.71$ | -0.45 | -0.28 | 1.07 |

Continued on next page...

| 32 | 0.002434 | 0.13 | 0.976 | 3.16 | 3.70 | 3.48 | 0.00 | -0.73 | 0.10 | 1.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 0.001438 | 0.22 | 1.145 | 3.53 | 4.57 | 4.46 | 0.00 | 0.04 | -0.04 | 1.00 |
| 32 | 0.001021 | 0.31 | 1.159 | 4.25 | 4.49 | 4.37 | 0.00 | 0.08 | -0.16 | 1.01 |
| 32 | 0.000791 | 0.40 | 1.435 | 4.79 | 4.92 | 4.81 | 0.00 | 0.27 | -0.03 | 1.01 |
| 32 | 0.000659 | 0.48 | 1.451 | 5.41 | 5.55 | 5.45 | 0.00 | -0.31 | -0.10 | 1.00 |
| 32 | 0.000565 | 0.56 | 1.328 | 6.94 | 7.97 | 7.88 | $-0.43$ | 0.29 | -0.19 | 1.05 |
| 32 | 0.000502 | 0.63 | 1.601 | 7.97 | 7.93 | 7.85 | 0.00 | 0.43 | -0.22 | 1.01 |
| 32 | 0.000459 | 0.69 | 1.328 | 9.90 | 13.94 | 13.78 | -1.41 | -1.00 | -0.34 | 1.15 |
| 32 | 0.000422 | 0.75 | 1.406 | 13.22 | 40.24 | 39.94 | -4.09 | 2.17 | -0.43 | 1.45 |
| 45 | 0.003423 | 0.13 | 0.935 | 3.66 | 3.79 | 3.59 | 0.00 | -0.65 | 0.06 | 1.00 |
| 45 | 0.002023 | 0.22 | 1.129 | 3.99 | 5.27 | 5.17 | 0.00 | -0.16 | -0.10 | 1.01 |
| 45 | 0.001435 | 0.31 | 1.165 | 4.74 | 5.11 | 5.01 | 0.00 | 0.14 | 0.11 | 1.00 |
| 45 | 0.001112 | 0.40 | 1.328 | 5.56 | 5.70 | 5.60 | 0.00 | 0.32 | -0.19 | 1.01 |
| 45 | 0.000927 | 0.48 | 1.441 | 6.40 | 6.87 | 6.79 | 0.00 | 0.15 | -0.21 | 1.01 |
| 45 | 0.000795 | 0.56 | 1.304 | 7.14 | 6.62 | 6.48 | 0.00 | 0.80 | -0.45 | 1.00 |
| 45 | 0.000706 | 0.63 | 1.444 | 8.93 | 8.48 | 8.38 | 0.00 | 0.80 | -0.08 | 1.00 |
| 45 | 0.000645 | 0.69 | 1.796 | 10.82 | 13.24 | 13.15 | $-0.75$ | -0.84 | 0.14 | 1.08 |
| 45 | 0.000593 | 0.75 | 1.544 | 13.35 | 20.68 | 20.54 | -1.39 | 1.61 | -0.14 | 1.14 |
| 60 | 0.004564 | 0.13 | 0.843 | 4.51 | 4.85 | 4.72 | 0.00 | -0.36 | 0.25 | 1.00 |
| 60 | 0.002697 | 0.22 | 1.002 | 4.92 | 4.90 | 4.79 | 0.00 | -0.31 | -0.00 | 1.00 |
| 60 | 0.001914 | 0.31 | 1.125 | 5.95 | 7.10 | 7.01 | -0.44 | 0.00 | -0.14 | 1.05 |
| 60 | 0.001483 | 0.40 | 1.402 | 6.97 | 7.61 | 7.52 | -0.24 | 0.35 | -0.10 | 1.03 |
| 60 | 0.001236 | 0.48 | 1.662 | 7.63 | 7.62 | 7.56 | 0.00 | 0.00 | -0.03 | 1.01 |
| 60 | 0.001060 | 0.56 | 1.445 | 8.67 | 8.51 | 8.43 | 0.00 | 0.52 | -0.21 | 1.00 |
| 60 | 0.000942 | 0.63 | 2.057 | 10.98 | 14.11 | 14.02 | 0.00 | 1.15 | $-0.22$ | 1.00 |
| 60 | 0.000860 | 0.69 | 1.570 | 12.47 | 16.00 | 15.92 | -1.13 | 0.01 | -0.18 | 1.11 |
| 60 | 0.000791 | 0.75 | 1.578 | 17.96 | 17.26 | 17.09 | 0.00 | -2.26 | 0.55 | 0.62 |
| 80 | 0.006085 | 0.13 | 0.981 | 4.86 | 5.97 | 5.86 | 0.00 | -0.46 | 0.19 | 1.00 |
| 80 | 0.003596 | 0.22 | 1.017 | 5.74 | 5.94 | 5.85 | 0.00 | -0.11 | -0.03 | 1.00 |
| 80 | 0.002552 | 0.31 | 1.204 | 6.80 | 6.70 | 6.62 | 0.00 | 0.00 | -0.14 | 1.00 |
| 80 | 0.001978 | 0.40 | 0.972 | 8.73 | 7.79 | 7.72 | 0.00 | 0.24 | -0.09 | 1.01 |
| 80 | 0.001648 | 0.48 | 1.272 | 9.43 | 8.73 | 8.63 | 0.00 | 0.71 | -0.35 | 1.02 |
| 80 | 0.001413 | 0.56 | 1.273 | 10.99 | 10.40 | 10.31 | $-0.34$ | 0.74 | -0.33 | 1.04 |
| 80 | 0.001256 | 0.63 | 1.281 | 14.59 | 12.61 | 12.54 | 0.00 | 0.94 | 0.57 | 0.88 |
| 110 | 0.008367 | 0.13 | 0.969 | 5.80 | 6.46 | 6.34 | 0.00 | -0.75 | 0.06 | 1.00 |
| 110 | 0.004944 | 0.22 | 1.120 | 6.33 | 7.05 | 6.97 | 0.00 | 0.32 | -0.20 | 1.00 |
| 110 | 0.003509 | 0.31 | 1.194 | 7.92 | 8.08 | 8.02 | 0.00 | -0.23 | -0.05 | 1.00 |
| 110 | 0.002719 | 0.40 | 1.073 | 9.71 | 8.40 | 8.34 | 0.00 | 0.00 | -0.02 | 1.00 |
| 110 | 0.002266 | 0.48 | 1.364 | 11.40 | 11.68 | 11.63 | 0.00 | 0.49 | -0.30 | 0.92 |
| 110 | 0.001942 | 0.56 | 1.381 | 17.15 | 21.36 | 21.35 | 0.00 | 0.00 | -0.50 | 0.44 |

Table 2: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=$ 318 GeV for the shifted-vertex region. Further details as described in caption of Table 1.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { MER } \end{gathered}$ | $\delta_{\text {stat }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\text {sys }} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\gamma p} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{E_{\text {had }}}$ <br> (\%) | $\delta_{\text {diff }}$ <br> (\%) | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0.000161 | 0.69 | 1.388 | 12.44 | 16.85 | 15.52 | -1.64 | -5.94 | -0.63 | 2.20 |
| 7 | 0.000148 | 0.75 | 1.183 | 8.53 | 14.60 | 13.55 | -3.93 | -2.81 | -0.59 | 2.44 |
| 9 | 0.000298 | 0.48 | 1.441 | 16.01 | 15.02 | 14.87 | 0.00 | 0.65 | -0.69 | 1.83 |
| 9 | 0.000255 | 0.56 | 1.187 | 8.38 | 9.18 | 8.90 | $-0.75$ | -0.69 | -0.37 | 1.97 |
| 9 | 0.000227 | 0.63 | 1.397 | 6.02 | 5.22 | 4.77 | -0.21 | -0.87 | $-0.51$ | 1.83 |
| 9 | 0.000207 | 0.69 | 1.149 | 5.30 | 8.94 | 8.16 | -1.96 | $-2.28$ | $-0.56$ | 1.99 |
| 9 | 0.000190 | 0.75 | 1.154 | 4.82 | 10.37 | 9.04 | -3.04 | -3.51 | -0.58 | 1.96 |
| 12 | 0.000614 | 0.31 | 1.051 | 12.15 | 11.90 | 11.74 | 0.00 | $-0.28$ | $-0.57$ | 1.86 |
| 12 | 0.000476 | 0.40 | 1.087 | 7.88 | 6.98 | 6.71 | -0.06 | 0.12 | -0.02 | 1.90 |
| 12 | 0.000397 | 0.48 | 1.317 | 4.79 | 4.40 | 3.97 | -0.27 | $-0.05$ | -0.25 | 1.87 |
| 12 | 0.000340 | 0.56 | 1.257 | 3.74 | 4.05 | 3.46 | -0.51 | -0.88 | -0.37 | 1.80 |
| 12 | 0.000302 | 0.63 | 1.248 | 3.73 | 4.44 | 3.75 | -1.05 | -1.18 | -0.39 | 1.72 |
| 12 | 0.000276 | 0.69 | 1.319 | 3.47 | 5.63 | 4.83 | -1.22 | -2.04 | -0.40 | 1.59 |
| 12 | 0.000254 | 0.75 | 1.359 | 3.69 | 7.36 | 6.42 | $-2.07$ | -2.51 | -0.54 | 1.42 |
| 17 | 0.002076 | 0.13 | 0.818 | 7.78 | 7.31 | 6.84 | 0.00 | -1.73 | -0.04 | 1.93 |
| 17 | 0.001227 | 0.22 | 1.110 | 5.27 | 5.47 | 5.12 | 0.00 | -0.36 | -0.09 | 1.87 |
| 17 | 0.000870 | 0.31 | 1.252 | 3.77 | 3.61 | 3.12 | 0.00 | -0.22 | -0.15 | 1.81 |
| 17 | 0.000675 | 0.40 | 1.208 | 3.18 | 3.01 | 2.45 | -0.08 | -0.09 | -0.19 | 1.74 |
| 17 | 0.000562 | 0.48 | 1.264 | 2.85 | 2.72 | 2.01 | -0.20 | -0.76 | -0.21 | 1.64 |
| 17 | 0.000482 | 0.56 | 1.306 | 2.69 | 2.83 | 2.23 | -0.53 | -0.55 | -0.28 | 1.54 |
| 17 | 0.000428 | 0.63 | 1.277 | 3.18 | 4.70 | 3.70 | -1.12 | -2.23 | -0.40 | 1.43 |
| 17 | 0.000391 | 0.69 | 1.208 | 3.59 | 4.91 | 4.06 | -1.46 | -1.95 | -0.39 | 1.26 |
| 17 | 0.000360 | 0.75 | 1.344 | 3.91 | 7.10 | 5.35 | -1.91 | -4.10 | -0.53 | 1.03 |
| 24 | 0.002930 | 0.13 | 0.921 | 2.40 | 3.21 | 2.61 | 0.00 | -0.65 | 0.00 | 1.75 |
| 24 | 0.001732 | 0.22 | 1.112 | 2.13 | 3.06 | 2.55 | 0.00 | -0.17 | -0.04 | 1.68 |
| 24 | 0.001229 | 0.31 | 1.128 | 2.10 | 2.41 | 1.81 | -0.03 | -0.13 | -0.04 | 1.59 |
| 24 | 0.000952 | 0.40 | 1.259 | 2.08 | 2.20 | 1.62 | -0.04 | -0.10 | -0.22 | 1.47 |
| 24 | 0.000794 | 0.48 | 1.287 | 2.20 | 2.15 | 1.64 | -0.22 | -0.27 | -0.16 | 1.34 |
| 24 | 0.000680 | 0.56 | 1.342 | 2.32 | 2.32 | 1.92 | -0.31 | -0.47 | -0.18 | 1.17 |
| 24 | 0.000605 | 0.63 | 1.269 | 2.98 | 3.39 | 2.77 | -0.82 | -1.44 | -0.29 | 1.01 |
| 24 | 0.000552 | 0.69 | 1.371 | 3.19 | 3.69 | 2.77 | -0.84 | -2.12 | -0.29 | 0.79 |
| 24 | 0.000508 | 0.75 | 1.386 | 3.85 | 5.67 | 4.74 | -1.93 | -2.35 | -0.37 | 0.56 |
| 32 | 0.003907 | 0.13 | 0.881 | 1.70 | 2.20 | 1.46 | 0.00 | -0.65 | 0.06 | 1.50 |
| 32 | 0.002309 | 0.22 | 1.018 | 1.76 | 2.37 | 1.90 | 0.00 | -0.19 | 0.02 | 1.40 |
| 32 | 0.001638 | 0.31 | 1.132 | 1.88 | 1.89 | 1.38 | -0.01 | -0.10 | -0.11 | 1.28 |
| 32 | 0.001270 | 0.40 | 1.226 | 2.11 | 2.17 | 1.83 | -0.07 | -0.23 | -0.14 | 1.13 |
| 32 | 0.001058 | 0.48 | 1.287 | 2.37 | 2.01 | 1.74 | -0.13 | -0.14 | -0.18 | 0.97 |
| 32 | 0.000907 | 0.56 | 1.242 | 2.68 | 2.60 | 2.27 | -0.42 | $-0.87$ | -0.15 | 0.79 |
| 32 | 0.000806 | 0.63 | 1.316 | 3.43 | 3.56 | 3.36 | $-0.77$ | $-0.57$ | -0.33 | 0.59 |
| 32 | 0.000736 | 0.69 | 1.296 | 3.85 | 3.72 | 3.50 | -0.92 | $-0.76$ | -0.24 | 0.38 |
| 32 | 0.000677 | 0.75 | 1.268 | 4.57 | 7.54 | 5.87 | -1.77 | -4.38 | -0.31 | 0.21 |

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| ---: | :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: |
| 45 | 0.005494 | 0.13 | 0.905 | 1.53 | 2.16 | 1.70 | 0.00 | -0.61 | 0.06 | 1.18 |
| 45 | 0.003247 | 0.22 | 1.031 | 1.71 | 2.12 | 1.83 | -0.01 | -0.10 | 0.04 | 1.06 |
| 45 | 0.002304 | 0.31 | 1.114 | 1.96 | 1.78 | 1.51 | -0.01 | -0.18 | -0.05 | 0.92 |
| 45 | 0.001786 | 0.40 | 1.165 | 2.32 | 1.81 | 1.63 | -0.02 | 0.10 | -0.17 | 0.76 |
| 45 | 0.001488 | 0.48 | 1.211 | 2.68 | 2.62 | 2.53 | -0.18 | -0.20 | -0.15 | 0.60 |
| 45 | 0.001275 | 0.56 | 1.306 | 2.92 | 2.30 | 2.22 | -0.26 | -0.26 | -0.22 | 0.41 |
| 45 | 0.001134 | 0.63 | 1.358 | 3.76 | 3.05 | 2.90 | -0.42 | -0.76 | -0.32 | 0.25 |
| 45 | 0.001035 | 0.69 | 1.426 | 4.08 | 3.42 | 3.20 | -0.58 | -0.99 | -0.28 | -0.11 |
| 45 | 0.000952 | 0.75 | 1.409 | 4.65 | 5.80 | 5.60 | -1.19 | -0.90 | -0.26 | -0.02 |
| 60 | 0.007326 | 0.13 | 0.822 | 1.78 | 1.84 | 1.51 | 0.00 | -0.64 | 0.16 | 0.82 |
| 60 | 0.004329 | 0.22 | 0.974 | 2.00 | 2.15 | 2.01 | 0.00 | -0.26 | 0.00 | 0.70 |
| 60 | 0.003072 | 0.31 | 1.100 | 2.29 | 1.82 | 1.73 | 0.00 | -0.13 | -0.04 | 0.56 |
| 60 | 0.002381 | 0.40 | 1.163 | 2.72 | 1.99 | 1.94 | 0.00 | -0.08 | -0.17 | 0.41 |
| 60 | 0.001984 | 0.48 | 1.106 | 3.30 | 2.28 | 2.25 | -0.05 | 0.22 | -0.14 | 0.27 |
| 60 | 0.001701 | 0.56 | 1.294 | 3.51 | 2.62 | 2.60 | -0.20 | 0.09 | -0.25 | 0.13 |
| 60 | 0.001512 | 0.63 | 1.345 | 4.41 | 3.47 | 3.38 | -0.33 | -0.71 | -0.18 | -0.04 |
| 60 | 0.001380 | 0.69 | 1.429 | 4.94 | 4.09 | 3.99 | -0.69 | -0.62 | -0.13 | -0.01 |
| 60 | 0.001270 | 0.75 | 1.436 | 5.53 | 5.07 | 4.76 | -0.79 | -1.55 | -0.15 | 0.00 |
| 80 | 0.009768 | 0.13 | 0.776 | 2.07 | 1.82 | 1.70 | 0.00 | -0.43 | 0.16 | 0.48 |
| 80 | 0.005772 | 0.22 | 0.953 | 2.27 | 2.22 | 2.18 | 0.00 | -0.09 | 0.11 | 0.37 |
| 80 | 0.004096 | 0.31 | 1.021 | 2.70 | 2.02 | 2.00 | -0.01 | 0.14 | 0.03 | 0.25 |
| 80 | 0.003175 | 0.40 | 1.114 | 3.22 | 2.54 | 2.54 | -0.01 | -0.14 | -0.03 | 0.14 |
| 80 | 0.002645 | 0.48 | 1.167 | 3.65 | 2.86 | 2.84 | -0.06 | 0.23 | -0.21 | -0.06 |
| 80 | 0.002268 | 0.56 | 1.172 | 4.21 | 3.32 | 3.29 | -0.39 | -0.20 | -0.07 | -0.01 |
| 80 | 0.002016 | 0.63 | 1.270 | 5.28 | 5.34 | 5.32 | -0.29 | -0.35 | -0.10 | -0.00 |
| 80 | 0.001840 | 0.69 | 1.294 | 5.83 | 4.44 | 4.26 | -0.57 | -1.06 | -0.27 | 0.00 |
| 80 | 0.001693 | 0.75 | 1.499 | 6.14 | 5.24 | 5.11 | -0.66 | -0.97 | -0.22 | 0.00 |
| 110 | 0.013431 | 0.13 | 0.702 | 2.43 | 2.18 | 2.10 | 0.00 | -0.54 | 0.16 | 0.18 |
| 110 | 0.007936 | 0.22 | 0.854 | 2.74 | 1.95 | 1.94 | 0.00 | -0.17 | 0.09 | -0.11 |
| 110 | 0.005632 | 0.31 | 0.981 | 3.15 | 2.11 | 2.10 | -0.02 | -0.14 | 0.02 | -0.04 |
| 110 | 0.004365 | 0.40 | 1.066 | 3.74 | 2.61 | 2.61 | -0.01 | 0.12 | 0.09 | -0.01 |
| 110 | 0.003638 | 0.48 | 1.163 | 4.36 | 2.97 | 2.96 | 0.00 | -0.12 | -0.12 | 0.00 |
| 110 | 0.003118 | 0.56 | 1.152 | 4.87 | 3.58 | 3.53 | -0.22 | -0.46 | -0.27 | 0.00 |
| 110 | 0.002771 | 0.63 | 1.191 | 6.43 | 4.67 | 4.62 | -0.37 | 0.41 | -0.29 | 0.00 |
| 110 | 0.002530 | 0.69 | 1.097 | 7.59 | 5.95 | 5.88 | -0.37 | -0.82 | -0.18 | 0.00 |

Table 3: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=$ 251 GeV for the central-vertex region. Further details as described in caption of Table 1.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { MER } \end{gathered}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{aligned} & \hline \delta_{\text {sys }} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\gamma p} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{E_{\text {had }}}$ (\%) | $\begin{gathered} \hline \delta_{\text {diff }} \\ (\%) \end{gathered}$ | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 0.000126 | 0.63 | 1.189 | 13.57 | 13.03 | 12.97 | 0.00 | -0.37 | -0.39 | 1.09 |
| 7 | 0.000278 | 0.40 | 1.123 | 12.50 | 13.23 | 13.19 | 0.00 | -0.63 | -0.05 | 0.74 |
| 7 | 0.000231 | 0.48 | 1.173 | 9.63 | 9.46 | 9.40 | -0.11 | -0.38 | 0.13 | 1.00 |
| 7 | 0.000198 | 0.56 | 1.229 | 9.74 | 14.02 | 13.93 | -0.93 | 0.54 | $-0.21$ | 1.12 |
| 7 | 0.000176 | 0.63 | 1.225 | 10.51 | 12.08 | 12.01 | -0.16 | -0.65 | $-0.35$ | 1.03 |
| 7 | 0.000161 | 0.69 | 1.650 | 11.15 | 11.36 | 11.17 | -0.38 | 1.67 | -0.36 | 1.07 |
| 7 | 0.000148 | 0.75 | 1.036 | 15.29 | 20.39 | 20.04 | -3.19 | -1.45 | $-0.38$ | 1.33 |
| 9 | 0.001099 | 0.13 | 0.685 | 12.88 | 13.68 | 13.65 | 0.00 | -0.50 | 0.00 | 0.66 |
| 9 | 0.000649 | 0.22 | 1.218 | 7.80 | 9.61 | 9.56 | 0.00 | 0.14 | -0.04 | 0.89 |
| 9 | 0.000461 | 0.31 | 1.107 | 6.82 | 7.52 | 7.45 | 0.00 | 0.00 | -0.01 | 1.00 |
| 9 | 0.000357 | 0.40 | 1.123 | 7.28 | 7.71 | 7.63 | $-0.05$ | -0.23 | $-0.38$ | 1.02 |
| 9 | 0.000298 | 0.48 | 1.220 | 7.54 | 6.97 | 6.88 | -0.28 | 0.26 | -0.17 | 1.04 |
| 9 | 0.000255 | 0.56 | 1.194 | 8.01 | 8.87 | 8.80 | -0.08 | -0.48 | $-0.13$ | 1.02 |
| 9 | 0.000227 | 0.63 | 1.506 | 9.27 | 8.12 | 7.96 | -0.12 | -1.18 | $-0.26$ | 1.02 |
| 9 | 0.000207 | 0.69 | 1.401 | 10.23 | 8.42 | 8.14 | -0.32 | -1.83 | $-0.26$ | 1.05 |
| 9 | 0.000190 | 0.75 | 0.995 | 13.85 | 13.68 | 13.51 | -1.52 | -1.06 | -0.12 | 1.17 |
| 12 | 0.001465 | 0.13 | 0.836 | 5.40 | 7.29 | 7.17 | 0.00 | -0.86 | 0.12 | 1.01 |
| 12 | 0.000866 | 0.22 | 0.979 | 5.50 | 6.86 | 6.78 | 0.00 | 0.39 | -0.12 | 1.01 |
| 12 | 0.000614 | 0.31 | 1.164 | 5.77 | 5.27 | 5.15 | -0.03 | 0.47 | 0.02 | 1.01 |
| 12 | 0.000476 | 0.40 | 1.073 | 7.20 | 7.23 | 7.13 | -0.40 | 0.44 | 0.09 | 1.05 |
| 12 | 0.000397 | 0.48 | 1.149 | 7.60 | 6.11 | 6.02 | 0.00 | -0.41 | -0.10 | 1.01 |
| 12 | 0.000340 | 0.56 | 1.330 | 7.79 | 7.24 | 7.14 | -0.08 | 0.60 | $-0.26$ | 1.02 |
| 12 | 0.000302 | 0.63 | 0.743 | 16.22 | 31.99 | 31.63 | -3.56 | -2.85 | $-0.43$ | 1.39 |
| 12 | 0.000276 | 0.69 | 1.367 | 11.48 | 9.41 | 9.33 | -0.39 | 0.50 | -0.09 | 1.04 |
| 12 | 0.000254 | 0.75 | 1.203 | 12.91 | 14.27 | 11.98 | -1.17 | -7.58 | $-0.25$ | 1.12 |
| 17 | 0.002076 | 0.13 | 0.884 | 5.33 | 4.95 | 4.84 | 0.00 | $-0.23$ | 0.08 | 1.00 |
| 17 | 0.001227 | 0.22 | 1.151 | 5.30 | 4.44 | 4.30 | 0.00 | -0.36 | -0.15 | 1.00 |
| 17 | 0.000870 | 0.31 | 0.993 | 6.54 | 4.80 | 4.69 | 0.00 | -0.14 | -0.18 | 1.00 |
| 17 | 0.000675 | 0.40 | 1.222 | 6.89 | 5.97 | 5.88 | -0.05 | -0.18 | $-0.17$ | 1.01 |
| 17 | 0.000562 | 0.48 | 1.238 | 8.04 | 7.08 | 6.98 | -0.15 | 0.63 | $-0.06$ | 1.03 |
| 17 | 0.000482 | 0.56 | 1.175 | 8.74 | 7.32 | 7.23 | 0.00 | -0.26 | -0.48 | 1.01 |
| 17 | 0.000428 | 0.63 | 1.331 | 12.02 | 9.87 | 9.74 | -0.41 | -1.04 | -0.42 | 1.05 |
| 17 | 0.000391 | 0.69 | 1.150 | 13.57 | 10.96 | 10.85 | $-0.94$ | -0.54 | $-0.22$ | 1.09 |
| 17 | 0.000360 | 0.75 | 1.563 | 14.57 | 14.14 | 14.01 | -1.28 | $-0.78$ | $-0.38$ | 1.13 |
| 24 | 0.002930 | 0.13 | 0.980 | 4.72 | 4.60 | 4.35 | 0.00 | -1.06 | 0.18 | 1.00 |
| 24 | 0.001732 | 0.22 | 0.872 | 5.77 | 4.24 | 4.11 | -0.02 | 0.10 | -0.13 | 1.01 |
| 24 | 0.001229 | 0.31 | 1.178 | 6.12 | 4.95 | 4.84 | 0.00 | 0.08 | -0.02 | 1.00 |
| 24 | 0.000952 | 0.40 | 1.247 | 7.10 | 6.54 | 6.44 | 0.00 | 0.35 | $-0.23$ | 1.01 |
| 24 | 0.000794 | 0.48 | 1.215 | 8.90 | 9.42 | 9.28 | $-0.97$ | -0.53 | $-0.24$ | 1.11 |
| 24 | 0.000680 | 0.56 | 1.428 | 9.22 | 9.54 | 9.43 | -0.21 | -1.07 | -0.03 | 1.03 |
| 24 | 0.000605 | 0.63 | 1.357 | 13.03 | 12.05 | 11.96 | $-0.80$ | 0.59 | -0.15 | 1.08 |
| 24 | 0.000552 | 0.69 | 1.491 | 12.90 | 11.69 | 11.63 | 0.00 | $-0.35$ | $-0.31$ | 1.01 |
| 24 | 0.000508 | 0.75 | 1.683 | 14.09 | 12.60 | 12.42 | $-0.34$ | -1.72 | -0.44 | 1.04 |

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| ---: | :---: | ---: | ---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 32 | 0.003907 | 0.13 | 0.907 | 5.60 | 4.71 | 4.55 | 0.00 | -0.71 | 0.10 | 1.00 |
| 32 | 0.002309 | 0.22 | 0.979 | 6.54 | 5.62 | 5.53 | 0.00 | -0.00 | 0.14 | 1.00 |
| 32 | 0.001638 | 0.31 | 1.217 | 7.15 | 5.64 | 5.55 | 0.00 | -0.22 | 0.14 | 1.00 |
| 32 | 0.001270 | 0.40 | 1.497 | 8.03 | 6.26 | 6.18 | 0.00 | 0.15 | -0.08 | 1.01 |
| 32 | 0.001058 | 0.48 | 1.088 | 11.17 | 7.59 | 7.51 | -0.14 | -0.15 | -0.33 | 1.02 |
| 32 | 0.000907 | 0.56 | 1.251 | 11.65 | 9.24 | 8.91 | -0.38 | -2.15 | -0.46 | 1.04 |
| 32 | 0.000806 | 0.63 | 1.702 | 13.24 | 10.56 | 10.48 | 0.00 | 0.81 | -0.00 | 1.01 |
| 32 | 0.000736 | 0.69 | 1.256 | 16.03 | 11.86 | 11.75 | -0.74 | -0.79 | -0.19 | 1.08 |
| 32 | 0.000677 | 0.75 | 1.415 | 19.40 | 15.37 | 15.28 | -1.12 | -0.62 | 0.02 | 1.11 |
| 45 | 0.005494 | 0.13 | 0.825 | 6.67 | 5.55 | 5.43 | 0.00 | -0.43 | 0.34 | 1.00 |
| 45 | 0.003247 | 0.22 | 0.817 | 7.86 | 5.52 | 5.43 | 0.00 | -0.11 | -0.03 | 1.00 |
| 45 | 0.002304 | 0.31 | 1.179 | 8.68 | 6.53 | 6.45 | 0.00 | 0.31 | 0.10 | 1.01 |
| 45 | 0.001786 | 0.40 | 1.250 | 9.95 | 8.48 | 8.40 | 0.00 | 0.46 | -0.07 | 1.01 |
| 45 | 0.001488 | 0.48 | 1.278 | 11.38 | 8.94 | 8.86 | 0.00 | -0.49 | -0.27 | 1.01 |
| 45 | 0.001275 | 0.56 | 0.985 | 15.16 | 11.62 | 11.53 | 0.00 | -1.07 | 0.04 | 1.01 |
| 45 | 0.001134 | 0.63 | 1.224 | 18.27 | 14.98 | 14.93 | 0.00 | 0.57 | -0.44 | 1.00 |
| 45 | 0.001035 | 0.69 | 1.057 | 19.17 | 11.42 | 11.33 | 0.00 | 0.70 | -0.72 | 1.01 |
| 60 | 0.007326 | 0.13 | 0.757 | 8.26 | 5.87 | 5.72 | 0.00 | -0.84 | 0.15 | 1.00 |
| 60 | 0.004329 | 0.22 | 0.865 | 9.15 | 6.65 | 6.56 | 0.00 | -0.15 | 0.26 | 1.00 |
| 60 | 0.003072 | 0.31 | 0.988 | 10.79 | 7.04 | 6.96 | 0.00 | 0.41 | 0.04 | 1.01 |
| 60 | 0.002381 | 0.40 | 1.522 | 11.40 | 9.82 | 9.72 | 0.00 | -0.96 | -0.10 | 1.01 |
| 60 | 0.001984 | 0.48 | 1.525 | 12.89 | 10.05 | 9.98 | 0.00 | -0.40 | -0.65 | 1.01 |
| 60 | 0.001701 | 0.56 | 1.342 | 15.74 | 10.89 | 10.82 | 0.00 | 0.48 | -0.41 | 1.02 |
| 60 | 0.001512 | 0.63 | 1.276 | 20.03 | 55.38 | 55.37 | 0.00 | 0.98 | 0.16 | 1.00 |
| 80 | 0.009768 | 0.13 | 0.674 | 9.72 | 7.14 | 7.07 | 0.00 | 0.16 | -0.05 | 1.00 |
| 80 | 0.005772 | 0.22 | 1.009 | 9.92 | 7.67 | 7.59 | 0.00 | 0.41 | -0.14 | 1.00 |
| 80 | 0.004096 | 0.31 | 1.159 | 12.17 | 9.56 | 9.49 | 0.00 | -0.62 | 0.03 | 1.00 |
| 80 | 0.003175 | 0.40 | 1.218 | 13.69 | 10.36 | 10.31 | 0.00 | 0.03 | -0.04 | 1.01 |
| 80 | 0.002645 | 0.48 | 1.276 | 17.62 | 17.22 | 17.13 | -0.81 | -1.17 | -0.17 | 1.09 |
| 80 | 0.002268 | 0.56 | 0.907 | 22.78 | 21.80 | 21.71 | -1.34 | 0.73 | 0.02 | 1.14 |
| 110 | 0.013431 | 0.13 | 0.710 | 11.63 | 8.30 | 8.21 | 0.00 | 0.49 | 0.40 | 1.00 |
| 110 | 0.007936 | 0.22 | 0.780 | 13.18 | 9.39 | 9.33 | 0.00 | 0.01 | 0.24 | 1.00 |
| 110 | 0.005632 | 0.31 | 0.932 | 14.68 | 10.20 | 10.08 | 0.00 | 1.15 | -0.30 | 1.01 |
| 110 | 0.004365 | 0.40 | 1.132 | 17.06 | 11.86 | 11.80 | 0.00 | 0.58 | -0.30 | 1.01 |
|  |  |  |  |  |  |  |  |  |  |  |

Table 4: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=$ 251 GeV for the shifted-vertex region. Further details as described in caption of Table 1.

| $\begin{gathered} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{gathered}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { LER } \end{gathered}$ | $\delta_{\text {stat }}$ <br> (\%) | $\delta_{\text {sys }}$ <br> (\%) | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \hline \delta_{\gamma p} \\ & (\%) \\ & \hline \end{aligned}$ | $\begin{array}{r} \delta_{E_{\mathrm{had}}} \\ (\%) \\ \hline \end{array}$ | $\delta_{\text {diff }}$ <br> (\%) | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0.000219 | 0.63 | 1.543 | 14.72 | 12.67 | 12.43 | $-0.63$ | -1.12 | $-0.80$ | 1.92 |
| 7 | 0.000200 | 0.69 | 1.204 | 9.26 | 9.98 | 9.34 | -1.51 | -2.24 | $-0.70$ | 2.14 |
| 7 | 0.000184 | 0.75 | 1.332 | 5.72 | 8.80 | 7.20 | -2.26 | -3.93 | $-0.69$ | 2.14 |
| 9 | 0.000370 | 0.48 | 1.259 | 11.80 | 8.72 | 8.50 | -0.28 | -0.16 | -0.35 | 1.89 |
| 9 | 0.000317 | 0.56 | 1.053 | 7.02 | 6.48 | 6.14 | $-0.27$ | -0.64 | -0.28 | 1.93 |
| 9 | 0.000282 | 0.63 | 1.294 | 4.58 | 4.44 | 3.57 | -0.41 | -1.77 | -0.42 | 1.87 |
| 9 | 0.000258 | 0.69 | 1.312 | 3.73 | 6.03 | 4.70 | -1.90 | $-2.50$ | -0.58 | 2.00 |
| 9 | 0.000237 | 0.75 | 1.318 | 3.22 | 6.47 | 4.55 | -2.13 | -3.58 | -0.65 | 1.82 |
| 12 | 0.001077 | 0.22 | 0.781 | 16.48 | 11.13 | 11.00 | 0.00 | 0.00 | 0.06 | 1.72 |
| 12 | 0.000765 | 0.31 | 1.219 | 8.12 | 6.18 | 5.88 | -0.10 | -0.15 | -0.15 | 1.89 |
| 12 | 0.000593 | 0.40 | 1.158 | 5.20 | 3.98 | 3.50 | 0.00 | -0.14 | -0.14 | 1.88 |
| 12 | 0.000494 | 0.48 | 1.214 | 3.64 | 3.41 | 2.82 | -0.26 | -0.32 | -0.20 | 1.85 |
| 12 | 0.000423 | 0.56 | 1.299 | 2.66 | 3.23 | 2.53 | -0.52 | -0.62 | -0.40 | 1.79 |
| 12 | 0.000376 | 0.63 | 1.278 | 2.71 | 4.11 | 3.33 | -0.89 | -1.36 | -0.45 | 1.72 |
| 12 | 0.000344 | 0.69 | 1.219 | 2.72 | 4.80 | 3.70 | -1.59 | -1.95 | -0.52 | 1.64 |
| 12 | 0.000316 | 0.75 | 1.357 | 2.74 | 6.03 | 3.86 | -1.98 | -3.88 | -0.58 | 1.44 |
| 17 | 0.002583 | 0.13 | 0.829 | 5.55 | 4.40 | 3.90 | 0.00 | $-0.73$ | 0.13 | 1.91 |
| 17 | 0.001526 | 0.22 | 1.035 | 3.84 | 4.04 | 3.57 | 0.00 | -0.15 | 0.02 | 1.87 |
| 17 | 0.001083 | 0.31 | 1.191 | 2.85 | 2.78 | 2.09 | -0.07 | -0.18 | -0.09 | 1.82 |
| 17 | 0.000840 | 0.40 | 1.218 | 2.35 | 2.41 | 1.65 | -0.05 | -0.19 | -0.18 | 1.73 |
| 17 | 0.000700 | 0.48 | 1.225 | 2.14 | 2.16 | 1.30 | -0.18 | -0.40 | -0.25 | 1.64 |
| 17 | 0.000600 | 0.56 | 1.303 | 1.96 | 2.22 | 1.41 | -0.44 | -0.58 | -0.30 | 1.53 |
| 17 | 0.000533 | 0.63 | 1.253 | 2.35 | 3.06 | 2.33 | $-0.79$ | -1.12 | $-0.32$ | 1.40 |
| 17 | 0.000487 | 0.69 | 1.274 | 2.51 | 3.38 | 2.34 | -1.10 | -1.75 | -0.42 | 1.22 |
| 17 | 0.000448 | 0.75 | 1.255 | 2.95 | 4.79 | 3.29 | -2.03 | -2.59 | -0.45 | 1.04 |
| 24 | 0.003647 | 0.13 | 0.887 | 1.80 | 2.71 | 1.95 | 0.00 | -0.69 | 0.07 | 1.74 |
| 24 | 0.002155 | 0.22 | 1.043 | 1.61 | 2.49 | 1.84 | -0.00 | -0.19 | -0.06 | 1.68 |
| 24 | 0.001529 | 0.31 | 1.094 | 1.56 | 2.03 | 1.26 | -0.03 | -0.11 | -0.08 | 1.59 |
| 24 | 0.001185 | 0.40 | 1.161 | 1.59 | 1.83 | 1.06 | -0.09 | -0.14 | -0.18 | 1.48 |
| 24 | 0.000988 | 0.48 | 1.201 | 1.66 | 1.92 | 1.29 | -0.18 | -0.38 | -0.21 | 1.35 |
| 24 | 0.000847 | 0.56 | 1.269 | 1.71 | 2.09 | 1.59 | -0.35 | -0.53 | -0.25 | 1.18 |
| 24 | 0.000752 | 0.63 | 1.264 | 2.18 | 2.46 | 1.78 | -0.68 | -1.15 | -0.30 | 1.01 |
| 24 | 0.000687 | 0.69 | 1.280 | 2.46 | 2.91 | 2.27 | -1.06 | -1.19 | $-0.33$ | 0.82 |
| 24 | 0.000632 | 0.75 | 1.273 | 2.82 | 4.72 | 3.05 | -1.83 | -3.03 | $-0.35$ | 0.56 |
| 32 | 0.004862 | 0.13 | 0.859 | 1.27 | 2.31 | 1.65 | -0.00 | -0.58 | 0.08 | 1.50 |
| 32 | 0.002873 | 0.22 | 0.990 | 1.31 | 2.24 | 1.74 | -0.00 | -0.19 | 0.01 | 1.40 |
| 32 | 0.002039 | 0.31 | 1.077 | 1.42 | 1.53 | 0.83 | -0.01 | -0.11 | -0.11 | 1.28 |
| 32 | 0.001580 | 0.40 | 1.168 | 1.57 | 1.70 | 1.24 | -0.06 | -0.20 | -0.15 | 1.13 |
| 32 | 0.001317 | 0.48 | 1.258 | 1.75 | 1.52 | 1.11 | -0.16 | -0.22 | -0.18 | 0.98 |
| 32 | 0.001129 | 0.56 | 1.248 | 1.97 | 1.76 | 1.45 | -0.35 | -0.47 | -0.18 | 0.79 |
| 32 | 0.001003 | 0.63 | 1.275 | 2.55 | 2.14 | 1.76 | -0.60 | -0.83 | -0.20 | 0.60 |
| 32 | 0.000916 | 0.69 | 1.398 | 2.67 | 2.45 | 1.95 | -0.61 | -1.27 | -0.26 | 0.39 |
| 32 | 0.000843 | 0.75 | 1.292 | 3.24 | 4.21 | 2.77 | -1.47 | -2.79 | -0.27 | 0.20 |

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| ---: | ---: | ---: | ---: | :---: | :---: | :---: | ---: | ---: | ---: | ---: |
| 45 | 0.006838 | 0.13 | 0.823 | 1.18 | 1.90 | 1.34 | 0.00 | -0.65 | 0.10 | 1.18 |
| 45 | 0.004040 | 0.22 | 0.944 | 1.30 | 2.06 | 1.75 | 0.00 | -0.14 | 0.00 | 1.06 |
| 45 | 0.002867 | 0.31 | 1.098 | 1.45 | 1.31 | 0.92 | -0.01 | -0.12 | -0.09 | 0.93 |
| 45 | 0.002222 | 0.40 | 1.109 | 1.74 | 1.30 | 1.03 | -0.06 | -0.10 | -0.11 | 0.77 |
| 45 | 0.001852 | 0.48 | 1.119 | 2.02 | 1.46 | 1.30 | -0.11 | -0.21 | -0.14 | 0.60 |
| 45 | 0.001587 | 0.56 | 1.255 | 2.18 | 1.60 | 1.39 | -0.16 | -0.60 | -0.23 | 0.42 |
| 45 | 0.001411 | 0.63 | 1.228 | 2.83 | 1.93 | 1.69 | -0.42 | -0.78 | -0.19 | 0.25 |
| 45 | 0.001288 | 0.69 | 1.284 | 3.15 | 2.68 | 2.33 | -0.74 | -1.07 | -0.20 | -0.11 |
| 45 | 0.001185 | 0.75 | 1.260 | 3.51 | 4.10 | 3.63 | -1.00 | -1.61 | -0.24 | -0.03 |
| 60 | 0.009117 | 0.13 | 0.761 | 1.35 | 1.61 | 1.23 | 0.00 | -0.58 | 0.16 | 0.83 |
| 60 | 0.005387 | 0.22 | 0.897 | 1.53 | 1.81 | 1.66 | 0.00 | -0.14 | 0.01 | 0.71 |
| 60 | 0.003823 | 0.31 | 1.009 | 1.75 | 1.20 | 1.06 | -0.01 | -0.04 | -0.07 | 0.57 |
| 60 | 0.002963 | 0.40 | 1.107 | 2.04 | 1.40 | 1.33 | -0.02 | -0.10 | -0.13 | 0.42 |
| 60 | 0.002469 | 0.48 | 1.153 | 2.34 | 1.62 | 1.57 | -0.11 | -0.17 | -0.15 | 0.28 |
| 60 | 0.002116 | 0.56 | 1.198 | 2.65 | 1.73 | 1.70 | -0.17 | -0.20 | -0.16 | 0.13 |
| 60 | 0.001881 | 0.63 | 1.198 | 3.46 | 3.24 | 3.12 | -0.59 | -0.58 | -0.16 | -0.05 |
| 60 | 0.001718 | 0.69 | 1.251 | 3.73 | 2.92 | 2.73 | -0.78 | -0.68 | -0.21 | -0.01 |
| 60 | 0.001580 | 0.75 | 1.213 | 4.27 | 3.66 | 3.40 | -1.11 | -0.80 | -0.15 | 0.00 |
| 80 | 0.012156 | 0.13 | 0.726 | 1.55 | 1.52 | 1.32 | 0.00 | -0.58 | 0.11 | 0.48 |
| 80 | 0.007183 | 0.22 | 0.917 | 1.69 | 1.61 | 1.56 | 0.00 | -0.13 | 0.05 | 0.37 |
| 80 | 0.005098 | 0.31 | 0.961 | 2.02 | 1.13 | 1.10 | -0.01 | -0.09 | -0.04 | 0.26 |
| 80 | 0.003951 | 0.40 | 1.043 | 2.38 | 1.31 | 1.30 | -0.03 | 0.04 | -0.04 | 0.15 |
| 80 | 0.003292 | 0.48 | 1.110 | 2.74 | 1.39 | 1.37 | -0.01 | -0.17 | -0.12 | -0.06 |
| 80 | 0.002822 | 0.56 | 1.124 | 3.09 | 2.19 | 2.17 | -0.15 | 0.19 | -0.11 | -0.01 |
| 80 | 0.002508 | 0.63 | 1.172 | 3.95 | 2.75 | 2.70 | -0.40 | 0.24 | -0.17 | 0.00 |
| 80 | 0.002290 | 0.69 | 1.095 | 4.51 | 3.29 | 3.22 | -0.56 | -0.16 | -0.26 | 0.00 |
| 80 | 0.002107 | 0.75 | 1.285 | 4.75 | 3.80 | 3.57 | -0.81 | -0.99 | -0.33 | 0.00 |
| 110 | 0.016714 | 0.13 | 0.679 | 1.80 | 1.75 | 1.62 | 0.00 | -0.62 | 0.15 | 0.19 |
| 110 | 0.009877 | 0.22 | 0.822 | 2.04 | 1.51 | 1.50 | 0.00 | -0.08 | 0.10 | -0.11 |
| 110 | 0.007009 | 0.31 | 0.890 | 2.42 | 1.18 | 1.18 | -0.01 | 0.11 | 0.03 | -0.05 |
| 110 | 0.005432 | 0.40 | 0.994 | 2.80 | 1.51 | 1.51 | 0.00 | -0.07 | -0.03 | -0.01 |
| 110 | 0.004527 | 0.48 | 1.117 | 3.19 | 1.73 | 1.71 | -0.10 | -0.15 | -0.13 | 0.00 |
| 110 | 0.003880 | 0.56 | 1.050 | 3.69 | 1.85 | 1.84 | -0.07 | -0.06 | -0.15 | 0.00 |
| 110 | 0.003449 | 0.63 | 1.199 | 4.68 | 3.84 | 3.81 | -0.31 | -0.33 | -0.16 | 0.00 |
| 110 | 0.003149 | 0.69 | 1.209 | 5.27 | 3.34 | 3.23 | -0.38 | -0.72 | -0.29 | 0.00 |
| 110 | 0.002897 | 0.75 | 0.940 | 9.18 | 6.18 | 6.10 | -0.94 | -0.17 | -0.31 | 0.00 |

Table 5: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=$ 225 GeV for the central-vertex region. Further details as described in caption of Table 1.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $y$ | $\begin{gathered} \tilde{\sigma} \\ \text { LER } \end{gathered}$ | $\begin{gathered} \delta_{\text {stat }} \\ (\%) \end{gathered}$ | $\begin{aligned} & \hline \delta_{\text {sys }} \\ & (\%) \\ & \hline \end{aligned}$ | $\delta_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \delta_{\gamma p} \\ & (\%) \end{aligned}$ | $\begin{array}{r} \delta_{E_{\mathrm{had}}} \\ (\%) \\ \hline \end{array}$ | $\delta_{\text {diff }}$ (\%) | $\delta_{\text {hits }}$ <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 0.000176 | 0.56 | 1.022 | 10.84 | 7.82 | 7.74 | -0.28 | -0.54 | -0.37 | 0.90 |
| 5 | 0.000157 | 0.63 | 1.011 | 10.04 | 9.90 | 9.76 | $-0.27$ | -1.09 | $-0.34$ | 1.10 |
| 5 | 0.000143 | 0.69 | 1.049 | 10.23 | 9.43 | 9.23 | -1.02 | -1.05 | -0.54 | 1.18 |
| 7 | 0.000446 | 0.31 | 0.952 | 15.23 | 10.10 | 10.08 | 0.00 | 0.43 | -0.26 | 0.35 |
| 7 | 0.000346 | 0.40 | 0.964 | 9.54 | 6.63 | 6.57 | 0.00 | 0.33 | $-0.07$ | 0.83 |
| 7 | 0.000288 | 0.48 | 0.870 | 7.67 | 6.58 | 6.49 | -0.18 | -0.18 | -0.21 | 1.03 |
| 7 | 0.000247 | 0.56 | 1.074 | 6.84 | 7.66 | 7.39 | $-0.63$ | -1.56 | -0.24 | 1.08 |
| 7 | 0.000219 | 0.63 | 1.114 | 7.79 | 7.90 | 7.73 | -0.39 | -1.14 | -0.21 | 1.06 |
| 7 | 0.000200 | 0.69 | 1.075 | 9.14 | 14.24 | 11.69 | -2.08 | -7.75 | -0.38 | 1.22 |
| 7 | 0.000184 | 0.75 | 1.334 | 9.00 | 11.40 | 11.11 | -1.64 | -1.51 | -0.41 | 1.18 |
| 9 | 0.001368 | 0.13 | 0.874 | 7.77 | 6.49 | 6.38 | 0.00 | -0.96 | 0.15 | 0.67 |
| 9 | 0.000808 | 0.22 | 0.940 | 5.96 | 5.58 | 5.50 | -0.02 | -0.21 | 0.03 | 0.91 |
| 9 | 0.000573 | 0.31 | 1.040 | 4.99 | 5.14 | 5.04 | 0.00 | 0.11 | 0.01 | 1.00 |
| 9 | 0.000444 | 0.40 | 1.130 | 4.96 | 5.81 | 5.71 | -0.09 | -0.30 | -0.17 | 1.02 |
| 9 | 0.000370 | 0.48 | 0.990 | 5.78 | 5.28 | 5.11 | $-0.67$ | -0.41 | -0.11 | 1.08 |
| 9 | 0.000317 | 0.56 | 1.282 | 5.28 | 4.79 | 4.66 | $-0.30$ | 0.09 | -0.22 | 1.03 |
| 9 | 0.000282 | 0.63 | 1.063 | 7.22 | 5.64 | 5.47 | $-0.52$ | -0.51 | -0.49 | 1.07 |
| 9 | 0.000258 | 0.69 | 1.211 | 7.75 | 7.06 | 6.62 | $-0.91$ | -1.97 | -0.36 | 1.10 |
| 9 | 0.000237 | 0.75 | 0.703 | 13.78 | 23.78 | 22.87 | $-6.15$ | -1.34 | -0.51 | 1.65 |
| 12 | 0.001823 | 0.13 | 0.812 | 3.86 | 4.46 | 4.29 | 0.00 | -0.64 | 0.10 | 1.00 |
| 12 | 0.001077 | 0.22 | 1.017 | 3.74 | 4.53 | 4.42 | 0.00 | -0.13 | 0.01 | 1.01 |
| 12 | 0.000765 | 0.31 | 0.966 | 4.34 | 3.78 | 3.63 | $-0.01$ | -0.31 | 0.00 | 1.01 |
| 12 | 0.000593 | 0.40 | 1.106 | 4.68 | 3.80 | 3.66 | -0.04 | -0.08 | -0.08 | 1.01 |
| 12 | 0.000494 | 0.48 | 1.067 | 5.47 | 4.82 | 4.67 | -0.43 | -0.17 | -0.23 | 1.05 |
| 12 | 0.000423 | 0.56 | 1.138 | 5.65 | 4.79 | 4.65 | -0.14 | -0.44 | -0.20 | 1.02 |
| 12 | 0.000376 | 0.63 | 0.926 | 8.40 | 7.00 | 6.84 | -0.95 | -0.41 | -0.17 | 1.10 |
| 12 | 0.000344 | 0.69 | 1.319 | 8.21 | 7.20 | 7.00 | -1.09 | -0.60 | -0.15 | 1.12 |
| 12 | 0.000316 | 0.75 | 1.275 | 9.10 | 7.45 | 7.11 | -1.12 | -1.49 | -0.40 | 1.12 |
| 17 | 0.002583 | 0.13 | 0.813 | 3.72 | 3.94 | 3.72 | 0.00 | -0.84 | 0.09 | 1.00 |
| 17 | 0.001526 | 0.22 | 0.901 | 4.04 | 3.75 | 3.60 | 0.00 | -0.30 | -0.08 | 1.01 |
| 17 | 0.001083 | 0.31 | 1.137 | 4.18 | 3.68 | 3.53 | 0.00 | -0.11 | -0.02 | 1.01 |
| 17 | 0.000840 | 0.40 | 1.108 | 5.00 | 4.00 | 3.87 | -0.02 | -0.14 | -0.15 | 1.01 |
| 17 | 0.000700 | 0.48 | 1.034 | 6.05 | 5.06 | 4.93 | -0.40 | -0.17 | -0.13 | 1.05 |
| 17 | 0.000600 | 0.56 | 1.257 | 6.07 | 4.58 | 4.46 | -0.14 | 0.14 | -0.08 | 1.02 |
| 17 | 0.000533 | 0.63 | 1.240 | 8.48 | 5.99 | 5.83 | -0.47 | 0.64 | -0.23 | 1.05 |
| 17 | 0.000487 | 0.69 | 1.076 | 10.76 | 12.02 | 11.56 | $-2.20$ | $-2.07$ | -0.23 | 1.23 |
| 17 | 0.000448 | 0.75 | 1.259 | 10.72 | 8.14 | 7.96 | -0.83 | -0.97 | -0.24 | 1.09 |
| 24 | 0.003647 | 0.13 | 0.834 | 3.46 | 3.63 | 3.43 | 0.00 | -0.60 | 0.22 | 1.00 |
| 24 | 0.002155 | 0.22 | 0.928 | 3.80 | 3.56 | 3.41 | 0.00 | -0.16 | -0.02 | 1.00 |
| 24 | 0.001529 | 0.31 | 1.030 | 4.38 | 3.30 | 3.14 | 0.00 | -0.23 | -0.03 | 1.01 |
| 24 | 0.001185 | 0.40 | 1.230 | 4.83 | 3.68 | 3.52 | 0.00 | -0.25 | -0.12 | 1.01 |
| 24 | 0.000988 | 0.48 | 1.125 | 5.90 | 3.96 | 3.82 | -0.03 | -0.05 | -0.30 | 1.01 |
| 24 | 0.000847 | 0.56 | 1.129 | 7.08 | 5.23 | 5.11 | -0.28 | -0.23 | -0.17 | 1.03 |
| 24 | 0.000752 | 0.63 | 0.979 | 9.91 | 7.61 | 7.47 | -0.86 | -0.25 | -0.26 | 1.10 |
| 24 | 0.000687 | 0.69 | 1.391 | 9.02 | 6.25 | 6.08 | -0.82 | -0.38 | -0.17 | 1.09 |
| 24 | 0.000632 | 0.75 | 1.274 | 10.03 | 6.32 | 6.17 | -0.28 | -0.78 | -0.34 | 1.03 |

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| 32 | 0.004862 | 0.13 | 0.815 | 4.02 | 3.69 | 3.47 | 0.00 | -0.72 | 0.08 | 1.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 0.002873 | 0.22 | 0.998 | 4.29 | 3.85 | 3.72 | 0.00 | -0.09 | $-0.07$ | 1.00 |
| 32 | 0.002039 | 0.31 | 0.982 | 5.32 | 3.49 | 3.34 | 0.00 | 0.12 | -0.04 | 1.01 |
| 32 | 0.001580 | 0.40 | 1.152 | 6.07 | 3.94 | 3.80 | $-0.03$ | 0.03 | -0.19 | 1.01 |
| 32 | 0.001317 | 0.48 | 0.991 | 7.94 | 6.62 | 6.50 | $-0.59$ | 0.24 | -0.19 | 1.07 |
| 32 | 0.001129 | 0.56 | 1.073 | 8.55 | 5.26 | 5.15 | -0.17 | 0.07 | $-0.07$ | 1.02 |
| 32 | 0.001003 | 0.63 | 1.028 | 11.08 | 5.88 | 5.77 | $-0.28$ | $-0.46$ | -0.18 | 1.03 |
| 32 | 0.000916 | 0.69 | 1.257 | 11.42 | 6.47 | 6.36 | -0.16 | 0.03 | -0.54 | 1.02 |
| 32 | 0.000843 | 0.75 | 1.692 | 11.29 | 8.11 | 8.02 | -0.42 | 0.22 | -0.23 | 1.05 |
| 45 | 0.006838 | 0.13 | 0.796 | 4.54 | 4.03 | 3.84 | 0.00 | -0.69 | 0.12 | 1.00 |
| 45 | 0.004040 | 0.22 | 0.885 | 5.25 | 3.89 | 3.76 | -0.04 | 0.09 | $-0.07$ | 1.01 |
| 45 | 0.002867 | 0.31 | 0.984 | 6.08 | 3.98 | 3.85 | 0.00 | 0.08 | -0.11 | 1.01 |
| 45 | 0.002222 | 0.40 | 1.020 | 7.32 | 4.49 | 4.37 | 0.00 | -0.10 | -0.22 | 1.01 |
| 45 | 0.001852 | 0.48 | 1.161 | 8.25 | 4.94 | 4.82 | 0.00 | 0.13 | -0.34 | 1.01 |
| 45 | 0.001587 | 0.56 | 1.405 | 8.57 | 5.22 | 5.11 | 0.00 | $-0.23$ | -0.10 | 1.01 |
| 45 | 0.001411 | 0.63 | 1.513 | 10.73 | 7.40 | 7.32 | $-0.13$ | 0.23 | -0.17 | 1.02 |
| 45 | 0.001288 | 0.69 | 1.530 | 13.06 | 10.24 | 9.92 | -1.08 | -1.97 | -0.45 | 1.11 |
| 45 | 0.001185 | 0.75 | 1.033 | 17.90 | 15.83 | 15.63 | -2.13 | 0.38 | -0.27 | 1.21 |
| 60 | 0.009117 | 0.13 | 0.708 | 5.71 | 3.80 | 3.58 | 0.00 | $-0.76$ | 0.18 | 1.00 |
| 60 | 0.005387 | 0.22 | 0.846 | 6.30 | 4.25 | 4.12 | 0.00 | -0.29 | -0.02 | 1.00 |
| 60 | 0.003823 | 0.31 | 1.013 | 7.24 | 4.32 | 4.19 | 0.00 | 0.27 | -0.05 | 1.01 |
| 60 | 0.002963 | 0.40 | 1.011 | 8.60 | 4.96 | 4.85 | 0.00 | 0.24 | -0.02 | 1.01 |
| 60 | 0.002469 | 0.48 | 1.069 | 10.47 | 6.06 | 5.98 | 0.00 | -0.13 | 0.07 | 1.00 |
| 60 | 0.002116 | 0.56 | 1.292 | 11.12 | 5.86 | 5.75 | 0.00 | 0.47 | 0.06 | 1.01 |
| 60 | 0.001881 | 0.63 | 0.948 | 18.67 | 25.30 | 25.16 | $-2.27$ | 0.40 | -0.15 | 1.24 |
| 60 | 0.001718 | 0.69 | 1.531 | 15.44 | 9.43 | 9.36 | 0.00 | 0.53 | -0.39 | 0.98 |
| 80 | 0.012156 | 0.13 | 0.706 | 6.48 | 4.22 | 4.03 | 0.00 | -0.68 | 0.15 | 1.00 |
| 80 | 0.007183 | 0.22 | 0.885 | 7.16 | 4.41 | 4.26 | 0.00 | -0.57 | -0.11 | 1.00 |
| 80 | 0.005098 | 0.31 | 0.879 | 8.60 | 5.12 | 5.02 | 0.00 | -0.24 | -0.03 | 1.00 |
| 80 | 0.003951 | 0.40 | 0.948 | 10.80 | 5.42 | 5.32 | 0.00 | 0.29 | -0.08 | 1.01 |
| 80 | 0.003292 | 0.48 | 1.058 | 11.86 | 5.91 | 5.79 | 0.00 | 0.63 | -0.19 | 1.01 |
| 80 | 0.002822 | 0.56 | 0.735 | 15.48 | 6.52 | 6.36 | 0.00 | 0.98 | -0.04 | 1.00 |
| 80 | 0.002508 | 0.63 | 0.871 | 21.91 | 9.78 | 9.68 | 0.00 | 1.05 | -0.43 | 0.80 |
| 110 | 0.016714 | 0.13 | 0.619 | 8.34 | 4.82 | 4.65 | 0.00 | $-0.73$ | 0.30 | 1.00 |
| 110 | 0.009877 | 0.22 | 0.872 | 8.60 | 5.29 | 5.19 | 0.00 | 0.00 | 0.13 | 1.01 |
| 110 | 0.007009 | 0.31 | 0.960 | 9.76 | 5.19 | 5.09 | 0.00 | 0.01 | 0.04 | 1.00 |
| 110 | 0.005432 | 0.40 | 1.016 | 11.45 | 6.37 | 6.28 | 0.00 | 0.23 | -0.16 | 1.00 |
| 110 | 0.004527 | 0.48 | 0.960 | 15.50 | 8.25 | 8.15 | 0.00 | 0.86 | -0.22 | 0.91 |

Table 6: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$ at $\sqrt{s}=$ 225 GeV for the shifted-vertex region. Further details as described in caption of Table 1.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\begin{gathered} \tilde{\sigma} \\ \text { ZEUS97 } \end{gathered}$ | $\overline{\tilde{\delta}_{\text {stat }}}$ <br> (\%) | $\tilde{\delta}_{\text {unc }}$ <br> (\%) | $\begin{aligned} & \tilde{\delta}_{\text {eID }} \\ & (\%) \end{aligned}$ | $\tilde{\delta}_{\text {dxdy }}$ <br> (\%) | $\begin{array}{r} \hline \tilde{\delta}_{e \theta} \\ (\%) \end{array}$ | $\begin{aligned} & \tilde{\delta}_{E_{e}} \\ & (\%) \end{aligned}$ | $\tilde{\delta}_{\text {hdFC }}$ <br> (\%) | $\tilde{\delta}_{\text {hdBC }}$ <br> (\%) | $\tilde{\delta}_{\text {hdRC }}$ (\%) | $\tilde{\delta}_{\text {hdF }}$ <br> (\%) | $\tilde{\delta}_{\text {Bhd }}$ <br> (\%) | $\begin{gathered} \hline \tilde{\delta}_{\gamma p} \\ (\%) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | 0.00025 | 1.214 | 1.60 | 0.90 | 1.64 | 0.56 | 0.00 | 0.59 | $-0.08$ | 0.21 | 0.39 | 0.24 | 1.61 | 0.20 |
| 9 | 0.00040 | 1.086 | 1.64 | 0.92 | 1.38 | 0.59 | 0.00 | -0.44 | 0.08 | 0.18 | $-0.36$ | 0.10 | 0.63 | -0.18 |
| 12 | 0.00033 | 1.270 | 2.61 | 0.96 | 2.21 | 1.26 | 0.00 | $-0.95$ | 0.22 | 0.29 | 0.98 | 0.54 | 6.78 | 0.24 |
| 12 | 0.00041 | 1.186 | 2.30 | 0.87 | 1.61 | 0.59 | 0.00 | 0.22 | -0.16 | -0.06 | 0.13 | 0.26 | 1.87 | -0.01 |
| 12 | 0.00054 | 1.106 | 2.30 | 0.87 | 1.61 | 0.59 | 0.00 | 0.22 | -0.16 | -0.06 | 0.13 | 0.26 | 1.87 | $-0.01$ |
| 17 | 0.00047 | 1.277 | 1.94 | 0.68 | 1.94 | 0.42 | 0.00 | -0.49 | 0.11 | 0.26 | 0.39 | 0.22 | 3.49 | 0.09 |
| 17 | 0.00076 | 1.150 | 1.94 | 0.68 | 1.94 | 0.42 | 0.00 | -0.49 | 0.11 | 0.26 | 0.39 | 0.22 | 3.49 | 0.09 |
| 24 | 0.00067 | 1.315 | 2.55 | 1.22 | 1.99 | 0.47 | 0.00 | $-1.02$ | 0.13 | 0.23 | 0.59 | 0.66 | 2.83 | $-0.01$ |
| 24 | 0.00082 | 1.289 | 2.20 | 0.65 | 1.54 | -0.12 | 0.00 | 0.27 | -0.09 | 0.35 | -0.10 | 0.25 | 0.63 | -0.49 |
| 24 | 0.00108 | 1.140 | 2.26 | 1.40 | 1.30 | 0.99 | 0.00 | -0.15 | -0.17 | 0.39 | $-0.55$ | 0.01 | 0.18 | 0.12 |
| 32 | 0.00089 | 1.351 | 1.48 | 0.66 | 1.89 | 0.36 | 0.01 | -0.95 | -0.02 | 0.04 | 0.32 | 1.05 | 1.44 | $-0.01$ |
| 32 | 0.00109 | 1.253 | 1.36 | 0.33 | 1.49 | 0.47 | 0.00 | -0.28 | -0.07 | 0.43 | -0.46 | 0.28 | 0.29 | 0.05 |
| 32 | 0.00143 | 1.199 | 1.63 | 0.43 | 1.29 | 0.55 | 0.00 | -0.47 | -0.15 | 0.32 | $-0.36$ | 0.14 | 0.03 | $-0.07$ |
| 45 | 0.00125 | 1.309 | 1.34 | 0.34 | 1.62 | 0.46 | 0.00 | $-0.73$ | 0.03 | 0.34 | $-0.78$ | 0.17 | 0.45 | $-0.01$ |
| 45 | 0.00153 | 1.143 | 1.70 | 0.39 | 1.37 | 0.49 | 0.00 | $-0.20$ | 0.09 | 0.56 | $-0.27$ | -0.12 | 0.15 | 0.09 |
| 60 | 0.00167 | 1.313 | 1.3 | 0.5 | 1.8 | 0.53 | -0.1 | -0.44 | 0.12 | 0.14 | -0.06 | 0.42 | 0.72 | $-0.02$ |
| 60 | 0.00204 | 1.222 | 1.49 | 0.46 | 1.48 | 0.38 | 0.00 | -0.23 | -0.16 | 0.35 | -0.45 | 0.42 | 0.36 | $-0.14$ |
| 60 | 0.00269 | 1.129 | 1.49 | 0.40 | 1.31 | 0.35 | 0.00 | 0.16 | $-0.07$ | 0.36 | -0.27 | $-0.17$ | 0.08 | $-0.14$ |
| 80 | 0.00222 | 1.200 | 1.98 | 0.35 | 1.81 | 0.20 | $-0.38$ | $-0.83$ | $-0.03$ | 0.36 | $-0.23$ | 0.23 | 0.47 | 0.03 |
| 80 | 0.00272 | 1.201 | 1.85 | 0.22 | 1.52 | 0.43 | -0.35 | 0.26 | 0.08 | 0.41 | -0.20 | -0.06 | 0.05 | 0.06 |
| 80 | 0.00359 | 1.116 | 1.68 | 0.56 | 1.32 | 0.15 | -0.06 | -0.74 | -0.09 | 0.13 | -0.17 | $-0.27$ | 0.00 | $-0.30$ |
| 110 | 0.00306 | 1.176 | 1.93 | 0.73 | 1.67 | 0.30 | 0.05 | -0.69 | -0.04 | 0.31 | -0.41 | 0.02 | 0.10 | $-0.03$ |
| 110 | 0.00493 | 1.030 | 1.72 | 0.50 | 1.41 | 0.54 | $-0.71$ | 0.09 | -0.16 | 0.37 | -0.22 | $-0.16$ | 0.08 | 0.04 |

Table 7: The reduced cross sections, $\tilde{\sigma}$, for the reaction $e^{+} p \rightarrow e^{+} X$, at $\sqrt{s}=$ 300 GeV measured in the years 1996 and 1997, after the adjustments of the binning for the $F_{L}$ extraction. The first two columns contain the bin centres in $Q^{2}$ and $x$, the next three contain the measured cross section, the statistical uncertainty and the uncorrelated systematic uncertainty, respectively. The final ten columns list the bin-to-bin correlated uncertainties from each systematic source. The normalisation uncertainty (see Section 8) is not included.

| $\begin{array}{r} Q^{2} \\ \left(\mathrm{GeV}^{2}\right) \end{array}$ | $x$ | $\begin{gathered} F_{2} \\ \text { (unconstrained fit) } \end{gathered}$ | $\begin{gathered} F_{2} \\ \text { (constrained fit) } \end{gathered}$ | $\begin{gathered} F_{L} \\ \text { (unconstrained fit) } \end{gathered}$ | $\begin{gathered} F_{L} \\ \text { (constrained fit) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | 0.00025 | $1.223_{-0.039}^{+0.045}$ | $1.259_{-0.033}^{+0.039}$ | ${ }_{-0.06}{ }_{-0.19}^{+0.22}$ | $0.00_{-0.00}^{+0.24}$ |
| 9 | 0.00031 | $1.325_{-0.129}^{+0.111}$ | $1.331_{-0.081}^{+0.111}$ | $0.25_{-0.42}^{+0.55}$ | $0.47{ }_{-0.41}^{+0.29}$ |
| 9 | 0.00040 | $1.085_{-0.033}^{+0.039}$ | $1.115_{-0.027}^{+0.033}$ | ${ }_{-0.08}^{-0.51}+$ | $0.00_{-0.00}^{+0.57}$ |
| 12 | 0.00033 | $1.319_{-0.057}^{+0.063}$ | $1.349_{-0.051}^{+0.063}$ | $0.15{ }_{-0.20}^{+0.20}$ | $0.29_{-0.16}^{+0.20}$ |
| 12 | 0.00041 | $1.181_{-0.039}^{+0.045}$ | $1.229_{-0.027}^{+0.039}$ | $-0.24_{-0.20}^{+0.26}$ | $0.00_{-0.00}^{+0.17}$ |
| 12 | 0.00054 | $1.139_{-0.039}^{+0.045}$ | $1.169_{-0.033}^{+0.033}$ | ${ }_{-0.09}{ }_{-0.40}^{+0.44}$ | $0.00_{-0.00}^{+0.46}$ |
| 17 | 0.00047 | $1.343_{-0.039}^{+0.039}$ | $1.361_{-0.033}^{+0.045}$ | $0.18{ }_{-0.11}^{+0.17}$ | $0.31_{-0.14}^{+0.12}$ |
| 17 | 0.00058 | $1.217_{-0.045}^{+0.057}$ | $1.271_{-0.027}^{+0.039}$ | $-0.20_{-0.20}^{+0.22}$ | $0.00_{-0.00}^{+0.15}$ |
| 17 | 0.00076 | $1.175_{-0.045}^{+0.033}$ | $1.199_{-0.027}^{+0.033}$ | ${ }_{-0.09}^{+0.35}$ | $0.00_{-0.00}^{+0.30}$ |
| 24 | 0.00067 | $1.373_{-0.027}^{+0.033}$ | $1.385_{-0.027}^{+0.033}$ | $0.21{ }_{-0.11}^{+0.13}$ | $0.29{ }_{-0.10}^{+0.12}$ |
| 24 | 0.00082 | $1.277_{-0.027}^{+0.033}$ | $1.289_{-0.027}^{+0.027}$ | $0.06{ }_{-0.13}^{+0.16}$ | $0.14{ }_{-0.12}^{+0.09}$ |
| 24 | 0.00108 | $1.193_{-0.027}^{+0.033}$ | $1.205_{-0.021}^{+0.033}$ | $0.09_{-0.22}^{+0.28}$ | $0.19_{-0.17}^{+0.19}$ |
| 32 | 0.00089 | $1.355_{-0.027}^{+0.027}$ | $1.361_{-0.021}^{+0.033}$ | $0.06{ }_{-0.11}^{+0.11}$ | $0.14{ }_{-0.11}^{+0.08}$ |
| 32 | 0.00109 | $1.271_{-0.021}^{+0.027}$ | $1.283_{-0.021}^{+0.027}$ | $0.13_{-0.11}^{+0.16}$ | $0.22_{-0.12}^{+0.12}$ |
| 32 | 0.00143 | $1.193_{-0.027}^{+0.027}$ | $1.205_{-0.021}^{+0.021}$ | ${ }_{-0.0}{ }_{-0.20}^{+0.23}$ | $0.00_{+0.00}^{+0.23}$ |
| 45 | 0.00125 | $1.319_{-0.027}^{+0.027}$ | $1.325_{-0.021}^{+0.027}$ | $0.12_{-0.10}^{+0.13}$ | $0.19_{-0.09}^{+0.12}$ |
| 45 | 0.00153 | $1.259_{-0.027}^{+0.027}$ | $1.271_{-0.021}^{+0.027}$ | $0.06{ }_{-0.16}^{+0.14}$ | $0.09_{-0.09}^{+0.11}$ |
| 45 | 0.00202 | $1.163_{-0.021}^{+0.027}$ | $1.169_{-0.021}^{+0.027}$ | $0.30_{-0.23}^{+0.20}$ | $0.35_{-0.21}^{+0.17}$ |
| 60 | 0.00167 | $1.319_{-0.021}^{+0.033}$ | $1.331_{-0.027}^{+0.027}$ | $0.12_{-0.11}^{+0.14}$ | $0.19_{-0.11}^{+0.12}$ |
| 60 | 0.00204 | $1.235{ }_{-0.021}^{+0.027}$ | $1.241_{-0.021}^{+0.027}$ | $0.22_{-0.17}^{+0.16}$ | $0.25_{-0.14}^{+0.16}$ |
| 60 | 0.00269 | $1.133_{-0.021}^{+0.027}$ | $1.145_{-0.015}^{+0.027}$ | $0.04{ }_{-0.23}^{+0.25}$ | $0.09_{-0.09}^{+0.21}$ |
| 80 | 0.00222 | $1.235_{-0.027}^{+0.033}$ | $1.247_{-0.027}^{+0.027}$ | $0.15{ }_{-0.14}^{+0.13}$ | $0.17{ }_{-0.11}^{+0.11}$ |
| 80 | 0.00272 | $1.223_{-0.021}^{+0.033}$ | $1.229_{-0.021}^{+0.033}$ | $0.45{ }_{-0.20}^{+0.14}$ | $0.44{ }_{-0.16}^{+0.19}$ |
| 80 | 0.00359 | $1.097_{-0.021}^{+0.027}$ | $1.103_{-0.015}^{+0.027}$ | $0.18{ }_{-0.23}^{+0.25}$ | $0.23_{-0.18}^{+0.20}$ |
| 110 | 0.00306 | $1.187_{-0.033}^{+0.033}$ | $1.199_{-0.027}^{+0.033}$ | $0.12{ }_{-0.17}^{+0.16}$ | $0.166_{-0.14}^{+0.12}$ |
| 110 | 0.00374 | $1.157_{-0.033}^{+0.033}$ | $1.163_{-0.027}^{+0.033}$ | $0.21_{-0.22}^{+0.20}$ | $0.20_{-0.14}^{+0.20}$ |
| 110 | 0.00493 | $1.037_{-0.021}^{+0.027}$ | $1.049_{-0.015}^{+0.027}$ | $-0.12_{-0.23}^{+0.31}$ | $0.00_{+0.00}^{+0.25}$ |

Table 8: Extracted values of $F_{2}$ and $F_{L}$ at $27\left(x, Q^{2}\right)$ points. Values are quoted for the unconstrained and constrained fits. The quoted uncertainties cover both the statistical and systematic sources. The normalisation uncertainty (see Section 9.1) is not included. For the constrained fits with mode at $F_{L}=0$, the upper uncertainties correspond to $68 \%$ probability limits.

| $Q^{2}$ |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
| $\left(\mathrm{GeV}^{2}\right)$ | $F_{L}\left(Q^{2}\right)$ | $F_{L}\left(Q^{2}\right)$ | $R\left(Q^{2}\right)$ | $R\left(Q^{2}\right)$ |
| 9 | $-0.040_{-0.167}^{+0.191}$ | $0.000_{-0.000}^{+0.135}$ | $0.01_{-0.13}^{+0.17}$ | $0.05_{-0.05}^{+0.17}$ |
| 12 | $-0.003_{-0.162}^{+0.181}$ | $0.000_{-0.000}^{+0.135}$ | $0.01_{-0.12}^{+0.17}$ | $0.05_{-0.05}^{+0.15}$ |
| 17 | $0.075_{-0.104}^{+0.130}$ | $0.126_{-0.102}^{+0.090}$ | $0.09_{-0.10}^{+0.11}$ | $0.14_{-0.11}^{+0.08}$ |
| 24 | $0.160_{-0.115}^{+0.095}$ | $0.184_{-0.078}^{+0.107}$ | $0.14_{-0.07}^{+0.11}$ | $0.17_{-0.08}^{+0.10}$ |
| 32 | $0.078_{-0.093}^{+0.093}$ | $0.102_{-0.076}^{+0.077}$ | $0.08_{-0.07}^{+0.08}$ | $0.09_{-0.05}^{+0.09}$ |
| 45 | $0.115_{-0.090}^{+0.111}$ | $0.138_{-0.075}^{+0.102}$ | $0.10_{-0.06}^{+0.10}$ | $0.12_{-0.08}^{+0.09}$ |
| 60 | $0.135_{-0.091}^{+0.132}$ | $0.179_{-0.102}^{+0.090}$ | $0.14_{-0.09}^{+0.10}$ | $0.16_{-0.09}^{+0.10}$ |
| 80 | $0.222_{-0.108}^{+0.127}$ | $0.244_{-0.099}^{+0.112}$ | $0.25_{-0.12}^{+0.12}$ | $0.25_{-0.11}^{+0.14}$ |
| 110 | $0.086_{-0.100}^{+0.154}$ | $0.101_{-0.079}^{+0.127}$ | $0.10_{-0.10}^{+0.16}$ | $0.12_{-0.09}^{+0.13}$ |

Table 9: Extracted values of $F_{L}$ and $R$ at $9 Q^{2}$ points. Values are quoted for the unconstrained and constrained fits. The quoted uncertainties cover both the statistical and systematic sources. The normalisation uncertainty (see Section 9.1) is not included. For the constrained fits with mode at $F_{L}=0$, the upper uncertainties correspond to $68 \%$ probability limits.

## ZEUS



Figure 1: Efficiency-corrected $Z_{\mathrm{vtx}}$ distribution for the $H E R, M E R$ and $L E R$ data sets. A sum of several contributions is fitted to the data: 6 Gaussian functions representing electron satellite peaks, 2 Gaussians representing proton satellite peaks and 2 Gaussians for central vertex events. The fitted individual contributions are shown (dotted, dash-dotted and dashed curves) as well as the total fit (solid curve).

## ZEUS



Figure 2: Detector-level distributions of the variables $E_{e}^{\prime}$ and $\theta_{e}$ for the $\left|Z_{\mathrm{vtx}}\right|<30 \mathrm{~cm}$ region for the HER, MER and LER data sets, compared to the combined MC predictions (MC DIS+BG). The background-only MC is labelled MC BG. The vertical dashed line indicates the lower limit on the scattered-electron energy used in this analysis. The $\theta_{e}$ distributions are shown for $E_{e}^{\prime}>6 \mathrm{GeV}$.

## ZEUS



Figure 3: Detector-level distributions of the variables $E_{e}^{\prime}$ and $\theta_{e}$ for the $30<Z_{\mathrm{vtx}}<100 \mathrm{~cm}$ region for the HER, MER and LER data sets, compared to the combined MC predictions $(M C D I S+B G)$. The background-only $M C$ is labelled $M C$ BG. The vertical dashed line indicates the lower limit on the scattered-electron energy used in this analysis. The $\theta_{e}$ distributions are shown for $E_{e}^{\prime}>6 \mathrm{GeV}$.

## ZEUS



Figure 4: The reduced cross sections, $\tilde{\sigma}$, for $\sqrt{s}=318 \mathrm{GeV}$, at 11 values of $Q^{2}$ as a function of $y$. The central- and shifted-vertex data results are shown as circles and squares, respectively. The inner error bars correspond to the statistical uncertainty, while the outer error bars represent the statistical and systematic uncertainties added in quadrature. NNLO QCD predictions from HERAPDF1.5 are also shown. The bands indicate the uncertainty in the predictions.

## ZEUS



Figure 5: The reduced cross sections, $\tilde{\sigma}$, for $\sqrt{s}=251 \mathrm{GeV}$, at 11 values of $Q^{2}$ as a function of y. The central- and shifted-vertex data results are shown as circles and squares, respectively. The inner error bars correspond to the statistical uncertainty, while the outer error bars represent the statistical and systematic uncertainties added in quadrature. NNLO QCD predictions from HERAPDF1.5 are also shown. The bands indicate the uncertainty in the predictions.

## ZEUS



Figure 6: The reduced cross sections, $\tilde{\sigma}$, for $\sqrt{s}=225 \mathrm{GeV}$, at 11 values of $Q^{2}$ as a function of $y$. The central- and shifted-vertex data results are shown as circles and squares, respectively. The inner error bars correspond to the statistical uncertainty, while the outer error bars represent the statistical and systematic uncertainties added in quadrature. NNLO QCD predictions from HERAPDF1.5 are also shown. The bands indicate the uncertainty in the predictions.

## ZEUS



Figure 7: Rosenbluth plots for the lowest-x points for each of the $9 Q^{2}$ values for which $F_{L}$ was measured. The reduced cross sections are shown as points, with error bars representing the combined statistical and systematic uncertainties. The line represents the result from the fit to the reduced cross sections (see Section 9), shown together with the $68 \%$ uncertainty band.

## ZEUS



Figure 8: $F_{L}$ and $F_{2}$ values as a function of $x$ for 9 values of $Q^{2}$, extracted from the unconstrained fit (see Section 9.2). The error bars on the data represent the combined statistical and systematic uncertainties. The error bars on $F_{2}$ are typically smaller than the symbols. NNLO QCD predictions from HERAPDF1.5 are also shown. The bands indicate the uncertainty in the predictions.

## ZEUS



Figure 9: Values of (a) $F_{L}$ and (b) $R$ as a function of $Q^{2}$. The ZEUS data, extracted from the unconstrained fit (see Section 9.2) are shown as filled circles, with error bars representing the combined statistical and systematic uncertainties. The values of $F_{L}$ at different $Q^{2}$ points are correlated, as well as the values of $R$. The shaded band labelled "ZEUS (overall $R$ )" represents the $68 \%$ probability interval for the overall $R$. The H1 data are shown as open circles with error bars representing the total uncertainties. The ZEUS and H1 points are measured at somewhat different x values. NNLO QCD predictions from HERAPDF1.5 are also shown. The bands indicate the uncertainty in the predictions.


[^0]:    ${ }^{1}$ Here and in the following the term electron denotes generically both the electron and the positron.

[^1]:    ${ }^{2}$ The ZEUS coordinate system is a right-handed Cartesian system, with the $Z$ axis pointing in the proton beam direction, referred to as the "forward direction", and the $X$ axis pointing towards the centre of HERA. The coordinate origin is at the centre of the CTD.

